ELMO BUMPY TORUS

Alternate Concepts Development Program

October 1978

Contract No. W-7405-eng-26
Prepared by the
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
operated by
UNION CARBIDE CORPORATION
for the
DEPARTMENT OF ENERGY
This document is based on the long-term research activities at Oak Ridge National Laboratory, especially that of R. A. Dandl who originated the EBT concept. Particular effort in assembling the material was given by J. D. Callen, R. A. Dandl, R. A. Dory, and C. L. Hedrick, and N. A. Uckan.

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The ELMO Bumpy Torus (EBT) program of experiment, theory, and reactor studies has been a remarkably successful one. In the five years since EBT-I began operating, work has progressed from demonstrating macrostability to an increasingly detailed understanding of transport properties. Collisionless scaling ($\tau_E$ increases with temperature) has been observed and the magnitude of the energy confinement time is consistent with neoclassical theory. Experiments on EBT-S are now being conducted at the increased magnetic field levels and higher microwave power and frequency made possible by a 28-GHz gyrotron development program. Initial results confirm our assumptions of neoclassical scaling. In conjunction with the experimental advances, EBT theory now has a well-developed transport theory which models the physics which we now think to be important: for example, it yields negative ambipolar electric fields which are consistent with those measured. Stability calculations continue to predict stable equilibrium with $\beta_{\text{ring}} \sim \beta_{\text{core}} \sim 20-40\%$.

Based on experiment and theory, projected reactor concepts are competitive economically with those of other geometries and they offer flexibility to deal with the critical reactor questions of maintenance, accessibility, fueling, ash removal, impurity control, and power handling. Technological demands are relatively modest and, with the exception of microwave power sources, can be met within existing programs. Moreover, rapid progress is being made in microwave tube development.

When the dimensionless plasma parameters required for an EBT reactor are examined, we find that, with the exception of $\beta$, the EBT-I experiment is already operating in the correct regimes.
Thus, it is possible to supply the necessary bridge to reactor levels with one major experiment. At the same time, this experiment should achieve in Phase I \( n \sim 5 \times 10^{13} \text{ cm}^{-3}, T_e \sim 3 \text{ keV}, T_i \sim 1 \text{ keV}, \) and \( \beta \sim 2\% \).

For Phase II with neutral injection we expect \( n \sim 10^{14} \text{ cm}^{-3}, T_e \sim 5-10 \text{ keV}, T_i \sim 5-10 \text{ keV}, \) and \( \beta \sim 5-10\% \). Performance at this level would provide a critical test of equilibrium, stability, and transport properties of EBTs and of our ability to heat and fuel a larger, denser plasma.

The successful operation of EBT-II will provide the physics base for a reactor-scale extension of the concept.

Finally, it should be noted that the operation of a large superconducting, microwave heated, steady-state device would benefit the entire magnetic confinement fusion community.
I. CONCEPT DEVELOPMENT AND EXPERIMENTAL RESULTS

For 18 years experiments at Oak Ridge National Laboratory (ORNL) and at other institutions have shown that electron cyclotron heating (ECH) is an efficient and reliable technique for creating and sustaining energetic plasmas. Oak Ridge ECH experiments were conducted in both simple (PTF and ELMO) and minimum B (INTEREM and IMP) mirror geometries. The minimum B experiments were directed towards providing a target plasma of sufficient quality ($n_e/a_o$) for effective neutral injection, while the simple mirror experiments were directed towards gaining a general understanding of heating, equilibrium, and stability in hot electron plasmas. In 1967 it was shown by R. A. Dandl and co-workers [1] that the plasma in ELMO was in the form of an annulus composed of very energetic electrons ($T \sim 0.5$-1 MeV) with beta values approaching unity. These rings were a few Larmor radii thick and were found to be macroscopically stable for sufficiently high filling pressures. Also during this period, more refined energy balance calculations made it clear that even with classical behavior, it would be very difficult to produce net power in a standard mirror configuration. Thus, it was highly desirable to utilize in some way the high beta annulus in a toroidal configuration.

A simple toroidal system (with only $B_n$) does not confine single particles. Several improved toroidal traps have been devised to provide confinement: those which use external windings to generate a rotational transform, as in stellarators; those using internal windings, such as Levitrons; those inducing plasma currents, as in tokamaks; and those using periodic spatial modulation of the magnetic field, the so-called bumpy torus. It should be noted that unlike a mirror, a bumpy torus is
a true toroidal device and energy and particles can be confined for many pitch angle scattering times. Also unlike the other toroidal devices mentioned, a bumpy torus relies on magnetic gradient drifts to provide particle confinement and thus invokes a different set of transport physics.

A fundamental problem arose however because a bumpy torus with only vacuum magnetic fields is MHD unstable and thus received relatively little attention. In 1967 R. A. Dandl conjectured that the high beta electron rings which were observed in ELMO could be formed in a bumpy torus and that the magnetic well which these rings create would stabilize the toroidal plasmas.

The first step advancing this concept was to establish that the relativistic electron rings could be formed in a nonaxisymmetric mirror with the coils oriented at an appropriate angle with respect to each other (10-20° for a reasonable number of sectors), and this was done on the canted-mirror experiment in 1971. This led to approval of EBT-I, whose construction was completed in late 1973. Figure 1 shows a picture of the device. Further details can be found in Attachments 1 and 2.

Very early experiments showed that the high beta rings did stabilize the toroidal core. Increasingly detailed measurements over the last four years have allowed us to reach a number of conclusions. They are (not necessarily in chronological order):

1. The high beta annuli do produce a macroscopically stable toroidal plasma with interesting parameters ($n \sim 1-2 \times 10^{12} \, \text{cm}^{-3}$, $T_e \sim 300-600 \, \text{eV}$, $T_i \sim 50-100 \, \text{eV}$, and $T_{\beta} \sim 3-8 \, \text{msec}$).

2. Well closed magnetic field lines are required for good confinement, but these can be easily obtained by a combination of careful construction and global correction.
Fig. 1. Photograph of EBT-I
3. Impurity levels in the toroidal core plasma are very low \( \left( \frac{n_{i}}{n_{e}} \approx 10^{-3}-10^{-4} \right) \) for carbon and aluminum. In addition, the poorly confined surface plasma appears to act as a "natural" divertor and effectively screens the toroidal plasma from incident impurities.

4. Plasmas can be produced in the "collisionless regime" \( \nu/\Omega < 1 \), where \( \nu \) is the scattering rate and \( \Omega \) is the poloidal precessional drift frequency, and in this regime electron energy confinement time increases with temperature.

5. The ambipolar electric field is negative (points radially inward) with \( \frac{e\phi}{kT} \approx 1 \). Details of these and other measurements, as well as the assumptions involved in the interpretations, are presented in Attachment 2.

Operation of EBT-I has been limited to magnetic fields of about 6.5 kG which correspond to the available 18-GHz microwave supplies. Microwave tube advances (by Varian Associates, Palo Alto, California, under subcontract to ORNL) have allowed an increase in the magnet field strength to 10 kG. Results from this experiment (called EBT-S for scale) at a power level of about 50 kW demonstrate the scaling arguments advanced later in this document. At the time of this writing, an improved tube is being installed which should produce power levels in excess of 100 kW. (It should be noted that this tube development has been constrained so that the 28-GHz tube is in fact prototypical of the 120-GHz tubes desired for EBT-II and other experiments. The success of this development bodes well for future development.) An additional modification of EBT-I, which should operate in 1979, involves the addition of so-called Aspect Ratio Enhancement (ARE) coils. These coils should allow a further improvement in the plasma parameters [2].
II. STATUS OF THEORY

EQUILIBRIUM

In the design phase of EBT-I (1971), questions of magnetic equilibria were raised concerning islands in the pressure surfaces and the effect of small field errors. The island questions [3] have been resolved by calculating 3-D tensor pressure equilibria. It is found that no islands occur provided small amounts of ellipticity, triangular, etc., are allowed in the pressure surfaces.

