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By
J. T. Hamrick

April 1980

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REDESIGN OF TURBINE PUMP IMPELLER
AND DIFFUSER USING HYDRODYNAMIC
DESIGN TECHNIQUES

FINAL REPORT

by

J. T. Hamrick

April 1980

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SUMMARY

From the results of a survey reported in 1976 it was indicated that the average operating efficiency of well irrigation pumps in the United States, including losses in the column pipe and line shaft, was 55.5 percent. From the quantity of irrigation water used in that year it was determined that an annual saving of approximately .13 quadrillion Btu which is the equivalent of 23.3 million bbls of oil could be saved if the average well pump system efficiency could be raised to 82 percent. The information presented in this report shows the method by which losses in a pumping system were reduced, and that it is possible to reach the goal of 82 percent system efficiency.

Hydrodynamic design methods which were used to analyze and modify a commercially available pump are presented. The results of tests with the pump are presented for which delivery losses were reduced by means of a packer at the pump and for which line shaft losses were reduced by means of a high strength line shaft. For a 500 gpm pump operating at a 100 ft depth the gains in system efficiency were 3 points with a packer, 0.6 points with a high strength line shaft, 3 points with polished impellers, and 1.6 points with modified impellers for an overall system efficiency of 82.6 percent in pumping water to the surface with a developed head of 116 feet at 500 gallons per minute. With pumps that are now commercially available, where the flow rate varies from that corresponding to maximum efficiency, the efficiency drops off. In meeting his requirements it is not always possible for the farmer to operate his system at the optimum flow rate. Methods of designing pumps that have a broader high efficiency range were explored, and a design approach for doing so is presented. The method was not evaluated experimentally.

The packer allows use of both column pipe and the annulus between casing and column pipe as a flow conduit. As a result, a smaller column pipe can be used with a substantial saving in cost of the pipe over the cost of the packer. In addition, the smaller and lighter weight column pipe allows installation and removal of the pump with on-the-farm tools. The method of installation and removal was repeatedly demonstrated and is reported herein. The use of a heat treated lighter weight shaft substantially reduces the cost of the line shaft. The fabrication cost of an efficient pump is no greater than that for one with low efficiency. The only item of increased cost is impeller polishing which is nominal. Overall, the adoption of the more efficient system as demonstrated will cost much less to purchase and install than presently used systems of the same capacity and head.

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REDESIGN OF TURBINE PUMP IMPELLER AND DIFFUSER USING HYDRODYNAMIC DESIGN TECHNIQUES

I INTRODUCTION

Multistage centrifugal pumps are used almost universally for pumping water from wells in the quantities needed for irrigation. They are normally referred to in the trade as turbine pumps. The existing commercial turbine pumps are economical to manufacture and are reliable. While some manufacturers have attempted to provide pumps of high efficiency, primary emphasis has been placed on low cost and reliability, because the previous low cost of fossil fuels resulted in reduced emphasis on efficiency by the user. In addition, pump installers have traditionally accepted a sizable reduction in efficiency due to losses through the column pipe which delivers water from the pump to the surface. Most turbine pumps are driven by line shafts from the surface. The line shaft bearing losses, while not as significant as column pipe losses, may possibly be reduced. The purpose of this research is to analyze existing pumping systems and to determine to what extent these losses can be reduced.

Presently, multistage efficiencies range from 65 to 76 percent for commercially available pumps. The efficiency of 76 percent can be increased to 80 percent by polishing impeller and diffuser surfaces and close fitting the impeller to the housing to reduce recirculation losses. For a 600 gpm pump of four stages or more the predicted attainable efficiency is 84 to 86 percent at optimum specific speed with polished impeller and diffuser surfaces. The traditionally accepted reduction in efficiency in selecting a column pipe has been 5 points. The loss due to line shaft bearings may be as much as 2 points.

The problem is one of increasing pump efficiency and reducing column pipe and line shaft losses. The target overall efficiency for a pump with 200 feet of lift and line shaft drive is 82 percent. The approach toward achieving this goal includes redesign of an existing pump, reducing the column pipe losses, and cutting line shaft losses in half. Centrifugal impeller analysis and design methods developed at the National Advisory Committee For Aeronautics Lewis Flight Propulsion Laboratories from 1948 to 1955 are

used to analyze existing impellers and to design more efficient configurations. The column pipe losses are reduced by sealing the well casing at the pump and using the well casing in addition to the column pipe as the conduit. The bearing losses are reduced by the line shaft using higher tensile strength material for the line shaft so that the diameter can be reduced.

Use of both casing and column pipe for delivery allows a selection of a small column pipe providing the extra benefit of reducing the weight of the system so that removing the pump from the well can be accomplished with lighter weight equipment than is normally required. Performance data for redesigned pump components, pump to casing packer seals, and high strength line shaft are presented and the results analyzed. An open line shaft was used with the system described herein. However, an enclosed line shaft could have been used equally well.

During the program, members of the turbine pump industry were sought out for advice. At the end of the program they were invited to view a demonstration of the results. There appears to be general acceptance by the industry of the results, and the probability of the demonstrated methods being applied by industry is good.

II DATA SPECIFICATION

The main components of the pumping operation are the turbine pump, conduit for delivery of water to the surface, the line shaft drive, and driver. The driver was not considered in the work reported herein. The pump, conduit for delivery to the surface, and the line shaft drive performances can be improved. From a survey reported in reference 1* it was indicated that in 1976 the average operating efficiency for irrigation pumps in the United States, including losses in the column pipe and line shaft, was 55.5 percent. Preliminary analyses of losses which caused this low efficiency indicate that the losses can be greatly reduced and that there is a possibility of raising the overall efficiency to 82 percent.

FACTORS TO BE ANALYZED

Of the three components, the one that lends itself best to theoretical analysis is the pump. Hydrodynamic design techniques are available, but have not been widely used among pump manufacturers. While knowledge of flow patterns inside centrifugal pump impellers is very important to the achievement of high efficiency, the subject is seldom discussed in engineering handbooks. The available hydrodynamic analysis methods are used in this research program and applied to modify an existing impeller design. The impeller to bowl seal is also considered as well as the diffuser and matching of the impeller to the diffuser. The losses in the column pipe and line shaft are analyzed and alternative designs presented.

PERFORMANCE DATA

The generation of performance data takes place in two steps. The first consists of assembly and test of an as-received commercial pump in a 100 ft. deep well at a flow rate of 600 gpm. The pump is driven by a modified line shaft and delivery to the surface is through both column pipe and casing. In the second step, the pump is modified to incorporate design refinements and retested in the same manner as in the first step. Losses in column pipes and by line shaft bearings are more easily assessed than impeller losses. Therefore, the main purpose of the part of the demonstration pertaining to improvements in the shafting and modified conduit is to demonstrate their func-

*A list of references is provided in Appendix 1.

tionality. Data derived from the demonstration are flow rate, work input and overall efficiency for the two sets of test conditions.

EXPERIMENTAL DESIGN

The schematic arrangement of the experimental system is shown in figure 1. Not shown in that figure is the 30 hp electric motor and torque measuring set up at the well head. Torque input of the motor is measured during all tests. The flow rate is measured in the surface delivery pipe. The head at inlet and outlet is measured with direct reading bourdon type pressure gages.

III DATA ANALYSIS

The data analysis consists of theoretical analysis and theoretical evaluation of the impeller, diffuser, surface finish, impeller to bowl seal, line shaft bearing losses, and column pipe losses. Discussion of theory is followed by application of the theory to a commercial pump.

