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**LASER INTERFEROMETRIC MEASUREMENT OF
SPHERICAL SURFACE CONTOURS**

By P. E. Klingsporn

Published February 1979

Topical Report

Prepared for the United States Department of Energy
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**Kansas City
Division**

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Topical Report
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A laser interferometer was made operational for alignment of highly reflective spheres in a laser surface inspection system and for semi-automated measurement of roundness and waviness amplitude of the spherical surfaces. The non-contact interferometric measurements of roundness agree well with conventional measurements made with a stylus. The theoretical foundation was developed for a computer program to perform semi-automated measurement of waviness wavelength associated with the measured waviness amplitude. Theory was developed for using the interferometer to position the center of a sphere on a rotation axis when the laser beam is accessible to only one-quarter revolution of the spherical surface about this axis.

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SUMMARY

A laser interferometric displacement-measuring system was made operational for measuring roundness and waviness amplitude on highly reflective spherical surfaces. The laser interferometer was interfaced with a computer to provide semi-automated measurements and data plots. The resolution of the interferometer is $\pm 0.025 \mu\text{m}$ ($\pm 1.0 \mu\text{in.}$) for waviness amplitude; however, the degree of temperature control of the overall system and the mechanical stability of the mechanism holding the sphere limit the resolution of the present system to $\pm 0.13 \mu\text{m}$ ($\pm 5 \mu\text{in.}$). The resolution of arc length along the surface over which this change in amplitude can be measured is approximately $16 \mu\text{m}$ ($625 \mu\text{in.}$), which is the diameter of the illuminated spot of laser light on the spherical surface. The non-contact interferometric measurements of roundness are in excellent agreement with conventional, stylus contact measurements on spheres as round as $0.33 \mu\text{m}$ ($13 \mu\text{in.}$) and as out-of-round as $10.2 \mu\text{m}$ ($400 \mu\text{in.}$). The theoretical basis was developed to write a computer program to provide semi-automated measurement of waviness wavelength associated with the waviness amplitudes measured in the present capability.

The laser interferometer represents only a portion of an overall semi-automated system for spherical surface inspection, the major portion of which involves the detection and measurement of pit and scratch defects. For surface inspection, it is necessary that the sphere be positioned accurately in a two-axis rotary fixture. The laser interferometer was made operational for accurate, non-contact positioning of spheres in the two-axis rotary fixture of the semi-automated inspection system for surface defects. Full 360° rotations of the sphere are executed about one axis, and use of the interferometer for locating the center of the sphere on this axis is analogous to that used with a conventional stylus indicator. However, only one-quarter revolution of the sphere can be executed about the second rotational axis because of restraints of the optical system. The theory was developed for use of the laser interferometer to locate the center of the sphere on the latter rotation axis even though the laser light beam can sample only one-quarter revolution of the sphere about this axis.

DISCUSSION

SCOPE AND PURPOSE

A laser interferometric displacement-measuring system was made operational for non-contact alignment of spheres in the two-axis rotary fixture of a laser surface inspection system. The laser interferometer was interfaced with a computer to provide semi-automated measurement of spherical surface roundness and waviness amplitude.

PRIOR WORK

A laser interferometer was used to study the feasibility of measuring waviness on the surface of a sphere, but, in that brief study, the sphere could be rotated about only one axis, and no attempts were made to use the interferometer to align the sphere on the axis or to measure the roundness of the sphere. All measurements made in the feasibility study were manual.

ACTIVITY

The semi-automated system for surface finish inspection and roundness and waviness measurement of highly reflective spheres is shown schematically in Figure 1. The sphere is positioned in the vacuum chuck of a precision two-axis rotary fixture which provides computer-controlled rotation of the sphere about its center. A laser interferometer is used for aligning the sphere in the system and for making roundness measurements. The laser interferometer represents only a portion of the overall system for semi-automated inspection of spherical surfaces. The portion of the system dealing with surface finish inspection has been described previously.¹ The present work deals with the laser interferometric displacement-measuring system and its use for aligning a sphere in the surface inspection system and for measuring the roundness and surface waviness of the sphere.

Principle of Operation

The laser interferometer displacement-measurement system, consisting of a laser head, single beam interferometer, and a display, provides a means of measuring displacements accurately in the microinch range. The principle on which the system operates and the manner in which it can be used to measure out-of-roundness and waviness

Figure 1. Semi-Automated System for Spherical Surface Finish Inspection and Roundness and Waviness Measurement, Schematic

on superfinished spherical surfaces can be understood by referring to Figure 2. In the laser head the light from a 70 μ W helium-neon laser (wavelength 6328 Å) is subjected to a permanent magnetic field, which causes the light to be split into two frequency components f_+ and f_- (separated by only 1.8 MHz), by the phenomenon of Zeeman splitting. A small amount of the energy in the two-component beam is diverted internally in the laser head and is used as a reference beam, but the remainder emerges as a collimated exit beam approximately 7.6 mm (0.3 in.) in diameter. The planes of polarization of the two frequency components are oriented at 90° to one another. The single beam interferometer contains waveplates P_1 and P_2 , which rotate the plane of polarization of a given frequency component by 90° upon double passage of the beam through the plate. Also, the mirrors M_1 and M_2 in the interferometer are coated and aligned relative to the exit beam from the laser in such a way that one of the components can be reflected but not transmitted, whereas the component with polarization differing by 90° will be transmitted but not reflected. Therefore, when the two-frequency beam enters the interferometer, one of the components (f_+ , for purposes of discussion) will be totally reflected by the mirror M_1 , but the component f_- with polarization plane differing by 90° will be totally transmitted. The reflected component f_+ passes through the quarter-wave plate P_1 , is retro-reflected by the "corner-cube" mirror C, and then passes again through P_1 . The double pass through P_1 rotates the polarization plane of f_+ by 90° and hence f_+ will be transmitted as it encounters M_1 . The orientation and coating of mirror M_2 are such that f_+ is totally reflected and therefore f_+ will return to the laser head, having undergone only reflections and transmissions internally in the interferometer optics. The component f_- of the beam incident on M_1 is transmitted and proceeds to the test surface, where it is retroreflected. When the component f_- returns to the mirror M_1 it has undergone two passages through the quarter-wave plate P_2 and hence its polarization plane is such that it is totally reflected from M_1 . It then passes to M_2 and back to the laser head. If the test surface is stationary relative to the single-beam interferometer, the frequency difference between the components f_+ and f_- returning to the laser head does not vary. However, if the test surface is moved toward or away from the interferometer, the retroreflected component f_- will vary in frequency because of the Doppler effect. The shifted frequency Δf will be counted by the counting circuitry of the laser head and will be converted to the cumulative change in distance of the test surface relative to the interferometer. The change in distance can be determined to within ± 25.4 nm (± 1 μ in.). A special resolution extender can be used to extend the resolution to ± 2.5 nm (± 0.1 μ in.), but measurements in this realm are not significant unless extreme care is taken to reduce temperature fluctuations and mechanical vibrations below the level of interference.

Figure 2. Laser Interferometric Displacement Measurement System, Principle of Operation

Optical Arrangement

The displacement-measuring system is commonly used for aligning precision machines in which the test surface either is a plane or a plane reflector can be mounted conveniently on the surface under test. In these cases the 7.6-mm-diameter parallel laser beam exiting from the single beam interferometer can be used in its parallel form provided the plane reflecting surface is sufficiently smooth that at least 5 percent of the beam energy is ultimately returned to the laser head counting circuitry. An

error signal will be triggered if less than 5 percent of the energy returns. The system's specifications state that a plane surface should have a 12- μ in.-rms surface finish or better.

If, as in the present application, the test surface is a sphere, it is clear that the 7.6-mm-diameter (0.3 in.) laser beam leaving the single beam interferometer cannot be simply directed on the spherical surface, because the reflected beam would diverge and very little light energy would return to the counting circuitry of the laser head. Two possible solutions can be obtained if a convergent lens is placed in the exit beam of the single beam interferometer. In one arrangement, shown in Case A of Figure 3, the convergent lens is used to focus the exit laser beam to a point on the surface of the sphere. In this arrangement it is easy to see that any given incident ray within the converging cone of light will be reflected at an angle equal to its incident angle and will return to the interferometer and eventually to the laser head. For the spherical surfaces in question, there is no problem concerning surface finish requisite for reflection because the measured AA values are 1 μ in. or better. The second possible arrangement is shown in Case B of Figure 3, in which the convergent lens is used to focus the laser beam to a virtual point at the center of the sphere. In the latter case each ray is incident normally on the spherical surface and hence is retroreflected and returns along its initial path.

