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BEAM TILT DUE TO LUMPED RF DISTRIBUTION*

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A lumped distribution of rf cavities leads to discrete energy gains of the electron and the positron beams in a storage ring, which in turn leads to a distortion in the closed orbits of the two beams. This effect has been studied by several authors.¹⁻³ For PEP, it is found to be a small effect.

There is another consequence of having a lumped rf distribution. It comes from a skewness of the synchrotron phase space ellipse and has nothing to do with the net energy gains in the cavities. If the rf cavities are distributed uniformly around the ring, the synchrotron x - δ phase space is an upright ellipse. If the cavities are lumped, however, the ellipse will be slightly tilted.⁴ The coupling between the horizontal coordinate x and δ then leads to a small tilting angle θ in the x - z projection of the beam distribution.[†] This small tilt must be compared with another small quantity, σ_x/σ_z , because if θ is not too small compared with σ_x/σ_z at the interaction points, there might be some reduction of luminosity. We studied the x - z tilt due to lumped rf distribution and found it is small compared with σ_x/σ_z for PEP, hence little loss of luminosity. Our results are presented in this note.

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[†] Coupling also produces a tilting angle in the y - z plane for the case with $\eta_y \neq 0$ at either the rf cavities or at the observation point.

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It is worth mentioning that the beam tilt considered here comes from single particle linear lattice. It does not involve lattice nonlinearities (except that orbit distortions in the nonlinear elements change the linear lattice parameters) or beam current dependent effects.

The skewness in the synchrotron phase space due to lumped rf distribution has been calculated by Helm and Rees.⁴ Their calculation, reproduced in the appendix, involves looking for an equivalent β -function in the synchrotron motion. If we observe the ellipse at a position right before the cavity, we find

$$\begin{aligned}\langle z^2 \rangle &= \left(\frac{\alpha L}{2\pi v_s} \right)^2 \langle \delta^2 \rangle \\ \langle z\delta \rangle &= -\frac{1}{2} \frac{\alpha L}{P} \langle \delta^2 \rangle\end{aligned}\tag{1}$$

where α is the momentum compaction factor; v_s and L are the synchrotron tune and the total path length of the ring; and P is the number of rf superperiods in the ring. (PEP has an approximate rf superperiod of 3.) We assume $v_s/P \ll 1$.

We can roughly estimate the x-z tilt by completely ignoring the betatron contribution to x and set $x = \eta\delta$ with η the dispersion function. The tilting angle θ in the x-z plane will be given by

$$\theta \cong -2\pi^2 v_s^2 \frac{\eta}{\alpha L P}\tag{2}$$

The ratio of θ to σ_x/σ_z is then

$$\frac{\theta}{\sigma_x/\sigma_z} \cong \frac{\pi v_s}{P}\tag{3}$$

If we take $v_s = 0.05$, $P = 3$, $\eta = 1m$, $\alpha = 0.003$ and $L = 2200m$, the beam tilt angle is found to be about 2.5 mrad. The ratio of θ to σ_x/σ_z is 1/20. Such a small tilt is not expected to do much harm on luminosity.

More accurate calculation of this effect can be obtained by running the computer program SLIM.⁵ The results are given in Figs. 1 to 6. We begin with a lattice with $\beta_x^* = 3.0m$, $\beta_y^* = 0.11m$, $v_x = 21.78$, $v_y = 18.72$, $v_s = 0.05$, $\eta_x^* = -0.51m$ (not a mini-B lattice). The x-z tilting angle is plotted in Fig. 1A. The value ranges between $\sim \pm 10$ mrad peak to peak, in order-of-magnitude agreement with the rough estimate mentioned above. This tilting angle is compared with σ_x/σ_z , which is plotted in Fig. 1B. The ratio between θ and σ_x/σ_z is about 1/20, again in agreement with the rough estimate. Table 1 lists the values of the relevant quantities for the 6 interaction points. The fact that the tilting angles are very small at the interaction points (giving θ to σ_x/σ_z ratio of about 1/300) is a result of the symmetry of the rf distribution. The fact that they are not exactly equal to zero is due to the fact that the symmetry is not perfect and that $\eta_x \neq 0$ at the cavities and at the interaction points. See Fig. 7A.