THEORETICAL ANALYSIS OF IMPELLERS

The ideal solution of flow through an impeller is one in which a true three dimensional solution of the viscous flow from hub to shroud and vane to vane can be achieved. A more practical if less sophisticated approach is one in which nonviscous, axially symmetrical conditions are assumed and a solution is first obtained in the hub-to-shroud plane followed by solutions in the vane to vane direction. The latter approach is often referred to as quasi three dimensional, and it is the one used herein. It is a modification of that presented in reference 2 which has been applied to both compressors and pumps. The method as used for pumps is contained in appendix 2. It does not take into account slip factor or misalignment of the fluid and vanes at the impeller inlet. Modification of the solution by the method to account for slip factor can be made by use of the results of solutions and equations presented by Stanitz in reference 3. Similar modification due to misalignment of flow at inlet can be made by using the work presented by Kramer in reference 4 as a guide. Optimum conditions of backward curvature for impellers have been determined by Stanitz and were published as a discussion of the work of reference 5. The methods and solutions of the foregoing

FIGURE 1. Schematic Drawing of Pump, Packer, Column Pipe and Line Shaft Arrangement.

references have been used to develop a more cohesive approach to analysis of pump impellers. A discussion of each aspect of the procedure follows.

Axially Symmetrical Solution

Application of the method which is described in detail in appendix 2 is initiated by estimating the position of streamlines between the hub and shroud in the hub-to-shroud or meridional plane. The curvature of these streamlines determines the average velocity distribution in the meridional plane for a given volume flow rate. Lines crossing the streamlines and normal to each streamline at the point of intersection are drawn in from hub to shroud at intervals from inlet to outlet. Velocity at the hub at each normal is estimated and using the method of appendix 2 the weight flow across the normal from hub to shroud is computed. If the weight flow obtained is not equal to the prescribed overall weight flow, the velocity at the hub is adjusted up or down as required and the solution repeated. This procedure is repeated for each normal. If the correct streamline distribution has been selected, the weight flow between each set of streamlines will be relatively uniform across normals from inlet to outlet. If it is not, new streamlines must be selected on the basis of the solution and the process repeated. Generally, for a constant overall weight flow across all normals from hub to shroud, a ten percent deviation from average across normals through sets of streamlines provides a usable solution. A computer program in which seventh order polynomials are used to represent the streamlines and iterative procedures are used for arriving at overall flow rates across normals has been written and demonstrated. Care must be exercised in the employment of polynomials in this manner. A suggested numerical routine for computer programming is provided in appendix 2.

Velocity Distribution on Vane Surfaces

The velocities on the vane surfaces may be determined from the vane pressure loading. The pressure differential between the driving and suction faces, assuming no slip and no misalignment between flow direction and vane at inlet, can be computed from equation 6 of appendix 2. The velocity differential can then be computed by use of equation 7. Using the velocities determined for the meridional plane and assuming a linear distribution of velocity from driving to suction face, the velocities on the driving and suction faces can be determined. The exact flow solutions by Stanitz in reference 3 show that the velocity distribution across the passage between vanes is approx-

imately linear except when eddies occur on the driving face. The velocity distributions obtained by the method of appendix 2 can be corrected for slip factor and misalignment at inlet as demonstrated in the following sections.

Slip Factor

For impellers with straight radial vanes the slip factor is defined as the ratio of the average absolute tangential velocity at the impeller tip to the tip speed of the impeller. For an impeller with an infinite number of vanes, the fluid would be perfectly guided by the vanes and there would be no slip. As the number of vanes decreases, the flow direction in the channels between vanes starts to deviate from the vane direction at a radius that can be estimated from the solutions by Stanitz in reference 3. Figure 2 shows a plot of radius at which deviation at midpassage begins versus the angle between vanes. The two points for 12° and 18° were derived from streamline distributions shown in reference 3. The third point, at zero angle between vanes, obviously falls at full radius. This curve provides an estimate of the radius at which unloading of the vane commences for a given impeller. For the solution of velocity distribution as provided in appendix 2 where unloading is not taken into account, it is important to know where unloading commences. It is also necessary to know the extent of slip at exit. Among the methods available for determination of slip factor, the method devised by Stanitz in reference 2 for radial vanes appears to be the most applicable. Wiesner in reference 6 applied the Stanitz slip factor directly to backward curved vanes without correcting for backward curvature and obtained poor correlation with test slip factors. The Stanitz slip factor equation for radial vanes is:

$$\mu = 1 - .315\sigma \text{ where } \sigma = \frac{2\pi}{N} \sin \alpha \quad (1)$$

Blade spacing for radial vanes is a function of N and for backward curved vanes it is a function of $N/\cos \beta$. A good approximation for backward curved vanes is:

$$\sigma = \frac{2\pi}{N} \cos \beta \sin \alpha \quad (2)$$

Table 1 shows a comparison of experimental slip factor for radial pumps listed in reference 6 and slip factor determined by the Stanitz equation taking into account backward curvature. Considering the possible error in computing the experimental slip factor the correlation is very good. Test

FIGURE 2. Radius At Which Slip Begins At Mid Passage Versus Angle Between Vanes. (From Modified Equation C-3 of Ref. 3)

TABLE. 1 Slip Factors for Pumps Listed In Reference 6 as Computed From Stanitz Method With Correction for Backward Curvature

Source From Ref.	Number Of Vanes	β (Equals $90 - \beta_2$ Of Ref. 6)	Stanitz Slip Factor Without Correction For Backward Curvature By Wiesner	Test Slip Factor As Shown In Ref. 6	Stanitz Slip Factor Corrected For Backward Curvature
Varley	5	75	.604	.824	.897
Wood	6	74.6	.670	.786	.912
Acosta, et al	5	73	.604	.832	.884
Peck	6	70	.670	.774	.887
Peck	6	70	.670	.868	.887
Kasai	7	70	.717	.925	.903
Kasai	7	64	.717	.838	.876
Varley	4	63	.505	.725	.775
Varley	5	63	.604	.765	.820
Varley	6	63	.670	.826	.85
Varley	8	63	.753	.910	.888
Varley	12	63	.835	.975	.925
Kasai	7	60	.717	.785	.859
Varley	5	50	.604	.75	.746

slip factors with asterisks in reference 6 were not included in table 1.

For an impeller with backward curved vanes the angle between vanes is equal to $360 \cos \beta / N$. Using the angle thus determined the radius at which slip begins can be approximated from figure 2. To determine the

backward component of relative velocity at the impeller exit, the backward component as determined in the solution from appendix 2 is added to the slip component as determined from the Stanitz equation corrected for backward curvature. The through flow component can be determined from the relative velocity obtained from appendix 2 using the vane outlet angle. With the backward flow component and through flow component, the average flow angle at the outlet can be determined. By constructing a curve tangent to a radius at which slip begins and to a line defining the flow angle at the tip radius the approximate flow angles at intermediate points can be approximated. The values of flow angle thus obtained can then be used in equation 6 of appendix 2 in place of the actual vane angles to determine vane loading in the slip region.

Misalignment of Flow and Vane at Inlet

In the solution of appendix 2 it is assumed that the flow angle of the fluid becomes the same as the flow angle of the vane immediately upon entry into the impeller. This is not a valid assumption and in order to assess the possible impact of a large misalignment a correction must be made to the solution. When the angle of misalignment is such that flow is directed at the driving face of the vane it is referred to as a positive angle of attack and when it is aimed at the suction face of the vane it is referred to as a negative angle of attack. Kramer, in reference 4, provides solutions for nonviscous flow at various angles of attack. Figure 3 shows a Kramer solution for a flow rate with a large negative angle of attack. The velocity ratio W , which is the ratio of the relative velocity to impeller tip speed, reaches a maximum of 1.35 at the nose of the vane on the driving face and remains higher than on the suction face for a small distance into the impeller. For a similar impeller, operating with 45 lbs. of air per sec. flow, measurements in the rotating passages showed large losses on the pressure face. The results of those tests are presented in reference 7. Positive angles of attack of the same magnitude did not result in large losses.