The primary difference between the two cases in Figure 3 is the size of the illuminated spot on the spherical surface. In Case A, the spot size is controlled mainly by diffraction limitations which dictate the minimum focussed spot size possible. If d is the $1/e^2$ diameter of the parallel laser beam (which in the present case is 7.6 mm), of wavelength λ , then the focussed spot radius ϵ for Case A in Figure 3 is

$$\epsilon = \frac{2f\lambda}{\pi d}, \quad (1)$$

where f is the focal length of the convergent lens. In Case B, the illuminated spot radius r on the sphere of radius R is

$$r = \frac{Rd}{2f}. \quad (2)$$

For given values of the spherical radius R , lens focal length f , and laser beam diameter d , the illuminated spot size on the sphere is much larger in Case B than in Case A. For example, if the typical values $R = 8.8$ mm, $d = 7.62$ mm, $f = 150$ mm, and $\lambda = 6328$ Å are used, Equation 1 gives a spot diameter $2\epsilon = 0.01586$ mm (624 μ in.) for Case A, whereas the spot diameter

CASE A

CASE B

Figure 3. Possible Optical Arrangements for Use of Laser Interferometric Displacement Measuring System on a Spherical Test Surface

$2r$ for Case B is 0.447 mm (0.0176 in.). For these typical values of R and f , the spot size in Case B is 28 times that in Case A. This means fewer samples are required for 100 percent inspection of a given sphere with the arrangement of Case B than with Case A. However, the resolution in Case B is much less than for Case A, because the arc length subtended on the sphere is much larger.

The preceding considerations are based on the assumption of a perfectly spherical surface which is aligned perfectly with its center at the virtual focus (Case B) or its surface at the focal point (Case A). One of the uses of the laser displacement measurement system is for alignment of the sphere in the two-axis rotary fixture. Even for the case of initial, coarse alignment, a sphere may be off-center as much as 100 μm (0.004 in.). The following analysis shows that Case B is not as practical as Case A because the positioning of the sphere relative to the light beam is much more critical than for Case A.

In the geometry of Case B, let $2r$ be the diameter of the illuminated spot on the surface of the sphere of radius R . When the sphere is properly aligned relative to the beam so that the light proceeds to a virtual focus at the center of the sphere, the point of incidence of each incident ray is at the same optical path distance from the center of the sphere. This symmetry is destroyed when the sphere is moved a distance X_0 relative to the beam, in a direction perpendicular to the axis of the beam, as shown in Figure 4a. Portions of the spherical surface within the illuminated spot are now closer to the interferometer than other regions. The maximum change Δ_L^c in distance for points within the illuminated spot can be determined from the geometry of Figure 4a, and the result, as shown in Appendix A, is found to be

$$\Delta_L^c = 2x_0 \frac{r}{R} \left(1 - \frac{r^2}{R^2} \right)^{1/2} \quad (3)$$

If the sphere is moved toward or away from the interferometer by a distance X_0 along the axis of the light beam as shown in Figure 4b, the analysis in Appendix A shows that the change Δ_θ^c in distance for points in the illuminated spot is given by

$$\Delta_\theta^c = \frac{1}{2} x_0 \frac{r^2}{R} \quad (4)$$

The center of the reflected beam returning to the lens of the interferometer will be deflected off-axis a distance y_d^c in the plane of the lens. For the case in Figure 4a, it is shown in Appendix A that this distance is given by

$$y_d^c = 2f \frac{x_0}{R} \left(1 - \frac{d^2}{4f^2} \right)^{1/2} \quad (5)$$

A. PERPENDICULAR TO INCIDENT LASER BEAM

B. ALONG AXIS OF INCIDENT BEAM

Figure 4. Geometry for Movement of Sphere by Distance x_0 for Case B (Figure 3)

where d is the diameter of the beam from the interferometer as it strikes the lens. The beam is not deflected off axis for movement of the type in Figure 4b.

For the arrangement in Case A in which the beam is focussed to a spot of diameter 2ϵ on the surface of the sphere, the maximum relative change in elevation of points in the illuminated spot on the surface when the sphere is moved a distance x_0 (Figure 5) is found in Appendix B to be

$$\Delta_{\perp}^S = 2x_0 \frac{\epsilon}{R} \quad (6)$$

The corresponding change when the sphere is moved toward or away from the interferometer along the axis of the beam does not pose a problem for Case A because the presence of aberration in even the most perfect of optical systems results in a relatively long cylindrical region at the focus of the lens. That is, the light is not brought to a perfect focus at a mathematical point at the surface, but there is an extended region which results in a relaxation of the accuracy with which the sphere needs to be positioned in the beam.

As the sphere is moved in a direction perpendicular to the beam axis, the reflected beam returning to the lens will be deviated by the amount

$$y_d^S = 2x_0 \frac{f}{R} \quad (7)$$

in the plane of the lens.

The quantities Δ^C and Δ^S in Equations 3, 4, and 6 represent a "warping" of the illuminated surface in the presence of the beam. This is much different from the intended use of the displacement-measuring system, in which the surface moves uniformly toward or away from the interferometer because the phase of the returning light is actually shifted by the amount Δ^C or Δ^S . The circuitry of the laser head registers changes in distance by counting the Doppler-shifted frequency difference between the beam reflected from the sphere and the reference beam, and then accumulating the shift in a fringe-count register, and finally converting the shift to microinches. In comparing the arrangements of Figure 3, it is convenient to express the "warping" in terms of $\lambda/2$, which is the distance between adjacent interference fringes. A shift of one fringe represents a change in distance of one-half the wavelength of the light which, for the helium-neon laser wavelength, is 3164 Å. That is, when Δ^C or Δ^S approaches this value, the fringe-counting circuitry will respond as if portions of the illuminated spherical surface moved toward (or away) by 12 μ in. more than the remainder of the illuminated spot on the surface. This can result in an ambiguity in the number of fringe shifts to be counted by the circuitry of the laser head and hence an error can be triggered. For example, for a sphere of radius $R = 8.89$ mm (0.35 in.) and illuminated spot

Figure 5. Geometry for Movement of Sphere by Distance x_0 for Case A (Figure 3)

diameters of $2r = 0.447$ mm and $2\epsilon = 0.0157$ mm in Cases B and A, respectively, the maximum distances x_0 which the sphere can be moved relative to the beam before the "warping" represents one-half fringe shift ($\Delta^C = \Delta^S = 12 \mu\text{in.}$) in Cases A and B can be determined from Equations 3 and 6. The results are

$$\text{Case A: } (x_0)_{\max} = 0.171 \text{ mm } (6.73 \times 10^{-3} \text{ in.})$$

$$\text{Case B: } (x_0)_{\max} = 0.006 \text{ mm } (239 \mu\text{in.}).$$

(8)

From the latter it can be seen that the sphere must be aligned very accurately in relation to the laser beam in the Case B arrangement of Figure 3 to avoid triggering a signal error in the displacement-measuring system. According to the calculations for the typical conditions considered, the sphere center must be coincident with the sphere rotation axis to within $6 \mu\text{m}$ ($239 \mu\text{in.}$) to avoid an error signal as the sphere is rotated. However, when the laser beam is focussed to the surface as in Case A, the foregoing calculations show that the sphere center can be misaligned as much as $170 \mu\text{m}$ (0.0067 in.), or 28 times that in Case B, before there is danger of triggering an error. These theoretical conclusions are supported by experimental results which showed that the Case B arrangement with a 17.8-mm-diameter sphere is highly unstable, compared with Case A, and results in frequent triggering of error signals in the displacement-measuring system.

When a sphere with center imperfectly aligned on the rotation axis by the amount x_0 is rotated, the apparent "warping" of the surface in the illuminated spot is given by the expressions in Equations 3, 4, and 6. The beam returning from the sphere to the lens of the interferometer (Figure 3) undergoes corresponding deflections, in the plane of the lens, as the sphere is rotated and these deflections are given by Equations 7 and 5 for Cases A and B, respectively. According to Equations 7 and 5, the maximum deflections, in the plane of the lens of the interferometer, corresponding to the maximum calculated values of misalignment in Equation 8, are 5.77 mm (0.227 in.) and 0.202 mm (0.008 in.) for Cases A and B, respectively. This shows that the arrangement of Case A in Figure 3, which is much less prone to triggering an error signal for a given value of x_0 than Case B, has the somewhat undesirable characteristic of producing a much larger off-axis deflection of the returning light beam, in the plane of the lens. However, this does not pose a problem for two reasons. First, the foregoing calculated deflections represent maximum values and, second, the deflection is not a problem as long as the return beam still enters the lens of the interferometer and returns to the counting circuitry of the laser head. The diameter of the 150-mm focal length lens in use at present is 25 mm and, therefore, the above maximum calculated deflection of 5.77 mm for Case A will still allow the return beam to enter this lens.

Roundness Measurement

The displacement measuring system was used to determine the roundness of spheres as round as $0.33 \mu\text{m}$ (13 $\mu\text{in.}$) and as out-of-round as $10 \mu\text{m}$ (400 $\mu\text{in.}$). Each sphere was positioned in the vacuum chuck of the air bearing spindle and was aligned with the wobble plate so that its center was as nearly centered on the spindle axis as possible. The maximum variation in reading of the interferometer as the sphere is rotated through one revolution gives the roundness for a great circle of the sphere because the illuminated spot on the sphere traces out a great circle as the sphere is rotated about its axis.

The laser interferometer is interfaced with the computer, and, when the sphere has been aligned in the fixture as described, the roundness measurements can be accomplished automatically by calling the appropriate program at the computer terminal. The computer samples the output of the laser interferometer for one complete rotation of the sphere and prints out the roundness for this great circle of the sphere. Actually, the number printed out includes not only out-of-roundness on the sphere great circle but also any misalignment of the sphere

center on the spindle axis. However, tests have shown that the latter effect is negligible when standard procedures are used for centering a sphere on the spindle axis. The agreement between the non-contact laser interferometer roundness measurements and conventional, stylus contact roundness measurements is very good.