The effects of error fields have been resolved by a rather comprehensive set of experiments. The central idea is that the most serious class of errors can be compensated by a simple set of coils. The experiments indicate a comfortably wide operating window exists in which error field effects are negligible, provided $\delta B/B \leq \rho_e/R_T$. This last inequality follows from comparing motion away from the ideal field line, $\vec{B}$, produced by error fields, $v_\parallel \delta \vec{B}$, with drift motion away from the field lines, $\vec{V}_D$ (fast in EBT), using the smallest gyroradius, $\rho_e$, and the largest scale length, $R_T$, the major radius.

STABILITY

Macroscopic stability has been a key issue for EBT from its inception and is described at some length in Appendix A. EBT-I demonstrated at modest values of toroidal beta, $\beta_T$, the soundness of the central idea of minimum average-B stabilization. An MHD analysis in which the hot electron rings were regarded as rigid indicated that macrostable equilibria can exist for much higher values of $\beta_T$. A recent theory in which the toroidal core plasma and hot electron ring plasma are treated on an equal footing fortifies the validity of these MHD ballooning mode calculations.
Since available microwave power and neoclassical transport limits \( \lambda_T \) (~1/3\% in EBT-I and ~1/2\% in EBT-S, a major design goal of EBT-II is to achieve the maximum \( \beta_T \) consistent with economic constraints. The primary stability result is that the toroidal beta, \( \beta_T \), can be comparable to that of the hot electron annuli, \( \beta_A \), depending somewhat upon the shapes of pressure profiles. While present heating and transport theory suggests that the profile shapes required for stability are automatically achieved, greater sensitivity to profile details could occur at higher values of \( \beta_T \). EBT-II should allow this issue to be addressed and permit studies of the optimum combination of low frequency microwave heating (profile heating) to achieve maximum performance.

**TRANSPORT**

There has been a recent rapid evolution in neoclassical theory for EBT which is discussed at length in the latter part of Appendix A, and is summarized here. By neoclassical we mean transport which depends upon differences in drift orbits produced by geometrical effects as distinct from classical effects in which the step size is a gyroradius.

One of the more interesting features of EBT transport is that the ambipolar electric field has both \( \mathbf{E} \times \mathbf{B} \) and electrostatic effects. A 1-D (radially resolved) fluid transport code has been developed which includes both effects [4]. The requirement of equal electron and ion particle fluxes to maintain charge neutrality yields a maximum of three roots or solutions for the electric field strength. Of these, only one root is stable and interesting. The stable root corresponds to the experimentally observed sign of the electric field.
The transport coefficients used in Ref. [4] only apply for relatively high collisionality. A new set of transport coefficients more appropriate for low collisionality has been used in the 1-D code [5]—Attachment 3. Figure 2 shows the recent 1-D transport code results as well as experimental data from EBT-I and EBT-S.

The basic mechanisms at work in determining the sign of the electric field are readily understood from a rough balance of the ion and electron particle fluxes. This turns out to be roughly equivalent to balancing the diffusion coefficients for the two species which scale as $\nu \langle (\Delta x)^2 \rangle$. Since the electron collision frequency, $\nu_e$, is much greater than $\nu_i$, the average step size for ions, $\langle (\Delta x)^2 \rangle_i$, must exceed that of the electrons. This is achieved by an interaction of $E \times B$ and $VB$ drifts to produce on the tail of the distribution crescent or banana shaped ion drift orbits in sufficient number and width ($\Delta x$) to increase $\langle (\Delta x)^2 \rangle$ for ions.

One of the natural parameters for measuring the neoclassical transport is collisionality, $\nu/\bar{\nu}$. In the past, theoretical speculation has been that EBT might have difficulty in achieving a collisionless regime $(\nu/\Omega)_e < 1$, but Fig. 2 clearly illustrates that $(\nu/\Omega)_e < 1$ in EBT-I and EBT-S for both theory and experiment.

**HEATING**

Electron cyclotron heating has proven to be an effective method of heating, and the microwave tube already developed for EBT-S (28 GHz) is prototypical of those projected for EBT-II and EBT reactor (~100 GHz). Microwave cutoff conditions, $\omega_{pe}/\omega_{ce} \leq 1$ limit the plasma density achievable with such heating ($n \sim 3^2$). Supplementary (lower frequency) heating is used to vary the axial and radial shapes of both the hot
Fig. 2. Comparison of EBT experimental data with 1-D transport. The results using the older transport coefficients (circular ion orbits) are also shown for comparison.
electron annuli and toroidal core plasma. This has proven desirable in EBT-I and is a way to achieve the lower values of collisionality required for EBT-I and EBTR.

A supplementary heating source which heats ions directly is required for advanced EBTs because this provides more direct control over the ion temperature. While several possibilities exist, neutral beam injection has been the most successful to date. The beam energy and plasma opacity must be matched so that $1 \leq n_{ac} \leq 4$.

**IMPUERITIES AND FUELING**

Plasma fueling and impurity control raise questions for all steady-state plasma confinement concepts. In the present EBT-I, plasma fueling is accomplished through neutral gas influx at the plasma boundary. Since EBT-I is transparent to wall-evolved neutrals, this edge fueling source is also a volume source, and hence plasma fueling is a relatively simple matter. In a reactor the EBT plasma will be denser ($n \geq 10^{14}$ cm$^{-3}$) and thicker ($a \geq 100$ cm). Then, this edge source of neutrals may or may not be sufficient. (In present tokamak experiments, an edge source is sufficient for reasons that are not completely understood, but are probably in part due to pinch effects.) In any case, such an EBT plasma can be fueled easily by pellets traveling at velocities of about $10^3$ m/sec, which have already been achieved. The viability of pellet fueling has already been demonstrated in tokamaks (in ISX-A), and there is no reason to believe it would not also work in the EBT concept.

Impurity control and fusion ash removal also raise questions for all steady-state or long-pulse fusion concepts because the energy and particle outflux from the plasma to the surrounding material walls will
surely produce impurities via sputtering, unipolar arcs, or other processes. These impurities must be pumped away, but since the plasma is the most effective pump in the device, one would expect some of the impurities to be ingested into the plasma. The question then is, to what level do they accumulate in the plasma in equilibrium. In EBT the electric field is radially outward outside the high \( \beta \) annuli and hence, such as to strongly repel impurities coming from the wall. Also, since most drift orbits outside the hot electron annulus intersect the wall, there is a natural divertor action at the plasma edge. The small fraction of impurities that do manage to tunnel through the potential barrier into the toroidal core plasma could accumulate there, but the following arguments suggest their density should remain small. In an EBT the ions are very nearly in a Boltzmann distribution. Thus, the equilibrium densities of impurities and fusion ash might be expected to be governed by the relationship of the potential at the plasma center to that at the plasma edge where the impurities are produced and both the impurities and fusion ash are pumped away. Since the potential is nearly the same at the center as it is at the edge, one would expect the relative (to hydrogenic ion density) impurity density in the center to equal to or, including divertor action and neutral penetration effects, perhaps less than that in the edge, and hence quite small. Indeed, in present EBT experiments the impurity density is observed to be very small \( (n_2/n_e \sim 10^{-4}) \) in the plasma center. While this qualitative picture has not been worked out in detail for a reactor, it gives some encouragement that impurities will not significantly inhibit the operation of an EBT reactor, providing there is enough pumping of impurities and fusion ash at the plasma edge.
III. SCALING—OVERVIEW

The EBT scaling laws are developed using basic physical arguments in Appendix B. These scaling laws agree with detailed theoretical calculations as well as with experimental observations in EBT-I and EBT-S. A basic premise of the simple scaling laws is that the energy lifetime is governed by diffusive neoclassical electron losses, so that for small collisionality

$$nT_e \sim A^2 T_e^{3/2},$$

where $A$ is the (magnet) aspect ratio.