There are no criteria for determination of the point at which the angle of flow becomes the same as that of the vane. However, as it is a problem that can be eliminated for the design flow by providing a zero or small positive angle of attack at inlet, the need for an accurate determination of the point at which guidance by the vane is reached is not essential for impeller design. The main concern is one of showing the approximate velocities on the vane at inlet for solutions where the flow is misaligned. This can be done

FIGURE 3. Contours of Constant Velocity Ratio W For Refined Solution In Nose Region. (From Reference 4)

by computing the through flow velocity ahead of the inlet and using the impeller speed to determine the relative inlet flow angle. By following a procedure similar to that used in accounting for slip factor, the flow angle distribution at inlet can be determined and the values of flow angle thus obtained used in equation 6 of appendix 2 instead of the vane angle.

Optimum Backward Curvature

At any relative outlet flow angle, β there is an optimum ratio of relative outlet velocity, q to impeller tip speed, U . Stanitz, in reference 5 derived the following equation:

$$\left(\frac{q}{U}\right)_{\text{opt.}} = \frac{1 - \cos \beta}{\sin \beta} \quad (3)$$

A plot of this equation over the range of β is shown in figure 4.

DIFFUSER ANALYSIS

The diffuser in a turbine irrigation pump serves the functions of converting some of the velocity head leaving the impeller into pressure, conducting the flow from the impeller exit to the inlet of the next stage, and turning the flow to the axial direction in preparation for entry into the next stage. Reference 8 presents data for a variety of diffuser configurations. From that data it can be concluded that if the angle of divergence exceeds the optimum angle of approximately 7° for diffusers of circular or square cross section, the efficiency of diffusion starts to drop off. With rectangular cross sections with two parallel walls the optimum angle of divergence is approximately 11° . For a constant distribution of velocity across the initial and final sections of the diffuser, reference 8 provides the following expression for diffuser efficiency, η :

$$\eta = \frac{p_2 - p_1}{\frac{1}{2} \rho \bar{u}_1^2 \left[1 - \left(\frac{A_1}{A_2} \right)^2 \right]} \quad (4)$$

Where p is pressure, ρ is mass density, $\bar{u}$ is average velocity, A is area, and the subscripts 1 and 2 refer to initial and final areas. For final to initial area ratios of 2.25 to 2.34 and divergence angles in the 7° to 10° range, efficiencies of .93 to .94 have been achieved. Efficiency drops off rapidly for angles of divergence above approximately 15° . Two sources of loss, over and above those normal for a diffusing section of pipe, are failure to match the diffuser vane at inlet with the incoming flow and

FIGURE 4. Optimum Ratio of Relative Outlet Velocity To Tip Speed as a Function of Relative Outlet Flow Angle.

turning the flow for entry into the next stage. For maximum overall efficiency at the design flow condition, the angle of entry into the diffuser vanes as well as into the impeller at inlet must be such that flow around a sharp corner does not occur. High velocity build up from flow around a sharp or rounded edge is followed by deceleration and usually, separated boundary layer. It is difficult to predict the exact angle of entry of a fluid into a set of diffuser vanes. The most reliable approach is to probe the flow out of an impeller with a direction sensitive instrument and use the information to set the vane inlet angle.

For most turbine pumps a sharp turning radius occurs in the transition zone between the impeller outlet and the inlet to the diffuser vanes. For turbine impellers which are steeply backward swept, the through flow component of velocity is low and this is the component that is most affected by the sharp turning. There are design procedures available such as that of reference 9, by which a passage can be designed to accelerate the flow on both inner and outer walls, thereby reducing the possibility of decelerating too rapidly on the inner wall, a condition that could result in boundary layer separation. Such a design would result in the requirement for more diffusion in the remaining portion of the passage.

SURFACE FINISH EVALUATION

Improvement of surface finish can be accomplished by either polishing or coating or a combination of the two. The vane surfaces of a fully shrouded impeller are not easily accessible and in most cases are left in the as cast condition. Machines which move an abrasive slurry through the passages have been effective in smoothing and polishing the vane surfaces. The general history of impeller polishing is that it increases efficiency from 1 to 4 points. The problem becomes one of economics. Machines for polishing with abrasive slurry are available commercially. One machine, manufactured by Sutton Tool Company of Sturgis, Michigan, is now being used by seven pump manufacturers in the United States.

A brief study of the problem indicates that polishing machines are designed to handle a wide variety of items. There was no indication that machines which utilize circulation of the slurry by the impeller being polished as the primary means of polishing have been considered as a means of reducing the cost of such machines.

Coating materials may also be used to provide smooth surfaces. There are a variety of coating materials and methods of application. Thin coatings such as Teflon are more effective if applied over smooth surfaces. Thicker coatings such as enamels and the epoxies or epoxies mixed with fillers can be used to provide smooth coatings on as-cast surfaces. Cast iron impellers are often enameled. Dip coating is usually required with fully shrouded impellers. Diffuser surfaces are also inaccessible. Most bowls which are made of cast iron and contain the diffuser have enameled passages.

In spite of the demonstrated effectiveness of providing smooth impeller surfaces, the practice of polishing impellers is not widespread in the industry and is often done only at customer request at extra cost.

EVALUATION OF IMPELLER TO BOWL SEALING METHODS

Flow of fluid from the impeller outlet between the impeller and the bowl which houses it can result in large losses. Figure 5 shows a typical passage configuration for an impeller and bowl at the impeller inlet.

One method of reducing the flow is to cut grooves in one of the annulus walls to form a labyrinth as shown in figure 5. As the water flows into each successive groove, it loses its velocity head and it may be presumed that the head used in computing velocity through each clearance would be the total head differential divided by the number of lands. That being the case, the flow through the clearance would be as shown in table 2.

TABLE 2. Flow Across Labyrinth With 0.009 Inch Clearance at 2.53 Inch Radius and 50 Ft Head Differential at 600 Gpm Flow.

No. of Lands	Flow Leakage Gpm	Percent of Pump Flow
1	16.43	2.74
2	11.62	1.94
3	9.48	1.58
4	8.21	1.37
5	7.35	1.22

It was assumed that the 0.009 inch clearance is the minimum that can be tolerated in a practical pumping application for the chosen clearance radius, and that some other type of sealing method would have to be used to drop the

FIGURE 5. Sketch Showing Passage Modification To Form Labyrinth For More Effective Resistance To Leakage.

flow rate much below the 1.22 percent shown for 5 lands. One seal manufacturer has proposed a floating seal made of carbon or hard rubber. If a seal of that type could allow reduction of the clearance to 0.002 inches, with a single groove the leakage would drop to .61 percent of the 600 gpm flow rate. With two lands on the floating seal, it would drop to .43 percent.

What the foregoing illustrates is that although there may be some poor assumptions in the calculations, with a labyrinth seal the benefits tend to level off with more than three lands and substantial benefits result if the clearance can be reduced to 0.002 inches or less.

Rubbing face seals can be effective in this application. However, they do not appear to be economical, and their vulnerability to abrasion by sand or other impurities in the water may preclude their general use.

LINE SHAFT BEARING LOSS EVALUATION

Catalogs supplied by manufacturers of centrifugal pumps provide loss information on both open line shafts which have water lubricated rubber bearings supported by spiders from the column pipe and enclosed line shafts supported by an inner column pipe with oil lubricated brass bearings. The traditional interval between bearings is 10 feet for open line shafts, and 5 feet for enclosed line shafts. The losses for them are stated to be equal for a given shaft diameter. Because of the experience that has been gained with existing bearing spacings, and the difficulty in making meaningful computations for the lightly loaded bearings in the line shaft assembly, there is some doubt that experimental results could be obtained that would be convincing enough to encourage the use of greater spacing intervals.