Comparisons were made between the non-contact laser roundness measurements and the stylus contact measurements on spheres as round as $0.33\text{ }\mu\text{m}$ ($13\text{ }\mu\text{in.}$) and as out-of-round as $10\text{ }\mu\text{m}$ ($400\text{ }\mu\text{in.}$). The laser interferometer roundness measurements agreed with the stylus measurements to within 50.8 to 127.0 nm on the same great circle of the round spheres. The non-contact laser roundness value was within 3 percent of the stylus-measured value obtained on the same great circle of the $10\text{ }\mu\text{m}$ out-of-round sphere. The results are summarized in Table 1.

When the spindle axis is oriented perpendicular to the axis of the laser interferometer light beam, full rotations of the sphere about the spindle axis provide roundness measurements on a great circle of the sphere lying in a plane perpendicular to the spindle axis. The turn-around fixture can be used to change the sphere orientation relative to the vacuum holder of the spindle, and roundness measurements can then be made on a different great circle of the sphere. Also, for any particular orientation of the sphere in the spindle vacuum holder, roundness measurements can be made on any minor circle, from equator to pole of the sphere, lying in a plane parallel to the great circle plane. This is accomplished by rotating the rotary table to achieve the desired angle between spindle axis and light beam axis prior to executing the full rotation about the spindle axis. The computer program for roundness measurement provides the capability, at the option of the operator, for roundness measurement on any desired number of minor circles from the equator to pole of the sphere.

The roundness of the sphere on three intersecting, mutually perpendicular great circles can be measured by making the appropriate rotations of the sphere in relation to the light beam of the laser interferometer. This can be understood with reference to Figure 6, in which the points A, B, C, D, E, and F are considered fixed on the surface of the sphere. Let the sphere be positioned arbitrarily in the vacuum holder of the air bearing spindle, and let the rotary table be oriented so that the spindle axis is perpendicular to the axis of the laser interferometer light beam. That is, the spindle axis and the light beam lie in a horizontal plane and intersect at right angles at the center of the sphere. As the sphere is rotated through 360° about the spindle axis, the laser interferometer light beam

Table 1. Comparison of Laser Non-Contact and Stylus Roundness Measurements on the Same Great Circles of Four Spheres

Sphere Diameter (mm)	Run	Roundness Measurement			
		Laser (μm)	($\mu\text{in.}$)	Stylus (μm)	($\mu\text{in.}$)
17.8	1	0.356	14	0.432	17
	2	0.305	12		
	3	0.356	14		
17.8	1	2.03	80	1.91	75
17.8	1	0.330	13	0.330	13
	2	0.279	11		
20	1	10.2	400	10.4	410

traces a great circle ABCD on the surface of the sphere, as shown in Figure 6. If the vacuum is now released on the spindle axis and transferred to the vertical axis of the "turn-around" fixture, the sphere will be held stationary on this axis. If the sphere is rotated counterclockwise through 90° about the vertical axis and the vacuum is again transferred so the sphere is secured in the air-bearing spindle holder, the light beam will trace the great circle EBFD on the surface of the sphere as the sphere is rotated through 360° about the spindle axis. The great circle EBFD will lie in a plane perpendicular to the plane of the first great circle ABCD, by virtue of the 90° rotation about the vertical axis. Now, the great circles ABCD and EBFD lie in vertical planes and the diameter EF lies along the axis of the light beam. If the plane of the first great circle ABCD is made horizontal by a clockwise 90° rotation of the sphere about the air-bearing spindle axis, the diameter DB will lie along the light beam axis. If the vacuum is transferred to the vertical axis of the "turn-around" fixture, the sphere can be rotated counterclockwise 90° about the vertical axis so that the diameter CA lies along the axis of the light beam. When the vacuum is transferred back to the air bearing spindle and the sphere is rotated through 360° about the spindle axis, the light beam will trace the great circle CEAF on the surface of the sphere. The plane of the latter great circle is perpendicular to the planes of the first two great circles.

Theoretical Basis for Semi-Automated Computer Measurement of Waviness Amplitude and Wavelength

The laser interferometric displacement measuring system is interfaced with the computer and is operational for semi-automated

measurement of spherical roundness on a great circle or any minor circle. The distinction between "out-of-roundness" and "waviness" is not a well defined one, although out-of-roundness is usually thought of as a departure from sphericity in which the amplitude variation is rather gradual and extends over an arc length representing a fairly large percentage of the spherical circumference. By contrast, the term "waviness" is usually used to describe contour defects or amplitude variations characterized by an arc length or "wavelength" very small compared with the circumference of the sphere. The present semi-automated roundness measuring system provides the capability of measuring waviness amplitudes but not the corresponding waviness wavelengths associated with these amplitudes. The theoretical framework for the writing of a computer program has been worked out and is described below, although the writing of the computer program has not been completed.

The design of the computer program is such that the operator will enter the radius R of the sphere, the illuminated spot diameter 2ϵ , and quantities A_{max} , A_1 , A_2 , A_3 , λ_1 , λ_2 and λ_3 which are code values related to the criteria for inspection for waviness amplitudes and the associated wavelengths, respectively. The sphere will be aligned in the system such that the inspection will begin on a great circle and proceed on minor circles in planes parallel to the great circle plane. The angle θ will represent the rotation of the sphere about the horizontal, air-bearing spindle axis and the angle ϕ will measure rotation of the sphere about the vertical rotary table axis. Full 360° rotations of the sphere about the spindle axis combined with a 90° rotation about the table axis will permit full inspection of one-half of the spherical surface. Inspection of the other half can be accomplished after first rotating the sphere 180° about the table axis with the aid of the "turn-around" fixture referred to in Figure 6. The major steps which the computer program must execute to analyze for waviness amplitudes and the associated wavelengths are given below.

1. Use the sensing signal from the rotating air-bearing spindle to determine the time for 10 complete revolutions of the sphere. From this, calculate the time t for one revolution of the sphere.
2. Calculate the number of data samples for the equatorial great circle from the equation

$$N_0 = \frac{\pi R}{\epsilon} \cos \left(\frac{2n\epsilon}{R} \right),$$

where R is the radius of the sphere, 2ϵ is the illuminated spot diameter, and the subscript "0" on N indicates that $N = 0$ for the first scan on the equatorial great circle. (That is, $N_0 = \pi R/\epsilon$ on the equator.)

3. Calculate the equatorial data-taking rate or number of readings per second from the equation

$$\text{Rate} = N_0/t .$$

The time interval between consecutive readings is

$$\text{Interval} = t/N_0 .$$

(It is assumed that the air-bearing spindle speed will be set such that the equatorial rate is approximately 200 data samples per second, or the interval = 0.005 second, which is the maximum updating rate of the interferometer.)

4. After completing the calculations in steps 1, 2, and 3, the computer should search for the spindle rotation signal and, upon receiving it, should begin taking interferometric readings at the equatorial rate calculated in step 3. Let the first interferometric reading on the equator be the zeroth reading and let it arbitrarily correspond to $\theta = 0^\circ$. The value θ for the k th reading is

$$\theta = \frac{k}{N_0} (360^\circ) \quad (\text{in degrees})$$

$$\theta = \frac{k}{N_0} 2\pi \quad (\text{in radians}).$$

In general, the arc length S on the sphere measured from the zeroth reading out to the k th reading on the n th minor circle which contains N readings is

$$S = R \frac{k}{N} 2\pi ,$$

where

$$N = \frac{\pi R}{\epsilon} \cos \left(\frac{2n\epsilon}{R} \right).$$

(On the equator $N = 0$, and so $N = N_0$, and $S = 2\pi Rk/N_0$). Consider the hypothetical series of consecutive interferometric readings ℓ versus angle θ and arc length S as shown in Figure 7. The first reading received by the computer at $\theta = 0^\circ$ is ℓ_0 . The next reading is ℓ_1 at $\theta_1 = 360^\circ/N_0$ and arc length $S_1 = 2\pi R/N_0$. ℓ_1 is greater than, less than, or equal to ℓ_0 . (If by chance $\ell_1 = \ell_0$, ignore ℓ_1 and ℓ_0 and proceed to ℓ_2 for the following analysis.) For the purposes of discussion, assume as

indicated in the Figure, that $l_1 > l_0$. This implies that the slope of reading l versus arc length S is >0 over the interval $S_0 \rightarrow S_1$ ($\theta_0 \rightarrow \theta_1$). Next the computer receives reading l_2 and finds (Figure 7 example) that $l_2 > l_1$ which again indicates that the slope of l versus S is >0 over the interval S_1 to S_2 . Likewise it finds that $l_3 > l_2$, $l_4 > l_3$, and $l_5 > l_4$ and consequently the slope remains positive over the corresponding arc length intervals $S_1 \rightarrow S_2$, $S_2 \rightarrow S_3$, $S_3 \rightarrow S_4$ and $S_4 \rightarrow S_5$. However, when the computer receives reading l_6 it finds that $l_6 < l_5$ and hence the slope over the interval $S_5 \rightarrow S_6$ is <0 . Thus, to the "left" of the position on the sphere represented by reading l_5 the slope of l versus S is >0 and to the "right" of this position the slope is <0 . The computer then can ascertain that reading l_5 corresponds to a relative maximum in the l versus S "curve". (The maximum is relative and not absolute because l_{17} , for example, exceeds l_5 .)