The electron collisionality $v/\Omega_e$ is proportional to $nR_B B/T_e^{5/2}$ so that

$$\frac{T_e T_e}{B} \sim \frac{A^2}{n} \frac{T_e^{5/2}}{T_e}$$

$$= A^2 \frac{(nR_B)}{(v/\Omega)}$$

[At higher collisionality than in EBT-I the right side has an additional form factor—$(1 + v^2/\Omega^2)$]. Notice that Fig. 2 of the last section plotted $T_e T_e / B$ vs $v/\Omega$ and that collisionless behavior is indicated by both theory and experiment.

Cutoff for microwave propagation ($\omega_p \leq \omega_c$) imposes another scaling consideration: $n < \text{constant } B^2$.

These scalings can be used to predict density and temperature within constraints of available power, credible collisionality, and microwave cutoff as illustrated in Appendix B. One of the more interesting aspects of this development of the scaling laws is that one can obtain $n \sim p^{1/2}$ which was observed experimentally. This empirical result has been used for several years to provide scaling laws for EBT operation;
for example, used to develop this same set of scaling laws which have been used in the conceptual design, Attachment 1, and the reactor study, Attachments 4 and 5.

IV. REACTOR EMBODIMENT

The EBT concept provides a flexible basis for a steady-state modular fusion reactor. The specific reactor advantages include: steady-state operation, potential for high-beta, large aspect ratio, modular construction, favorable geometry for ease of maintenance, modest technology requirements, high Q-value, and good economics. The details and essential features of the EBT reactor are described in greater length in Attachments 4 and 5.

As mentioned in previous sections, the experimental results from EBT-I were very encouraging and motivated detailed reactor studies and the plans for constructing larger scale EBTs. A dimensionless parameter scaling gives credibility to the extrapolation for future systems. A comparison of the parameters of EBT-I and those projected for EBT-II and for an EBT reactor yields the important result that most of the dimensionless parameters (see Appendix C) which influence stability and transport are either the same or in the direction of increased stability and/or improved transport: annulus beta ($\beta_A$); ratio of gyroradii to various lengths $[(c/L)_{e,i}$ with $L = a, (d \ln \Omega/\delta r)^{-1}, (d \ln n/\delta r)^{-1}...]$; field errors ($\delta B/B$); aspect ratio ($A$); ratio of cold to hot electron density ($n_c/n_h$); ratio of electron to ion temperature ($T_e/T_i$); ratio of electron plasma frequency to cyclotron frequency ($\omega_p/\omega_c$); and ratio of ambipolar potential to temperature ($e\Phi/T$). The approximate constancy of these dimensionless parameters, which are important for stability and
transport is very encouraging, and the extrapolation of the present experimental results to EBTR has more certainty than might be guessed from the change in magnitude in T, n, and t. The beta of the toroidal core plasma which is projected to be >20% in reactor and 5-10% in EBT-II compared to 0.3-0.5% in EBT-I, and, to a lesser extent, the collisionality are factors which change appreciably. Their importance and uncertainties for reactor scale plasmas are expected to be resolved in the proposed program.

V. PROOF OF PRINCIPLE—EBT-II

"Proof of principle" involves several factors, and each must be considered in developing a program which will provide information at a level which will significantly impact the course of magnetic fusion. The principal factors are those dimensionless parameters (\( \omega_{pe}/\omega_{ce} \), \( \rho/a \), \( \beta \), \( \nu/\Omega \), etc.), absolute parameters (T, n, \( n_\tau \), etc.), and technology (wall loading, duty cycle, heating technology, magnet technology etc.). The program we are presenting seeks appropriate levels of achievement in all three areas.

From the viewpoint of dimensionless parameters, as was presented in the previous section, the present experiment is remarkably close to a reactor. The notable exception is beta. EBT-I has achieved betas of \( \sim 0.3\% \), while for a reactor it is desirable to have betas greater than 20%. Thus, EBT-II has been designed to achieve betas in the 5-10% range. Attaining higher values would be too expensive to justify, and lower ones would not provide conclusive information. Betas at this level appear to require direct ion heating in addition to the ECH. We
have chosen neutral injection for this purpose on the basis that it is
the most proven (with the exception perhaps of ECH) and best understood
technique, and it is undesirable to complicate a "mainline" confinement
experiment any more than is necessary. It would be desirable, however,
to have an evaluation of alternates on the same time scale as EBT-II.
At betas of 5-10% we expect to be able to study the interaction between
the toroidal core plasma and the high beta rings and explore the validity
that the "rigid ring" model which predicts stability for $\beta_{\text{core}} \sim 0.02_{\text{ring}}$.

Present experiments have $T's \sim$ hundreds of eV, $n \sim 1-2 \times 10^{12}$ cm$^{-3}$,
and $T \sim 1-10$ msec, while reactors require $\sim 10$ keV, $\sim 10^{14}$ cm$^{-2}$, 2 sec,
respectively. Achieving EBT-II parameters of $T \sim 3-5$ keV, $n \sim 2-5 \times 10^{13}$ cm$^{-3}$,
and $T \sim 0.1$ sec will give us confidence in extrapolating to a reactor
and will allow resolution of the critical issues.

Finally, the EBT-II design represents a significant step towards
establishing a technology base. EBT-II will be a steady-state, superconducting
device with steady-state heating systems, both microwave and neutral
beams. The microwave system for EBT-II is based on a 200-kW, 120-GHz
gyroklystron which is the same module size and frequency as is assumed
in the EBT reactor studies.