The materials that have traditionally been selected for bearings appear to be optimum, and it may be concluded that the present bearing arrangements can not be greatly improved without installing permanently lubricated ball or roller bearings, a move that may not be economical and would require a large amount of development.

It appears that the largest reductions in bearing losses can be made by reducing shaft diameter. The friction loss increases as the square of the diameter. The friction losses for a 1.5 in. diameter shaft are 2.25 times those for a 1 in. diameter shaft based on manufacturers data.

Steels currently used for line shafts have half the strength of heat treated 4300 series steels which are tough and have high fatigue strength and have

been used in the aircraft industry for over 40 years. They can be used in irrigation pump line shafts to save energy and at the same time reduce the initial cost of the system. Use of smaller diameter shafting should require no change in pumping hardware stocks as bearing systems for both oil and water lubricated line shafts are advertised as being carried in stock down to 3/4 in. diameter.

COLUMN PIPE LOSS EVALUATION

A column pipe head loss in the vicinity of 6 feet per 100 feet of column has traditionally been accepted in irrigation practice even though it amounts to 4 to 6 points in pump system efficiency depending upon the installation. This practice was established in times when the need for energy conservation was not as obvious as in 1978. However, many wells have been drilled to accept pumps and column pipes based upon that practice.

Pump manufacturers catalogs provide adequate information on selection of column pipes. Therefore, the primary concern is not determination of column pipe losses, but provision of adequate flow cross section to minimize losses. One way to reduce losses is to seal the space between the pump and well casing so that in addition to flowing through the column pipe, the water can flow up through the space between the column pipe and casing. This arrangement is as illustrated in figure 1. The size of the column pipe is determined by the cross section of metal needed to support the pump and head of water rather than from flow considerations.

APPLICATION OF THEORY TO A COMMERCIAL PUMP

Pumps from two prominent manufacturers were given a preliminary evaluation of which one was chosen for full analysis and experimental evaluation. The analysis and recommendations for modification for the pump that was chosen are included in this section. A cross section outline of the pump is shown in figure 6.

Results of Impeller Flow Analysis

The method of reference 2 as modified in Appendix 2 was applied to the impeller for a flow rate of 550 gpm and 1760 rpm. A meridional-plane, cross sectional view of the impeller with streamlines drawn in is shown in figure 7. Velocity distributions on the vanes were determined for all surfaces of revolution identified by the streamlines including the hub and shroud. Corrections

FIGURE 6. Cross Sectional View of Lower Half of Impeller and Diffuser in Place.

FIGURE 7. Impeller Streamline Distribution in Meridional Plane for Turbine Pump.

were made for slip factor and misalignment at inlet. Velocity distributions for the hub, shroud, and streamtube 3 are shown in figure 8. The velocity is higher on the suction face than on the driving face for all three at inlet with the difference decreasing noticeably from hub to shroud. This indicates that the angle of attack is positive at the 550 gpm flow rate from hub to shroud. Plots of the pressure differential from driving to suction face for the same three streamtubes are shown in figure 9. The pressure differential provides a more direct indication of vane loading. For a given inlet pressure the pressure distribution on the vane can be determined. Pressures so determined generally agree with the experimental values.

Numerical Solution

To start the solution, seven streamlines were drawn including hub and shroud. Lines normal to the streamlines were drawn in starting with ten positions along the shroud between the inlet and outlet. The streamline positions were estimated by keeping approximately constant annular area between streamlines from inlet to outlet. Each normal was drawn in by means of a partially reflecting mirror by lining up the streamline in front of and behind the mirror with the reflection and joining the normals for one streamline to the normal for the next midway between streamlines.

The vane angle β as defined in appendix 2 was determined in the following manner. Symbols not defined in this section are defined in appendix 2. For the impeller being analyzed, the vane cross section lying on a plane normal to the axis of rotation is the same at the intersections of all planes normal to the axis from shroud inlet to hub outlet. The angle between the vane and a line radiating from the centerline on a plane normal to the centerline is defined as β_1 . Referring to figure 10, surface b-c-e lies on a plane normal to the axis of rotation. Surface a-c-d lies on a streamline surface of revolution.

$$\tan \beta_1 = \frac{c}{b}, \tan \beta = \frac{c}{a}, \text{ and } \cos (\alpha_1 - \alpha_2) = \frac{b}{a}$$

From these relationships:

$$\beta = \arctan \left(\tan \beta_1 \cos (\alpha_1 - \alpha_2) \right) \quad (5)$$

The foregoing, together with vane thickness, defined the conditions adequately enough to analyze the flow of water through the impeller rotating at

FIGURE 8. Velocity Distribution on Vanes at Hub, Shroud, & Streamtube 3 for Original Impeller Design. Flow Rate 550 Gpm.

FIGURE 9. Pressure Differential From Driving to Suction Face of Vanes at Hub, Shroud, & Streamtube 3 for Original Impeller.

FIGURE 10. Sketch Showing Relationship of Vane Angle on Streamline Surface of Revolution to Impeller Vane Angle on Plane Normal to Axis of Rotation.

1760 rpm. In the use of the equations in appendix 2, the component of the blade force normal to the streamlines was not significant enough to be taken into account and was neglected. There was no preswirl of the fluid at inlet. The computer program written in APL language as suggested in appendix 2 was used to obtain a solution which neglected slip factor effect and misalignment at inlet. Corrections were made as demonstrated by the following numerical procedures for the shroud streamline.

Slip Factor Effect At Shroud

The following outlet conditions were used to compute slip factor:

β_1	-66
α_1 , degrees	90
α_2 , degrees	78
q from equations in appendix 2	21.8

angular velocity, radians/sec.	184.3
radius, r, ft.	.297
No. of vanes	6

From the above, the following were determined:

$$\beta = \arctan [\tan \beta_1 \cos (\alpha_1 - \alpha_2)] = -65.526$$

Backward velocity, u due to vane backward sweep

$$= q \sin \beta = -19.84 \text{ ft/sec}$$

Tip speed, $U = \omega r = 54.737 \text{ ft/sec}$

$$\sigma = \frac{2\pi}{N} \cos \beta \sin \alpha = .4244$$

Slip factor $\mu = 1 - .315 \sigma = .87$

Backward velocity due to slip

$$= (1 - .87) (U) = 7.12 \text{ ft/sec}$$

Total relative backward component of velocity

$$= u - \text{slip} = -26.956 \text{ ft/sec}$$

The through flow component of velocity

$$v = q \cos (-65.526) = 9.03 \text{ ft/sec}$$

The average angle of flow at the outlet which takes into account the slip:

$$\beta_f = \arctan \frac{u - \text{slip}}{v} = -71.48^\circ$$

The average relative velocity at outlet which takes into account the slip:

$$q_f = \frac{v}{\cos \beta_f} = 28.43 \text{ ft/sec}$$

Using β_f as thus determined and the value of β at the radius at which slip begins the intermediate values of β can be estimated. The angle between vanes equals $360 \cos \beta / N = 22^\circ$. From figure 2 the approximate radius ratio at which slip begins is .81. For the impeller under evaluation, at that radius ratio $\beta = -65.53^\circ$. An approximation of the intermediate values may be made as shown in the plot on figure 11. To draw the curve a spline with weights so arranged that the spline was held tangent for the values of β at $r = .24 \text{ ft}$ with two weights and allowed to float at $r = .297 \text{ ft}$ under a single weight. The values of q corresponding to the β values taken from figure 11 between radii of .24 and .30 ft were determined as follows. The through flow velocity, v at each radius was determined from the value of q obtained in the solution by the equations of appendix 2 and the vane angle β . The component v thus obtained was used with the value of β from figure 11 to obtain the new value of q . The new values of β , q and component u were then used in equation

(6) of appendix 2 to correct for slip at the outlet. For all streamlines, the Δp dropped to zero at the outlet.

