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THE TMX SUPERMODEL

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B. G. Logan

Introduction

The TMX Supermodel is an attempt to synthesize all presently known experimental observations and theoretical scaling laws concerning particle and energy losses into a comprehensive zero-dimensional description of plasma confinement in the center cell and plugs of TMX. A list of important loss processes and physical effects included in the present state of evolution of the supermodel is given in Table I. I say "present state of evolution" because that part of the supermodel concerning the plugs has not yet been fully tested against all past 2XIIB data, and the center cell part is yet to be compared with future TMX experiments. I would encourage all TMX physicists to participate in checking, modifying and improving the supermodel so that it can be a tool everyone can use in evaluating TMX data as it comes in. With 145 equations at present, the supermodel is too unwieldy to be easily used until it is coded on some computer (being analytic, however, it should run fast). With some approximations that can be made in some limiting cases, the supermodel might be condensed to run on an HP calculator. Of course we all would prefer a simpler model to describe our individual diagnostic data, in the belief that nature loves simplicity. However, past attempts to use a simple rate equation with one beam trapping term and one particular loss term like quasilinear loss, drag, or purely radial diffusion, while perhaps successful in predicting effect A or effect C with appropriate adjusted coefficients, always fail to agree with other observations B, D, E, F, etc. Of course no model will predict every observation, no matter how complex. However, we know from the long list of important processes in Table I, most of which have been observed in 2XIIB at

one time or another, that the real experimental situation is complex, and will be especially so in TMX with the addition of the center cell. I am convinced it will take many different loss processes as in the supermodel to make a reasonable (better than 50%) prediction of several parameters simultaneously measured, such as density, average ion energy, T_e , and loss current, etc. Thus I think the effort in evolving the supermodel will ultimately be fruitful.

One may ask why then bother with a zero-dimensional model when codes exist to calculate many individual effects in Table I in exquisite detail. The problem is that those codes will apply only in special cases. We have a Monte-Carlo neutral transport code for calculating beam trapping, a Superlayer code to calculate finite orbit effects, a 2-D bounce averaged Fokker Planck code for classical scattering and drag effects, a 1-D quasilinear code that calculates R. F. losses self-consistently, an analytic tandem rate code (TAMRAC) which calculates classical multispecies Pastukhov losses, and a 2-D radial transport code now under development for center cell losses. These codes, specialized to one or two effects, can be applied to TMX only in a piecemeal fashion. As a good example, we have seen graphs of energy confinement product $(n\tau)_E$ plotted for 2XIIB data along with some theoretical model as a function of T_e , as though T_e were an independent variable. In fact, T_e was often varied by beam current, which also varied density, beta, R. F. fluctuation level, angular spreading, and charge-exchange losses on gas. To make a proper interpretation of such plots we simply have to take into account the effect of all of those changes on T_e . And we can't afford to wait years for a 3-D coupled quasilinear-Fokker-Planck-radial-transport code to be developed before making a reasonable assessment of all these effects. Instead, the supermodel will be most useful when testing the effects of some new, but rudimentary, loss terms that may be necessary to explain some observed

experimental scalings. While other important losses besides the new ones should be retained, it is of little use to calculate the other loss terms with separate codes that are more accurate than is consistent with the new scaling terms. At this point for example, we know radial loss in the plugs will be important at high R. F. level, but since we will probably only be able to measure a global-average radial loss from charge-exchange on gas, that important term in the supermodel will appear as some τ_{radial} or $\tau_{\text{cx gas}}$, averaged over the plug volume. The main value of the specialized transport codes will be to construct more refined analytic scaling laws which can then be fed into the supermodel. A good example of this would be if Tom Rognlien's Monte-Carlo code gives us a better scaling of the T_e depression in the center cell, and the potential well depth ϕ_c , as a function of the center cell radiation power.

Since the supermodel is zero-dimensional, its predictions for parameters like T_e based on volume-averaged input power and volume-averaged losses should only be construed as average values of such parameters, and so for example we might expect a local T_e value measured by Thomson scattering on axis to be somewhat higher than the model prediction. To the extent radial diffusion of particles and energy is important in the plugs and center cell, we may find local values to be in closer agreement with average values. Since many of our diagnostics such as beam attenuation and charge-exchange are line-integral, we should find even better agreement in the parameters they measure.

At the present time the supermodel includes auxiliary ion heating in the center cell through an unspecified term P_{ic} which could represent R.F. heating either by ICRH or naturally via plug fluctuations. Present estimates show center cell ion heating by ω_{ci} fluctuations from the plugs to be negligible due to the short resonant zone widths on the center cell side of the inner mirrors, but this conclusion depends on the fluctuation

spectrum being very narrow. This topic needs more careful consideration.

Although much of steady-state 2XIIB operation used Titanium washer-stack guns in a steady streaming mode, they are intended to be used only for initial startup of one or both plugs in TMX, with stream-stabilization of the plugs provided in steady state by the center cell ion losses. In spite of much experience in their use, we still today do not know very much about a washer gun's effect on electron energy balance, its total output of plasma stream, or how efficiently a stream gun plasma penetrates a hot plasma. We do know they put out considerable amounts of titanium impurity, which may cause substantial radiation loss if trapped electrostatically in the center cell. Therefore, because (1) we do not know how to model a streaming gun, and (2) we do not intend or desire to use them in steady state in TMX, there is no attempt to include them in the supermodel. If someone could develop an adequate model for the washer gun streaming plasmas, it would still be useful to apply such a washer gun model in the plug part of the supermodel to examine past 2XIIB experiments which employed washer guns. Unless this can be done, however, the supermodel plug description will be of limited use for the bulk of 2XIIB operation.

Even if it should turn out that T_{ec} (center cell) < T_{ep} (plugs) due to excessive radiation loss in the center cell, there will be a power flow in electron energy between the plugs and center cell, and so T_e anywhere in the plasma will be affected by what goes on in both the plugs and the center. The biggest uncertainties in the supermodel concerning electron power balance are (1) shorting of the sheaths due to cathode spots (unipolar arcs on the end walls) and (2) radial transport of electrons and/or electron cross field thermal conductivity. Cathode arcs, by shorting out sheaths, would generally cause electron thermal conductivity loss to the end walls. While there are no existing theories on cathode arc formation, experimental evidence with Langmuir probe shorting in the end fans of 2XIIB

indicates higher probability of shorting at higher densities (10^{13} - 10^{14} at the probe). Although we expect low external densities at the wall in TMX due to expansion of the field lines, the area of the end fans are larger and that may increase the probability of cathode arcs. In the absence of an appropriate theory, we need an experimental method of detecting cathode arcs. Bill Condit suggests photographing the end walls, since cathode arcs are very bright, localized light-emitting areas. On the question of radial electron transport, a preliminary theoretical assessment by Ron Cohen⁽¹⁾ of the worst electron particle transport by resonant or neoclassical diffusion is that such processes are considerably slower than similar estimates of resonant transport by ions in either the plugs or center cell, mainly because the electrons drift at only the same velocity as the center cell ions, while they have a much smaller step size because of their smaller gyroradius. In addition there will likely be a radial gradient in plasma potential which also tends to hold electrons in radially as well as longitudinally. Therefore in the electron particle balance which follows it is assumed that all electrons escape axially even though a considerable fraction of the ions in the center cell and also in the plugs may escape radially. (In the context of the global, zero-D supermodel, radial ion loss in the plugs just means transport out far enough in radius to charge-exchange on gas, whereas in the center cell radial transport which redistributes plasma within the radius of the gas box limiter, causing only a radially-redistributed end loss, is of no consequence in the zero-D supermodel, which averages end loss over the radius.) Even though there may be negligible radial particle transport for the electrons, there might still be significant radial electron thermal conductivity to the gas box limiters, just as small fluctuation levels in Tokamaks can apparently cause anomalously short electron radial energy confinement times compared to radial particle confinement times. Although no appropriate theory for

fluctuation-enhanced cross field thermal conductivity exists for TMX, we can measure temperature rise on the gas box limiters, so this effect is included in the supermodel as an experimentally determined term P_{eKL} .

The model assumes that the plugs are equal in density and energy so that all quantities in the model with subscript p refer to both plugs. For steady state, both plugs require streaming ion losses from the center cell to suppress DCLC fluctuations, while electrostatic losses of ions from the center cell will be controlled by the smaller of the two potential barriers, if they are different. Therefore if we find one plug substantially lower in density than the other ($\geq 10\%$), and center cell losses coming out primarily one end, we should take the time to carefully increase beam current into the weaker plug, or reduce beam injection into the denser plug, to restore balance in the end losses. Certainly this can be done to the extent that trapped beam power, line density and diamagnetic signals are equal within an eyeball look at scope traces. Thus setting the plug's volume, density and drag power input to electrons to be equal in the model is at least consistent with the accuracy of the supermodel in predicting global-average quantities. A potentially large error could be made in estimating center cell confinement or plug R.F. diffusion lifetime on the basis of end loss measurements at only one end, since there could be substantially different end losses at the other end due to the sensitivity in electrostatic confinement with the ratio of plug density to central density $[\tau_c \approx (n_p/n_c)^{T_{ec}/T_c}]$. Therefore one must always measure the total end loss at both ends, and use the smaller of any two plug densities in the model.

In the model which follows, these general rules apply: (1) until more information is available from experiment, only densities are radially averaged, and for gaussian profiles in both the plug (in r and z) and center cell (in r) (same gaussian 1/e radii mapping along field lines), and only

for loss processes like drag which are density dependent. If and when we measure adequate radial profiles of T_e , or density profiles other than gaussian, we can easily change the numerical factors arising from the volume integrals. For example, for any density-dependent loss process the appropriate density to use in computing its τ is the peak density divided by $2\sqrt{2}$ for the plugs or just 2 for the center cell. The center cell volume V_c and plug volumes V_p are defined so that multiplying by the respective peak densities gives the total number of particles in each. Thus for plugs with gaussian $1/e$ scale lengths r_p and l_p , $V_p = \pi r_p^2 \sqrt{\pi} l_p$, and for the center cell with gaussian radial scale length r_c and uniform density in z , $V_c = \pi r_c^2 L_c$, where $L_c = \int dz B_{cl}(z=0)/\bar{B}_{cl}(z) \approx 300$ cm (plug-to-plug). (2) Unless otherwise specified, densities and betas referred to are the peak values. (3) All energies and potentials are given in keV, densities in cm^{-3} , B fields in kG, and lengths in cm. All particle and energy loss terms are multiplied by the electron charge so that all particle input or loss terms are in amperes equivalent, and so that all energy input or loss terms are in kW units. (4) Whenever an existing theory and experimental observation are in substantial agreement on some effect A, the theory scaling is preferable to use over an empirical scaling for that particular term, if the theory scaling is more complete in describing effect A as a function of more variables than have yet been tested in the empirical scaling. (5) When an existing theory and experimental observation are in substantial disagreement, an empirical fit to the experimental observations is always preferred in the supermodel. We want the supermodel to be the best updated description of what the experiment is telling us about scaling, not just a theoretical tool. The experiment tests theory, not vice versa. This is an important choice to make in the area of quasilinear theory and end loss scaling. We have observed nearly empty loss cones in 2XIIB (albeit with dominant fluctuation

losses), in contrast to DCLC theory which demands a 50% full loss cone under any conditions. Also DCLC theory requires higher cyclotron harmonics to warm up the stream ions, while as far as we can tell the fundamental is the only harmonic we see with sufficient amplitude to explain the observed ion energy diffusion (See Turner's analysis).⁽²⁾ Thus we rely on Molvik's plots of ω_{ci} fluctuation amplitude versus measured end loss current, however meager they are, to relate fluctuation-caused losses to the amount of stream current. (6) In the case no previously tested theory or experimental scaling exists to describe a given observation, we are all free to cobble up some plausible or even implausible scaling term to plug into the supermodel. This is really the proper function of the supermodel - to test new scaling laws explaining new data, within a framework retaining previously tested scalings.

Table 1

Physical Processes Included in the Supermodel

(January 30, 1979)

Plug Ions

1. Ion-Ion angular scattering
2. Electron Drag
3. R.F. diffusion into the Loss cone
4. Charge exchange off the beam
5. Angular spreading by instability
6. Radial losses equivalent to charge exchange on gas
7. Beam trapping efficiency including beam aiming, cold ion fractions and ionization of cx neutrals

Electrons

1. Heating by plug ion drag and auxiliary heating
2. Gas recycling in the end tanks
3. Space-charge-limited secondary electron emission
4. Radiation losses by impurities in the center cell
5. Convection losses through sheath potentials
6. Equilibration losses to cold ions in the center cell
7. Ionization energy losses (gas and beams)
8. Radial electron thermal conductivity to the gas box

Center Cell Ions

1. Input by equilibration with electrons and auxiliary heating
2. Charge-exchange energy losses on gas
3. Pastukhov and collisional loss out of potential well
4. Radial diffusion to gas box by resonant ions.
5. Two-component operation with beam injection

Independently-Controlled Experimental Variables

(1) Incident plug beam current I_{bp} (each plug)	(Amps)
(2) Incident center cell beam current I_{bc}	(Amps)
(3) Total center cell gas feed current I_g	(Amps)
(4) Auxiliary electron heating power P_e	(kW)
(5) Auxiliary center cell ion heating power P_{ic}	(kW)
(6) Plug plasma radius r_p (beam aiming)*	(cm)
(7) Center Cell plasma radius r_c (gas box limiter) ⁺	(cm)
(8) Solenoid field B_c	(kG)
(9) Plug midplane field B_p	(kG)
(10) Plug mirror ratio R_p	

*Other things besides beam aiming can affect the plug radius, such as radial distribution of end loss from the center cell, charge-exchange erosion on gas, fluctuation level (radial diffusion), and gyroradius size. However, beam aiming is the most effective handle on controlling the plasma radius for any set of conditions.