Expected parameters (and their comparison with EBT-I/S), costs, and
schedules for EBT-II are shown in Tables 1, 2, and 3. Additional details
concerning EBT-II can be found in Attachment 1. A brief description of
the major subsystems is given in Table 4.
# Table 1. Machine and plasma parameters for EBT-I, EBT-S, and EBT-II

<table>
<thead>
<tr>
<th>MACHINE</th>
<th>PAST (EBT-I EXPERIMENTS CONCLUDED NOVEMBER 1977 (MEASURED))</th>
<th>PRESENT (EBT-S EXPERIMENTS BEGIN SUMMER 1978 (ANTICIPATED))</th>
<th>FUTURE (EBT-II EXPERIMENTS BEGIN POSSIBLY FY 1982 (ANTICIPATED))</th>
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<tr>
<td>MAGNETIC FIELD (MIDPLANE, MIRROR)</td>
<td>0.45 T, 0.9 T</td>
<td>0.7 T, 1.4 T</td>
<td>3.0 T, 6.0 T</td>
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<tr>
<td>MAGNETIC FIELD POWER</td>
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<td>12 MW</td>
<td>1.3 kW (REFRIGERATION)</td>
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<td>NONE</td>
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<td>520 cm</td>
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<td>MAJOR RADIUS</td>
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<td>18.8 cm</td>
<td>28 cm</td>
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<td>COIL MEAN RADIUS</td>
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<td>8.1</td>
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<tr>
<td>MICROWAVE POWER, CONTINUOUS WAVE (CW)</td>
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<td>1.6 MW, 120 GHz</td>
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<td>n_e</td>
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<tr>
<td>T_e</td>
<td>100-200 keV</td>
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<td>500-2000 keV</td>
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<td>(\beta_{\text{max}})</td>
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<td>10-40%</td>
<td>10-50%</td>
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<tr>
<td>n_e</td>
<td>1.4 x 10^{12} cm^{-3}</td>
<td>2.6 x 10^{12} cm^{-3}</td>
<td>~5 x 10^{13} cm^{-3}</td>
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<tr>
<td>T_e</td>
<td>150-600 eV</td>
<td>300-900 eV</td>
<td>~5 x 10^{13} cm^{-3}</td>
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<tr>
<td>T_i</td>
<td>70-150 eV</td>
<td>100-200 eV</td>
<td>3.8 keV</td>
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<tr>
<td>(\beta_{\text{max}})</td>
<td>0.2-0.6%</td>
<td>~0.5%</td>
<td>3 keV</td>
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<td>MIDPLANE MINOR RADIUS</td>
<td>10 cm</td>
<td>10 cm</td>
<td>~1%</td>
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<td>EFFECTIVE ASPECT RATIO</td>
<td>8:1</td>
<td>8:1</td>
<td>~&lt;17 cm</td>
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<tr>
<td>VOLUME</td>
<td>400 liters</td>
<td>400 liters</td>
<td>~&lt;20.1</td>
</tr>
<tr>
<td>(n_r)</td>
<td>5 x 10^{10} sec cm^{-3}</td>
<td>~10^{11} sec cm^{-3}</td>
<td>~&lt;2000 liters</td>
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<tr>
<td>n_e</td>
<td>5 x 10^{13} cm^{-3}</td>
<td>5 x 10^{13} cm^{-3}</td>
</tr>
<tr>
<td>T_e</td>
<td>3 keV</td>
<td>3 keV</td>
</tr>
<tr>
<td>T_i</td>
<td>1 keV</td>
<td>1 keV</td>
</tr>
<tr>
<td>(\beta_{\text{max}})</td>
<td>~&lt;17 cm</td>
<td>~&lt;17 cm</td>
</tr>
<tr>
<td>MIDPLANE MINOR RADIUS</td>
<td>~&lt;20.1</td>
<td>~&lt;20.1</td>
</tr>
<tr>
<td>EFFECTIVE ASPECT RATIO</td>
<td>~&lt;2000 liters</td>
<td>~&lt;2000 liters</td>
</tr>
<tr>
<td>VOLUME</td>
<td>~&lt;2000 liters</td>
<td>~&lt;2000 liters</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>PHASE 2</th>
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<tbody>
<tr>
<td>WITH ARe</td>
</tr>
<tr>
<td>n_e</td>
</tr>
<tr>
<td>T_e</td>
</tr>
<tr>
<td>T_i</td>
</tr>
<tr>
<td>(\beta_{\text{max}})</td>
</tr>
<tr>
<td>MIDPLANE MINOR RADIUS</td>
</tr>
<tr>
<td>EFFECTIVE ASPECT RATIO</td>
</tr>
<tr>
<td>VOLUME</td>
</tr>
</tbody>
</table>
Table 2. EBT program costs  
(FY 1979 dollars in thousands)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
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<tbody>
<tr>
<td><strong>EBT-II Device</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Engineering</td>
<td>2,000</td>
<td>3,400</td>
<td>2,500</td>
<td>2,000</td>
<td>0</td>
<td>0</td>
<td>9,500</td>
</tr>
<tr>
<td>Construction</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Special facilities</td>
<td>1,700</td>
<td>18,100</td>
<td>8,400</td>
<td>4,400</td>
<td>0</td>
<td>0</td>
<td>32,600</td>
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<tr>
<td>Utilities</td>
<td>2,900</td>
<td>2,600</td>
<td>1,200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6,700</td>
</tr>
<tr>
<td><strong>Subtotal</strong></td>
<td>3,700</td>
<td>24,400</td>
<td>13,500</td>
<td>7,600</td>
<td>0</td>
<td>0</td>
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<td>Escalation(^d)</td>
<td>0</td>
<td>2,400</td>
<td>2,800</td>
<td>2,500</td>
<td>0</td>
<td>0</td>
<td>7,700</td>
</tr>
<tr>
<td>Contingency at 25%(^b)</td>
<td>800</td>
<td>6,200</td>
<td>3,700</td>
<td>2,400</td>
<td>0</td>
<td>0</td>
<td>13,100</td>
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<td><strong>TOTAL</strong></td>
<td>4,500</td>
<td>33,000</td>
<td>20,000</td>
<td>12,500</td>
<td>0</td>
<td>0</td>
<td>70,000</td>
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<tr>
<td><strong>Operating Costs</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Research and Development</td>
<td>1,800</td>
<td>2,100</td>
<td>2,200</td>
<td>1,600</td>
<td>0</td>
<td>0</td>
<td>7,700</td>
</tr>
<tr>
<td>EBT Theory</td>
<td>480</td>
<td>625</td>
<td>800</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>4,905</td>
</tr>
<tr>
<td>EBT-S/A Operations</td>
<td>2,385</td>
<td>2,500</td>
<td>2,500</td>
<td>700</td>
<td>0</td>
<td>0</td>
<td>8,085</td>
</tr>
<tr>
<td>EBT-II Operations</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2,000</td>
<td>6,200</td>
<td>6,400</td>
<td>14,600</td>
</tr>
<tr>
<td>Supporting Experiments (ELMO)</td>
<td>100</td>
<td>300</td>
<td>400</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>800</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>4,765</td>
<td>5,525</td>
<td>5,900</td>
<td>5,300</td>
<td>7,200</td>
<td>7,400</td>
<td>36,090</td>
</tr>
<tr>
<td>Capital Equipment (for Development)</td>
<td>200</td>
<td>200</td>
<td>150</td>
<td>150</td>
<td>0</td>
<td>0</td>
<td>700</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>4,965</td>
<td>5,725</td>
<td>6,050</td>
<td>5,450</td>
<td>7,200</td>
<td>7,400</td>
<td>36,790</td>
</tr>
</tbody>
</table>

\(^a\) 10% escalation rate for FY 1980, FY 1981, and FY 1982.  
\(^b\) Based on experience and DOE-ORO guidance.  
\(^c\) Does not include cost of EBT-S operations.
Table 3. EBT-II schedule