FIGURE 11. Plot of Estimated Average Flow Angle β In Slip Region.

Angle of Attack at Inlet at Shroud

The following inlet conditions were used to evaluate flow at inlet:

β_1 , degrees	-65.55
α_1 , degrees	90.0
α_2 , degrees	24.2
q from equations in appendix 2	42.0
angular velocity, radians/sec	184.3
radius, r , ft	.1764
No. of vanes	6
blade factor, f	.815

From the above, the following were determined:

$$\beta = \arctan [\tan \beta_1 \cos (\alpha_1 - \alpha_2)] = -42.0$$

$$v = q \cos \beta = 31.21 \text{ ft/sec}$$

The above value of v is that just inside the impeller vanes. To obtain

an estimated value of v_0 , the velocity just upstream of the vane inlet:

$$v_0 = v_f = 25.4 \text{ ft/sec}$$

The vane velocity at inlet is equal and opposite the relative tangential velocity u of the fluid. If there is no preswirl as in this case:

$$U = -\omega r = -32.5 \text{ ft/sec} = u$$

The relative angle of flow at inlet

$$\beta_0 = \arctan \frac{u}{v_0} = -52.0^\circ$$

and

$$q_0 = \frac{v_0}{\cos 51.99} = 41.25 \text{ ft/sec}$$

At the shroud, there is an angle of attack equal to 10.0 degrees which means that the flow is directed toward the driving face of the vane, a condition that resulted in a lower velocity on the vane driving face than on the suction face. In the absence of more definitive guide lines, it was assumed that the fluid would not be perfectly guided by the vanes until the fluid had moved to a point at which the angle of the vane, would coincide with the flow angle if the vane had been cut back to that point. New values of β , q , and u at intermediate points were then estimated in the same manner that was used in the slip region. The angle of attack at the hub was found to be 12.4 degrees following the same procedure.

There is a note of caution regarding the use of the foregoing method. With substantial blockage by the vanes at inlet, the fluid is accelerated as it moves into the vanes, and it is this accelerated condition of flow that is used in application of the equations in appendix 2. How the lower velocities upstream of the vanes will affect hub-to-shroud distributions of flow is not easy to determine. In the foregoing treatment of flow at inlet, it has been assumed to be small.

Comparison of Outlet Flow Angle With the Optimum

The velocity q_f as computed at the shroud outlet was 28.43 ft per second at an impeller tip speed of 54.74 ft/sec which gives a q_f/U ratio of .52 at the outlet flow angle, $\beta_f = -69.21^\circ$. Figure 4 shows that the optimum ratio for q/U at -71.48° is approximately .72. Viscosity was not taken into account in the solution, but it is well known that with turbulent, viscous flow in a passage, the center velocity varies between approximately 1.08 and 1.28 times the average velocity. If the higher value of 1.28 is assumed for flow away from the impeller surface the q_s/U ratio for the impeller with water would be

approximately .66. Thus, the backward curvature of the vane at the shroud for 550 gpm is less than optimum. At the hub the β_f was computed to be -73.3° and q_f to be 22.55 ft/sec which gives a q_f/U ratio of .412. The optimum q_s/U is .74. Viscous effects raise the ratio to .54 which is still below the optimum. The difference in velocity across the outlet can result in mixing losses which are discussed in the next subsection.

Mixing Losses Due to Velocity Variation at Outlet

Since the mechanical work imparted to the fluid is equal to the difference in moment of momentum between the fluid leaving and entering the impeller, a velocity differential from hub-to-shroud will result in losses if there is no preswirl at inlet to offset the difference.

The tangential component of velocity leaving the impeller is:

$$V = U + (u - \text{slip})$$

At the shroud with no viscous effects and including slip:

$$V = 27.78 \text{ ft/sec and at the hub}$$

$$V = 32.41 \text{ ft/sec.}$$

For nonviscous flow the head

$$H = \frac{UV}{g}$$

For the tip velocity U , of 54.74 ft/sec the head at the shroud is 45.65 ft. and at the hub 55.13 ft. This large differential in head results in a lower than necessary efficiency. Applying the same correction for viscous effects to q as in the preceding section and using the same slip value as for nonviscous flow the value of V at the shroud becomes:

$$V = U + [1.28 (-21.4) - 6.57] = 20.78 \text{ ft/sec}$$

and at the hub

$$V = U + [1.28 (-15.82) - 6.57] = 27.92 \text{ ft/sec}$$

The head rise at the shroud would then be 35.44 ft and at the hub 47.50 ft.

The overall head rise based on an average of the values at the hub and shroud would be 41.43 ft. The experimental value of head rise for the pump at 550 gpm is 37.5 ft. per stage for a 4 stage pump.

Possible Impeller Modifications To Improve Efficiency

The foregoing analysis provides a basis for modification of the impeller to develop higher efficiency. While the method is not exact, deficiencies in the impeller design are pinpointed and the direction in which modifications should be applied is provided. It is estimated that through modification of the impeller, two or three points increase in efficiency can be achieved.

Modification will consist of cutting back the outlet edges of the impeller vanes. It is probable that slowing down of the water along the inner and outer walls of the inlet annulus will result in a positive angle of attack at inlet at the walls with a larger negative angle of attack at mid blade height.

Modification of the Vane Outlet Edge

To provide equal development of head at the hub and shroud at the outlet, it would be necessary to either increase the radius of the impeller at the shroud or decrease the radius at the hub. For a stock impeller, the hub would have to be reduced. Following the same procedure as in the section on mixing losses due to velocity variation at outlet, it was determined that the radius on the hub at which the head at the hub would equal the theoretical 35.44 ft. head at the shroud is 3.3 inches. The original and cut-back edges at outlet are shown in figure 12. A cut back of the impeller trailing edge in a straight line from the existing shroud tip to the 3.3 inch radius at the hub should result in higher impeller efficiency, assuming that the angle of entry into the diffuser vanes is not made worse in the process. Using a viscosity factor of 1.28 the new ratio of q_5/U would be .62 which is closer to the optimum value of .73. Thus the cut back helps to optimize impeller outlet conditions from standpoint of efficiency with the only major drawback being a reduction in head rise through the impeller.

FIGURE 12. Cross Sectional View of Meridional Plane of the Impeller Showing Original and Cut Back Edge of the Impeller Vanes.

ROLE OF PRESWIRL IN BROADENING IMPELLER EFFICIENCY RANGE

It is possible to use preswirl vanes to hydrodynamically shift the inlet flow angle so that it will more nearly coincide with the impeller vane inlet angle as the flow rate changes.

FIGURE 13. Vector Diagram of Flow Relative to Rotating Impeller Vane at Inlet With Component V_s Generated by Preswirl Vanes.

The diagram of figure 13 represents the velocity vectors at the inlet to an impeller for which vanes upstream of the impeller have imparted a tangential component of velocity, V_s to the incoming flow. In the diagram V_s is opposite in direction to the vane motion and the fluid has an additional motion relative to the rotating vane equal to U . The effective tangential velocity of the fluid relative to the impeller then becomes $V_s + U$. The fluid approaches the impeller in the axial direction with a velocity V_z . The angle of approach to the impeller vane then may be defined as:

$$\beta_0 = \arctan \frac{V_s + U}{V_z}$$

Example of Preswirl Use

For any given exit angle for the preswirl vanes, the ratio V_z to V_s will be approximately constant over a given flow range. If for example the exit angle for the preswirl vanes is -45° , V_z will be equal to V_s if the fluid is

perfectly guided by the vanes.