As the computer continues, it finds that the slope is also <0 over the consecutive intervals $S_6 \rightarrow S_7$ and $S_7 \rightarrow S_8$. However, the computer finds that the slope changes to >0 for the interval $S_8 \rightarrow S_9$ and consequently the computer can determine that the position on the sphere represented by reading l_8 corresponds to a relative minimum in the "curve" of l versus S , because to the left of l_8 the slope is <0 but to the right of l_8 it is >0 . (The minimum is only a relative minimum and not an absolute one because l_{25} , for example, is less than l_8 .)

As the computer continues comparing consecutive readings in this way it will determine that the next change in sign of slope is represented by the reading l_{12} and that l_{12} corresponds to a relative maximum. Now, l_5 and l_{12} are adjacent relative maxima which contain a single relative minimum (l_8) in between, and the computer can recognize this as the presence of a one-cycle "wave" over the arc length interval from S_5 to S_{12} . The computer should assign to this wave a peak-to-peak amplitude A given by

$$A = \left| \frac{l_5 + l_{12}}{2} - l_8 \right| ,$$

and it should assign a "wavelength" λ of magnitude

$$\lambda = R \frac{2\pi}{N_0} [(k=12) - (k=5)] = \frac{14\pi R}{N_0}$$

Moreover, the computer should store or "remember" this wave if $A > A_{\max}$ and should ignore it if $A < A_{\max}$. If

Figure 7. Hypothetical Interferometric Reading λ Versus Angle θ and Arc Length S on a Sphere, Illustrating the Theory for Computer Measurement of Waviness Amplitude and Associated Wavelength

$A > A_{\max}$, the computer should store the wave amplitude, wavelength, and angular position. To continue in the example of Figure 7, the next wave the computer detects is that represented by the interferometric readings ℓ_{12} , ℓ_{14} , and ℓ_{17} . The amplitude and wavelength are

$$A = \left| \frac{\ell_{12} + \ell_{17}}{2} - \ell_{14} \right|$$

$$\lambda = R \frac{2\pi}{N_0} (17 - 12) = \frac{10\pi R}{N_0}.$$

The concepts of relative maxima and relative minima in the readings of ℓ versus S provide a pictorial way of considering waviness defects. The interferometric readings of ℓ which separate regions of one slope sign from the opposite slope sign provide the key for characterizing waviness in terms of the "peaks", or relative maxima, and "valleys" or relative minima. In general, the computer needs three readings corresponding to adjacent slope sign changes for classifying a one-cycle wave. For example, if the computer finds that readings ℓ_m , ℓ_p , and ℓ_w represent adjacent slope sign changes, corresponding to angular positions θ_m , θ_p , and θ_w , respectively (corresponding arc length positions S_m , S_p , and S_w), then the computer should assign to these readings a one-cycle wave of amplitude

$$A = \left| \frac{\ell_m + \ell_w}{2} - \ell_p \right|$$

and wavelength

$$\lambda = \frac{2\pi R}{N} (w - m),$$

where m , p , and w are integers and $m < p < w$.

5. The computer should halt data taking when the cumulative number of samples reaches N_0 as calculated in step 2. The computer should then store and list the absolute maximum reading $\ell_{\max}$ and the absolute minimum reading $\ell_{\min}$, and the corresponding angles $\theta_{\max}$ and $\theta_{\min}$.
6. The computer should also store and list the waves which have amplitudes and wavelengths in the ranges specified by the following read-in values:

$$(A_{\max} \rightarrow A_1, 0 \rightarrow \lambda_1)$$

$$(A_1 \rightarrow A_2, \lambda_1 \rightarrow \lambda_2)$$

$$(A_2 \rightarrow A_3, \lambda_2 \rightarrow \lambda_3)$$

Also, the computer should list the angle ϕ corresponding to the minor circle or "ring" on which these waves are detected. For the first ring or equator, $\phi = 0$. In general, for the n^{th} ring, the corresponding value of ϕ is

$$\phi = \frac{2n\epsilon}{R} \frac{180^\circ}{\pi} .$$

7. Next, the computer should give the command to index the rotary table.
8. Repeat steps 2 through 7, but set $n = 1$ in step 2 and use the corresponding calculated values in the steps of the sequence.
9. Repeat steps 2 through 7, but set $n = 2$ in step 2 and observe the corresponding calculated values in the steps of the sequence.
10. Continue the sequence referred to in steps 8 and 9 until a value of n is reached for which

$$\frac{2\epsilon n}{R} \frac{180^\circ}{\pi} \geq 90^\circ .$$

This ends the inspection for the hemisphere.

In considering the data from the hemisphere as a whole, the computer should be programmed to correlate a waviness condition from one minor circle or "ring" to another, where such correlation exists. A waviness condition of amplitude A and wavelength λ , located at angle θ on a given ring is considered to be related to a waviness of amplitude A' , and wavelength λ' located at angle θ' on adjacent rings, provided

$$A' \approx A$$

$$\theta' \approx \theta$$

$$\theta' \approx \lambda .$$

Therefore, the computer should consider the data as a whole and provide a "two-dimensional" description of a surface waviness condition in terms of the two angles θ and ϕ (Figure 8).

Sphere Alignment on Spindle Axis of Two-Axis Rotary Fixture

The two-axis rotary fixture was designed to provide a rotation of the sphere about two perpendicular, intersecting axes while maintaining the sphere center fixed in space. When a sphere is first positioned in the fixture, the sphere center is not coincident

Figure 8. Geometry Illustrating the Spherical Three-Dimensional Characteristics of a Surface Waviness Defect

with either the horizontal spindle axis or the vertical table axis, but positioning devices are provided for achieving this. A wobble plate is used to locate the sphere center on the spindle axis, and x- and y- movements in the plane of the rotary table are used to move the air bearing spindle and sphere as a unit to make the sphere center coincident with the vertical table axis.

During use of the system, full 360° rotations of the sphere are made about the horizontal air-bearing spindle axis, and the laser interferometric displacement readings are used to provide a quantitative measure of the wobble plate movements needed to locate the sphere center on the spindle axis. For this portion of the alignment, the table is positioned so that the spindle axis is perpendicular to the axis of the light beam from the single beam interferometer (Figure 9). As the sphere is rotated about the spindle axis, the laser interferometer will measure the change in distance relative to a point on the great circle of the sphere. For a perfect sphere with center positioned on the

Figure 9. Geometry for Centering the Sphere on the Spindle Axis (The sphere center is at O and the spindle axis is perpendicular to the plane of the page at S. A wobble plate provides movement of the sphere center in the plane of the page.)

spindle, the laser interferometer reading should not change as the sphere is rotated through a complete revolution. However, when a sphere is first positioned in the vacuum chuck of the spindle, the interferometer may change as much as several thousandths of an inch as the sphere is rotated. The way in which the laser readings can be used to center the sphere on the spindle axis can be understood by considering four points A, B, C, and D, separated by 90° on the great circle as shown in Figure 9. Let d_A and d_C be the laser readings when points A and C, respectively, are illuminated by the beam from the interferometer. If the sphere is placed in a stationary position with either point A or point C illuminated by the beam, and the wobble plate is adjusted so that the interferometer gives a reading equal to the average of the algebraic sum of d_A and d_C , then the sphere diameter BD should intersect the spindle axis. Next, let d_B and d_D be the readings when points B and D, respectively, are illuminated by the interferometer beam. If the wobble plate is again adjusted for the sphere positioned with either point B or D in the path of the beam so that the interferometer gives a reading equal to the average of the algebraic sum of d_B and d_D , the sphere diameter AC will be made to intersect the spindle axis. Therefore, since the

first adjustment placed diameter BD on the spindle axis, the sphere center should now be coincident with the spindle axis. In practice, if the two motions of the wobble plate are not independent, the two-step procedure described above may have to be repeated.

Sphere Alignment on Rotary Table Axis

When Laser Light Beam Intersects the Table Axis

The final step in positioning the center of the sphere at the intersection of the spindle and table axes is to move the spindle bearing and sphere as a unit until the center of the sphere coincides with the table axis. In principle, this problem is identical to that of positioning the sphere center on the spindle axis. In the latter case, the sphere position must be adjusted in the vertical plane, whereas in the former case adjustments must be made in the horizontal plane. This procedure is not as straight-forward as that for placing the sphere center on the spindle axis alone because the location of optical components in the system restricts the rotational motion of the rotary table to approximately 120° or less. Thus, the procedure outlined above for centering the sphere on the spindle axis cannot be used because the sphere on the table cannot be rotated through a sufficiently large angle to provide interferometer readings for opposite ends of a given sphere diameter, let alone opposite ends of two orthogonal diameters. Therefore, a method requiring only a 90° rotation about the vertical table axis is described below for centering the sphere on the table axis.

