SOLVING THE FOKKER-PLANCK EQUATION ON A MASSIVELY PARALLEL COMPUTER

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ABSTRACT
The Fokker-Planck package FPPAC has been converted to the Connection Machine 2 (CM2). For fine mesh cases the CM2 outperforms the Cray-2 when it comes to time-integrating the difference equations. For long Legendre expansions the CM2 is also faster at computing the Fokker-Planck coefficients.

BACKGROUND
When modeling magnetically confined plasmas, if the charged particle distribution functions are not Maxwellian and if a knowledge of those distribution functions is important, kinetic equations must be solved. Examples include the heating of a fusion plasma by energetic neutral beams and the production of current in a tokamak by the application of microwaves. The kinetic equation of relevance is the Boltzmann equation with Fokker-Planck collision terms.

The Fokker-Planck equation, as it is more commonly called, is a highly nonlinear second order partial differential equation in six phase space variables and time. In the case of spatial homogeneity and azimuthal symmetry, the phase space dimensionality can be reduced to two velocity space coordinates - speed and pitch angle. The coefficients of the Fokker-Planck equation may be written in terms of moments of the various charged particle distribution functions and computed using a Legendre expansion technique.

The Fokker-Planck package FPPAC[1], which solves the complete nonlinear multispecies Fokker-Planck collision operator for a plasma in two-dimensional velocity space, has been rewritten for the Connection Machine 2 (CM2). The CM2 consists of up to 65,536 single bit processors, each with up to 32 kbytes of random access memory. Groups of 32 processors share Weitek floating point hardware. Arithmetic is carried out using either 32 or 64 bits. The CM2 is SIMD; that is, all processors performing an operation must perform the same operation. A Fortran compiler with parallel extensions is available. The CM2 is front-ended typically by either a Vax or a Sun-4.

COMPUTATIONAL MODEL
The conversion of FPPAC has involved reallocation and restructuring of variables and replacement of Cray-optimized algorithms with highly parallel
ones. The main procedures which have been modified are the computation of the Fokker-Planck coefficients and the solution of the difference equations.

The computation of the Fokker-Planck coefficients involves a number of steps:

(a) computation of the Legendre projections of the distribution functions.  
(b) computation of the moments of the Legendre projections.  
(c) computation of the Legendre projections of the coefficient pieces.  
(d) synthesis of the Legendre series.

Steps (a) and (d) (which turn out to be the most time-consuming) are cast in terms of matrix multiplication. Step (c) involves linear combinations of the various moments. A typical term in step (b) involves computing all partial sums of a series of numbers. On the CM2 this is carried out most efficiently using a "scan"; the total number of arithmetic operations is greater (than on a Cray), but the number of parallel steps is smaller.

The time-integration is carried out using either implicit operator splitting or an alternating direction implicit method. Either scheme involves the solution of parallel tridiagonal systems. On the Cray, vectorization over the direction orthogonal to the sweep is carried out. On the CM2, parallelization is performed over that direction but additionally, a cyclic reduction procedure is employed in the direction along the sweep. The number of steps in the cyclic reduction procedure may be minimized by performing odd and even reductions simultaneously, thus obviating the need for backfilling. The general tridiagonal solver of the Connection Machine Scientific Software Library (CMSSL) has recently been implemented.

RESULTS:  
The cases presented here are carried out using the Connection Machine facilities at NASA Ames and at the ACL at Los Alamos. Results are compared with a single processor of a static memory Cray-2 at NERSC.

The first case considered has a single species and a mesh of 128 points in v and 64 points in \( \theta \); the Fokker-Planck coefficients use a 5-term Legendre expansion. Using 32-bit arithmetic and one eighth (8K processors) of a CM front-ended by a Vax-6320, the time advancement takes 8 times longer on the CM than on the Cray-2 and the coefficient computation takes 200 times longer. The extraordinary time to compute the coefficients is because of the smaller degree of parallelism for that phase of the calculation and the fact that the CM matrix multiply is tuned for very large matrices. Using 64 Legendre polynomials, the time to compute the coefficients reduces to 48 times as long on the CM, the main culprit being the relatively slow matrix multiply.
The mesh is now expanded to 512 points in v and 256 points in \( \theta \), with 64 Legendre polynomials. Using one quarter (16K processors) of a CM, the time advancement is only 27% slower than on the Cray-2 and the coefficient computation executes 6 times as fast as on the Cray-2. This vast improvement over the earlier case is due to the much larger degree of parallelism and to the fact that the matrix multiply executes much more efficiently. (The CM performs floating point arithmetic most effectively when the virtual processor (VP) ratio, that is, the ratio of the degree of parallelism to the number of physical processors, is high. To convey the dependence of CM matrix multiply performance on size, on one quarter of a CM two matrices of order 1024 get multiplied at a rate of 577 Mflops, whereas multiplication of order-64 matrices executes at only 10 Mflops; on the Cray-2 the numbers are 453 and 431 Mflops, respectively.)

The timings quoted above are based on 32-bit arithmetic with a Vax front end, using a Fortran parallel cyclic reduction routine. For 64-bit arithmetic with a Sun-4 front end, using the CMSSL general tridiagonal solve, the coefficients computation (for the finer mesh, long Legendre series case) executes 6 times faster on one quarter of the CM than on the Cray-2 (single processor) and the time-advancement executes at roughly the same speed. When factoring in the cost, the CM is clearly the cheaper way to go.

**TOWARD THE FUTURE**

FPPAC is a relatively simple but representative Fokker-Planck code, whereas a state-of-the-art Fokker-Planck code such as CQL3D[2] includes averaging over the particle trajectory (bounce-averaging) and radial (flux-surface) dependence. The higher dimensionality allows an even higher degree of parallelism. Hence the time-advancement, when using an alternating direction method, should execute quite favorably on the CM. Many problems demand a greater degree of implicitness and could make good use of a parallelized preconditioned conjugate gradient (PCG) technique; such solvers for the CM are a topic of current research. The biggest question regarding the viability of nonlinear Fokker-Planck calculations on the CM is whether or not the coefficients computation could be sped up for cases involving a small to moderate number of Legendre polynomials.

Present day Fokker-Planck calculations typically involve the application of radio frequency waves. The algorithms used to compute the actual ray tracing and the resulting rf diffusion coefficients are both highly parallel, so that a state-of-the-art code such as CQL3D is likely to run well when ported to a massively parallel environment.

**SUMMARY**

The Connection Machine has been shown to be advantageous for fine resolution problems with extensive Legendre polynomial expansions. For the fine mesh case reported here, the coefficients computation executes 6
times faster (using 64-bit arithmetic) on the Connection Machine than on the Cray-2, and the time-integration executes at roughly the same speed. Assuming a Connection Machine to cost roughly one quarter as much as a Cray-2, this translates into factors of 24 and 4 cost-performance in favor of the CM for the coefficients and time-advancement, respectively. At coarser resolution the cost-performance of the time-advancement is still likely to be favorable on the CM; however for short Legendre series the present implementation of the coefficients computation executes much too slowly on the CM and is much better suited to the Cray.

Further information on this study can be found in Ref. [3].

REFERENCES

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