For this case:

$$\beta_0 = \arctan \left(1 + \frac{U}{V_z} \right) \quad (6)$$

and for a range of values of V_z/U the angle, β_0 varies as shown on the top curve in figure 14. The remaining curves show the variation in β_0 for V_s equal .5 V_z and zero preswirl. For a given impeller rotational speed, the term V_z/U is an indicator of flow rate change. It can be seen from the curves of figure 14 that the greater the amount of preswirl the smaller the change in β_0 with change in flow rate. For example, with V_s equal V_z the angle decreases from -71.8° to -63.6° when V_z/U goes from 0.5 to 1.0, a twofold increase in flow. For the case of no preswirl, the angle decreases from -63.5° to -45° , a much greater difference. The losses at inlet should not vary significantly over the flow range represented by a V_z/U range of .5 to 1 for the case with preswirl with a vane angle of -68° for V_s equal V_z , but losses probably would vary greatly if an average angle of -54° should be chosen for the case with no preswirl.

FIGURE 14. Plot of Inlet Flow Angle Versus Ratio of Axial Inlet Velocity to Vane Velocity for Various Amounts of Preswirl and No Preswirl.

Creation of Preswirl

For an impeller with large negative inlet vane angles such as -68° as discussed in the previous section, the number of vanes can be reduced to as few as needed to relieve the entrance blockage, and vanes can be added as needed downstream inside the impeller to satisfy loading requirements.

In the annulus ahead of the impeller the provision of vanes for preswirling the fluid presents a greater problem. The large number of vanes required to assure adequate guidance of the fluid can result in blockage and distortion of the flow. One method of obtaining preswirl without blocking the area is to swirl the flow with vanes placed at a larger radius upstream. As the fluid moves to a smaller radius, the moment of momentum is preserved and the swirl rate is increased, a factor that has to be taken into account in the design of the vanes. A problem that was considered in the use of the concept involved the possible decay of the swirl as the fluid moved to the smaller radius. It is a complex problem which cannot be wholly solved by experimentation. The problem lends itself to theoretical analysis and a mathematical model embracing the problem was devised. The method of approach to the problem and numerical results are included in appendix 3.

Possible Effects of Preswirl on Performance

If V_s is negative, as it would be if directed as shown in figure 13, the net result would be an increased head rise over what would exist if there were no preswirl and a reduced rate of increase in angle of attack as the flow rate is decreased. However, these effects are produced at the expense of higher inlet velocity and increased vane loading. The published performance of one commercially available pump is shown in figure 15. The curve shows

FIGURE 15. Performance Curve for Commercial Pump. Broken Lines Show Total Head That Theoretically Can be Achieved With Preswirl. The Value of V_z/U is .39.

a decrease of approximately 20 points in efficiency as the flow rate drops from 600 to 300 gpm. If this drop is primarily due to the increasing inlet angle of attack, the use of preswirl can be beneficial. If it is assumed that with preswirl the increase in difference of moment of momentum between the fluid entering and leaving the impeller correspondingly increases the total head per stage the total head would be as shown by the broken lines in figure 15 which represent the new head for each ratio of V_s/V_z shown. In addition to providing more favorable angles of attack as the flow decreases, the total head remains more nearly constant. This eliminates or greatly reduces the throttling losses which normally accompany a cut back in flow with a conventional pump. In arriving at an optimum system a reduction in the impeller diameter, rpm, or number of stages can be considered. With a reduction in rpm, the friction losses would also be reduced.

The effect of increased velocity at inlet can be qualitatively assessed since friction losses vary approximately as the square of the velocity.

From figure 13:

$$\left(\frac{W}{W_0}\right)^2 = 1 + \frac{2 \left(\frac{V_z}{U}\right) \left(\frac{V_s}{V_z}\right) + \left(\frac{V_z}{U}\right)^2 \left(\frac{V_s}{V_z}\right)^2}{\left(\frac{V_z}{U}\right)^2 + 1} \quad (7)$$

where $W_0 = W$ at $V_s = 0$

The ratio of losses with various values of $\frac{V_z}{U}$ and $\frac{V_s}{V_z}$ are shown in figure 16.

FIGURE 16. Curves Showing Qualitative Friction Loss Increase Near Impeller Inlet With Changes in Axial Inlet Velocity and Preswirl.

These curves should be viewed in a qualitative sense only as the losses can vary as low as the velocity to the 3/2 power (Ref. 11 page 52) and with pre-swirl, the rpm can be reduced for a given required head.

A qualitative evaluation of the effect of increased vane loading is less easily accomplished. Theoretical optimization of both velocity effects and loading with respect to reduction in angle of attack followed by experimental evaluation will be required.

In a practical situation, if the farmer decides to cut back the flow from a well to a given irrigation piping arrangement, there is no alternative but to throttle the flow. It can be seen in figure 15 that a cut back in flow from 600 gpm to 300 gpm would require a throttling loss of 12 ft of head per stage. With this throttling loss, the pump would be operating at approximately 46 percent efficiency. This assumes that operation is with a constant speed drive. With a variable speed drive the speed could be reduced and from data not shown in figure 15 the efficiency would drop down to approximately 67 percent. While this drop of 17 points in efficiency is less costly, it is still significant. What this demonstrates is that even with installation of a variable speed drive, the major assured gain is in elimination of throttling losses. The pump may still operate at a flow which is in the low efficiency range at a reduced flow rate.

Another factor which should be considered is specific speed of the pump. The specific speed, $N_s = \text{Rpm} \sqrt{\text{gpm}} / H^{3/4}$, where H is the rise in head in feet. For the pump of figure 15 at 600 gpm the specific speed is 3200. From established curves of efficiency versus specific speed, the maximum efficiency for a 600 gpm single stage pump occurs at a specific speed of approximately 2500. For $V_s/V_z = 1$ the head is such that the rpm could be reduced to 1330 to achieve the head rise of 32 feet. The specific speed would drop to 2400 which is closer to the optimum. Thus, the use of preswirl, at least in this case, provides a more favorable specific speed as the speed is reduced. It should be noted that for the curve at $V_s/V_z = 2$ the falling head below approximately 450 gpm could lead to instability of flow, and the use of $V_s/V_z = 1.5$ may be more practical.

NUMERICAL ANALYSIS OF THE DIFFUSER

A cross sectional view of one centrifugal pump stage is shown in figure 17. The lower half of the cross section shows the original diffuser, and the upper half the modified diffuser. A diffuser is normally comprised of a vaneless transition section between the impeller outlet and the vane inlet, and the vaned portion. For the original pump there is no vaneless transition section. The diffuser vanes extend to the impeller outlet with allowance for running clearance only between the impeller and vanes. According to the flow solution at the impeller outlet there would be an abrupt increase in the flow area as the fluid entered the diffuser vanes. One method of modifying the diffuser section to correct the problem is to cut back the vanes until the flow angle matches that of the vanes if an analysis of the flow shows that it is possible. An analysis of the flow was made and is discussed in the next section.

Analysis Of Flow

The cut back impeller as shown in figure 12 provides a constant moment of momentum from hub to shroud for fluid leaving the impeller exit at a flow rate of 600 gpm. If it is assumed that the angular momentum of the fluid is preserved, as it passes through the vaneless section of the diffuser, flow direction can be determined, and stream filament techniques can be used to provide a flow solution in that section. Setting ω equal to zero in equation (3) of appendix 1, it becomes:

$$q = q_0 e^{\int_0^n \alpha dn}$$

In the absence of vanes, the angle, β becomes the angle between the flow direction and axial-radial plane on the streamline surface of revolution. The product of the tangential component of the velocity at the outlet and the radius r , remain constant. Using that component and the through flow component as computed from streamline spacing, the flow direction at any point can be determined for a given set of streamlines. In the absence of a computer program, a rapid method of determining r_c at any normal, for a given streamline, is to use a transparent overlay of concentric circles to match and determine the curvature.