Consider Figure 10, in which the sphere and table are viewed directly from above. The sphere of radius R has its center at O and it is assumed that O lies on the spindle axis SS' as a result of the alignment procedure described earlier. The beam from the interferometer lies in the horizontal plane through the center of the sphere and strikes the spherical surface at point P. The axis of the beam is along the line PT, which is assumed to intersect the table axis. The center of the sphere is at an arbitrary distance $(B^2 + E^2)^{1/2}$ from the table axis T, where B is the displacement of the spherical center from the table axis in the direction of the light beam axis and E is the displacement in a direction perpendicular to the light beam axis. These conditions specify the initial orientation of the sphere relative to the table axis and beam axis. When the table is rotated clockwise through an angle ϕ about its center T as shown, the point P' on the sphere will then lie on the axis of the light beam. The corresponding change H in laser interferometer reading will be

$$H = TP - TP' \quad (9)$$

$$= B + (R^2 - E^2)^{1/2} - x$$

Figure 10. Top View of Sphere of Radius R With Center O on Spindle Axis SS' but Displaced Relative to Table Axis, Which is Perpendicular to Page at T (The point P' will be illuminated by the beam when the table is rotated clockwise through the angle ϕ .)

When the law of cosines is applied to triangle OP'T, the distance X can be related to R, E, B, and ϕ by the relation

$$\begin{aligned} R^2 &= x^2 + E^2 + B^2 - 2x(E^2 + B^2)^{1/2} \cos \left[\frac{\pi}{2} - \phi - \tan^{-1} \left(\frac{B}{E} \right) \right] \\ &= x^2 + E^2 + B^2 - 2x(E^2 + B^2)^{1/2} \sin \left[\phi + \tan^{-1} \left(\frac{B}{E} \right) \right]. \end{aligned} \quad (10)$$

The latter can be solved for X to give

$$\begin{aligned} x &= (B^2 + E^2)^{1/2} \sin \left[\phi + \tan^{-1} \left(\frac{B}{E} \right) \right] \\ &+ \left\{ (B^2 + E^2) \sin^2 \left[\phi + \tan^{-1} \left(\frac{B}{E} \right) \right] + R^2 - B^2 - E^2 \right\}^{1/2}. \end{aligned} \quad (11)$$

When the sine factors in Equation 11 are expanded and the resulting expression for X is substituted into Equation 9, the following relation is obtained for the change H in reading of the interferometer as a function of the sphere radius R, the off-center distances B and E, and the angle ϕ through which the table is rotated clockwise in Figure 10.

$$\begin{aligned} H &= B + (R^2 - E^2)^{1/2} - E \sin \phi - B \cos \phi \\ &- [R^2 - (E \cos \phi - B \sin \phi)^2]^{1/2} \end{aligned} \quad (12)$$

The angular positions of the relative minima or maxima of H versus the angle ϕ , for given values of R, B, and E can be obtained by equating the first derivative of H with respect to ϕ to zero and solving for the corresponding angle ϕ_m . The first derivative of H with respect to ϕ is

$$\begin{aligned} \frac{dH}{d\phi} &= -E \cos \phi + B \sin \phi + [R^2 - (E \cos \phi - B \sin \phi)^2]^{-1/2} \\ &\quad \times (E \cos \phi - B \sin \phi)(-E \sin \phi - B \cos \phi). \end{aligned} \quad (13)$$

From Equation 13 it can be seen that $\frac{dH}{d\phi} = 0$ if $B \sin \phi_m - E \cos \phi_m = 0$, or

$$\tan \phi_m = E/B. \quad (14)$$

If $B \sin \phi_m - E \cos \phi_m \neq 0$, then $dH/d\phi = 0$ if $R^2 = E^2 + B^2$. However, this is not a physically meaningful solution because it holds only when the table axis T intersects the circumference of the sphere. In practice, E and B are both very small compared with the sphere radius R.

Some plots of the theoretical change H in interferometer reading versus the table rotation angle ϕ are shown in Figure 11 for a

$$H = B + (R^2 - E^2)^{1/2} - E \sin \theta - B \cos \theta - [R^2 - (E \cos \theta - B \sin \theta)^2]^{1/2}$$

Figure 11. Theoretical Change H in Laser Interferometer Reading Versus Clockwise Rotation Angle ϕ of the Rotary Table, for Various Values of B and E in Figure 10

17.8-mm-diameter sphere off-centered at various positions relative to the table axis T in Figure 10. Some notes should be made in regard to the signs of the off-center distances B and E. Equations 12 through 14 were derived on the assumption that B and E are positive for the case in Figure 10, that is, for B and E in the fourth quadrant of the x_0 - y_0 axis system centered at the spherical center O. This results in a positive value for the angle ϕ_m in Equation 14 and indicates that the change H in interferometer reading will exhibit a minimum or maximum for $0^\circ < \phi < 90^\circ$, because the tangent of an angle in this interval is positive. Likewise, when B and E are both negative, corresponding to the point T lying in the second quadrant in Figure 10, the value of ϕ_m in Equation 14 is positive and hence H will exhibit a minimum or maximum for $0^\circ < \phi < 90^\circ$. If either B or E is negative, corresponding to point T lying in the first or third quadrants in Figure 10, then the tangent of ϕ_m in Equation 14 is negative. For the latter two cases, H versus ϕ does not display a relative minimum or maximum in the interval $0^\circ < \phi < 90^\circ$ because $\tan \phi$ is positive in this interval. These conclusions are verified by the characteristics of the various plots shown in the examples of Figure 11.

The results of Equations 13 and 14 indicate that H will exhibit a minimum (or maximum) when the table is at the angle ϕ_m given by

$$\phi_m = \tan^{-1}(E/B) .$$

To make the spherical center coincident with the table axis, it is necessary to know both E and B and to adjust the sphere and spindle accordingly, relative to the table. However, Equation 14 provides a key to a somewhat experimental method based on knowledge of the ratio of E to B, without knowing the magnitude of either. That is, if the interferometer reading H shows a minimum or maximum as the table is rotated from $\phi = 0^\circ$ through $\phi = 90^\circ$, then the ratio of E to B is $\tan \phi_m$ where ϕ_m is the angle at which the minimum or maximum is observed. Thus, to proceed with the experimental method, the spindle and sphere would be moved toward or away from the interferometer along the line SS' by an amount B' somewhat less than the observed H ($\phi = 90^\circ$) reading. Next, the spindle and sphere would be moved from left to right or vice versa in Figure 10 by the amount

$$E = B \tan \phi_m ,$$

which tends to position the sphere center closer to T, along the line joining O and T. If the reading H still exhibits a minimum or maximum in the table angular interval $0^\circ < \phi < 90^\circ$, this procedure can be repeated again to move point O closer to T.

If the sphere radius R , the angle ϕ_m at which H exhibits a minimum or maximum, and the value of H at ϕ_m are known or measured experimentally, then the values of B and E in Figure 10 can be determined by solving Equations 12 and 14 simultaneously. That is, when ϕ is equal to ϕ_m , Equation 12 gives

$$H(\phi_m) = B + (R^2 - E^2)^{1/2} - E \sin \phi_m - B \cos \phi_m \\ - [R^2 - (E \cos \phi_m - B \sin \phi_m)^2]^{1/2}, \quad (15)$$

and when E is eliminated by substituting from Equation 14, the following quadratic in B is obtained.

$$B^2[(1 - \tan \phi_m \sin \phi_m - \cos \phi_m)^2 + \tan^2 \phi_m] - 2(1 - \tan \phi_m \sin \phi_m - \cos \phi_m) \\ [H(\phi_m) + R] B + [H(\phi_m) + R]^2 - R^2 = 0. \quad (16)$$

The two roots of this quadratic are

$$B = \frac{T [H(\phi_m) + R]}{T^2 + \tan^2 \phi_m} \pm \left[\frac{T^2 (H(\phi_m) + R)^2}{(T^2 + \tan^2 \phi_m)^2} - \frac{(H(\phi_m) + R)^2 - R^2}{T^2 + \tan^2 \phi_m} \right]^{1/2}, \quad (17)$$

where $T = 1 - \tan \phi_m \sin \phi_m - \cos \phi_m$. Only one of the two values for B in Equation 17 will be meaningful in any given situation in which the change in interferometer reading H versus ϕ exhibits a minimum (or maximum) at the angle ϕ_m . However, the extraneous root should be easily recognized in any given case. The corresponding value of E is then obtained from

$$E = B \tan \phi_m,$$

where B is the physically meaningful root in Equation 17.

As an example, assume that the plot with solid dots in Figure 11 is an experimentally generated curve of the change H in interferometer reading versus angle ϕ . This curve exhibits a minimum at the angle

$$\phi_m = 26.6^\circ \quad (18)$$

and the value of H at this angle is

$$H(26.6^\circ) = -11.8 \times 10^{-6} \text{ in.} \quad (19)$$

If the latter values for ϕ_m and $H(\phi_m)$ are substituted into Equation 17, the following two values for the off-center distance B are obtained:

$$B_1 = 99.9 \times 10^{-6} \text{ in.}$$

$$B_2 = -0.313 \text{ in.}$$

The second value of B is not physically meaningful because if B had this value intuition shows that H versus ϕ would show a much larger variation than the assumed experimental curve in Figure 11. However, the first value of B is in fact the value used to generate the curve in Figure 11. The corresponding value of E from Equation 14 is

$$\begin{aligned} E &= (99.9 \times 10^{-6} \text{ in.}) \tan(26.6^\circ) \\ &= 50.0 \times 10^{-6} \text{ in.} \end{aligned}$$

which also agrees with the value of E used in generating the assumed experimental curve of Figure 11.