⁺For gas box limiters mapping to a plug radius less than r_p . For gas box limiters mapping greater than r_p , as expected in initial operation in TMX, the center cell plasma radius should be limited by end loss around the boundary of the plug to approximately that radius which maps to r_p , determined by conservation of magnetic flux.

There are six species in the supermodel: (1) electrons, (2) hot plug ions (deuterons) (3) plug impurities (as a group) (4) cold center cell ions (deuterons from gas feed), (5) hot center cell ions (beam injected deuterons), and (6) center cell impurities (as a group). Allowance is made for use of hydrogen rather than deuterium if desired, in the gas feed and/or beams.

Thus included are 4 rate equations for the densities (or total number of particles) of electrons, hot plug ions, cold center cell ions (deuterons), and hot center cell ions, and 4 rate equations for their respective average energies (or total energy content in the volumes). Since the impurity ion equilibration time with the cold center cell ions will generally be quite short in comparison to the cold ion lifetime, the impurity ions are lumped together with the cold ions in the rate equation for cold ion energy balance. The impurity buildup in the center cell and plugs is described by a set of additional particle rate equations, one for each type of impurity, which are given simply by the source term for each impurity and whatever losses each has. As there are no models at present for the time-dependent impurity sources (with the exception of oxygen from the neutral beams), the impurity source terms must await experimental measurement.

Neutral Beam Trapping

In the calculation of neutral beam trapping in the plugs and center cell of TMX, it is important to take into account a cold ion fraction for which trapping is by charge-exchange as well as by ionization. This is especially true in the center cell, where beam injection gives rise to a two-component distribution, but is also true for the plugs in cases with large gas feed currents, and even with small gas feed currents because the center-cell plasma "leaning" on the inner side of the plugs can be trapped by the part of the plug beams.

For the general case of thick plasmas, ionization of fast neutrals from charge exchange off the beam must be taken into account. In the limit of infinite line density, the total ionization fraction does not approach unity because of a back scattered fraction of charge-exchanged atoms. For neutral beams with energies less than 40 keV, the backscattered fraction is

approximately constant at 50%, so in the following formulas the effect of energy dependence on the backscattered fraction can be neglected, permitting the use of a simple analytic fit for the secondary neutral ionization fraction $f_{\text{ion}}^{(?)}$.

Paul Drake⁽³⁾ has noted that in 2XIIIB, plasmas have typically contained up to 5% of hot, partially ionized, trapped oxygen impurities which originated from dirty arc chambers in the neutral beams, and that those impurities can significantly increase the fraction of trapped beam because of large charge-exchange cross-sections of deuterium beam atoms on partially ionized oxygen (typically several times larger than deuterium-deuterium charge-exchange cross sections). The same situation would be true for beam trapping in the center cell if significant impurity fractions exist there. As long as the impurity fractions are only a few percent, we can calculate their effect on beam trapping to satisfactory accuracy by simply treating the impurities as an additional cold deuterium density equal to the electron density component associated with their charge, and then use only cross-sections for ionization and charge-exchange for deuterium. The fact that the total cold ion density defined this way would be greater than the sum of impurity densities and cold deuterium densities due to the multiple charge of the impurities would be mostly offset by the larger charge-exchange cross-section of the impurities. This approximation is further justified in as much as the charge-exchange cross sections for multiply-ionized impurities are rather poorly known.

Plug Beam Trapping

The fraction of incident plug neutral beams ionized by either ionization collisions or by charge-exchange off cold ions (including impurities), is given by

$$f_{ion p} = (1 - e^{-\alpha p}) \left(\frac{\langle \sigma v \rangle_{ion p}}{\langle \sigma v \rangle_{atn p}} \right) \left[\left(1 + \frac{\langle \sigma v \rangle_{cxbp}}{\langle \sigma v \rangle_{ion p}} f_{ion p}^{(2)} \right) + \frac{\langle \sigma_{cx} \cdot v \rangle_{pC}}{\langle \sigma v \rangle_{ion p}} (1 - f_{ion p}^{(2)}) \left(\frac{\bar{n}_{pC}}{\bar{n}_p} \right) \right], \quad (1)$$

and the fraction of incident plug neutral beams escaping the plasma as charge-exchanged neutrals is given by

$$f_{cxbp} = (1 - e^{-\alpha p}) (1 - f_{ion p}^{(2)}) \left[\frac{\langle \sigma v \rangle_{cxbp}}{\langle \sigma v \rangle_{atn p}} - \frac{\langle \sigma_{cx} \cdot v \rangle_{pC}}{\langle \sigma v \rangle_{atn p}} \left(\frac{\bar{n}_{pC}}{\bar{n}_p} \right) \right], \quad (2)$$

where the electron density in the plug averaged over the beam footprint (taking gaussian beam width $\Delta z = \text{plug scale length } l_p$) is

$$\bar{n}_p = \bar{n}_{pC} + \frac{\sqrt{\pi}}{2} n_{pH}, \quad (3)$$

with the peak hot deuterium plug ion density being n_{pH} , and with

$$\bar{n}_{pC} = 1/4 n_{cC} + n_s + \sum_i \frac{\sqrt{\pi}}{2} n_{zpi} z_{pi} \quad (4)$$

being the cold ion density averaged over the beam footprint. The cold ion density presented to the plug beams has three parts: the cold ion density n_{cC} in the center cell having a gradient into the inner half of the plugs (hence the 1/4 factor), a streaming ion component n_s from center cell

losses streaming all the way through the plugs, and a collection of impurities n_{zpi} (mainly oxygen) with charge Z_{pi} trapped in the plug by injection from the plug neutral beams. Since the cold center cell ion density is attenuated by the plug ambipolar potential down to the escaping density n_s at the plug midplanes, the peak midplane electron density is given by

$$n_p = n_{pH} + n_s + \sum_i n_{zpi} Z_{pi} \quad (4a)$$

In eqs. (1) and (2), the $\langle \sigma v \rangle$'s are averaged over the hot and cold plug ion components:

$$\langle \sigma v \rangle_{ion p} = \langle \sigma v \rangle_e + \frac{(\sigma_i \cdot v)_{pC} \bar{n}_{pC} + (\sigma_i \cdot v)_{pH} n_{pH} \frac{\sqrt{\pi}}{2}}{\bar{n}_p} \quad (5)$$

$$\langle \sigma v \rangle_{cxbp} = \frac{(\sigma_{cx} \cdot v)_{pC} \bar{n}_{pC} + (\sigma_{cx} \cdot v)_{pH} n_{pH} \frac{\sqrt{\pi}}{2}}{\bar{n}_p} \quad (6)$$

$$\langle \sigma v \rangle_{atnp} = \langle \sigma v \rangle_{ion p} + \langle \sigma v \rangle_{cxbp} \quad (7)$$

where $\langle \sigma v \rangle_e$ is the electron impact ionization rate and where the individual $\sigma \cdot v$ products are obtained from Fig. 1 as follows: $\sigma_i \cdot v$ products for ion impact ionization and $\sigma_{cx} \cdot v$ products for charge-exchange which have subscripts pC are evaluated from Fig. 1 at an effective collision energy equal to the average incident beam energy $\bar{E}_{bp}$, and those with subscripts pH are evaluated at an average collision energy equal to the sum of average beam energy and average hot plug ion energy $\bar{E}_{bp} + \bar{E}_{pH}$. The

total attenuation factor α_p in eqs. (1) and (2) is given by

$$\alpha_p = f_{bp} (\int ndl)_{op}^{<cv>} \text{atnp} / v_{bp} \quad (8)$$

where $f_{bp}(\int ndl)_{op}$ is the average electron line density over the beam footprint, and v_{bp} is the beam velocity averaged over the full, half and third energy components of all the plug beams. The average beam line density factor f_{bp} is given by

$$f_{bp} = \frac{(\int ndl)_p}{(\int ndl)_{op}} = \frac{\int_0^\infty dy \int_0^\infty dz (\int ndl)_p(y,z) e^{-(z/\Delta z)^2} e^{-\left(\frac{y-y_0}{\Delta y}\right)^2}}{(\int ndl)_{op} \int_0^\infty dy \int_0^\infty dz e^{-(z/\Delta z)^2} e^{-\left(\frac{y-y_0}{\Delta y}\right)^2}}, \quad (9)$$

where $(\int ndl)_{op}$ is the central electron line density of the plug,

$$(\int ndl)_p(y,z) = (\int ndl)_{op} \sqrt{1 - (y/r_p)^2} e^{-(z/l_p)^2}, \quad (10)$$

for beams aimed at y_0 above the axis, with focused gaussian widths Δz in the horizontal direction and Δy in the vertical direction. r_p and l_p are the 1/e radius and half length for gaussian plug profiles. For head-on injection, ($y_0 = 0$), $\Delta z \approx l_p$, and $\Delta y \ll r_p$, we have approximately

$$f_{bp} \approx \frac{1}{\sqrt{2}} [1 - 1/4 \left(\frac{\Delta y}{r_p}\right)^2] \approx 0.69 \quad \text{for } \Delta y \approx 3 \text{ cm}, \quad (11)$$

$$r_p = 10 \text{ cm}.$$

The fraction $f_{ion\ p}^{(2)}$ of charge-exchanged neutrals from the beam which are ionized before final escape from the plasma (after 2nd, 3rd, 4th ...etc. cx events) is given by the following empirical fit to Barry Stallard's buildup code⁽⁴⁾ for a pencil beam:

$$f_{ion\ p}^{(2)} = 0.58 (1 - e^{-0.93 \alpha_p^2 \langle \sigma v \rangle_{ion\ p}^2 / \langle \sigma v \rangle_{atnp}^2})^{1/2} \quad (12)$$

The effect of secondary ionization of cx neutrals being less outside the midplane, or for beam aimings off axis, should be largely accounted for by the fact that α_p is an average attenuation factor containing the average line density factor f_{bp} . While this empirical fit (eq. 11) for $f_{ion\ p}^{(2)}$ used in eq. (1) for $f_{ion\ p}$ (with $\bar{n}_{pC} = 0$) reproduces Barry Stallard's buildup code to within a few percent, it gives $f_{ion\ p}$ 13% lower than the upper-limit trapping fraction at line density $2 \times 10^{15} \text{cm}^{-2}$ given by Bill Turner's trapping code (see Fig. 2). At line densities below 10^{15}cm^{-2} , eq. (1) agrees with either code better than 5%. Since neither code calculates secondary ionization of charge-exchanged neutrals in 3D (x, y, and z), eq. 11 is probably just as good as either code, for cases of zero cold ion fraction, $\bar{n}_{pC} = 0$. It should be noted that the secondary neutral ionization fraction $f_{ion\ p}^{(2)}$ is applied in this model to charge-exchange neutrals arising from cold ions as well as hot ions, a simplifying but crude approximation. As $f_{ion\ p}^{(2)}$ was fit to ionization only of hot charge-exchange neutrals in Stallard's code, the ionization fraction of colder charge-exchanged neutrals will be underestimated. For moderate line densities $\leq 10^{15} \text{cm}^{-2}$ and small cold ion fractions, the underestimate in total trapping fraction $f_{ion\ p}$ will be small. Note that $f_{ion\ p}^{(2)} \rightarrow 0.58$ in the limit of infinite line density, as it should because of backscattered, hot charge-exchanged neutrals.