Using the solution of reference 9 as a guide, the inner wall of the vaneless section was drawn as shown on figure 17. Estimated streamlines were drawn in using spacing intervals proportional to those for the solution of reference 9. The first solution showed the estimate of streamline spacing to be adequate-

FIGURE 17. Cross Sectional View of Impeller and Diffuser In Place. Portion Below Centerline Shows Original Configurations and Above Centerline Shows Modified Configurations.

ly accurate to provide the needed flow solution. The streamline distribution is shown on figure 18, and the main velocities for the streamtubes bounded by the streamlines are shown in figure 19. There are no significant flow decelerations on the walls, and any boundary layer build up should result in an acceleration of the main stream with an accompanying restriction of further boundary layer growth. The inner ring which will be contoured to provide the new inner wall shape for the vaneless section can be made of either plastic or metal. If metal, it will have to be made in two parts to pass through the bored hole in the diffuser housing.

Vaned Section Analysis

The primary functions of the vanes for the example pump are to turn the flow for entry into the next stage and to serve as struts to support the bearings for the impeller shaft. The area normal to the flow direction at the modified vane inlet is approximately 82 percent of that at exit. The walls formed by the vanes converge at both hub and shroud from inlet to exit. There is a divergence of approximately 3 degrees from hub to shroud. While the vaned section does not provide a significant amount of diffusion, it does serve to turn the flow from the larger radius required for discharge of the impeller to the smaller radius required by the impeller inlet at the next stage and the turning appears gradual enough to incur minimum losses. Thus, except for the transition section from the impeller to the diffuser, the original design appears adequate. At the point to which it is proposed to cut back the vanes as shown in figure 17 the average vane angle β is approximately 57° . The corresponding flow angle at the inner wall ahead of the vanes is 66° and at the outer wall it is 69° . Taking vane blockage into consideration, it is 61° at the inner wall and 65° at the outer wall. The vanes have adequate thickness to allow shaping at inlet for smooth entry of fluid from the vaneless section. However, the vanes at that point are not easily accessible for machining, and it may not be physically possible to make the needed changes without modifying the pattern and casting a new bowl.

FIGURE 18. Streamline Distribution for Vaneless Section of Modified Diffuser.

FIGURE 19. Velocity Distribution In Modified Diffuser Between Impeller Outlet and Vaned Section. Flow Rate, 550 Gpm.

IV TECHNICAL RESULTS

The first objective of the pump tests was to demonstrate that a commercial pump installed in a well which is cased to the bottom can deliver water to the surface at 82 percent or higher efficiency. A second objective was to demonstrate the capability to remove and replace the pump with on-the-farm tools. The methods by which this was accomplished are described herein.

APPARATUS, INSTRUMENTATION, AND PROCEDURE

The test apparatus consisted of a 124 ft. deep well with 10" diameter casing, a commercial pump rated at 600 gpm, a water well packer, a 4 inch column pipe, an open line shaft, and a cradle mounted 30 hp electric motor. The well, which was drilled in granite below approximately 17 feet, was not capable of producing 600 gallons of water per minute. To provide the water needed for the demonstration, the well was bored 16 inches in diameter and cased down to the granite with a 16 inch casing, thereby providing an annulus around the 10 inch casing whereby water could be returned to the bottom of the well for recirculation. The apparatus arrangement at the well, above ground is shown in figure 20. Referring to figure 20, the tank at left

FIGURE 20. Above Ground Apparatus For Demonstration Well System.

serves as a reservoir for the recirculated water which reenters the annulus

around the casing through the boxlike structure which encompasses the well casing. The electric motor that sits above the well casing is mounted on bearings to allow torque measurements. A schematic drawing of the bearing arrangement is shown in figure 21. The pump is driven by a 3/4 inch diameter high strength open line shaft with water lubricated bearings

FIGURE 21. Schematic Drawing Of Motor And Bearing Mount For Torque Measurement.

at 10 ft. intervals. A manufacturer's drawing of a typical line shaft with coupling and bearing with spider mounts for the bearing at the column pipe coupling is shown in figure 22. This arrangement is identical to that for the 4 inch column pipe used in the demonstration system. Water moves up through the casing and column pipe. The top five foot section of column pipe with tee at the top for venting the water into the well casing is shown attached to the motor base in figure 23. Water from the casing discharges

FIGURE 22. Drawing Of Open Line Shaft And Bearing Arrangement In Column Pipe.

FIGURE 23. Four Inch Column Pipe Attached To Motor Base.

into the pipe at the lower right. The water is conducted through that pipe into the smaller horizontal pipe through the flow meter and throttle valve. The throttle valve, which is located just above the tank, is used to adjust the flow rate through the pump.

The pump, packer, and inlet screen are shown in figure 24. Water discharges from the top of the packer into the 4 inch column pipe and through support struts into the casing. The packer is inflated with 100 psi air through an air line from the surface. The bottom of the packer is threaded

FIGURE 24. View Of Packer, Pump, And Screen.

to fit the standard top component of the pump and the top is threaded to fit a standard 4 inch column pipe coupling. The overall length is five feet.

To install or remove the pump from the well a three ton gear chain hoist was mounted on a tripod above the well as shown in figure 25. The

FIGURE 25. View Of Tripod And Chain Hoist Above Well.

tripod consists of three 4 inch standard pipes 20 feet long drilled at the top for a threaded 1 inch diameter rod. These items are considered to be generally available or easily procurable for farm use. Chain wrenches were used to connect the four inch pipes. The impeller and diffuser configurations were modified as shown in figures 12 and 17. The impeller to bowl sealing arrangement was modified to agree with the configuration shown in figure 5. A photograph of the revised impeller is shown in figure 26. The backing ring

FIGURE 26. Photograph Of Modified Impeller.

which was machined from polyethylene is shown in figure 27. The pump bowl

FIGURE 27. Photograph Of Impeller To Bowl Seal Backing Ring.

was modified to accept the backing ring and a snap in retaining ring. Flow was measured by means of a turbine type flow meter, torque with a 5 foot arm and spring scale, and inlet and outlet head by means of bourdon pressure gages. Motor rpm was measured by means of a magnetic pick up and electronic counter. An attempt was made to obtain measurements by means of instruments that are currently available to agricultural extension agents and others that may engage in determination of efficiency for the user. For that reason, the measurement of electrical input to the motor was considered as a means of determining input power. However, it was determined that even with a motor calibrated at the manufacturer's plant, the possible error in power measurement was plus or minus two percent or greater.

Head rise across the pump was measured over a pump flow range from approximately 125 to 630 gpm at intervals of approximately 50 gpm. The motor speed varied from 1777 rpm at maximum flow rate to 1787 rpm at minimum flow rate. Flow rate was controlled by the valve downstream of the flow meter. Tests were made with the pump as received, after polishing, after impeller modification, and after modification of diffuser and seal ring.

ACCURACY OF MEASUREMENT

The nominal precision of readings were as follows:

	<u>Percent</u>
Suction head	$\pm .5$
Discharge head	$\pm .7$
Torque	± 1.0
Motor rpm	$\pm .2$
Flow rate	$\pm .16$

Calibrations on all instruments were traceable to the Bureau of standards. The reading precision for flow rate does not necessarily reflect the accuracy as the flow meter was calibrated by measuring flow into a tank over a period of time. The flow rate which was linear with rpm of the turbine was determined with an accuracy of ± 0.7 percent. Averages for a number of tests were used to arrive at the final values of efficiency and head. That approach together with the smoothness of curves plotted through the data points gives indication that the efficiency values are accurate within plus or minus one efficiency point.