When the Laser Light Beam Does Not Intersect Table Axis

The results of the preceding analysis are valid also for the case in which the laser interferometer light beam does not intersect the table axis, provided the perpendicular distance from the beam axis to the table axis is small compared with the radius of the sphere. This can be understood by considering Figure 12 which is the same as Figure 10 except that the axis of the light beam is displaced from the table axis T by a perpendicular distance L . When the sphere and rotary table rotate clockwise through an angle ϕ , the point P' on the circumference of the sphere will be in the path of the light beam and point C will coincide with point C' . Therefore, the change H' in laser interferometer reading for rotation of the rotary table through angle ϕ is

$$H' = PC' - P'C, \quad (20)$$

where $P'C \equiv X'$. The distance X' can be expressed in terms of E , B , L , and ϕ by applying the law of cosines to triangle $OP'O'$ in Figure 12. If the angle W is defined as

$$W \equiv \phi + \tan^{-1}(B/E), \quad (21)$$

then

$$R^2 = (X' + L \tan W)^2 + \left[(B^2 + E^2)^{1/2} + \frac{L}{\cos W} \right]^2 - 2(X' + L \tan W) \left[(B^2 + E^2)^{1/2} + \frac{L}{\cos W} \right] \cos \left(\frac{\pi}{2} - W \right). \quad (22)$$

When the latter is expanded, reduced, and solved for X' , the result is

$$X' = E \sin \phi + B \cos \phi + [R^2 - \{L + (E \cos \phi - B \sin \phi)\}^2]^{1/2}, \quad (23)$$

and when this is combined with the geometry of Figure 12, the expression for H' in Equation 20 can be written

$$H' = B + [R^2 - (E+L)^2]^{1/2} - E \sin \phi - B \cos \phi - \{R^2 - [L + (E \cos \phi - B \sin \phi)]^2\}^{1/2}. \quad (24)$$

When $L = 0$ the latter reduces to the expression given in Equation 12.

The change in interferometer reading with rotation angle ϕ can be determined by taking the first derivative of H' in Equation 24 with respect to ϕ , which gives

$$\frac{dH}{d\phi} = -E \cos \phi + B \sin \phi + \frac{[L + (E \cos \phi - B \sin \phi)](-E \sin \phi - B \cos \phi)}{\{R^2 - [L + (E \cos \phi - B \sin \phi)]^2\}^{1/2}}. \quad (25)$$

The sine and cosine functions cannot exceed unity and the distances L , E , and B are all much smaller than the radius R of the sphere. The result in Equation 25 can therefore be written

$$\frac{dH}{d\phi} \approx -E \cos \phi + B \sin \phi - \frac{(L + E - B)(E + B)}{R}. \quad (26)$$

But, $L, E, B \ll R$ and, therefore,

$$\left| \frac{1}{R} (L + E - B)(E + B) \right| \ll \left| -E \cos \phi + B \sin \phi \right|,$$

which allows Equation 26 to be written

$$\frac{dH}{d\phi} = -E \cos \phi + B \sin \phi. \quad (27)$$

The curve of H' versus ϕ will have a relative or maximum when $dH'/d\phi = 0$ and, according to Equation 27, this gives

$$\tan\phi_m = E/B, \quad (28)$$

which is the same relation obtained earlier in Equation 14. This shows that Equation 14 holds even when the light beam axis does not intersect the rotary table axis, provided that L , E , and B are small compared with the radius R of the sphere.

If $u \ll a$, the approximation

$$(a^2 + u^2)^{1/2} \approx a \left(1 + \frac{u^2}{2a^2} \right) \quad (29)$$

holds and hence Equation 24 can be approximated by

$$H' \approx B - \frac{E^2}{2R} - \frac{EL}{R} - E\sin\phi - B\cos\phi + \frac{(E\cos\phi - B\sin\phi)^2}{2R} + \frac{L(E\cos\phi - B\sin\phi)}{R} \quad (30)$$

for B , E , $L \ll R$. In the same manner, Equation 12 can be approximated by

$$H \approx B - \frac{E^2}{2R} - E\sin\phi - B\cos\phi + \frac{1}{2R} (E\cos\phi - B\sin\phi)^2, \quad (31)$$

which gives the change H in interferometer reading versus ϕ for the case in which the light beam axis intersects the table axis. From Equations 30 and 31 the changes in interferometer readings H and H' for $L = 0$ and $L \neq 0$, respectively, are related to one another by the expression

$$H' = H - \frac{L}{R} [E(1 - \cos\phi) + B\sin\phi] \quad (32)$$

As the table is rotated from $\phi = 0^\circ$ to $\phi = 90^\circ$ the first term in the square brackets varies from zero to a maximum value of E and the second term varies from zero to a maximum of B and hence the largest value of the combination is $E + B$, which gives

$$H' \approx H - \frac{L}{R} (E + B) \quad (33)$$

But, L , E , and B are much less than R ; thus, Equation 33 gives

$$H' \approx H \quad (34)$$

An intuitive but revealing way of understanding that H' and H are very nearly equal for any angle ϕ when $L \ll R$ can be seen from the two radicals appearing in each of the two equations for H and H' in Equations 12 and 24. When $L = 0$, each radical in the expression for H' is equal to the corresponding radical in the expression for H . For a non-zero value of R , each radical in Equation 24 is somewhat less than its corresponding value for $L = 0$. However, the expression for H' in Equation 24 involves the difference between the two radicals and therefore the slight changes introduced in the radicals for $L \neq 0$ should approximately cancel out for values of L small compared with R .

The results of the analysis associated with Equations 20 through 34 show that the earlier Equations 12, 14, and 17, derived for the case $L = 0$ in which the light beam axis intersects the table axis are also valid for the case in which the light beam axis does not intersect the table axis, provided that L is small compared with the sphere radius R . These conclusions can be demonstrated by a specific example. Table 2 gives a comparison of exact calculations of H' and H from Equations 12 and 24 for $B = 2.54 \mu\text{m}$ (100 $\mu\text{in.}$), $E = 1.27 \mu\text{m}$ (50 $\mu\text{in.}$), $R = 8.89 \text{ mm}$ (0.35 in.), and various values of L ranging from 10.2 μm (400 $\mu\text{in.}$) to 254 μm (10,000 $\mu\text{in.}$), for various values of the rotation angle ϕ . For this range in L the greatest difference between H' and H is (0.11 μm) 4.3 $\mu\text{in.}$ and occurs for $\phi = 90^\circ$. This would not be bothersome from a practical point of view because the use of Equations 12, 14, and 17 relies on the value of H at a value of ϕ in the interval $0^\circ < \phi < 90^\circ$. In fact, from Equation 14 the important value of H is the value at the angle ϕ_m at which H has a relative minimum or maximum. In the earlier example for $L = 0$ and $B = 2.54 \mu\text{m}$ (100 $\mu\text{in.}$), $E = 1.27 \mu\text{m}$ (50 $\mu\text{in.}$) the angle ϕ_m was 26.6° (see Figure 11) and $H(\phi_m) = -0.299 \mu\text{m}$ (-11.8 $\mu\text{in.}$). Similar graphs of H' versus ϕ were constructed for the values of L , B , E , and R given in Table 2. For this range in values of L , each curve exhibited a relative minimum at $\phi = 26.6^\circ$ and the corresponding values of H' at $\phi = 26.6^\circ$ were very nearly equal to the value at 26.6° when $L = 0$. The maximum difference between H' (26.6°) and H (26.6°) is (0.038 μm) (1.5 $\mu\text{in.}$), for $L = 0.25 \text{ mm}$ (0.010 inch).

Tilting of Rotary Table Axis

The presence of the airbearing spindle, motor drive, and slides on the rotary table top results in a non-symmetric distribution of mass. As a result, it is believed that the rotary table vertical axis does not remain truly vertical as the table is rotated, but instead precesses about the true vertical. Therefore, as the table is rotated, the perpendicular to the table top generates an imaginary cone characterized by a very small half-angle. The vertex of this cone can be considered to lie in the table top or at some point below this plane, depending upon the resultant axis about which the table "tilts" because of the non-symmetric distribution of mass on its top. The tilting

Table 2. Comparison of Exact Calculation of H' With Exact Calculation of H for Various Values of ϕ and L, for the Given Values of R = 8.89 mm, B = 2.54 μ m, and E = 1.27 μ m