Central Cell Beam Trapping

For beam trapping in the center cell, we can use the same formulas eqs. (1) through (12) by replacing subscripts p for the plugs with subscripts c for the center cell, and using appropriate beam and hot ion energies for the center cell. The electron density in the center cell would be given by (no average over z necessary for center cell beams)

$$n_c = n_{cC} + n_{cH} \quad (13)$$

where n_{cH} is the hot deuterium ion component created by beam injection into the center cell, and where the cold ion density in the center cell is defined as

$$n_{cC} = n_{cCd} + \sum_i n_{zci} Z_{ci} \quad (14)$$

where n_{cCd} is the center cell cold deuterium ion density generated by the gas feed, and where the n_{zci} are the impurities with charge state Z_{ci} trapped in the center cell. Because of their high Z, the impurities are confined for times long compared to the cold deuterons at a common temperature T_c . Because in general the ratio n_{cC}/n_c will be large (≈ 1) in two component operation, there will be few backscattered charge exchange neutrals, so we take the fraction of secondary charge-exchange neutral ionization to be unity in the limit of large line density:

$$f_{ion c}^{(2)} = (1 - e^{-0.93 \alpha_c^2 \langle \sigma v \rangle_{ion c}^2 / \langle \sigma v \rangle_{atnc}^2})^{1/2} \quad (15)$$

Because the effective center cell plasma length L_c will be much larger than the beam width, we can take the center cell line density z-variation $e^{-(2z/L_c)}$ to be constant at unity over the horizontal scale length of

the beam, so that

$$(fndl)_c(y,z) = (fndl)_{oc} \sqrt{1 - (y/r_c)^2}.$$

Electron Energy Rate Equation

The overall electron power flow determines T_e in the supermodel. According to the discussion in the introduction, we take all electrons to escape axially, while allowing for some cross-field thermal conductivity loss. We assume that conditions are such that T_e is high enough for electron mean-free-paths to be longer than the solenoid length, so that collisionless convective electron energy loss scaling will hold everywhere. Roughly, this means electron temperatures above 80 eV for solenoid densities of 10^{13} cm^{-3} . We also assume radiation loss P_{rad} due to impurities trapped in the center cell to be small enough to keep electron mfp's long there. For small depressions in T_e in the center cell due to radiation loss, the difference in electron temperatures between the center cell (T_{ec}) and the plugs (T_{ep}) can be given by a balance of heating of center cell electrons passing through hotter plug electrons, with the radiation loss P_{rad} minus beam heating of electrons in the center cell:

$$3 \left[n_c \sqrt{\frac{T_{ec}}{\phi_c}} \frac{B_p(1 - \beta_p/4)}{B_M} \right] (n_p v_p) (T_{ep} - T_{ec}) q = P_{rad} - \frac{qn_{CH}V}{\tau_{Dc}} (\bar{E}_{ph}^{-3/2} T_{ec}) \quad (17)$$

$5 \times 10^8 \phi_c^{3/2}$

where $B_p(1 - \beta_p/4)$ is the beta-depressed plug field and B_M the mirror field. Since P_{rad} in general peaks around $T_{ec} = 20$ eV, it will be important to minimize impurities so that at least the outer shells of the impurities can be ionized by available beam drag power (≈ 1 MW max), allowing buildup to a high T_{ec} . Some preliminary estimates by Drake⁽⁴⁾ indicate it will be necessary to start with less than 3% titanium in the center cell

for drag power to exceed the peak radiation losses at 20 eV. Assuming we will be able to solve this initial buildup problem, P_{rad} will in general decrease with T_{ec} above 20 eV as fast or faster than the drag power, allowing steady state operation with $T_{ec} \geq 80$ eV.

Since initial operation of TMX will not employ grids to suppress secondary electrons at the walls, it would be prudent to assume space-charge-limited flow rates of secondary electrons at a density n_{ext} at the end walls.⁽⁵⁾ Since gas recycling raises n_{ext} above the streaming ion density n_s , it effectively amplifies the space-charge-limited secondary flow rate. We can experimentally determine n_{ext}/n_s as a measure of gas recycling. By convention, electron energy loss per ion lost is described by a parameter η which includes the secondary emission and gas recycling energy losses. In keeping track of the components of electron current originating from center cell and plug neutral beams, one must use only the part of those beam's ionization which results in net input of free electrons, since charge-exchange of those beams off a cold ion component, while ionizing the beam, does not deposit a free electron in the process. With this in mind the overall electron energy rate equation can be written:

$$\frac{d}{dt} (3/2 q n_c T_{ec} V_c + 3q n_p T_{ep} V_p) =$$

$$\frac{2q n_{pH} V_p}{\tau_{Dp}} (\bar{E}_{pH} - 3/2 T_{ep})$$

$$+ \frac{q n_{cH} V_c}{\tau_{Dc}} (\bar{E}_{cH} - 3/2 T_{ec})$$

$$+ P_e$$

$$\begin{aligned}
& - \frac{q n_{cC} (Z^2/M)_{\text{eff}} v_c}{\tau_{Dc}} (3/2 T_{ec} - 3/2 T_c) \\
& - (I_g f_{ion g} + I_{bc} f_{ion c}^*) n_c T_{ec} \\
& - 2 I_{bp} f_{ion p}^* n_p T_{ec} \\
& - P_{\text{rad}} \\
& - P_{eK1}
\end{aligned} \tag{18}$$

The first and second terms in eq. (18) are the power inputs to electrons by drag from hot deuterium ions at densities n_{pH} and n_{cH} , average energies $\bar{E}_{pH}$, and $\bar{E}_{cH}$, and drag times τ_{Dp} and τ_{Dc} in the plugs and center cell, respectively. The third term on the R.H.S. of eq. (18), P_e , is the auxiliary electron heating power (such as ECRH, if any). The loss terms on the R.H.S. of eq. 18 are described in order as the equilibration losses to cold ions and impurities in the center cell, energy losses of $n_c T_{ec}$ per electron born at potential ϕ_e in the center cell, energy losses of $n_p T_{ec}$ per electron born at potential ϕ_p in the plugs, radiation losses in the center cell P_{rad} , and radial electron thermal conductivity losses to the gas box limiters P_{eK1} . Here we have made the approximation that the kinetic energy loss of electrons escaping to the end walls is characteristic of T_{ec} in the center cell rather than T_{ep} in the plugs, for either electrons originating from beam ionization in the plugs or for electrons originating from ionization in the center cell. This is because plug electrons, as they diffuse upward in energy in climbing out of the plugs' potential well, necessarily must become electrons passing through the

center cell before they have reached the end walls, and then their thermal temperature will equilibrate to the center cell temperature. We have neglected electron equilibration losses to the cold ion component n_{pC} in the plugs because the plug impurity ions are in fact hotter than T_{ep} , and because the streaming density n_s in the plugs will in general be quite small for minimum gas feeds. We neglect radiation loss by impurities in the plugs because we expect it will be small in comparison to radiation loss in the center cell, and also because that radiation loss (of order 50 kW per plug) is largely offset by the hot oxygen drag power by beam injection into the plugs ($\approx 5\%$ of 1 MW ≈ 50 kW), which we have also neglected (n_{pH} and E_{pH} are meant to be for hot deuterium plug ions only).

The drag times in eq. (18) are given by

$$\tau_{Dp} = \frac{2 \times 10^{13}}{\ln \Lambda_1(c)} \left(\frac{M_{ip}}{M_d} \right) \left(\frac{2\sqrt{Z}}{n_p} \right) T_{ep}^{3/2} \quad (19)$$

$$\tau_{Dc} = \frac{2 \times 10^{13}}{\ln \Lambda_1(c)} \left(\frac{M_{ic}}{M_d} \right) \left(\frac{2}{n_c} \right) T_{ec}^{3/2} \quad (20)$$

$$\ln \Lambda_1(p, c) = \ln (5.7 \times 10^{13} T_{ep, c} / \sqrt{n_{p, c}}) \quad (21)$$

and where we have included the mass factors ($M_{ip, c}/M_d$) to allow for hydrogen injection, if desired. Because impurities trapped in the center cell equilibrate faster with electrons than cold deuterium ions due to their higher Z , the factor

$$(Z^2/M)_{\text{eff}} = \frac{n_{cCd} + \sum_i n_{zci} Z_{ci}^2 (M_d/M_{zci})}{n_{cc}} \quad (22)$$

must be included in the electron-cold ion equilibration term.

In eq. 18, $f_{ion g}$ is the fraction of gas feed current which is ionized. We generally think $f_{ion g} \approx 1$, but we should allow for $f_{ion g} < 1$ (even though it may be difficult to resolve $f_{ion g} < 1$ from radial loss determined with particle balance measurements). The beam ionization fractions associated with electron deposition are those given by eq. 1 minus the charge-exchange part on cold ions:

$$f_{ion p}^* = f_{ion p} - (1 - e^{-\alpha_p}) \left[\frac{(\sigma_{cx} \cdot v)_p C}{\langle \sigma v \rangle_{atnp}} (1 - f_{ion p}^{(2)}) \left(\frac{\bar{n}_p C}{\bar{n}_p} \right) \right] \quad (23)$$

$$\text{and } f_{ion c}^* = f_{ion c} - (1 - e^{-\alpha_c}) \left[\frac{(\sigma_{cx} \cdot v)_c C}{\langle \sigma v \rangle_{atnc}} (1 - f_{ion c}^{(2)}) \left(\frac{n_c C}{n_c} \right) \right] \quad (24)$$

The eta values for electron currents from the plugs and center cell are given by

$$\eta_p = \left[\epsilon + \phi_p + \frac{T_{ec}}{1 - \lambda_{SCL}} \left(\frac{n_{ext}}{n_s} \right) \left(\frac{B_M}{B_w} \right) \right] / T_{ec} \quad (25)$$

$$\eta_c = \left[\epsilon + \phi_e + \frac{T_{ec}}{1 - \lambda_{SCL}} \left(\frac{n_{ext}}{n_s} \right) \left(\frac{B_M}{B_w} \right) \right] / T_{ec} \quad (26)$$

where B_w is the field at the end wall, $\epsilon = 30$ eV is the ionization energy loss, λ_{SCL} is a space-charge-limited secondary electron multiplier given by

$$\lambda_{SCL} \approx \left[0.15 \ln(b) + 0.656 \right]^{1/4}, \quad (27)$$

with

$$b \equiv \frac{T_{ec}}{E_i(\text{wall})} = \frac{T_{ec}}{\phi_p + E_s}, \quad (28)$$

$$\text{and } \bar{E}_S \approx \frac{T_c(I_g f_{ion g} + I_{bc} f_{ion c}^* + \bar{E}_{Lp}(2I_{bp} f_{ion p}^*))}{I_g f_{ion g} + I_{bc} f_{ion c}^* + 2 I_{bp} f_{ion p}^*} \quad (29)$$

being the average kinetic energy of all escaping ions at the plug midplanes. $\bar{E}_{Lp}$ is the escape energy of plug ions, to be derived later. Here we approximate all of the center cell trapped beam to be thermalized, although some of the trapped center cell beam ions scatter out the center cell mirrors before thermalization. Eq. 27 is an empirical fit to a graph (Fig. 3), taken from Gary Porter's memo⁽⁵⁾ on secondary electron emission.

The amplification of secondary electron emission by gas recycling appears in eqs. (25) and (26) for n_p and n_c as the factor $(n_{ext}/n_s)(B_M/B_W)$. The external cold plasma density at the walls

$$n_{ext} = n_s \left(\frac{B_W}{B_M} \right) + n_{recycle} \quad (30)$$

is equal to the sum of the streaming ion density n_s at the peak mirror field B_M ,

$$n_s = \frac{(1/2 I_g f_{ion g} + 1/2 I_{bc} f_{ion c}^* + I_{bp} f_{ion p}^*) B_M}{\pi r_p^2 B_p (1 - \beta_p/4) \left[\frac{2(\phi_p + \bar{E}_S)}{M_i} \right]^{1/2}}, \quad (31)$$

reduced by a factor (B_w/B_M) because of expansion to a low field B_w at the wall, and the cold ion density n_{recycle} due to ionization of gas at the wall, which is governed by the rate equation:

$$\left(\frac{n_{\text{recycle}}}{\tau_{\text{ext}}}\right) = n_{\text{ow}} \left[n_{\text{recycle}} + n_s \left(\frac{B_w}{B_M}\right) \right] \langle \sigma v \rangle_e, \quad (32)$$

$$\text{or } \left(\frac{n_{\text{recycle}}}{n_s}\right) \left(\frac{B_M}{B_w}\right) = \frac{n_{\text{ow}} \langle \sigma v \rangle_e \tau_{\text{ext}}}{1 - n_{\text{ow}} \langle \sigma v \rangle_e \tau_{\text{ext}}}, \quad (33)$$

$$\text{where } n_{\text{ow}} \approx 2 \times 10^9 \left(\frac{I_g}{2} f_{\text{ion } g} + \frac{I_{bc}}{2} f_{\text{ion } c} + I_{bp} f_{\text{ion } p} \right) \text{cm}^{-3} \quad (34)$$

is the equilibrium gas density at the end walls (from Atkinson's pumping calculation for the TMX end dump tanks with titanium-gettered LN cooled liners), and where

$$\tau_{\text{ext}} \approx \frac{L_{\text{fan}}}{v_{\text{sound}}} \approx \frac{300 \text{ cm}}{\sqrt{T_{\text{ec}}/M_d}} \approx \frac{1.3 \times 10^{-5}}{\sqrt{T_{\text{ec}}}} \text{ sec} \quad (35)$$

is the lifetime of a cold external ion generated by recycling. Thus we have

$$\left(\frac{n_{\text{ext}}}{n_s}\right) \left(\frac{B_M}{B_w}\right) = \frac{1}{1 - n_{\text{ow}} \langle \sigma v \rangle_e \tau_{\text{ext}}}, \quad (36)$$

which blows up when $n_{\text{ow}} \langle \sigma v \rangle_e \tau_{\text{ext}} \rightarrow 1$. When this happens n_{ext} increases to a high density limited only by cross-field diffusion. This is possibly an explanation of why n values of more than a hundred were often observed in 2XIIB.