**Critical path.**
Table 4. EBT-II major subsystems

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Description</th>
<th>Development Status</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Magnetics</strong></td>
<td>NiTi superconducting stored energy ~100 MJ</td>
<td>State of the art</td>
</tr>
<tr>
<td><strong>Heating</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ECH</td>
<td>2.4 MW (12 units) of 60-70 and 120-GHz power</td>
<td>Prototypical 28-GHz units operated at &gt;100 kW. Well-defined development program with high probability of success. ~1 MW produced by USSR for short pulses at ~90 GHz</td>
</tr>
<tr>
<td>Neutral beams</td>
<td>1 MW at 20 keV</td>
<td>Modest development required. Decrease in power from PLT should allow DC.</td>
</tr>
<tr>
<td>Vacuum Vessel</td>
<td>Conventional water-cooled aluminum O-ring construction</td>
<td>State of the art</td>
</tr>
<tr>
<td><strong>First Wall</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coating and Limiter</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>Vacuum Pumping</td>
<td>Cryogenic pumps</td>
<td>State of the art</td>
</tr>
<tr>
<td><strong>Fueling</strong></td>
<td>Initially by simple gas bleeds as in EBT-S. Pellet fueling can be used if necessary for routine operation or specific experiments</td>
<td>Pellet injectors are operational at required levels now</td>
</tr>
<tr>
<td><strong>Diagnostics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_e$</td>
<td>Thomson scattering, x-rays</td>
<td>Conventional</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Charge exchange neutrals</td>
<td>Conventional</td>
</tr>
<tr>
<td>Impurities</td>
<td>Optical and VUV spectroscopy</td>
<td>Conventional</td>
</tr>
<tr>
<td>$n_e$</td>
<td>Microwave interferometers</td>
<td>Conventional</td>
</tr>
<tr>
<td>Space Potential</td>
<td>Heavy ion probe</td>
<td>Will require heavier probe ions and higher energies but is straightforward</td>
</tr>
<tr>
<td>$B$</td>
<td>Usual array of diagnostic loops and magnetic probes</td>
<td>Conventional</td>
</tr>
</tbody>
</table>
VI. EBT PROGRAM

The similarity of the dimensionless parameter of EBT-I/S to a reactor-grade plasma and the relatively modest technology requirements of the concept allow a compact program (Fig. 3). We believe that if EBT-II performs as expected, there will be a sufficient physics base to initiate the construction of a reactor-scale experiment based on the EBT concept. This device (EBT Test Facility, EBT-TF) could be D-T (as opposed to D-D) if the remote handling technology is available and if the gain from having D-T is sufficiently high to justify the additional cost and complexity given the experience gained from TFTR D-T operation.

It is desirable to have a broadly based understanding of EBTs at the test reactor decision point, and it would be beneficial to have greater participation by the theoretical community in performing the necessary computations and analyses to support the experiments. In addition to confinement experiments, synchrotron measurements on the still existing ELMO device are desirable in order to validate our assumptions concerning classical radiation losses in the reactor power balance.

Finally, it should be noted that a second EBT experimental program is now being initiated at the University of Nagoya under the leadership of Hideo Ikegami. We are in close contact with this effort, and it is intended that there be active exchanges between the two programs and that their plans be coordinated.
Fig. 3. Schematic EBT program plan
REFERENCES


[2] We view this as a confirmatory rather than a fundamental test of EBT physics. Results will be available on a time scale which can influence the detailed design of EBT-II.


APPENDIX A — STATUS OF THEORY

The present theoretical understanding of EBT derives from research in: magnetic equilibria, particle orbits, heating, stability, and transport. Prior to operation of EBT-I the theoretical emphasis was mostly upon stability questions. Early results from EBT-I indicated the soundness of the basic EBT premise that the hot electron rings can provide macro-stability. Accordingly the emphasis in theoretical research passed to neoclassical losses since early estimates indicated that EBT could be dominated by such losses. Here we concentrate upon these two areas of emphasis: stability and transport. The remaining topics will be woven into the fabric of the discussion of these two topics.

STABILITY

The early theory of macrostability in EBT was based on simple arguments involving average minimum-\(B\) properties produced by the hot electron rings. More elaborate theoretical treatments of stability have confirmed this basic notion while placing limits on the toroidal core plasma \(\beta\) which can be attained for a given ring \(\beta\). The stability results are conveniently described in terms of the three plasma components: the toroidal core plasma, the cold edge plasma, and the hot electron rings. (See Fig. A.1)

**Toroidal Core Plasma**

The macroscopic stability of the toroidal core has been examined using a modified energy principle [1]. In this approximation the hot electron rings take part in forming the magnetic equilibrium but are regarded as rigid for the low frequency range associated with macroscopic oscillations of the toroidal core plasma. By neglecting certain positive definite terms
in the energy principle, an Euler equation is developed which is an ordinary second order differential equation (integro-differential equation in all its generality) with arc length along field lines being the independent variable.

This analysis provides a sufficient condition for stability. To examine its consequences, magnetic equilibria were generated which had finite toroidal core beta as well as finite annular beta. Care was taken to avoid the mirror instability (lack of magnetic equilibria) associated with the anisotropic hot electron annuli (rings). For fixed toroidal and annular beta's, the stability condition was examined on each of a closely spaced grid of field lines using a shooting code. The magnitude of the toroidal core beta was varied until the stability condition ceased to be satisfied on at least one of the field lines. This procedure was then repeated for a different magnitude of the hot electron ring $\beta$.

When altering the magnitude of the toroidal and ring betas, the shapes of the two pressure (or beta) profiles were held fixed. Figure A.2 indicates the radial variation of the pressure profiles used and the effect on $U = \frac{\Omega}{B}$ and $B$ in the midplane. Figure A.3 shows the stability boundaries for two ring pressure profiles of different axial extent ("short" and "long").

There are two important features of the stability boundary. It will be noticed that stability exists only for finite values of annular $\beta$. This value of $\beta_{\text{annulus}}$ is roughly that which is required to produce a definite minimum in $B$ and depends upon the width of the annuli and the position of the center of the annulus. For EBT-I and EBT-S this value of $\beta$ lies in the range 5 to 15%. Experimentally it is observed that the fluctuation level decreases rapidly as this boundary is crossed. Indeed the transition between
the cold and noisy C-mode of operation and the quiet T-mode with substantial toroidal core plasma appears to be associated with the forming of a distinct minimum in $\beta$.

The second feature of the stability boundaries occurs for high values of $\beta$. At these values of $\beta_{\text{annulus}}$, $\beta_{\text{core}}$ can become comparable to $\beta_{\text{annulus}}$ before instability can occur. This feature of the stability boundaries is less dependent upon details of the radial profiles than the low $\beta$ features. It is, however, somewhat dependent upon the axial extent of the hot electron annuli. Longer annuli create a longer minimum $\beta$ region and hence create a longer "average" minimum $\beta$ (more alteration of $U \equiv \int d\ell / B$).

We have found that good qualitative understanding of these numerical results can be obtained by considering the simple criteria for stability.

\[
(p' + \gamma p \frac{U'}{U} ) (U' - p' \int \frac{d\ell}{B^3}) > 0
\]

Here prime denotes radial ($\psi$) derivatives. The first factor governs stability at smaller minor radii where $U' > 0$: the second factor governs stability in the region where the radial derivative of $U$ (or $B$ in the mid-plane) is reversed. It is worth pointing out that no advantage was taken of the first factor when developing the $\beta_{\text{core}}$ vs $\beta_{\text{annulus}}$ stability boundary shown in Fig. A.3. (The toroidal core pressure is flat where $U' > 0$ in Fig. A.2.) Nonetheless, shapes qualitatively like those shown in Fig. A.1 are permissible - the pressure can rise markedly in regions where strong positive gradients occur in $U$ (strong negative gradients in $B$ midplane).