Angle ϕ (Degrees)	H L=0 (μ m)	H'			
		L=10.1 μ m (μ m)	L=40.6 μ m (μ m)	L=0.127mm (μ m)	L=0.254mm (μ m)
0	0	0	0	0	0
10	-0.182	-0.183	-0.184	-0.188	-0.195
20	-0.279	-0.282	-0.284	-0.295	-0.307
30	-0.295	-0.297	-0.302	-0.315	-0.335
40	-0.222	-0.224	-0.231	-0.250	-0.277
50	-0.066	-0.068	-0.076	-0.10	-0.134
60	0.170	0.167	0.157	0.130	0.089
70	0.478	0.475	0.462	0.432	0.386
80	0.848	0.843	0.833	0.798	0.747
90	1.27	1.27	1.25	1.22	1.16

effect will be a maximum when the axis of the air bearing is aligned co-linearly with the axis of the light beam, which corresponds to $\phi = 0^\circ$. Let Γ_0 be the magnitude of the tilt along the axis of the light beam at $\phi = 0^\circ$. If $\Gamma_0 \ll R$ and B and $E \ll R$, then the tilting effect is negligible at $\phi = 90^\circ$. This can be understood by the following reasoning. The tilting effect of magnitude Γ_0 for $\phi = 0^\circ$ causes a change in distance between sphere and interferometer, at $\phi = 90^\circ$, given by

$$\Gamma_{90} = R [1 - (1 - \Gamma_0^2/R^2)^{1/2}].$$

For Γ_0 as large as 0.01 R, the latter relation shows that Γ_{90} is only 0.5 percent of the tilt at $\phi = 0$. Therefore, the effect of the tilt on the interferometer reading is Γ_0 at $\phi = 0^\circ$ and is essentially zero at $\phi = 90^\circ$. As the rotary table rotates through the angle ϕ in Figure 10, the change in interferometer reading corresponds to the component $\Gamma_0 \cos \phi$ which clearly reduces to zero at $\phi = 90^\circ$. Reference to Figure 10 shows that tilt of the rotary table axis can be accounted for in Equation 12 by replacing the quantity B with $B + \Gamma_0 \cos \phi$, which gives

$$H = B + \Gamma_0 \cos \phi + (R^2 - E^2)^{1/2} - E \sin \phi - (B + \Gamma_0 \cos \phi) \cos \phi - \left\{ R^2 - [E \cos \phi - (B + \Gamma_0 \cos \phi) \sin \phi]^2 \right\}^{1/2}. \quad (12A)$$

Comparison of Experiment with Theory

Several experiments were performed with an 18-mm-diameter sphere in which the rotary table was rotated clockwise from $\phi = 0^\circ$ to $\phi = 90^\circ$ for various small but arbitrary values of the misalignment factors B and E. The readings H of the interferometer were recorded versus the angle ϕ for each run. In all cases the values of B and E were typical but small compared with the radius R of the sphere. Also, the misalignment L of the light beam relative to the center of the table was very small compared with R and, therefore, Equation 12 is applicable.

The angle ϕ_m at which the interferometer displayed a relative minimum was determined from each curve, along with the value H_m of the interferometer reading at this angle. The values ϕ_m and H_m were used to calculate the corresponding misalignment factors B and E, for each run, from Equations 14 and 17. These results, given in Table 3, do not take into account the tilt factor which is known to exist in the present case.

A convenient method of analyzing experimental data for non-zero values of B, E, and Γ_0 rests upon the fact that the two radicals in Equation 12A are each virtually equal to R for B, E, $\Gamma_0 \ll R$ and therefore the radicals cancel one another and the theoretical change in interferometer reading H can be written as

$$H = (B - \Gamma_0 \cos \phi) (1 - \cos \phi) - E \sin \phi. \quad (12B)$$

According to the latter equation, the theoretical values of the interferometer readings at $\phi = 30^\circ$, 45° , 60° , and 90° , for example, are

$$\begin{aligned} H_{30} &= \left(B - \frac{\sqrt{3}}{2} \Gamma_0 \right) \left(1 - \frac{\sqrt{3}}{2} \right) - E/2 \\ H_{45} &= \left(B - \frac{\sqrt{2}}{2} \Gamma_0 \right) \left(1 - \frac{\sqrt{2}}{2} \right) - \frac{\sqrt{2}}{2} E \\ H_{60} &= \left(B - \frac{1}{2} \Gamma_0 \right) \left(\frac{1}{2} \right) - \frac{\sqrt{3}}{2} E \\ H_{90} &= B - E \end{aligned} \quad (12C)$$

When the experimentally determined interferometer readings H_{30} , H_{45} , H_{60} , and H_{90} are substituted into Equation 12C, four sets of

Table 3. Determination of B, E, and Γ_0 (μm)

Run	From Equation 12C			From Equation 12C ($\Gamma_0=0$)		From Equations 14 and 17	
	B	E	Γ_0	B	E	B	E
1	2.65	3.20	3.96	3.31	4.62	3.51	4.84
2	2.52	3.35	3.68	3.25	4.71	3.45	4.93
3	1.07	1.16	3.07	1.56	2.25	1.78	2.44
4	1.98	-0.892	3.61	2.63	0.419		
5	2.93	0.759	2.43	3.41	1.66	1.50	0.958
6	2.33	0.508	3.84	2.93	1.86	1.17	1.17
7	2.37	1.18	3.66	3.66	2.58	0.174	2.01
8	2.48	1.83	3.28	3.36	3.16	2.16	2.66
9	1.97	2.53	3.27	2.64	3.76	2.58	3.82
10	1.84	2.40	3.34	2.53	3.66	2.23	3.57
11	2.63	3.20	3.61	2.98	4.36	3.40	4.68
12	3.07	37.3	2.83	3.57	4.76	3.25	4.64
13	3.15	12.7	4.26	3.55	14.1	5.54	15.2
14	1.61	1.23	3.45	1.76	2.26	1.91	2.45
15	2.56	1.32	4.52	3.73	3.12	1.97	2.34
16	-0.086	-0.241	3.63	0.384	0.996	0.795	1.32

three simultaneous equations are obtained from which the values of misalignment B, E, and the tilt factor Γ_0 can be determined. This was done for each experimental run and the values of B, E, and Γ_0 are given in Table 3.

It was pointed out earlier that the values of B and E determined from Equations 14 and 17 were based on the assumption that $\Gamma_0 \equiv 0$. For comparison sake, B and E were also determined from Equation 12C for the case in which the tilt was assumed to be zero. These values of B and E for each run also are given in Table 3.

Representative plots of the theoretical curves of H versus angle ϕ are shown in Figures 13, 14, and 15 for runs 4, 5, and 13, respectively. The experimentally measured values of interferometer readings H are shown also in the figures and the agreement between theory and experiment is very good. Linear regressions were performed on the data of experimental versus theoretical points for each run listed in Table 3, and the standard errors and correlation coefficients are given in Table 4. The maximum standard error among the runs is $0.189 \mu\text{m}$ ($7.5 \mu\text{in.}$). In only one case is the correlation lower than 95 percent. (Perfect agreement between theory and experiment would correspond to a correlation of 1.0, or 100 percent.)

Table 4 also shows the standard error for theory versus experiment for the case in which B and E were determined from Equation 12C under the assumption that the tilt factor Γ_0 is zero. It is seen that the standard error for each run is higher than the corresponding error when the tilt is taken into account. This indicates the reality of a tilt factor for the case investigated and, from the data in Table 3, the magnitude of the tilt is $\Gamma_0 = 3.3 \mu\text{m}$ ($130 \mu\text{in.}$).

The last column in Table 4 gives the standard errors for each run for the case in which the tilt Γ_0 was assumed to be zero and the values of B and E were determined from the measured angle ϕ of the minimum and the corresponding reading H_m of the interferometer. It is seen that the standard errors of the latter data are consistently higher than those of the first two sets, given in the fourth and fifth columns. The reason for this is not only that Γ_0 is neglected but also the fit between theory and experiment is better when B and E are determined from four measurements (H_{30} , H_{45} , H_{60} , H_{90} in Equation 12C) than when B and E are determined from only ϕ_m and H_m in Equations 14 and 17. Therefore, for those applications in which B, E, and Γ_0 are much smaller than the spherical radius R and the two radicals in Equation 12A cancel one another with sufficient accuracy, it is advisable to determine B and E from equations such as 12C rather than from Equations 14 and 17.

Figure 13. Theoretical and Measured Interferometer Readings Versus Rotation Angle for Run 4, Table 3

ACCOMPLISHMENTS

A laser interferometer was made operational for aligning spheres in the laser surface inspection system and for semi-automated measurement of roundness and waviness amplitude of spherical surfaces. Theory was developed for the writing of a computer

Figure 14. Theoretical and Measured Interferometer Readings Versus Rotation Angle for Run 5, Table 3

program to provide semi-automated measurement of waviness wavelength associated with the measured waviness amplitude. The theory was developed for using the interferometer to position the center of a sphere on an axis of rotation for the case in which the sphere can only be rotated through one-quarter turn about the desired axis. The method applies equally well for the use of a conventional stylus indicator.