The radiation loss in the center cell is given by

$$P_{\text{rad}} = V_c \left(\frac{n_c}{2} \right) \sum_i n_{zci} L_{zci} \times 10^{-10} \text{ (kW)} \quad (37)$$

where the radiation rates L_{zci} are given by D. E. Post⁽⁶⁾ in ergs per electron per ion $\text{cm}^{-3}\text{sec}^{-1}$. See Fig. 4 for the radiation rates of titanium, oxygen, nitrogen and carbon as functions of T_{ec} . These radiation rates are those occurring in coronal equilibrium, so they apply only for times longer than a recombination time of the final ionization state, which has to be in equilibrium with ionization of the next lower ionization state. One should check, for T_{ec} values higher than a few hundred eV, that such recombination or ionization times are not longer than the total 50 millisecc of available beam injection in TMX. Besides graphs of ionization rates, Post⁽⁶⁾ gives the average charge state $\langle Z \rangle$ for a given impurity ($\langle Z \rangle = Z_{ci}$ here) as a function of T_{ec} , as well as polynomial fits to both the ionization rates and average charge which can be used to compute both the radiation loss P_{rad} and the impurity contribution to electron density in eq. (14).

Electron Particle Balance

We assume quasi-charge neutrality, so that eqs. (4a) and (13) for the electron densities are satisfied at all times, given by the rate equations for the various ion densities. Balance of ion and electron losses thus determines the plasma potentials in TMX. We assume $n_c^2 V_c \gg n_p^2 V_p$, so regardless of where electrons are injected (plugs or center cell), electron scattering in the solenoid dominates over scattering in the plugs in determining the center cell potential ϕ_e . Then overall quasi-charge neutrality requires (see Appendix A 1 in the TMX proposal)

electron loss = total ion loss, at any instant in time

$$\frac{q[(1/2)n_c^2 \frac{v_c}{g(R_c)} + (1/\sqrt{2}) n_p^2 \frac{v_p}{(\pi\tau)_{ee}}]}{\left(\frac{T_{ec}}{\phi_e}\right) \exp - \left(\frac{T_{ec}}{\phi_e}\right)} =$$

$$I_g f_{ion g} + I_{bc} f_{ion c}^* + 2 I_{bp} f_{ion p}^* \quad (38)$$

$$\text{where } (\pi\tau)_{ee} = \frac{5.6 \times 10^8}{1 + Z_{eff}} T_{ec}^{3/2} \quad (39)$$

is the product of electron density and scattering time on electrons, deuterons and impurity ions in the center cell with effective Z

$$Z_{eff} = \frac{\sum_i n_{zci} Z_{ci}^2}{\sum_i n_{zci} Z_{ci}} \quad (40)$$

The function

$$g(R) \equiv \sqrt{\pi} (2R + 1) \ln (4R + 2)/4R \quad (41)$$

is evaluated at the radial-averaged center cell mirror ratio (from the center cell to the plug midplanes)

$$R_c = \frac{\bar{B}_{pi}}{\bar{B}_{ci}} \quad (42)$$

where we have used the "short-fat" approximation for the reduced field in

the plugs

$$\bar{B}_{pi} = B_p (1 - \bar{\beta}_p/2), \quad (43)$$

and the "long-thin" approximation for the reduced field in the center cell

$$\bar{B}_{ci} = B_c \sqrt{1 - \bar{\beta}_c}. \quad (44)$$

We take the radial-average betas to be 1/2 of their peak values ($\bar{\beta}_p = 1/2 \beta_p$, $\bar{\beta}_c = 1/2 \beta_c$), with the peak betas given by

$$\beta_p = 4 \times 10^{-14} \left(\frac{n_{pH} \bar{E}_{pH\perp} + n_p T_{ep} + n_{Op} \bar{E}_{pH}}{B_p^2} \right), \quad (45)$$

including any hot oxygen density n_{Op} from beam injection, with $\bar{E}_{pH\perp}$ the perpendicular component of the average hot ion energy $\bar{E}_{pH}$, and

$$\beta_c = 4 \times 10^{-14} \left[\frac{n_c T_e + (n_{cCd} + \sum_i n_{zci}) T_c + n_{cH} \bar{E}_{cH\perp}}{B_c^2} \right], \quad (46)$$

with $\bar{E}_{cH\perp}$ the perpendicular component of the average hot ion energy in the center cell.

To keep the net flow of electrons between the plugs and the center cell nearly zero (except for the sources $I_{g\text{ion } g} + I_{bc\text{f}^*\text{ion } c}$ in the center cell) while maintaining quasi-charge neutrality, the denser plugs must be more positive than the center cell by an amount ϕ_c given by the Boltzman factor:

$$\phi_c = T_{ep} \ln (n_p/n_c), \quad (47)$$

where we use the plug electron temperature T_{ep} as Rognlien's Monte Carlo calculations show that ϕ_c is insensitive to the colder T_{ec} on the low density side. The total plug potential ϕ_p is then given by

$$\phi_p = \phi_e + \phi_c. \quad (48)$$

Conservation of Magnetic Flux

Conservation of magnetic flux determines the center cell plasma radius r_c in terms of the plug plasma radius r_p , when r_c is not limited to a smaller radius by the gas box limiters:

$$\pi r_c^2 B_c \sqrt{1 - \beta_c/2} = \pi r_p^2 B_p (1 - \beta_p/4), \quad (49)$$

$$\text{or } r_c = r_p \left[\frac{B_p (1 - \beta_p/4)}{B_c \sqrt{1 - \beta_c/2}} \right]^{1/2}, \quad (50)$$

using the fields averaged over the plasma radii.

Plug Ion Particle Rate Equation

The plug ion particle rate equation determines the hot ion plug density n_{pH} in the supermodel. We take any radial losses within the volume of the plugs to be included in the charge-exchange loss time τ_{cxgp} on gas, of plug ions at the plug surface, which we will give an expression for later. Thus we define the plug hot ion particle rate equation to be

$$\frac{d}{dt}(qn_{pH}V_p) = I_{bp}f_{ion p} - \frac{q n_{pH}V_p}{\tau_p} = I_{bp}f_{ion p} - \frac{q n_{pH}V_p}{\tau_{lcp}} - \frac{q n_{pH}V_p}{\tau_{cxgp}}, \quad (51)$$

where τ_p is the total hot-plug-ion particle lifetime and τ_{lcp} the plug ion particle lifetime by diffusion into the loss cone, enhanced by charge-exchange off the beam according to Rensink's memo⁽⁷⁾:

$$\tau_{lcp} = \tau_{cxbp} (e^{\tau_{lcp}^*/\tau_{cxbp}} - 1), \quad (52)$$

with τ_{lcp}^* the diffusion time into the loss cone in the absence of charge exchange off the beam, and where the beam charge-exchange time in the plugs is given by

$$\tau_{cxbp} = \frac{q n_{pH} V_p}{I_{bp} f_{cxbp}}. \quad (53)$$

For τ_{lcp}^* we include diffusion into the loss cone by classical ion-ion angle-scattering and by ion cyclotron fluctuations as well as electron drag, even though Rensink's calculation⁽⁷⁾ considered only drag. Assuming the overall shape of the hot-ion distribution function is not drastically different when angle-scattering and R. F. is included, the enhancement of plug ion particle lifetime by charge exchange off the beam should be approximately the same for all three loss processes. Loss by radial diffusion, or charge-exchange of hot ions on gas is not enhanced by charge-exchange off the beam since the cold ions created by the charge-exchange on gas are expelled by the plug potential in a transit time

short compared to the charge-exchange time on the beams τ_{cxbp} . Thus we write

$$\tau_{\text{icp}}^{-1} = \tau_{\text{Sp}}^{-1} + \left(\tau_{\text{Dp}} \ln \frac{\bar{E}_{\text{bp}}}{\bar{E}_{\text{Lp}}} \right)^{-1} + \tau_{\text{RF}}^{-1} \quad (54)$$

where τ_{Sp} is the angle-scattering loss time in the plugs, $\tau_{\text{Dp}} \ln (\bar{E}_{\text{bp}}/\bar{E}_{\text{Lp}})$ the loss time by drag, and τ_{RF} the loss time by RF diffusion into the loss cone. For the classical losses by angle-scattering and drag, we use the model developed by Rensink and Logan⁽⁹⁾ which predicts 2D Fokker-Planck calculations within 5%. The scattering time for hot deuterium plug ions is given by

$$\tau_{\text{Sp}} = \frac{3.9 \times 10^{12}}{\ln_{ii}^A(p)} \left(\frac{M_{\text{ip}}}{M_{\text{d}}} \right)^{1/2} \left(\frac{2\sqrt{2}}{n_{\text{pH}} + \sum_i n_{\text{zpi}} Z_{\text{pi}}^2} \right) \bar{E}_{\text{bp}}^{3/2} \text{Log}_{10} R_{\text{eff}}, \quad (55)$$

$$\text{where } \ln_{ii}^A(p, c) = \ln \left[1.4 \times 10^{15} \sqrt{\left(\frac{M_{\text{ip},c}}{M_{\text{d}}} \right) \left(\frac{\tau_{\text{e}} \bar{E}_{\text{p},cH}}{n_{\text{p},c}} \right)} \right] \quad (56)$$

is the coulomb logarithm for ion-ion scattering,

$$R_{\text{eff}} = \frac{R_{\text{p}} \sin^2 \bar{\theta}_{\text{p}}}{1 + \Delta \phi_{\text{p}} / \bar{E}_{\text{bp}}} \quad (57)$$

is the effective plug mirror ratio ($R_{\text{eff}} \rightarrow 1$ on the loss boundary),

$$R_{\text{p}} = R_{\text{pvac}} / (1 - \beta_{\text{p}}/4), \text{ (radial average)} \quad (58)$$

is the beta-enhanced plug magnetic mirror ratio,

$$\text{and } \Delta\phi_p = \text{lesser of } \begin{cases} \tau_e \ln \left(\frac{n_p}{R_p n_s} \right) \\ \phi_p \end{cases} \quad (59)$$

is the potential drop between the plug midplanes and the outer mirror throat as seen by confined plug ions. The lifetime by electron drag is the second term in eq. (54), with τ_{0p} given by eq. (19).

For the lifetime by R. F. energy diffusion we adopt the model proposed by Turner⁽²⁾:

$$\tau_{RF} = \frac{\bar{E}_{bp}^2}{2 D_E (1/2 \bar{E}_{bp})} \quad (60)$$

$$\text{where } D_E = \frac{\omega_{ci}}{\Delta\omega_{ci}} \omega_{ci} |\tilde{\phi}_{RF}|^2 J_1^2(k_{\perp} \rho_i) \frac{(\text{KeV})^2}{\text{sec}} \quad (61)$$

$$\text{with } J_1^2(k_{\perp} \rho_i) \approx \left(\frac{2}{\pi} \right) \left(\frac{1}{4.5} \right) \left(\frac{E_{pH1}}{E} \right)^{1/2}, \text{ for } k_{\perp} \rho_i \gg 1, \quad (62)$$

is the quasilinear energy diffusion coefficient for fundamental ion cyclotron fluctuations, at $E = \bar{E}_{bp}/2$, with $\tilde{\phi}_{RF}$ given in keV. Molvik has made several plots of $\tilde{\phi}_{RF}$ in relative units as a function of transmitted stream current j in 2XIIB, of which three typical examples are shown in Figs. 5 a,b,c. Although there is substantial scatter in the data, two features are generally exhibited in such plots: (1) $\tilde{\phi}_{RF}$ usually decreases

roughly exponentially as the transmitted stream current is increased, and (2) there is often an apparent saturation in $\tilde{\phi}_{RF}$ as the transmitted stream current density is reduced nearly to zero (there are no data points at strictly zero current because there is at least a minimum current due to escaping plug ions). A simple functional form for $\tilde{\phi}_{RF}$ suggested by such data is

$$\tilde{\phi}_{RF} = A e^{-(j/j_s)^2} \quad (63)$$

where j is the transmitted stream current density (including any plug ion current) and A is the saturation amplitude, which can be determined by the observation that at or near the minimum stream current, end loss energy scans usually show an energy spread in the escaping ions consistent with a mean ambipolar hole energy $\bar{E}_h$ given by

$$\bar{E}_h = \bar{E}_{LP} = \sqrt{\frac{\bar{E}_{LP}^2 + \bar{E}_{LP} \Delta\phi_p}{R_p}}, \quad (64)$$

where $\bar{E}_{LP}$ is the average plug ion loss energy (See Fig. 6), for those ions diffusing into the loss cone. $\bar{E}_h$ approaches the ambipolar cutoff energy $\Delta\phi_p/(R_p - 1)$ in the limit of extreme drag, but is substantially larger due to classical ion-ion scattering even when $T_{ep} = 10^{-2}\bar{E}_{bp}$. The actual enlarged hole energy $\bar{E}_h$ is the one most important for characterizing the DCLC - driving portion of the plug ion distribution. At zero or minimum stream then, the fluctuation amplitude is determined by the energy diffusion coefficient at the hole energy $\bar{E}_h$, being sufficient to either heat up an external stream ion to $\bar{E}_h$, or diffuse escaping plug ions over the same