We note that the mathematical existence of stable magnetic equilibria does not guarantee that they will occur in a given experiment. The heating and transport mechanisms play a strong role in determining the shapes of the profiles. The heating and transport mechanisms seem to automatically give
the types of radial profiles required for macroscopic stability of the
toroidal core plasma. For example the boundary between the T-mode and the
M-mode in EBT can probably be altered by using a wider variety of the low
frequency (profile heating) microwave. This should create longer (less
anisotropic) pressure profiles for the hot electron rings.

**Low Frequency Dissipative Modes**

Since drift dissipative modes play an important role in tokamak
instability theory we have carried out an initial study of similar modes for
EBT [2]. When unstable, these modes have a higher growth rate than the ideal
MHD modes. However, the stability boundaries are similar to those obtained
for MHD so that they do not appear to pose a new threat to stability.

**Macroscopic Stability of the Hot Electron Rings**

The experimentally observed macroscopic stability of the hot electron
annuli in ELMO and the Canted Mirror Facility was one of the first questions
examined theoretically in the EBT program. Even though the expected fre-
quency range suggested that ordinary MHD would not be applicable, an extensive
study of tensor pressure MHD (guiding center) equilibria and stability was
carried out. Within this formalism it was found that ballooning modes of
the toroidal rings were always unstable, contrary to experimental observation.

This first approach was informative, because it laid the ground work
for the extension of the analysis. The mechanisms for instability present
in MHD were incorporated into a more appropriate Vlasov treatment which was
capable of dealing with frequencies in excess of the ion cyclotron fre-
quency [3]. The electrons are so hot that their drift frequencies ($\omega_e$ and $\omega_D$)
are comparable to or exceed the (microscopic) ion cyclotron frequency.

This work was superceded by that of Dominguez and Berk [4] who pointed
out the omission of a term in the original analysis and emphasized the
stabilizing role played by the cold electrons of the toroidal core plasma in addition to the role played by the high drift velocities of the hot electrons.

**Integrated Stability Picture**

The picture that emerges from the various stability calculations is that there is a symbiotic relation between the toroidal core plasma and the hot electron annuli. The hot electrons provide the minimum B necessary to stabilize the toroidal core plasma. The toroidal core plasma provides the cool electrons necessary to stabilize the hot electron rings. The cold outside edge exists beyond the minimum B region and is noisy (as is the toroidal core in the C-mode).

Recently, Nelson has formulated a Maxwell-Vlasov model which includes both core and annulus plasma in a slab approximation. It yields the rigid rings results in the low frequency limit and is equivalent to the formulation of Dominguez and Berk for the high frequencies associated with the hot electrons.

The first focus for this work has been to examine the validity of the rigid ring model. The dispersion relation at low frequencies takes the form

\[ a\omega^2 + bw + c = 0 \]

with \( a\omega^2 + c = 0 \) being the equivalent rigid ring dispersion relation. The coefficient \( b \) is proportional to the ratio of the hot electron density to the toroidal core density, \( \delta \). The coefficients \( a \) and \( c \) also contain modifications of order \( \delta \). \( \delta \) is required to be less than unity for stability of the hot electron rings and is observed to be small experimentally (\( \delta \approx 0.1 \)). A small \( \delta \) analysis of the dispersion relation reveals that the additional
destabilizing terms are smaller than those associated with the rigid ring
assumption by the factor $\delta_{\text{core}}$. Since $\beta_{\text{core}} \leq 30\%$ even in a reactor, these
modifications are only a few percent, which lends confidence to the validity
of the rigid ring model.
TRANSPORT

In the absence of losses induced by instabilities the dominant loss mechanisms of the toroidal core plasma are those associated with neo-classical processes. The neoclassical processes may be subdivided into two categories: diffusive transport losses and direct losses (unconfined drift orbits). The direct losses are not important because unconfined drift orbits only occur at high energy — on the tail of the distribution. Here we concentrate on the dominant diffusive losses.

Neoclassical transport coefficients for EBT tend to be more complicated than those for tokamaks because they depend upon many more parameters. The fundamental reason for this is that the guiding center orbits in EBT are strongly influenced by the ambipolar electric field and by the alteration of magnetic gradients produced by the high beta electron rings. Thus, in addition to collisionality, the transport coefficients depend upon eφ/kT as well as several scale lengths which vary with minor radius. Fortunately it is possible to make estimates and determine scaling of the most important plasma properties using an approximate version of the most important transport coefficient.

To resolve more detailed questions requires more accurate calculation of the transport coefficients. In the past few years there has been considerable evolution in the detailed expressions for the transport coefficients. This development is reaching a plateau with most of the physics clearly identified. Fortunately the magnitude of the transport coefficient which dominates the energy life time, the electron thermal conductivity, has remained relatively invariant. Thus approximate estimates of n, T, and τ have remained roughly invariant.
Before considering a simple model which can be used to project EBT-II performance, we summarize the central ideas and developments of EBT transport theory. The fundamental idea for neoclassical transport in EBT is that the diffusion coefficient (and other transport coefficients such as electron thermal conductivity) scale as

\[ \Delta x \sim \frac{V_y}{\Omega} \]

where \( V_y \) is the vertical drift produced by toroidal curvature and \( \Omega \) is the poloidal drift frequency produced by the bumpy magnetic field. That is,

\[ V_y = \frac{T}{(eB)} \]

\[ \Omega = \frac{T}{(cBR_Br)} \]

where \( T \) is the temperature, \( e \) is the electric charge, \( B \) is the magnetic field, \( R \) is the major radius, \( r \) is the minor radius and \( R_B = \left[ \frac{d\ln B}{dr} \right]^{-1} \) is the average scale length in the bumpy magnetic field. Combining these expressions yields

\[ D \sim \nu r^2 \left( \frac{R_B}{R} \right)^2 \]

Using \( 1/\tau \sim D/a^2 \) (\( a \) = plasma minor radius) and \( \nu \sim n/T^{3/2} \) for coulomb collision we can estimate the particle life time.

\[ \tau \sim \frac{T^{3/2}}{n} \left( \frac{a}{r} \right)^2 \left( \frac{R}{R_B} \right)^2 \]

The energy life time may be estimated from the electron heat conductivity (\( \kappa \sim D/n \)) and one obtains

\[ n\tau_E \sim T^{3/2} \left( \frac{a}{r} \right)^2 \left( \frac{R}{R_B} \right)^2 \]
It is important to note that a/r \sim 1 while R/R_0 \sim A = magnetic aspect ratio >> 1. Thus

\[ nT_e \sim A^2 T_e \frac{3}{2} \]

We now outline some of the details and their consequences which have been omitted from the above analysis. We begin with the ambipolar electric field. Because of the EBT's bumpiness (lack of axisymmetry) self-collisions are dominant. In the absence of an electric field \Delta x is the same for ions and electrons so that the ratio of electron to ion diffusion particle losses is large because \( \nu_e/\nu_i \gg 1 \). This cannot persist in steady state because of charge conservation. An electric field must build up to retard the electron particle losses and/or enhance the ion particle losses to produce ambipolar particle transport.

There are two basic mechanisms involving the electric field which can influence the loss rates: electrostatic and \( \mathbf{E} \times \mathbf{B} \) effects. The electrostatic mechanism manifests itself in fluid theory as a particle flux proportional to the electric field (and the mobility). By itself, this mechanism would lead to outward pointing electric fields (to push out ions and retard electrons). However, the experimental observation is that the electric field points inward in the central portion of the toroidal core plasma [5].