Figs. 2A and 2B show the same quantities as Figs. 1A and 1B except that the symmetry in the rf distribution has been removed. (See Fig. 7B, which is a rather extreme case.) The value of σ_x/σ_z in this case remains almost identical to that shown in Fig. 1B, but the tilt angle in x-z plane has increased appreciably. This again is in agreement with Eq. (3) since now with an rf superperiod of 1, P has decreased by a factor of 3. Relevant quantities at the interaction points are listed in Table 1. The ratio of θ to σ_x/σ_z has become larger, being equal to 1/8 at one of the interaction points.

We have studied the effect of the random distribution of vertical dipole field errors on the beam tilt. The vertical closed orbit is corrected using an rms scheme, yielding an orbit distortion of $\Delta y_{rms} = 1.23\text{mm}$ after correction. Sextupole fields are linearized around the closed orbit. The tilting angle in x-z plane and σ_x/σ_z are shown in Figs. 3A and 3B, respectively. These quantities remain similar to those of Figs. 1A and 1B, except that the interaction point now has somewhat larger values of θ to σ_x/σ_z ratios (see Table 1) since the rf symmetry has been partially destroyed. The rf distribution for this case is that of Fig. 7A.

The field errors also introduce a finite tilt in the y-z plane. The values of y-z tilt and σ_y/σ_z are shown in Fig. 4A and 4B. The θ to σ_y/σ_z ratio is about 1/15 both in the ring and at the interaction points.

The previous case is repeated using an asymmetric rf distribution of Fig. 7B. Results of this calculation are presented in Figs. 5A, 5B, 6A and 6B. Values at the interaction points are given in Table 1.

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APPENDIX

Consider one rf superperiod of the ring. The matrix that transforms the vector

$$\begin{bmatrix} x \\ z \end{bmatrix} \quad (A1)$$

across an rf cavity is

$$T_{\text{cav}} = \begin{bmatrix} 1 & C \\ 4\pi^2 \nu_s^2 / \alpha L P & 1 \end{bmatrix} \quad (A2)$$

where ν_s = synchrotron tune; L = circumference, α = momentum compaction factor, P = number of rf superperiods.

We assume there is one and only one cavity per rf superperiod. The transformation through the ring part of the superperiod is

$$T_{\text{ring}} = \begin{bmatrix} 1 & -\alpha L / P \\ 0 & 1 \end{bmatrix} \quad (A3)$$

The total transformation from the entrance of the cavity to the entrance of the cavity of the next superperiod is obtained by multiplying the two matrices (A2) and (A3):

$$T = T_{\text{ring}} T_{\text{cav}} = \begin{bmatrix} 1 - 4\pi^2 \nu_s^2 / P^2 & -\alpha L / P \\ 4\pi^2 \nu_s^2 / \alpha L P & 1 \end{bmatrix} \quad (A4)$$

This matrix is then compared with the general symplectic form

$$\begin{bmatrix} \cos \bar{\mu} + \bar{\alpha} \sin \bar{\mu} & \bar{\beta} \sin \bar{\mu} \\ -\frac{1}{\bar{\beta}} (1 + \bar{\alpha}^2) \sin \bar{\mu} & \cos \bar{\mu} - \bar{\alpha} \sin \bar{\mu} \end{bmatrix} \quad (A5)$$

In Eq. (A5), $\bar{\mu}$ is a phase advance; $\bar{\alpha}$ and $\bar{\beta}$ are the machine functions at the entrance of the cavity; they are equivalent to the familiar α - and β -functions of the betatron motions but now they are defined in the synchrotron dimension of motion. Resulting from the comparison we have*

$$\begin{aligned} \bar{\mu} &\approx -2\pi v_s / P \\ \bar{\alpha} &\approx \pi v_s / P \\ \bar{\beta} &\approx \frac{\alpha L}{2\pi v_s} \end{aligned} \quad (A6)$$

We have assumed $v_s \ll 1$ in deriving Eq. (A6).