energy downward, in one transit time $l_p/\bar{v}_{11}$:

$$\sqrt{2} D_E(\bar{E}_h) l_p/\bar{v}_{11} \approx \bar{E}_h \quad (65)$$

$$\text{whence } A^2 \approx \frac{2}{(4.5)^2} \bar{E}_h \bar{E}_{pHL} \left(\frac{\bar{v}_{11}}{l_p} \right) \left(\frac{\Delta\omega_{ci}}{\omega_{ci}} \right) \left(\frac{1}{\omega_{ci}} \right), \quad (66)$$

where we have used the small argument approximation to the Bessel function $J_1^2(x) \approx 1/4 x^2$ in evaluating D_E at $\bar{E}_h$. Expressed in terms of $\bar{E}_h$ and the mean parallel velocity at the midplane $\bar{v}_{11}$, the measured stream current density at which $\tilde{\phi}_{RF}$ has dropped to $1/e$ of its saturated amplitude is typically

$$j_s \approx 0.1 q \left(\frac{\bar{E}_h}{\bar{E}_{pHL}} \right) n_{pH} \bar{v}_{11} (0.4 r_p/a_{ip})^{-4/3} \quad (67)$$

where we have added the factor $(0.4 r_p/a_{ip})^{-4/3}$, with

$$a_{ip} = 6.46 \left(\frac{M_{ip}}{M_d} \right)^{1/2} \frac{\bar{E}_{pHL}}{B_p (1-\beta_p/4)} \text{ cm}, \quad (68)$$

to reflect the expected reduction in this stabilizing stream current density for larger plasmas. The $4/3$ exponent is a fit for r_p/a_{ip} in the range of 2 to 10. Thus using eqs. 61 through 67, we obtain

$$\tau_{RF} = \frac{\pi}{\sqrt{2}} \left(\frac{4.5}{2} \right)^3 \left(\frac{\bar{E}_{bp}}{\bar{E}_h} \right) \left(\frac{\bar{E}_{bp}}{\bar{E}_{pHL}} \right)^{3/2} \left(\frac{l_p}{\bar{v}_{11}} \right) e^{2(j/j_s)^2} \quad (69)$$

$$\text{where } (j/j_s)^2 \approx 10 \frac{(1/2 I_{q \text{ ion } g}^f + 1/2 I_{bc \text{ ion } c}^{f*} + I_{bp \text{ ion } p}^{f*})^2}{(q \pi r_p^2 n_{pH} \bar{v}_{11})^2}.$$

$$\left(\frac{\bar{E}_{pHL}}{\bar{E}_h} \right)^2 \left(\frac{r_p}{a_{ip}} \right)^{8/3} \quad (70)$$

$$\text{and } \bar{v}_{11} = \sqrt{\frac{\bar{E}_s}{M_{ip}}} \quad (71)$$

We turn now to calculation of the average plug ion loss energy $\bar{E}_{Lp}$ for the ions diffusing into the loss cone (not those charge-exchanging on gas). We extend the expression obtained in ref. 9 for $\bar{E}_{Lp}$ with classical scattering and drag to include R. F. diffusion in a straightforward manner:

$$\begin{aligned} \bar{E}_{Lp} = & \frac{\bar{E}_{bp} (0.6 \tau_{Sp})^{-1}}{(0.6 \tau_{Sp})^{-1} + (\tau_{Dp} \ln \frac{\bar{E}_{bp}}{\bar{E}_{Lp}})^{-1} + \tau_{RF}^{-1}} \\ & + \frac{E_{CDp} (\tau_{Dp} \ln \frac{\bar{E}_{bp}}{\bar{E}_{Lp}})^{-1} + \tau_{RF}^{-1}}{(0.6 \tau_{Sp})^{-1} + (\tau_{Dp} \ln \frac{\bar{E}_{bp}}{\bar{E}_{Lp}})^{-1} + \tau_{RF}^{-1}} \\ & + \frac{E_{CRF} \tau_{RF}^{-1}}{(0.6 \tau_{Sp})^{-1} + (\tau_{Dp} \ln \frac{\bar{E}_{bp}}{\bar{E}_{Lp}})^{-1} + \tau_{RF}^{-1}}, \end{aligned} \quad (72)$$

$$\text{where } E_{CDp} = \frac{\Delta\phi_p}{R_p \sin^2 \bar{\theta}_p - 1} \quad (73)$$

is the drag cutoff point,

$$\text{and } E_{CRF} = \frac{\Delta\phi_p + \sqrt{(\Delta\phi_p)^2 + 4(R_p - 1)R_p \bar{E}_{pH11}^2}}{2(R_p - 1)} \quad (74)$$

is the point on the loss boundary where an ion would cross if it were diffusing straight down in energy by RF fluctuations (see Fig. 6). In the limit of high T_e , no drag, and no RF diffusion, $\bar{E}_{Lp} \rightarrow \bar{E}_{bp}$, the mean injection energy, as it must by energy conservation.

The mean pitch angle $\bar{\theta}_p$ for hot confined plug ions can be considered as the sum of average injection angle and instability-induced angular spreading:

$$\bar{\theta}_p = \bar{\theta}_{injp} + \Delta\theta_{spreading}(p) \quad (75)$$

Thus the perpendicular and parallel components of average plug ion energy are related by

$$\frac{\bar{E}_{pH\perp}}{\bar{E}_{pH11}} = \tan^2 \bar{\theta}_p \quad (76)$$

Measurements of changes in the beam charge-exchange flux at 90° compared with density changes have been used to infer the mean parallel ion energy $\bar{E}_{pH11}$ in 2XIIB⁽¹⁰⁾. Other diagnostics such as the diamagnetic loop can

give the mean perpendicular ion energy $\bar{E}_{pH1}$ once the plasma length is known. Without any angular-spreading instability, a mean injection angle $\bar{\theta}_{inj p} = 7^\circ$ in TMX will alone give a parallel ion energy component $\bar{E}_{pH1} \approx 400$ eV at $\bar{E}_{ip} = 26$ keV, which is small. However, if $\bar{E}_{pH1}/\bar{E}_{pH} \approx 10 - 20\%$ results from an angular spreading instability as observed in 2XII B, the average loss energy $\bar{E}_{Lp}$ and hole energy $\bar{E}_h$ can be substantially increased according to eq. 72.

The charge-exchange lifetime on cold gas in the plugs can be written in a form convenient to experimental measurement as

$$\tau_{cxqp} = \frac{\eta_{pH} V_p}{S_p \phi_{cxgp}} \quad (77)$$

$$\text{where } S_p = 4\pi r_p l_p \sqrt{1 - \beta_p^2/4} \ln \left[\frac{(\text{fnd1})_{op}}{3.5 \times 10^{13}} \right] \text{ cm}^2 \quad (78)$$

is the effective plug surface area which is opaque to 1 eV gas neutrals, and where ϕ_{cxgp} is the average flux of energetic charge-exchange neutrals from cold gas, averaged over that surface. Under "best" vacuum conditions as in 2XII B and for times long compared to equilibration of gas from the neutral beam sources throughout the vacuum system ($t \geq 5$ ms), we can estimate a flux (11)

$$\phi_{cxgp} \approx 10^{14} Q_{\text{gas}} I_{bp} \text{ (cm}^{-2}\text{sec}^{-1}\text{)} \quad (79)$$

where Q_{gas} is a vacuum quality factor which is unity if TMX has as low a background gas flux as in the best operation of 2XII B. For $t < 5$ ms, see Fig. 7 for ϕ_{cxgp} as a function of $\int I_{bp} dt$. $Q_{\text{gas}} > 1$ may result in TMX

if some of the beams contribute less than normal beam current, if titanium walls saturate, if LN liners are not cooled, or if there are vacuum leaks. The flux ϕ_{cxgp} should be measured directly by secondary-emission detectors and/or bolometers at several points in z and at several angles, since vacuum conditions can be so variable. Even with moderate $Q_{gas} \approx 2 - 3$, one can estimate from eqs. 77 - 79 that most of the trapped beam current was lost by charge-exchange on cold gas in many 2XIIB experiments. Under such conditions the increase in effective plasma surface area S_p with increasing line density (the logarithm factor in eq. (78)) may be at least one contribution to the observed saturation in line density. The logarithm factor in the effective surface area accounts for the fact that the radius r_p' beyond which the residual line density is 10^{13} cm^{-2} (one mean-free-path for 1 eV neutrals) and the length z' out to which the fan remains opaque to neutral gas, both increase as the line density increases for fixed gaussian scale lengths r_p and l_p . Diagnostics such as Lyman alpha emission as a function of radius could be used to locate the radius of this critical surface experimentally.

Condensing some of the previous formulas for plug ion lifetime, the following representations may be convenient:

$$\tau_p = \tau_{lcp}^* \frac{\sigma_p}{\ln \left[1 + \left(\frac{\sigma_p}{1 - \sigma_p \frac{\tau_{cxbp}}{\tau_{cxgp}}} \right) \right]} \quad (80)$$

and

$$\tau_{lcp} = \tau_{lcp}^* \frac{\left(\frac{\sigma_p}{1 - \sigma_p \frac{\tau_{cxbp}}{\tau_{cxgp}}} \right)}{\ln \left[1 + \left(\frac{\sigma_p}{1 - \sigma_p \frac{\tau_{cxbp}}{\tau_{cxgp}}} \right) \right]} \quad (81)$$

$$\text{where } \sigma_p = \frac{f_{\text{cxbp}}}{f_{\text{ionp}}}, \quad (82)$$

and where, using eqs. (53) and (77), we have

$$\sigma_p \frac{\tau_{\text{cxbp}}}{\tau_{\text{cxgp}}} = 1.6 \times 10^{-5} Q_{\text{gas}} \left(\frac{S_p}{f_{\text{ionp}}} \right), \quad (83)$$

which we note is independent of beam current. When σ_p ($\tau_{\text{cxbp}}/\tau_{\text{cxgp}}$) approaches unity, $\tau_{\text{cxgp}} \ll \tau_{\text{icp}}$, and there is only a small margin of trapped beam current beyond what is lost by charge-exchange on gas, to make up for diffusion losses into the loss cone. This margin cannot be increased by simply increasing the beam current, since the gas flux is proportional to beam current, as well as the trapped current. Increasing f_{ionp} helps, and were it not for the delay in gas coming from the beam source modules, (see Fig. 7), buildup would not be possible (when f_{ionp} is initially small). In going from a line density of $5 \times 10^{14} \text{ cm}^{-2}$ to $2 \times 10^{15} \text{ cm}^{-2}$, the effective surface area S_p would increase by a factor of 1.7, while the increase in f_{ionp} would be about a factor of 3. Thus plasmas with higher line density are generally more tolerant of cold background gas. Unless gettered walls saturate with operation of more beams (increases in Q_{gas}), increasing beam current is still the best protection against cold gas losses (via increases in line density), in spite of increases in S_p with line density. The saturation in line density often observed in the buildup of 2XIIB plasmas would have to be caused by decreases in τ_{icp} with increasing line density, either from increasing

electron drag or from increases in

R. F. losses (reduced stream penetration), in addition to increases in S_p .

Since an end loss analyzer on the magnetic axis should not receive radial plug ion losses by charge-exchange on gas, the component of plug ion losses into the loss cone could be experimentally determined as

$$\tau_{lcp} = \frac{q n_{pH} \sqrt{\pi} l_p}{J_{ELA}(D)} \left[\frac{B_w}{B_p (1 - B_p/2)} \right] \quad (84)$$

where J_{ELA} is the component of analyzer current density (at the wall) coming from the plug. R.F. fluctuation levels would have to be sufficiently small so that plug ion losses at energies $\bar{E}_{Lp} + \phi_p$ could be distinguished from center cell ions escaping at energy $T_c + \phi_p$ by end loss energy scans. If this can be done, measurement of τ_{lcp} would compliment the measurement of radial charge exchange losses in checking particle and energy balance in the plugs.