Figure 15. Theoretical and Measured Interferometer Readings Versus Rotation Angle for Run 13, Table 3

Table 4. Comparison of Theory With Experiment

Source of Theory Compared With Experimental Data*						
Run	Equation 12C		Equation 12 ($\Gamma_0 = 0$)		Equations 14, 17	
	σ	γ	σ	γ	σ	γ
1	0.128	0.9900	0.302	0.9423	0.290	0.9467
2	0.165	0.9840	0.307	0.9432	0.292	0.9489
3	0.140	0.9588	0.259	0.8503	0.239	0.8745
4	0.137	0.9917	0.183	0.9849		
5	0.100	0.9942	0.119	0.9917	0.208	0.9746
6	0.098	0.9925	0.156	0.9808	0.559	0.7143
7	0.176	0.9751	0.204	0.9665	0.602	0.6522
8	0.189	0.9698	0.233	0.9537	0.549	0.7086
9	0.162	0.9763	0.282	0.9265	0.310	0.9089
10	0.184	0.9677	0.298	0.9126	0.351	0.8680
11	0.149	0.9837	0.290	0.9364	0.139	0.9575
12	0.142	0.9867	0.241	0.9611	0.310	0.9340
13	0.119	0.9995	0.274	0.9975	0.242	0.9981
14	0.114	0.9725	0.230	0.8814	0.228	0.8844
15	0.130	0.9892	0.272	0.9511	0.645	0.6834
16	0.134	0.9059	0.272	0.5177	0.227	0.6943

* σ is standard error in μm ; γ is the correlation coefficient.

FUTURE WORK

No further development work is planned for use of the laser interferometer in semi-automated measurement of roundness and waviness amplitude. Upon completion of work in progress on a computer program for semi-automated measurement of waviness wavelength, no further work is planned. However, further work will be required on temperature control and mechanical stability of the holder for the sphere if waviness inspection needs which require the full capability of the present laser interferometer for waviness amplitude resolution arise.

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APPENDIX A

DERIVATION OF EQUATIONS 3, 4, AND 5 OF TEXT

Equation 3

The expression for $\Delta_{\perp}^C$ in Equation 3 can be derived with the aid of Figure A-1 which shows a detailed view of the sphere and incident and reflected laser beams for the case in which the laser beam from the interferometer is initially focussed to a virtual point at the center of the sphere. From the geometry of Figure A-1, the quantity $\Delta_{\perp}^C$ can be expressed in terms of R and the angles α , θ , and ψ as

$$\begin{aligned}\Delta_{\perp}^C &= R \sin \left[\pi - \psi - \left(\frac{\pi}{2} - \alpha \right) \right] - R \sin \left[\pi - \theta - 2\alpha - \left(\frac{\pi}{2} - \alpha \right) \right] \\ &= R [(\cos \psi - \cos \theta) \cos \alpha + (\sin \psi + \sin \theta) \sin \alpha].\end{aligned}\quad (A-1)$$

The angles ψ and θ are related to x_0 , R, and α by the relations

$$\sin \psi = \frac{x_0}{R} \cos \alpha \qquad \sin \theta = \frac{x_0}{R} \cos \alpha \quad (A-2)$$

and these can be used to eliminate ψ and θ in Equation A-1 to give

$$\Delta_{\perp}^C = 2R \cos \alpha \sin \alpha \frac{x_0}{R} \quad (A-3)$$

The radius r of the illuminated spot on the sphere is related to the subtended angle α and the radius R of the sphere by the relation

$$\sin \alpha = \frac{r}{R}, \quad (A-4)$$

and, therefore, Equation A-3 can be written

$$\Delta_{\perp}^C = 2x_0 \frac{r}{R} \left(1 - \frac{r^2}{R^2} \right)^{1/2}, \quad (A-5)$$

which is Equation 3 of the text.

Equation 5

The movement of the center of the sphere a distance x_0 in a direction perpendicular to the axis of the incident laser beam causes the reflected beam to be deflected off axis by an angle ϕ relative to the incident beam. At distance L from the surface of the sphere, the deflection y_d^c , or perpendicular distance from the deflected beam center to the center of the incident beam is

$$y_d^c = L \tan \phi . \quad (A-6)$$

From the geometry of Figure A-1, the angle ϕ , measured from the center of the deflected, reflected beam to the center of the incident beam, is

$$\phi = \frac{1}{2} [(2\theta + \alpha) + (2\psi - \alpha)] = \theta + \psi . \quad (A-7)$$

From the law of sines applied to Figure A-1, the relations

$$\frac{\sin \psi}{x_0} = \frac{\sin \left(\frac{\pi}{2} - \alpha \right)}{R} \quad \frac{\sin \theta}{x_0} = \frac{\sin \left(\frac{\pi}{2} + \alpha \right)}{R} \quad (A-8)$$

are obtained, and for $x_0 \ll R$ these yield

$$\psi \approx \frac{x_0}{R} \cos \alpha \quad \theta \approx \frac{x_0}{R} \cos \alpha . \quad (A-9)$$

When Equation A-9 is substituted into Equations A-6 and A-7, the deflection y_d^c can be written

$$y_d^c = L \tan \left(2 \frac{x_0}{R} \cos \alpha \right) . \quad (A-10)$$

If f is the focal length of the lens and d the diameter of the laser beam at the plane of the lens, then the deflection y_d^c in the plane of the lens at $L = f$ can be obtained from Equation A-10 as

$$y_d^c = f \tan \left[2 \frac{x_0}{R} \left(1 - \frac{d^2}{4f^2} \right)^{1/2} \right], \quad (\text{A-11})$$

and for $x_0 \ll R$ this can be written as

$$y_d^c = 2f \frac{x_0}{R} \left(1 - \frac{d^2}{4f^2} \right)^{1/2}, \quad (\text{A-12})$$

which is Equation 5 of the text.

Equation 4

Equation 4 of the text can be derived with the aid of Figure A-2. When the sphere is moved a distance x_0 , the point of incidence on the surface at the center of the incident beam also moves a distance x_0 , but the point of incidence at the periphery of the illuminated spot moves a distance

$$R \cos \alpha - x \cos \alpha,$$

and hence the quantity Δ_ϕ^c in Equation 4 of the text is

$$\Delta_\phi^c = x_0 - (R \cos \alpha - x \cos \alpha). \quad (\text{A-13})$$

For $x_0 \ll R$ and $d \ll f$, the law of sines gives

$$\frac{x}{\alpha - \phi} = \frac{x_0}{\phi} = \frac{R}{\alpha}. \quad (\text{A-14})$$

When the angles α and ϕ are eliminated in Equation A-14 and the results are combined with Equation A-13, the expression for Δ^c can be written

$$\Delta_\phi^c = x_0 (1 - \cos \alpha). \quad (\text{A-15})$$

But, $\sin \alpha = r/R$ and for $r \ll R$, the approximation

$$\cos \alpha = (1 - \sin^2 \alpha)^{1/2} = \left(1 - \frac{r^2}{R^2} \right)^{1/2} \approx \left(1 - \frac{r^2}{R^2} \right)$$

holds, and Equation A-15 becomes

APPENDIX B

DERIVATION OF EQUATIONS 6 AND 7 OF TEXT

Equation 6

The quantity $\Delta_{\perp}^S$ in Equation 6 of the text can be derived with the aid of Figure B-1, which shows the case in which the laser beam from the interferometer is focussed to a spot of diameter 2ϵ on the surface of the sphere when the sphere is initially perfectly aligned in the beam with its center at O. When the center of the sphere is displaced a distance x_0 in the direction perpendicular to the incident beam, the quantity $\Delta_{\perp}^S$ referred to in the text can be expressed in terms of the geometry of Figure B-1 as

$$\begin{aligned}\Delta_{\perp}^S &= R\cos\theta_1 - R\cos\theta_0 \\ &= R \left\{ [1 - (x_0 - \epsilon)^2 R^{-2}]^{1/2} - [1 - (x_0 + \epsilon)^2 R^{-2}]^{1/2} \right\} . \quad (B1)\end{aligned}$$

For x_0 and $\epsilon \ll R$, which is typically the case, the radicals can be expanded in Equation B-1 and all terms of order higher than the first can be neglected. When this is done, Equation B-1 reduces to

$$\Delta_{\perp}^S = 2x_0 \frac{\epsilon}{OR} , \quad (B2)$$

which is Equation 6 of the text.

Equation 7

Equation 7 of the text also can be derived with the aid of the geometry shown in Figure B-1. The incident rays 1 and 2 represent the outer boundary of the converging cone of incident light on the surface of the sphere. When the sphere is displaced a distance x_0 in a direction perpendicular to the fixed incident beam, the incident rays will, upon reflection, be deflected into the paths shown in rays 1' and 2'. The reflected rays 1' and 2' make angles

$$\begin{aligned}\theta_1 &= \gamma - 2x_0/R \\ \theta_2 &= \gamma + 2x_0/R , \quad (B3)\end{aligned}$$

respectively, with the axis of the incident beam. If ϕ is the angle which the center of the reflected beam makes with the axis of the incident beam, then the deflection y_d^S of the reflected beam in the plane of the lens of the interferometer is

Figure B-1. Geometry for the Derivation of Equation 6 of the Text

$$y_d^S = f \tan \Phi . \quad (B-4)$$

The angle Φ between the center of the reflected beam and the center of the incident beam is

$$\begin{aligned} \Phi &= \frac{1}{2} \left[\gamma + 2 \frac{x_0}{R} - \left(-\gamma + 2 \frac{x_0}{R} \right) + \left(-\gamma + 2 \frac{x_0}{R} \right) \right] \\ &= 2x_0/R , \end{aligned} \quad (B-5)$$

and for $x_0 \ll R$, Equation B-4 gives

$$\begin{aligned} y_d^s &= f \tan \left(2 \frac{x_0}{R} \right) \\ &\approx 2f \frac{x_0}{R} . \end{aligned} \tag{B-6}$$

The latter is Equation 7 of the text.

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