The effect of \( \hat{\mathbf{E}} \times \hat{\mathbf{B}} \) drifts can be seen in a primitive way if we write

\[ D \sim \nu \Delta x^2 \sim \nu v_y^2 / \Omega^2 \]
results of the 1-D code for both sets of transport coefficients as well as the experimentally observed points [7]. The agreement between the more recent theoretical results and experiment is striking.

These preliminary transport coefficients were developed as part of a more comprehensive theoretical study which is just now yielding numerical results. From studying guiding center drift surfaces in electric fields like those observed experimentally and using numerical magnetic fields obtained from the Oak Ridge 3-D EBT equilibrium code, it became apparent that the major contribution to the average ion neoclassical step size comes from a small class of ions on the tail of the distribution. The drift surfaces for this class of ions have crescent or banana shaped cross-sections. (The topology of the adiabatic invariant $J$ for fixed $\varepsilon$ and $\mu$ is the familiar "volcanic banana" [8].)

In previous calculations these ions are treated as if they were highly displaced circles a process which grossly overestimates their contribution to the average neoclassical step size. That is, the transport coefficients were treated roughly as

$$D = \langle \nu \, \frac{V_y^2}{\Omega^2} \rangle$$

where $\langle \nu \, \frac{V_y^2}{\Omega^2} \rangle$ indicates a velocity space average and the constituents $\nu, V_y$ and $\Omega$ depend upon the velocity space coordinates ($\varepsilon$ and $\mu$). The resonance in velocity space where $\Omega = 0$ was only broadened or resolved by collisional effects. At low collisionality finite crescent width is the dominant broadening effect and can be treated approximately by writing
with

\[ \Omega = \frac{1}{r} \left( \frac{T}{eBR} + \frac{E}{B} \right) = \frac{T}{eBRr} \left( 1 + \frac{eER}{T} \right) \]

For the experimentally observed sign of the electric field \( eER_B \) is negative for ions and positive for electrons. (The sign of \( E \) and \( R_B \) are both reversed in the reversed magnetic gradient region — so this analysis works there as well.) Since \( \Delta x \sim 1/\Omega \) we see that the electric field increases the ion step size, \( \Delta x \), and decreases the electron step size (since \( e > 0 \) for electrons). Thus \( E \times B \) effects enter in a very strong way by effecting the neoclassical step size of both species.

A 1-D fluid transport code has been developed which includes \( E \times B \) and electrostatic effects [6]. The requirement of equal electron and ion particle fluxes in order to maintain charge neutrality admits a maximum of three roots or solutions for the electric field strength. Only one of the three roots is stable [see Fig. A.4] The stable root corresponds to the experimentally observed sign of the electric field.

The transport coefficients (particularly those for ions) used in Ref. [6] only apply for relatively high collisionality \( (\nu/\Omega > 1) \) and yield results which one might term collisional (life time decreases with decreasing collision frequency). At lower collisionality, the details of particle orbits must be modeled more accurately when calculating the transport coefficients than was done when carrying out the calculation of the transport coefficients used in Ref. [6]. More recently a set of transport coefficients which emphasizes the details of ion particle orbits (but makes some questionable approximations for analytic convenience) has been utilized in the Oak Ridge 1-D transport code. Figure (A.5) shows the
\[
D \sim <v V_y^2/(\Omega^2 + \Omega_{\text{eff}}^2 + v_{\text{eff}}^2)>
\]

where \(\Omega_{\text{eff}} = |2V_y/d\Omega/dr|^{1/2}\).

For low but somewhat higher collisionality (\(\Omega_{\text{eff}} < v < \Omega\)), Hazeltine and Krall [9] pointed out that a second broadening mechanism exists. Diffusion of sharp velocity space gradients in the first order distribution function constitute an enhanced collisional broadening mechanism. By utilizing the fact that the kinetic equation for the first order distribution function looks roughly like a driven Airy equation, one can estimate the enhancement of collisional broadening by

\[
v_{\text{eff}} \sim \Omega_0 (v/\Omega_0)^{1/3}
\]

where \(\Omega_0\) is the thermal value of \(\Omega\) at \(v_|| = 0\). This leads to the approximation

\[
D \sim <v V_y^2/(\Omega^2 + \Omega_{\text{eff}}^2 + v_{\text{eff}}^2)>
\]

For minor radii where the electric field and gradients in B are small, the collisional broadening tends to dominate the broadening due to crescent orbits (\(v_{\text{eff}} > \Omega_{\text{eff}}\)) for the collisionality observed in EBT-I and EBT-S. However, in the regions where the electric field and gradients of B are large (in the vicinity of the hot electron rings) crescent orbit broadening is comparable or dominates. In these later regions the thermal value of \(\Omega\) is large and hence the transport coefficients are small. It is just these regions at the edge of the toroidal core where the "insulation"
is good and one expects gradients in density and temperature to be strong (as in the wall of a building). Moreover, it is only in these regions that macroscopic stability permits strong gradients in density and temperature.

There are also direct losses to the wall caused by unconfined particle drift orbits [10]. By analogy to mirror machines one might (correctly) guess that pitch angle scattering would lead to losses roughly proportional to $v \log_{10} (\text{mirror ratio})$. However, since this class of particles only occurs for high energy, there is another important form factor. Since these particles are on the tail of the distribution function, the direct losses scale as

$$v \log_{10} (\text{mirror ratio}) \exp \left( -\frac{\varepsilon}{\varepsilon_c} \right)$$

where $\varepsilon$ is the energy and $\varepsilon_c$ is a critical energy determined by both single particle and collisional effects ($\varepsilon_c \geq 10T$ for EBT). Scattering in energy leads to a similar loss rate $v \sqrt{\varepsilon} \exp \left( -\frac{\varepsilon}{\varepsilon_c} \right)$.

Finally, it is possible that microinstabilities could play a role in the transport mechanisms of EBT. However the reasonable agreement of EBT-I and EBT-S results with neoclassical theory suggests that microinstabilities do not presently play a dominant role.
REFERENCES (APPENDIX A)


[10] The examination of this question was stimulated by questions from D. E. Baldwin and M. N. Rosenbluth in 1977. The examination of particle orbits which clarified this question played a key role in developing the theory of the more dominant diffusive losses.
Fig. A.1. Illustration of densities and temperatures for annulus and toroidal core plasmas. This configuration is macrostable.
Fig. A.2. Effect of hot electron annuli on $B$ and $U = dl/B$. 
Fig. A.3. Maximum stable toroidal beta as a function of $\beta_{\text{annulus}}$ for a long annulus and for a short annulus.
ELECTRIC FIELD DETERMINED BY AMBIPOLARITY AND STABILITY OF TRANSPORT EQUILIBRIUM

1. UNSTABLE ROOT BECAUSE $E \sim \nabla n, \ D \sim E^{-2} \ (\nabla n)^{-2}$
2. UNSTABLE BECAUSE $E$ FIELD VARIATION AWAY FROM EQUILIBRIUM EXACERBATES CHARGE IMBALANCE
3. ONLY STABLE ROOT

ELECTRON AND ION PARTICLE FLUX VERSUS AMBIPOLAR ELECTRIC FIELD.