Plug Ion Energy Rate Equation

The plug ion energy rate equation determines the average hot plug ion energy $\bar{E}_{pH}$ in the supermodel. Using the confinement times derived in the previous section on plug particle balance we have

$$\frac{d}{dt} (q n_{pH} \bar{E}_{pH} V_p) =$$

$$I_{bp} f_{ion p} (\bar{E}_{bp}) + I_{bp} f_{cxbp} (\bar{E}_{bp} - \bar{E}_{pH})$$

$$- \frac{q n_{pH} V_p}{\tau_{Dp}} (\bar{E}_{pH} - 3/2 T_e)$$

$$\begin{aligned}
& - \frac{q n_{pH} v_p}{\tau_{lcp}} (\bar{E}_{Lp} - \Delta E) \\
& - 1/2 I_g f_{ion g} \left(\frac{\tau_{cCz}}{\tau_{cC}} \right) (\Delta E) \\
& - q S_p \Phi_{cxgp} (\bar{E}_{cxgp})
\end{aligned} \tag{85}$$

$$\text{where } \Delta E = \bar{E}_h e^{-(j/j_s)^2} \tag{86}$$

is the energy change of an escaping ion by R. F. diffusion while in transit, according to eq. 65. For ions escaping from the plug, ΔE is subtracted from $\bar{E}_{Lp}$ as the perpendicular energy of the escaping ions is reduced. For streaming ions coming through the plug from the center cell, R. F. heating represents energy carried out of the confined hot plug ion distribution via the fluctuations, so ΔE multiplies the fraction of center cell current passing through the plugs. That fraction (τ_{cCz}/τ_{cC}) will be described in the next section. The average energy of a plug ion lost radially by charge-exchange on cold gas, $\bar{E}_{cxgp}$, can in principle be measured directly by a neutral analyzer just after beam turn off. Generally, $\bar{E}_{cxgp}$ appeared to be 10 to 30% greater than $\bar{E}_{pH1}$ as determined by the analyzer when the beams were on in 2XIIB. This conclusion is tentative in that the $\langle v \rangle$'s which went into the computer reduction of the energy channels are somewhat different for charge-exchange on gas than for charge-exchange on beams, but until we make a better measurement in TMX we can simply take

$$\bar{E}_{cxgp} \approx 1.2 \bar{E}_{pH1} \tag{87}$$

Note that since it is quite likely that the plug energy and angular distribution are not separable, the average ion energy measured by a neutral analyzer at 90° , $\bar{E}_{NA} \approx \bar{E}_{pH\perp}$. Thus if we take the time-averaged parallel energy component $\bar{E}_{pH\parallel}$, as determined by Nexsen⁽¹⁰⁾ according to his empirical scaling (See Fig. 8)

$$\frac{\bar{E}_{pH\parallel}}{\bar{E}_{pH}} \approx 6 \times 10^{-24} (n_p I_{bp})^{1.4}, \quad (88)$$

then we could experimentally determine $\bar{E}_{pH}$ as

$$\bar{E}_{pH} = \bar{E}_{pH\perp} + \bar{E}_{pH\parallel} = \frac{\bar{E}_{NA}}{1 - 6 \times 10^{-24} (n_p I_{bp})^{1.4}}. \quad (89)$$

One possible plug ion energy loss that is not included in eq. 85 is the wave energy that may be convected out of the plugs into the center cell by high frequency convective loss cone modes. The convection of wave energy itself is generally thought to be sufficient to keep the amplitude of these modes small, especially for short plugs in terms of l_p/a_i as in TMX; however, an estimate of this energy loss process specifically for TMX has not yet been made.

Center Cell Cold Ion Rate Equation

The center cell particle rate equation for the cold deuteron component which is maintained largely by the gas feed determines the steady state center cell cold deuteron density n_{cCd} . The presence of impurity ions enhances the Pastukhov-type losses of the cold deuterons by increasing their scattering rate. There are three sources of cold deuterons: (1) the gas

feed, (2) cold gas charge-exchange of hot plug ions on the inside half of the plugs (if the gas box limiters map to a radius $\geq r_p$), and (3) thermalization and charge-exchange on gas of hot deuterons from center cell beams, if any. There are also three types of losses considered (1) Pastukhov and the so-called collisional loss through the electrostatic barriers of the plugs, (2) radial loss by resonant ion drift motion and (3) charge-exchange of the cold deuterons on the part of the plug neutral beams on the inner side of the plugs, and on the center cell beams. We consider a beam injected deuteron in the center cell to become a cold deuteron once its energy either drags or gas-charge-exchanges below ϕ_c . Thus the cold deuteron particle balance is given by

$$\begin{aligned}
 \frac{d}{dt}(q n_{cCd} V_c) &= I_g f_{ion g} + I_{bp} f_{ion p} \left(\frac{\tau_p}{\tau_{cxgp}} \right) + I_{bc} f_{ion c} \left[1 - \frac{\tau_{cH}}{\tau_{Sc} G(\sigma_c)} \right] - \frac{q n_{cCd} V_c}{\tau_{cC}} \\
 &= I_g f_{ion g} + I_{bp} f_{ion p} \left(\frac{\tau_p}{\tau_{cxgp}} \right) + I_{bc} f_{ion c} \left[1 - \frac{\tau_{cH}}{\tau_{Sc} G(\sigma_c)} \right] \\
 &\quad - \frac{q n_{cCd} V_c}{\tau_{cCz}} - \frac{q n_{cCd} V_c}{\tau_{cCr}} - 2 I_{bp} f_{cxpc} - I_{bc} f_{cxcc} \quad , \quad (90)
 \end{aligned}$$

where τ_{cC} is the total center cell cold deuteron particle lifetime, τ_{cCz} is the center cell cold deuteron end loss lifetime by scattering out of the potential well, τ_{cCr} is the cold deuteron lifetime against radial loss to the gas box limiters,

$$f_{cxpc} = (1 - e^{-\alpha p})(1 - f_{ion p}^{(2)}) \left[\frac{\langle \sigma_{cx} \cdot v \rangle p_C}{\langle \sigma v \rangle a n p} \left(\frac{1/4 n_{cC}}{\bar{n}_p} \right) \right] \quad (91)$$

$$\text{and } f_{\text{CXCC}} = (1 - e^{-\alpha c})(1 - f_{\text{ion } c}^{(2)}) \left[\frac{(\sigma_{\text{CX}}^{\text{v}})_{\text{CC}}}{\langle \sigma v \rangle_{\text{atnc}}} \left(\frac{n_{\text{CCd}}}{n_c} \right) \right] \quad (92)$$

are the fractions of plug beams and center cell beams, respectively, which charge-exchange on the cold deuterons, $(\tau_p/\tau_{\text{CXgp}})$ is the fraction of plug beam ions charge-exchanging on gas on the inside halves of the plugs, and where $1 - \tau_{\text{CH}}[\tau_{\text{SG}}(\sigma_c)]$ is the fraction of center cell beams which thermalizes, or which does not scatter out of mirrors as hot ions. Rognlien and Cutler¹² have shown that the center cell electrostatic confinement time is larger than the Pastukhov lifetime $\tau_{\phi p}$ when the mean-free-path for a deuteron at energy ϕ_c to scatter into a center cell loss cone angle becomes less than the center cell length, roughly

$$\frac{\tau_{ii} \bar{v}_{ic}}{R_c L_c} \left(\frac{\phi_c}{T_c} \right) \leq 1. \quad (93)$$

Their results for electrostatic lifetime at different values of the mean-free-path parameter (eq. 93) suggests the following general expression for τ_{CCZ} (See Fig. 9):

$$\tau_{\text{CCZ}} = \tau_{\phi p} + \tau_{\phi c} \quad (94)$$

$$\text{where } \tau_{\phi p} = \tau_{ii} g(R_c/2) (\phi_c/T_c) \exp(\phi_c/T_c) \quad (95)$$

is the collisionless Pastukhov scaling with

$$\tau_{ii} = \frac{4 \times 10^{11}}{\ln \Lambda_{ii}(c)} \left(\frac{2}{n_c} \right) \left(\frac{M_{ic}}{M_d} \right)^{1/2} \frac{T_c^{3/2}}{z_{\text{eff}}}, \quad (96)$$

and where

$$\tau_{\phi C} = 2 R_c \left(\frac{L_c}{\bar{v}_{ic}} \right) \exp \left(\frac{\phi_c}{T_c} \right) \quad (97)$$

is their so-called collisional scaling, with

$$\bar{v}_{ic} = \sqrt{\frac{8 T_c}{\pi M_{ic}}} \approx 3.6 \times 10^7 \left(\frac{M_d}{M_{ic}} \right)^{1/2} T_c^{1/2} \text{ cm/sec} \quad (98)$$

Note that the ratio of $\tau_{\phi p}/\tau_{\phi c}$ is just the mean-free-path condition eq. (93), so that eq. (94) gives a smooth transition from collisionless Pastukhov scaling to collisional scaling at the right density regime. Using eq. 47 for ϕ_c , eq. 94 for τ_{cCz} can be written

$$\tau_{cCz} = \left(\frac{M_{ic}}{M_d} \right)^{1/2} \left[\frac{8 \times 10^{11} T_c^{1/2} T_e g(R_c/2) \ln(n_p/n_c)}{\ln A_i(c) n_c Z_{eff}} + \frac{5.7 \times 10^{-8} R_c L_c}{\sqrt{T_c}} \right] \left(\frac{n_p}{n_c} \right)^{T_{ec} T_c} \quad (99)$$

For the radial loss time to the gas box limiters by the mechanism of resonant ion drifts, we have approximately

$$\tau_{cCr} = \frac{r_{cl}^2}{D_{res}} \quad (100)$$

where r_{cl} is the radius of the gas box limiter, and where the radial diffusion coefficient for resonant ions is

$$D_{res} = (\delta r)^2 \frac{\bar{v}_{ic \parallel}}{L_c} \quad (101)$$

where the mean parallel velocity is

$$\bar{v}_{ic \parallel} = 1/2 \bar{v}_{ic} \quad (102)$$

and where the radial step size per bounce is given by

$$\delta r = a_{ic} \left(\frac{r_{c1}}{L_t} \right) \quad (103)$$

with a_{ic} the center cell ion gyroradius in the vacuum center cell field, and with L_t the transition length ≈ 100 cm according to Foote's Mafco II studies of TMX drift orbits. Thus we have

$$\tau_{cCr} = \left(\frac{L_t}{a_{ic}} \right)^2 \left(\frac{L_c}{\bar{v}_{ic \parallel}} \right) = 1.4 \times 10^{-9} L_t^2 L_c B_c^2 \sqrt{M_d/M_{ic}} T_c^{-3/2} (\text{sec}). \quad (104)$$

By varying the distribution of solenoid coil currents, Foote has shown one can vary L_t by a factor of 2, and hence τ_{cCr} by a factor of 4.

Impurity Rate Equations

Center Cell Impurities

Impurities in the TMX center cell may arise from an initial inventory

of streaming plasma trapped during the buildup process ($n_{\text{Czi}}(0)$), from bombardment of the gas box limiters and center cell vacuum walls, and from injection of neutral beams either in the plugs and center cell. We know from experiments in 2XII B that substantial oxygen impurities can be injected by neutral beams, for which we can give approximate formulas. At the present time we do not have formulas for the other impurity sources since they are complicated and uncertain. For example, the trapping of streaming gun plasma during buildup depends on the method of buildup, i.e., we can buildup one plug first, start one gas feed, and then after a purging period start the other plug. Hopefully this will minimize the initial impurity level, but we can't be sure what the residual impurity level will be. As for impurities arising from bombardment of the walls and gas box limiters, such impurities would have to gain sufficient energy $Z_{\text{ci}} \phi_e$ to penetrate the center cell plasma potential, and it is not clear how they could get such energies. Since oxygen injected in the plugs is ultimately lost by angle-scattering and drag into the loss cone, and since the plug potential going away from the center cell (ϕ_p) is larger than going toward the center cell (ϕ_c), most of the plug injected oxygen will preferentially be lost to the end walls, except for a small fraction f_{Opc} which suffer single (90°) collisions with other oxygen impurities, into the center cell side loss cone:

$$f_{\text{Opc}} = 1/2 \frac{\tau_{\text{DOp}} \ln \left(\frac{\bar{E}_{\text{bp}}}{Z_{\text{Op}} \Delta \phi_p} \right)}{\tau_{90^\circ(\text{Op})}} \approx 4 Z_{\text{Op}}^2 \left(\frac{n_{\text{Op}}}{n_p} \right) \left(\frac{M_0}{M_d} \right)^{1/2} \left(\frac{T_{\text{ep}}}{\bar{E}_{\text{bp}}} \right)^{3/2}, \quad (105)$$

where $\tau_{\text{DOp}} \propto T_{\text{ep}}^{3/2} Z_{\text{Op}}^{-2}$ is the plug oxygen drag time and where

$\tau_{90^\circ(\text{Op})} \propto \bar{E}_{\text{bp}}^{3/2} Z_{\text{Op}}^{-4}$ is the single-collision 90° scattering time

for plug oxygen impurities with density n_{Op} , mean charge state Z_{Op} and

energy $\bar{E}_{bp}/2$. Thus if we define f_{0b} as the ratio of injected oxygen current to deuterium current in the beams (incident currents I_{bp} and I_{bc} are taken to be incident deuterium currents), and approximate the trapping efficiency for oxygen in the form of accelerated water to be unity, the rate equation for center cell oxygen impurity density n_{0c} is given approximately by

$$\frac{d}{dt}(q n_{0c} V_c) = 2 I_{bp} f_{0b} f_{0pc} + I_{bc} f_{0b} + S_{0c} - \frac{q n_{0c} V_c}{\tau_{0cz}} - \frac{q n_{0c} V_c}{\tau_{0cr}}, \quad (106)$$

where S_{0c} is any other external source of oxygen, and where the electrostatic confinement time of the center cell oxygen is given to sufficient accuracy by

$$\tau_{0cz} = \tau_{cCz} \left(\frac{M_0}{M_d} \right)^{1/2} \exp [(Z_{0c} - 1)(\phi_c/T_c)], \quad (107)$$

where Z_{0c} is the average charge state in the center cell after thermalization of the oxygen to the cold deuterium component temperature T_c (the average charge-state as a function of T_{ec} can be taken from D. E. Post⁽⁷⁾) and where the radial loss time, assuming the oxygen impurity is also stochastic as is the cold deuterium component:

$$\tau_{0cr} = \tau_{cCr} Z_{0c}^2 \left(\frac{M_d}{M_0} \right)^{1/2}. \quad (108)$$

Since the exponential factor with Z_{0c} in eq. 107 is much stronger than the Z_{0c}^2 factor in eq. (108), radial loss will usually be the only way for

oxygen to escape, unless one plug is lowered in density to try to purge impurities out one end.