Let the beam distribution in z - δ phase space be given by

$$\exp \left\{ -\frac{1}{2W} \frac{z^2 + (\bar{\alpha}z + \bar{\beta}\delta)^2}{\bar{\beta}} \right\} \quad (A7)$$

with W the emittance. The rms values of the beam distribution are

$$\begin{aligned} \langle z^2 \rangle &= \bar{\beta} W \approx \frac{\alpha L}{2\pi v_s} W \\ \langle z\delta \rangle &= -\bar{\alpha} W \approx -\frac{\pi v_s}{P} W \\ \langle \delta^2 \rangle &= \frac{1 + \bar{\alpha}^2}{\bar{\beta}} W \approx \frac{1}{\bar{\beta}} W = \frac{2\pi v_s}{\alpha L} W. \end{aligned} \quad (A8)$$

* Mathematically, the negative sign in $\bar{\mu}$ ensures $\bar{\beta}$ to be positive. Physically it is a consequence of the negative longitudinal mass.

Eq. (1) follows directly from Eq. (A8). If we ignore betatron contribution to x and set $x = \eta\delta$, the tilt angle in the x - z plane is determined by

$$\tan 2\theta = \frac{2\langle zx \rangle}{\langle z^2 \rangle - \langle x^2 \rangle} \quad (\text{A9})$$

which for small v_s gives Eq. (2).

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Fig. 1. Tilt Angle in x-z Plane and σ_x/σ_z for PEP

(The rf distribution is that of Fig. 7A. No field errors are included in the lattice.)

Fig. 1A

Fig. 1B

Fig. 2. Tilt Angle in x-z Plane and σ_x/σ_z for PEP

(The rf distribution is that of Fig. 7B. No field errors are included in the lattice.)

Fig. 2A

Fig. 2B

Fig. 3. Tilt Angle in x-z plane and σ_x/σ_z for PEP

(The rf distribution is that of Fig. 7A. Field errors are included in the lattice with $\Delta y_{rms} \approx 1.23\text{mm}$ after correction.)

Fig. 3A

Fig. 3B

Fig. 4. Tilt Angle in the y-z Plane and σ_y/σ_z for PEP
(Conditions are the same as those of Fig. 3.)

Fig. 5. Tilt Angle in x-z Plane and σ_x/σ_z for PEP

(The rf distribution is that of Fig. 7B. Field errors are included.)

Fig. 6. Tilt Angle in y-z Plane and σ_y/σ_z for PEP
(Conditions are the same as those of Fig. 5.)

Fig. 7. Two rf Distributions Studied for PEP

(The synchrotron phase advances for the whole ring are kept the same for both cases.)

Fig. 7A

Fig. 7B

Table 1.

	<u>Tilt xz (mrad)</u>	<u>Tilt yz (mrad)</u>	<u>σ_x/σ_z (mrad)</u>	<u>σ_y/σ_z (mrad)</u>
rf dist. = Fig. 7A	0.137	0	38.9	0
No field errors	0.138	0	38.9	0
	0.142	0	38.9	0
	0.133	0	38.8	0
	0.140	0	38.9	0
	0.143	0	38.8	0
rf dist. = Fig. 7B	1.82	0	39.1	0
No field errors	0.150	0	38.4	0
	-1.56	0	39.1	0
	4.60	0	39.4	0
	0.146	0	38.7	0
	-4.34	0	39.4	0
rf dist. = Fig. 7A	0.102	0.254	34.5	3.9
With field errors	-0.378	0.213	36.9	3.9
	0.745	-0.030	35.7	4.2
	0.055	-0.285	34.4	4.0
	-0.342	-0.250	36.1	4.0
	0.769	0.0012	36.6	4.0
rf dist. = Fig. 7B	1.89	0.219	34.7	3.9
With field errors	-0.433	0.164	36.2	3.9
	-1.01	-0.088	35.8	4.2
	4.61	-0.252	35.0	4.0
	-0.385	-0.164	35.7	4.0
	-3.81	0.002	37.0	4.0