Fig. A.4.
Fig. A.5. Comparison of EBT experimental data with 1-D-transport. The results using the older transport coefficients (circular ion orbits) are also shown for comparison.
APPENDIX B — SCALING

Primitive theoretical considerations suggest that, depending on plasma parameters, there should be two regimes for plasma transport in EBTs. When Coulomb scattering occurs on a faster rate ($v$) than the poloidal precessional frequency ($\Omega$), then $v/\Omega >> 1$ and the plasma is collisional and (for example) the electron thermal conductivity will scale like $T^{7/2}$ ($\tau \propto T^{-7/2}$). The present experiment [1], however, operates in a regime where $v/\Omega < 1$, and as shown by simple arguments in the previous appendix, in this regime the losses scale like $T^{-3/2}$, thus we expect collision plasmas to have confinement times which increase with temperature ($\tau \sim T^{3/2}$). This is precisely what is observed in the experiment. According to theory the quantity $(\tau \cdot T)/B$ should be constant when we operate the same device at different magnetic field (and microwave heating frequency), but at fixed $v/\Omega$. This behavior has also been seen in the EBT-S experiment. The magnetic field in EBT-I was increased from 6.5 to 10 kG and the microwave frequency, from 10 to 28 GHz for the scale modification. Transport coefficients calculated from more refined models than discussed here show the same scaling. Thus, the EBT scaling is based on experiment, primitive physics arguments, and detailed theory.

At this point, electron thermal transport has been emphasized. It should also be noted that one-dimensional simulations utilizing the full transport matrix have been performed. These computations self-consistently calculate the ambipolar electric field and yield electron temperatures and densities consistent with those observed experimentally. The measured ion temperatures appear to be roughly a factor of 2 higher than the predicted values. However, the level of understanding, both experimentally
and theoretically, of ion transport is not yet as complete as that of the electrons. Relatively simple arguments suggest that for electron cyclotron heated experiments, $T_i/T_e$ should be constant because both the ion heating and loss terms scale as $n^2/T^{1/2}$ for plasmas in the collisionless regime.

The scaling laws follow from the previously developed expressions for the lifetime and collisionality (c.f., Appendix A).

$$nT_e \sim A^2 T^{3/2}$$  \hspace{1cm} (1)

and

$$\nu/\Omega = C_1 n (rR_B)/(T^{5/2})$$  \hspace{1cm} (2)

Implicit here is that $eA^2/T_e, T_i/T_e$, and profile shapes are approximately constant from one machine to the next so that it is permissible to drop factors like $(1 + e E R_B/T)$ in the collisionally, $\nu/\Omega$.

Inserting relation (1) into power balance, $P \sim nT/T$, and making use of Eq. (2) we obtain

$$P = C_2 \frac{\nu nT^2}{\Omega BR_c r} \frac{1}{A^2}$$  \hspace{1cm} (3)

Equations (2) and (3) can be solved for $n$ and $T$ in terms of power density and collisionality $\nu/\Omega$:

$$n = \frac{P^{5/3}}{(\nu/\Omega)^{1/3}} \frac{1}{C_1} \left( \frac{C_1}{C_2} \right)^{5/9} (rR_B)^{1/9} A^{10/9}$$  \hspace{1cm} (4)

$$T = \frac{P^{2/9}}{(\nu/\Omega)^{4/9}} \left( \frac{C_1}{C_2} \right)^{2/9} (rR_B)^{4/3} A^{4/9}$$  \hspace{1cm} (5)

This casts $n$ and $T$ in a form which clearly allows one to study effects imposed by any restrictions on power and collisionality.

We also wish to impose limits on the density imposed by microwave cutoff: $\omega_d \sim \omega_c \geq \omega_{pe}$. This may be written as
Notice that we have utilized two constraints (power and collisionality) as equations and imposed a third constraint (microwave cutoff) as an inequality. The choice of which constraints to treat as equations and which as inequalities depend upon which feature one wishes to emphasize. Depending on the choice made, the resulting equations will of course look different, which has caused some confusion in the past.

Fortunately it is possible to convey most of the information on a single plot. If we define

\[ N = \frac{n}{C_3 B^2}, \quad (7) \]

then the microwave cutoff condition becomes simply

\[ N < 1. \quad (8) \]

Defining a scaled power density by

\[ \tilde{P} = \frac{P}{(C_1 C_3)^{9/5} C_1 \left( r R B \right)^{1/5}} \]

and a scaled temperature by

\[ \tilde{T} = \frac{T}{(C_1 C_2 r R B^3)^{2/5}} \]

then Eqs. (4) and (5) become

\[ N = \frac{\tilde{P}^{5/9}}{(\nu/\Omega)^{1/9}} \quad (11) \]
\[ \tilde{T} = \frac{\tilde{P}^{2/9}}{(\nu/\Omega)^{4/9}} \quad (12) \]

Figure B1 shows contours of constant \( N \) and \( T \) in \( (\tilde{P}, \nu/\Omega) \) space. Note that for fixed \( \nu/\Omega \), increasing \( \tilde{P} \) increases \( N \) and \( \tilde{T} \) until microwave cutoff occurs. Conversely, keeping \( \tilde{P} \) fixed and decreasing \( \nu/\Omega \) also causes increased \( N \) and \( \tilde{T} \) until microwave cutoff is reached.
Fig. B1. Contours of constant scaled density, $N$, and temperature, $\tilde{T}$. The axes are collisionality, $\nu/\Omega$, and scaled power density, $\tilde{\rho}$. 
Equations (11) and (12) can be manipulated in several ways. For example, we may use (12) to obtain

\[ \sqrt[3]{N} = \frac{g_{1/2}}{T^{9/4}} \]  

so that (11) becomes

\[ N = \frac{g_{1/2}}{T^{1/4}} \]  

This last dependence of density on power was observed experimentally in EBT-I and formed the basis for an earlier variant of this same set of scaling laws.

REFERENCES

### APPENDIX C: DIMENSIONLESS PARAMETERS

<table>
<thead>
<tr>
<th>Dimensionless Parameters</th>
<th>EBT-I</th>
<th>EBT-II</th>
<th>EBTR</th>
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<tr>
<td>Annulus beta β&lt;sub&gt;A&lt;/sub&gt;</td>
<td>10-40%</td>
<td>10-50%</td>
<td>20-50%</td>
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<td>Toroidal core beta β&lt;sub&gt;T&lt;/sub&gt;</td>
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<td>Ratio of gyroradii to plasma radius ρ&lt;sub&gt;e/a&lt;/sub&gt;</td>
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<td>~3-5 x 10&lt;sup&gt;-4&lt;/sup&gt;</td>
<td>10&lt;sup&gt;-4&lt;/sup&gt;</td>
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<td>Ratio of cold (toroidal) to hot (annulus) electron density n&lt;sub&gt;e/n&lt;/sub&gt;&lt;sub&gt;h&lt;/sub&gt;</td>
<td>~10</td>
<td>~&gt;10</td>
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<td>Effective aspect ratio A&lt;sub&gt;eff&lt;/sub&gt;</td>
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<td>20:1-40:1</td>
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<td>Field errors δB/B</td>
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<td>~&lt;10&lt;sup&gt;-6&lt;/sup&gt;</td>
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<td>Ratio of electron plasma frequency to cyclotron frequency ω&lt;sub&gt;pe/ωce&lt;/sub&gt;</td>
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<td>Collisionality</td>
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<td>(v/Ω)&lt;sub&gt;i&lt;/sub&gt;</td>
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<td>0.1-0.01</td>
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