For other center cell impurity densities n_{zci} , the rate equations would have a similar form

$$\frac{d}{dt}(q n_{zci} V_c) = S_{zci} - \frac{q n_{zci} V_c}{\tau_{zcz}} - \frac{q n_{zci} V_c}{\tau_{zcr}} \quad (109)$$

$$\text{with } \tau_{zcz} = \tau_{cCr} \left(\frac{M_{zci}}{M_d} \right)^{1/2} \exp [(Z_{ci} - 1)(\phi_c/T_c)] \quad (110)$$

$$\text{with } \tau_{zcr} = \tau_{cCr} Z_{ci}^2 \left(\frac{M_d}{M_{zci}} \right)^{1/2} \quad (111)$$

The sources S_{zci} will have to be measured by EUV spectroscopic techniques⁽³⁾ for each impurity.

Plug Oxygen Impurity

Since oxygen from beam injection will likely be the only impurity with significant effect on the plug beam trapping efficiency, plug electron density to deuteron density ratio, and deuteron scattering time, we include here the rate equation for determining the oxygen concentration n_{Op} in the plugs:

$$\frac{d}{dt} (q n_{Op} V_p) = I_{bp} f_{Ob} - \frac{q n_{Op} V_p}{\tau_{DOp} \ln \left(\frac{E_{bp}}{Z_{Op} \Delta \phi_p} \right)} - \frac{q n_{Op} V_p}{\tau_{SOp}} \quad (112)$$

where the drag and angle-scattering losses are approximated to sufficient accuracy as

$$\tau_{00p} = 10^{12} \left(\frac{M_0}{M_d} \right) \left(\frac{2\sqrt{2}}{n_p} \right) \frac{T_{ep}^{3/2}}{Z_{0p}^2} \quad (113)$$

$$\text{and } \tau_{50p} \approx 1.8 \times 10^{11} \left(\frac{M_0}{M_d} \right)^{1/2} \left(\frac{2\sqrt{2}}{n_{pH} Z_{0p}^2 + n_{0p} Z_{0p}^4} \right) \bar{E}_{bp}^{3/2} \log_{10} R_{p0\text{eff}}, \quad (114)$$

$$\text{where } R_{p0\text{eff}} = \frac{R_p \sin^2 \bar{\theta}_p}{1 + Z_{0p} \Delta\phi_p / \bar{E}_{bp}}. \quad (115)$$

D. E. Post's coronal equilibrium calculations⁽⁷⁾ cannot be used for Z_{0p} since charge-exchange by the neutral beam is more important than recombination in limiting the highest charge state for oxygen. EUV measurements of the various charge-states of oxygen in the plugs should instead be used to find the average charge state Z_{0p} . For present purposes we can dispense with a rate equation for the plug oxygen energy, taking it to be $\bar{E}_{pH}$.

Center Cell Cold Ion Energy Rate Equation

The center cell cold ion energy rate equation determines the cold ion temperature T_c . Since impurities in the center cell equilibrate well with the cold deuterons, we consider the electron and hot ion energy inputs into all the cold ions, including impurities, to be available to supply the combined energy losses of the cold deuterons and impurity ions:

$$\frac{d}{dt} (3/2 q n_{cC} T_c V_c) =$$

$$\begin{aligned}
 P_{ic} + \frac{q n_{cC} (Z^2/M)_{\text{eff}} V_c}{\tau_{Dc}} (3/2 T_{ec} - 3/2 T_c) + \frac{q n_{cH} V_c}{\tau_{HC}} (\bar{E}_{cH} - 3/2 T_c) \\
 - \frac{q n_{cCd} V_c}{\tau_{cCz}} \left[\phi_c + \left(\frac{T_c \tau_{\phi P} + 2 T_c \tau_{\phi C}}{\tau_{cCz}} \right) \right] \\
 - \sum_i q \frac{n_{zci} V_c}{\tau_{zcZ}} (Z_{ci} \phi_c + 2 T_c) \\
 - \frac{q n_{cCd} V_c}{\tau_{cCr}} (3/2 T_c) \\
 - \sum_i q \frac{n_{zci} V_c}{\tau_{zcr}} (3/2 T_c) \\
 - \frac{q n_{cCd} V_c}{\tau_{cC}} (3/2 T_c) \frac{f_{cxgc}}{f_{ion g}}, \quad (116)
 \end{aligned}$$

where P_{ic} is any auxiliary cold ion heating such as by ICRH,

$$\tau_{HC} \approx \frac{5 \times 10^{11}}{n_{cH}^{1/2} (c)} \left(\frac{2}{n_{cC} (Z^2/M)_{\text{eff}}} \right) \left(\frac{M_{icH}}{M_d} \right) \bar{E}_{cH}^{3/2} \quad (117)$$

is the ion drag rate, or equilibration rate of hot ions (either injected

deuterons or protons, as indicated by mass M_{iCH}) on cold deuterons and impurities, and where $(f_{cxgc}/f_{ion g})$ is the ratio of charge-exchange of cold deuterons on gas to ionization of gas in the gas feeds. Since the gas feeds are located in the inner mirror throats where the fans are thin, we may approximate

$$\frac{f_{cxgc}}{f_{ion g}} \approx \frac{\langle \sigma v \rangle_{cxg}}{\langle \sigma v \rangle_e} \quad (118)$$

where $\langle \sigma v \rangle_{cxg}$ is the charge-exchange rate at the cold ion temperature T_c and where $\langle \sigma v \rangle_e$ is the electron impact ionization rate (ionization by cold ion impact would be negligible in the gas feed). The average energy loss $3/2 T_c$ per ion for radial loss to the limiters and per charge-exchange event in the gas feeds may be an overestimate since there will likely be a radial cold ion temperature gradient.

Center Cell Hot Ion Density Rate Equation

The center cell hot ion density rate equation determines the hot ion component density n_{cH} in the center cell arising from beam injection into the center cell. Since there is generally a large mirror ratio and a net potential well in the center cell, beam injected ions in the center cell tend to form a monotonically decreasing distribution function with increasing energy, with the low energy end of the beam-injected ion distribution merging smoothly with the Maxwellian cold ion distribution. This fact has been amply demonstrated by 2-D Fokker-Planck calculations of appropriate two-component regimes for TMX by Marvin Rensink. Since there is no loss cone "hole" in the hot ion distribution, we can neglect loss-cone instabilities and any diffusion they would cause; i.e., we can treat the hot ion losses as classical angle scattering, electron and cold ion drag, with

charge-exchange on the center cell beams and on cold gas. For this purpose we can apply the classical parts of the plug ion model developed in the previous sections to the injected center cell hot ions. We assume a "hot" ion becomes a "cold" ion if its energy falls below ϕ_c . We assume the initial injection angle in the center cell is close enough to 90° that the injected ions are sufficiently adiabatic for an initial slowing down time. For 20 keV beams in $B_c = 2\text{ kG}$ and $\theta_{inj} \geq 60^\circ$, Tibro calculations by Foote have shown sufficient adiabaticity to justify neglect of nonadiabatic losses of hot plug ions. Then we can write the center cell hot ion density rate equation as

$$\frac{d}{dt}(q n_{CH} V_c) = I_{bc} f_{ion c} - \frac{q n_{CH} V_c}{\tau_{CH}} - \frac{q n_{CH} V_c}{\tau_{cxgc}}, \quad (119)$$

$$\text{where } \tau_{CH}^{-1} = \left\{ \tau_{Sc}^{-1} + \left[\tau_{Dc} \ln \left(\frac{\bar{E}_{bc}}{\bar{E}_{Lc}} \right) \right]^{-1} + \left[\tau_{HC} \ln \left(\frac{\bar{E}_{bc}}{\bar{E}_{Lc}} \right) \right]^{-1} \right\} G^{-1}(\sigma_c), \quad (120)$$

$$\text{where } \tau_{Sc} = \frac{3.9 \times 10^{12}}{\ln \frac{1}{f} (c)} \left(\frac{M_{iCH}}{M_d} \right)^{1/2} \left(\frac{2}{n_c Z_{eff}} \right) \bar{E}_{bc}^{3/2} \text{Log}_{10} R_{eff c}, \quad (121)$$

$$R_{eff c} = \frac{B_M \sin^2 \bar{\theta}_c}{B_c \sqrt{1 - \beta_c/2} (1 - \phi_c/\bar{E}_{bc})}, \quad (122)$$

$$\bar{E}_{Lc} = \frac{\bar{E}_{bc} (0.6 \tau_{Sc})^{-1}}{(0.6 \tau_{Sc})^{-1} + \left[\tau_{Dc} \ln \left(\frac{\bar{E}_{bc}}{\bar{E}_{Lc}} \right) \right]^{-1} + \left[\tau_{HC} \ln \left(\frac{\bar{E}_{bc}}{\bar{E}_{Lc}} \right) \right]^{-1}}$$

$$+ \frac{\phi_c \left\{ \left[\tau_{Dc} \ln \left(\frac{\bar{E}_{bc}}{\bar{E}_{Lc}} \right) \right]^{-1} + \left[\tau_{Hc} \ln \left(\frac{\bar{E}_{bc}}{\bar{E}_{Lc}} \right) \right]^{-1} \right\}}{(0.6 \tau_{Sc})^{-1} + \left[\tau_{Dc} \ln \left(\frac{\bar{E}_{bc}}{\bar{E}_{Lc}} \right) \right]^{-1} + \left[\tau_{Hc} \ln \left(\frac{\bar{E}_{bc}}{\bar{E}_{Lc}} \right) \right]^{-1}}, \quad (123)$$

$$\bar{\theta}_c = \bar{\theta}_{inj}(c) + \Delta\theta_{spreading} \text{ (center cell)}, \quad (124)$$

$$\text{and } G(\sigma_c) = \frac{\left(\frac{\sigma_c}{1 - \sigma_c \frac{\tau_{cxbc}}{\tau_{cxgc}}} \right)}{\ln \left[1 + \left(\frac{\sigma_c}{1 - \sigma_c \frac{\tau_{cxbc}}{\tau_{cxgc}}} \right) \right]}, \quad (125)$$

$$\sigma_c \equiv \frac{f_{cxbc}}{f_{ion c}}, \quad (126)$$

$$\tau_{cxbc} = \frac{q n_{cH} V_c}{I_{bc} f_{cxbc}}, \quad (127)$$

$$\tau_{cxgc} = \frac{n_{cH} V_c}{S_c \phi_{cxgc}}, \quad (128)$$

$$\text{and } S_c \approx 2\pi r_c L_c \sqrt{\ln \left[\frac{(fnd1)_{oc}}{3.5 \times 10^{13}} \right]}. \quad (129)$$

The flux ϕ_{cxgc} must be determined by measurements of high energy charge-exchange neutral fluxes in the center cell. In general it should be

a function of I_{hc} and I_g , if $f_{ion g} < 1$.

The fraction of injected hot ions in the center cell which will scatter out of the center cell mirrors before thermalizing is given by $\tau_{cH} / [\tau_{Sc} G(\sigma_c)]$.

Center Cell Hot Ion Energy Rate Equation

The center cell hot ion energy rate equation determines the average energy of the hot ion component created by beam injection into the center cell. Here again we follow the classical part of the plug ion model to determine the average loss energy of the beam injected deuterons (Eq. 123). Since we have arbitrarily divided the center cell deuteron distribution into a hot component for $E_{cH} > \phi_c$ and a cold component for $E_{cC} < \phi_c$, we take the loss energy of a hot ion which thermalizes to be ϕ_c , and we take $\bar{E}_{cH}$ to be the average hot ion energy down to ϕ_c . Thus the hot ion energy rate equation in the center cell becomes:

$$\begin{aligned}
 d/dt (q n_{cH} \bar{E}_{cH} V_c) &= I_{bc} f_{ion c} (\bar{E}_{bc}) \\
 &+ I_{bc} f_{cxbc} (\bar{E}_{bc} - \bar{E}_{cH}) \\
 &- \frac{q n_{cH} V_c}{\tau_{cH}} (\bar{E}_{Lc}) - \frac{q n_{cH} V_c}{\tau_{cxgc}} (\bar{E}_{cxgc}) \\
 &- \frac{q n_{cH} V_c}{\tau_{Dc}} (\bar{E}_{cH} - 3/2 T_{ec}) - \frac{q n_{cH} V_c}{\tau_{HC}} (\bar{E}_{cH} - 3/2 T_c), \quad (130)
 \end{aligned}$$

where $\bar{E}_{cxgc}$ is the measured average energy of hot center cell ions lost by charge-exchange on gas.

Volume Dynamics

The center cell volume V_c and plug volumes V_p are included in the d/dt operators of the preceeding rate equations for densities and energies, because in general they may vary considerably during buildup, under changing beta and vacuum conditions:

$$V_p(t) = \pi^{3/2} r_p^2(t) l_p(t) \quad , \quad (131)$$

$$\text{and } V_c(t) = \pi r_c^2(t) L_c(t). \quad (132)$$

The plug radius r_p is controlled in part by beam aiming, which is constant during a shot, but is also affected by fluctuation level (radial losses) and by charge exchange on gas which will vary during a shot. At any point in the buildup, the critical radius r_p' to which gas neutrals penetrate

$$r_p' \approx r_p \sqrt{\ln \left[\frac{(fndl)_{op}}{3.5 \times 10^{13}} \left(\frac{r_p}{r_p'} \right) \right]} \quad (133)$$

is determined by a quasi-steady-state balance between radial diffusion by charge-exchange on beams and R.F. fluctuations, and erosion by charge-exchange on gas. That balance between outward radial diffusion and inward gas charge-exchange erosion at the critical radius r_p' can be written as

$$-(D_{cxb} + D_{QLr}) \left. \frac{\partial n_p}{\partial r} \right|_{r_p'} = \phi_{cxgp} \quad , \quad (134)$$

where D_{cxb} is the diffusion coefficient for charge exchange on beams and D_{QLr} is the quasi-linear radial diffusion coefficient, both evaluated at the average loss energy $\bar{E}_{cxgp}$ measured for charge exchange on gas, and at the critical radius r_p' . For a gaussian radial density profile, and using $(\int ndl)_{op} = n_p \sqrt{\pi} r_p$, with eq. (133), eq. (134) can be solved for the 1/e radius r_p :

$$r_p = \left[4 \times 10^{13} \left(\frac{D_{cxb} + D_{QLr}}{\phi_{cxgp}} \right) \left(\frac{r_p'}{r_p} \right) \right]^{1/2}, \quad (135)$$

where r_p'/r_p is a slowly varying function of the line density according to eq. (133).

Adopting Futch's model⁽¹³⁾ for the diffusion coefficients at $E = \bar{E}_{cxgp}$ and $r = r_p'$, we have

$$D_{cxb} = 2.4 \times 10^{14} \frac{j_{bo} \sigma_{cxb}}{q \omega_{ci}} \left(\frac{M_d}{M_{ip}} \right) (\bar{E}_{cxgp}) \left(\frac{\Delta y}{r_p' + y_0} \right) \frac{cm^2}{sec} \quad (136)$$

$$\text{where } j_{bo} \approx \frac{I_{bp}}{4 \Delta z \Delta y} \quad (137)$$

is the current density of beam neutrals in the incident beam footprint of gaussian widths Δz and Δy ,

$$\sigma_{cxb} = \frac{(\sigma_{cx} \cdot v)_{pH}}{\bar{v}_{bp}} \quad (138)$$

is the beam-to-hot-ion charge exchange cross section,

$$\omega_{ci} = 4.8 \times 10^6 \left(\frac{M_d}{M_{ip}} \right) B_p (1 - \beta_p/4) \quad (139)$$

is the beta-depressed fundamental ion cyclotron frequency, and where the $\pm$ signs in the beam aiming factor $\Delta y/(r \pm y_0)$ correspond to diamagnetic (+) and paramagnetic (-) aimings, respectively, at a beam offset y_0 . The quasilinear radial diffusion coefficient is given by

$$D_{QLr} \approx \left(\frac{k_{\perp}}{\omega_{ci}} \right)^2 \left(\frac{\tilde{\phi}_{RF}}{M_{ip}} \right)^2 \left(\frac{r_p}{r_p'} \right)^2 \frac{j_1^2(k_{\perp} \rho_i)}{\Delta \omega_{ci}} \left(\frac{\text{cm}^2}{\text{sec}} \right), \quad (r_p' > r_p) \quad (140)$$

where the perpendicular wavenumber experimentally is

$$k_{\perp} \rho_i (E = \bar{E}_{pH}) \approx 4.5, \quad (141)$$

and $(\Delta \omega_{ci})^{-1}$ is the correlation time. Using the fluctuation amplitude $\tilde{\phi}_{RF}$ given by eqs. (63), (66) and (70) as a function of the end loss current, and using eq. (139) for ω_{ci} , the quasilinear radial diffusion coefficient, evaluated at $E = \bar{E}_{cxgp}$, becomes

$$D_{QLr} \approx 2.9 \left(\frac{M_{ip}}{M_d} \right) \bar{E}_h \left(\frac{\bar{E}_{pHL}}{\bar{E}_{cxgp}} \right) \left(\frac{v_{11}}{r_p} \right) \left(\frac{r_p}{r_p'} \right)^2 \frac{e^{-2(j/j_s)^2}}{B_p^2 (1 - \beta_p/4)^2} \frac{\text{cm}^2}{\text{sec}}, \quad (142)$$

while using eq. (139) for ω_{ci} and eq. (137) for j_{b0} in the charge-exchange diffusion coefficient gives

$$D_{cxb} = 1.6 \times 10^{19} \left(\frac{M_{ip}}{M_d} \right) \left(\frac{\bar{E}_{cxgp} I_{bp} C_{cxb}}{\Delta z B_p^2 (1 - \beta_p/4)^2} \right) \left(\frac{1}{r_p^2 + y_0} \right) \frac{\text{cm}^2}{\text{sec}} \quad (143)$$

Note that D_{QLr} as well as $\tilde{\phi}_{RF}$ (eq. 69) is independent of the correlation time $(\Delta\omega_{ci})^{-1}$.

Very little is known about the factors which control the plasma axial scale length l_p . Experimentally, we usually observe that the axial plasma scale length l_p is approximately the beam horizontal scale length Δz , except that it becomes 1/3 shorter than Δz when beta is very high ($\beta_p = 2$). Thus we can take

$$l_p = \Delta z (1 - \beta_p/6) \quad (144)$$

for the time being. We know theoretically that the equilibrium axial scale length should depend on the anisotropy $\bar{E}_{pH11}/\bar{E}_{pH}$ of plug ions which can be measured, but we do not yet have a good analytic formula relating the plasma length to the anisotropy at high beta. Note that Δz and l_p may increase in time if the beam defocuses in time, as is frequently seen on beam attenuation.

For gas box limiters mapping to a radius greater than r_p , the center cell radius should follow r_p in time through eq. (50). Otherwise r_c would be constant at the gas box limiter value. While the center-cell cold plasma extends from plug-to-plug at a fixed 640 cm, the effective center cell full length L_c which gives the equivalent cylindrical volume at πr_c^2 cross section can vary in time according to the center cell beta $\beta_c(z, t)$:

$$L_c(t) = \int_{\text{plug}}^{\text{plug}} dz \bar{B}_{ci}(0,t) / \bar{B}_{ci}(z,t) \quad (145)$$

where the radially averaged internal center cell field $\bar{B}_{ci}(z,t)$ is given by eq. 44 using $B_c(r,t)$ from eq. 46 with the vacuum center cell field $B_c(z)$ given by Mafco (Fig. 10, for example). In eq. 46 for β_c , the cold plasma components can be taken to be axially uniform in density and temperature. The hot component pressure goes to zero at the inner mirror throats. The axial profile of the hot ions will depend on the self-consistent field profile due to the finite beta, and also on the angle of injection of the center cell beams, as well as any angular spreading instability like the AIC mode. Thus at high beta the hot ion axial length must be measured or calculated on Superlayer. To the extent the center cell field is uniform, and we try injection at 26° off perpendicular, the hot ion profile should largely fill the center cell length at low to moderate betas. As the orbits of the hot ions are large, the hot ion radial profile may be broader than the cold ions, offsetting the hot ions' shorter length to give approximately the same volume (V_c) for the hot ions.

Concluding Remarks

While all the formulas presented in the supermodel could benefit from more refined theories, and empirical scalings could be sharpened with more experimental data, the following topics are the ones most crudely treated in the supermodel (and some may ultimately prove unsatisfactory) and the ones most in need of improvement: (1) cross-field electron thermal conductivity. There is no theory or formula for P_{ek} yet. It's a purely experimental input term at this point. (2) The effects of R.F. fluctuations on center cell ion heating (contribution to P_{ic} ?), and perhaps more importantly on center cell ion confinement (R.F. plugging?) (3)

The dependence of R.F. amplitude $\tilde{\phi}_{RF}$ on the center cell losses. In particular we need a theory giving the saturation value of the fluctuation amplitude at higher T_e (larger ambipolar hole), in the limit of zero end loss current. Molvik's 2XIIB data (Fig. 5) is limited to $T_e < 140$ eV, and that data which suggests that the R.F. amplitude saturates at a value required to warm up a stream ion to the ambipolar hole may not remain valid at much higher T_e values which may occur in TMX. (4) Resonant radial diffusion of center cell ions depends on T_e and radial position. We have assumed the worst-case resonant transport will always occur, but in fact conditions required for resonant transport ($\omega_{drift} \approx \omega_{bounce} \Rightarrow \omega_{collision}$) may not always be present, and may not apply to the higher density central core of the plasma where electric field drifts and B drifts may be weak. (5) Impurity diffusion into the center cell from walls and gas box edges. We have no way to estimate these impurity sources yet. (6) Axial hot ion lengths (gradient scale lengths in z) for the plugs and for the hot ion component in the center cell. Finite beta is important. We need a theory which predicts 2XIIB lengths satisfactorily, to apply to TMX. Angular spreading instabilities, little understood at present, may be important in determining the axial lengths.

Beyond these immediate questions, what should we expect from the supermodel once applied to TMX? We would be naive to expect good agreement in every parameter predicted by the supermodel and measured in the experiment, even after the formulas for presently-known loss processes (Table I) have been checked and validated. An obvious discrepancy between the supermodel and some experimental observation, after all known loss processes are thoroughly accounted for, would motivate a search for some new loss process to account for the discrepancy. I'm talking about a large discrepancy such as the one we found between beam-power trapped and

calorimeter end loss power in 2XIIB. We would probably be wasting our time trying to resolve such basic discrepancies by refining our present formulas with large computer codes. We should first ask the question: Do we have all the physics to describe TMX? This then, should be the function of the supermodel - to help bring to light loss processes of which we are not yet aware.

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FIGURE CAPTIONS

- Fig. 1 $\langle \alpha v \rangle_e$ (curve labeled ei) as a function of T_e in eV (upper scale), $\sigma_{cx} \cdot v$ (curve labeled cx) and $\sigma_i \cdot v$ (curve labeled di) as a function of deuterium ion energy E_D in eV. For hydrogen, use $E_D = 2 E_H$ to obtain $\sigma_{cx} \cdot v$ and $\sigma_i \cdot v$.
- Fig. 2 Comparison of Bill Turner's trapping code for 2XIIB with the analytic trapping formula in the supermodel. (For zero cold fraction n_{pc}/n_p .)
- Fig. 3 Space-charge-limited secondary electron multiplier λ_{SCL} versus $T_e/\bar{E}_i$ at the wall.
- Fig. 4 Cooling rates L_{zci} for various impurities from D. E. Post (PPPL-1352).
- Fig. 5 R. F. fluctuation amplitude $\tilde{\phi}_{RF}$ in arbitrary units as a function of end loss analyzer current in 2XIIB (from Molvik).
- Fig. 6 Critical ion energy components and cutoffs in plug velocity space.
- Fig. 7 Energetic charge-exchange flux from background cold gas Φ_{cxgp} as a function of $\int I_{bp} dt$ in 2XIIB.
- Fig. 8 Angular spreading $(\bar{E}_{pH11}/\bar{E}_{pH})$ as a function of $n_p I_{bp}$.

in 2X11B (from Nexsen).

Fig. 9 Monte-Carlo calculation of $n\tau$ in the transition mean-free-path region between collisionless Pastukhov scaling and collisional loss scaling (from Rognlien and Cutler).

Fig. 10 Ax: B_z field in the center cell of TMX versus z .

Fig. 1. $\langle \sigma v \rangle_e$ (curve labeled ei) as a function of T_e in eV (upper scale), $\sigma_{cx} \cdot v$ (curve labeled cx) and $\sigma_i \cdot v$ (curve labeled di) as a function of deuterium ion energy E_D in eV. For hydrogen, use $E_D = 2 E_H$ to obtain $\sigma_{cx} \cdot v$ and $\sigma_i \cdot v$.

Fig. 2. Comparison of Bill Turner's trapping code for 2XIR with the analytic trapping formula in the supermodel. (For zero cold fraction n_{pc}/n_p .)

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Fig. 8. Angular spreading (E_{pH11}/E_{pH}) as a function of $n_p I_{bp}$

Fig. 9. Monte-Carlo calculation of n in the transition mean-free-path region between collisionless Pastukhov scaling and collisional loss scaling (from Rognlien and Cutler).

Fig. 10. Axial B_z field in the center cell of TMX versus z .

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