EXPERIMENTAL RESULTS AND DISCUSSION

Tests were performed under field conditions at the well shown in figure 20. Mechanical performances of the packer and high strength line shaft were demonstrated, and modifications to the pump were evaluated in a working situation. The results of the tests are presented in this section.

Packer Performance

The packer, which was pressurized by 100 psi air performed effectively. The packer and pump were installed in the well 6 times and removed 5 times without any problems. The rubber boot which seals the packer to the casing was not designed to support the downward hydraulic pressure. Therefore, it is necessary to provide a column pipe large enough to support the weight of the pump and force of the water column. The 4 inch column pipe used in this application can support a pump and 250 ft head of water in a sixteen inch diameter casing based on the ASTM pressure rating. The practical rating for well application is undoubtedly much greater. For a larger pump the main problem would be line shaft size. While spider bearings can be made for the largest shaft needed for a pump for a 16 inch casing, they may not be stock items. Use of high strength shaft can eliminate the problem to a large extent. The head loss in a column pipe and casing may be approximated from the pump manufacturer's literature. The following example is given:

According to the manufacturer's hydraulic friction loss chart a standard 4 inch column pipe at 45 gpm with a 3/4 inch line shaft has a .54 ft loss in head per 100 feet. A standard 10 inch pipe with a 4 inch enclosed line shaft tube has a .54 ft loss per 100 ft at 505 gpm. The implied result is that the approximate head loss for the combined column pipe and casing at 550 gpm is .54 ft per 100 ft.

The head loss for the nominal standard 6 inch column pipe and one inch open line shaft at 550 gpm is 4.4 ft per 100 ft. Thus a reduction in friction loss of 3.86 ft per 100 ft was effected by use of the packer. That amounts to three efficiency points for the system.

Line Shaft Performance

The 3/4 inch diameter line shaft used in the application was made of 17 - 4 PH steel alloy. Data in the manufacturer's catalog indicated that the strength of a 3/4 inch diameter carbon steel shaft as normally furnished could be marginal and that use of a one inch diameter steel shaft was indicated.

According to the manufacturer's data, the loss per 100 feet of 3/4 inch shaft rotating at 1760 rpm is 0.32 hp per 100 feet and for a 1 inch shaft it is 0.53 hp per 100 feet. The saving of .21 hp amounts to 0.6 efficiency points. No problems were encountered with handling the 3/4 inch shafting in shipping or installation. Based on spot surveys in the San Joaquin valley in the Hanford and Chowchilla areas of California large numbers of wells have casings 14 to 16 inches in diameter, line shafts 1 to 1.5 inches in diameter, and flow rates from 1000 to 2000 gpm. The use of 4300 series heat treated steel shafting would allow use of commercially available bearings in a 4 inch column pipe for those applications where 1.5 inch cold rolled shafts are now used, and at lower cost. Thus, most of the wells in that area could effectively use a packer system. There is one note of caution. In some of the wells of the spot survey, the casings were perforated at intermediate reservoir levels. The use of the packer would be complicated in those wells unless the upper level were unproductive enough to allow sealing off the perforations with a light weight intermediate packer.

Pump System Performance

The pump was first tested as received from the manufacturer with unpolished impellers. It was then tested after the following modifications in succession:

1. All four impellers polished.
2. Impeller cut back at outlet as shown in figure 12.
3. Impeller cut back, diffuser modified as shown in figure 18, and impeller to bowl seal modified to conform with the shape shown in figure 5.

Performance Of System With Pump As Received

The system includes the well casing, column pipe, line shaft and pump. The system performance with the as-received pump is shown in figure 28. The maximum efficiency of 78 percent occurred at 500 gpm with a total developed head of 132 ft for the four stages. With the nominal losses from casing, column pipe, and line shaft taken into account, the pump efficiency alone is 79.3 percent. The pump manufacturer obtained an efficiency of 80.2 percent at 521 gpm. During tests in the well, rock particles that were washed down from the walls of the return annulus around the casing were injected into the pump; enlarging the impeller to bowl seal clearance during the test. This could have caused a reduction in efficiency. In addition, the corrections for system

FIGURE 28. System Performance With Commercial Turbine Pump As Received From Manufacturer.

losses are nominal only. As a result the manufacturer's performance results for the pump are probably more accurate. After the pump was tested, it was returned to the manufacturer for polishing of the impellers. The damaged impellers were replaced, restoring the impeller to bowl clearance to the original value.

Performance Of System With Polished Impellers

The pump with polished impellers as assembled by the manufacturer was tested in the system. The polishing was accomplished with a commercially available production type machine which uses an abrasive slurry as the polishing medium. The system performance is shown in figure 29. The maximum system efficiency of 81 percent occurred at 500 gpm with a total developed head of 141 ft. With the nominal losses from casing, column pipe, and line shaft taken into account, the pump efficiency alone is 82.6 percent.

FIGURE 29. System Performance With Commercial Turbine Pump With Polished Impellers.

Performance Of System With Impellers Cut Back

The vanes only were cut back as shown in figure 12 to equalize work input to the fluid. The hub was left intact to provide a fairing between the impeller outlet and the diffuser. The system performance with cut back vanes is shown in figure 30. The maximum system efficiency of 82.6 percent occurred at 500 gpm with a total developed head of 116 ft. As pointed out previously, it is convenient to reduce the diameter of an existing impeller at the hub to achieve equal work input across the outlet, whereas increasing the diameter at the shroud is not. However, the head rise across the impeller is reduced, making it necessary to increase the number of stages to achieve the same overall head as was achieved before cutting back. In this case, five stages would be required instead of four. That is a small price to pay for the increase of 1.5 points in efficiency. The system efficiency of 82.6 percent thus achieved in pumping to the surface is higher than the goal of 82 percent. With the nominal losses from the casing, column pipe, and line shaft taken into account, the

pump efficiency alone is 84.5 percent.

FIGURE 30. System Performance With Commercial Turbine Pump With Vanes Cut Back At Outlet.

Performance Of System With Modified Impeller, Seal, And Diffuser

The impeller as previously cut back was tested with a modified diffuser and the impeller to bowl seal configuration shown in figures 5, 26, and 27. The performance results are shown in figure 31. The maximum efficiency of the system is 78 percent at 500 gpm and a total developed head of 110 feet. With the nominal losses from the casing, column pipe, and line shaft taken into account, the pump efficiency alone is 80 percent. The system efficiency is the same as for the as-received pump with unpolished impellers and the pump alone efficiency is 0.7 points higher. The reason for the higher pump efficiency for the modified pump is that the shaft and column pipe losses are a higher percentage of the total head for the modified pump. The lower system efficiency is attributed in part to the inability to modify the diffuser vanes

FIGURE 31. System Performance With Commercial Turbine Pump With Modified Impeller, Seal, and Diffuser.

to accept the flow at the angle needed for smooth entry into the vanes, inability to achieve a smooth and dimensionally correct contour on the modified bowl, and inability to correctly align all four impellers with the diffuser inlet. It appears that the manufacturer has correctly assessed the problems of aligning impellers with a dimensionally correct diffuser configuration and has chosen the presently used configuration as being less sensitive to misalignment. As previously stated, the diffuser vanes in the as-received configuration extend to the impeller inlet as shown in figure 6. The average β of the vanes is approximately 65° which is within approximately 2 degrees of the computed flow angle of the fluid at the impeller exit. Therefore, while it may be possible in the long run to improve the diffuser by providing a closer fit for the impeller either by using the configuration shown in figure 18 or a hydrodynamically more nearly correct passage configuration in which the diffuser vanes extend to the impeller outlet, the pump would be more sensitive to impeller alignment. It appears from the results obtained that a smooth angle of entry into the vanes is more important than achieving a match-

ing impeller and diffuser configuration in the meridional plane. This is an area that needs more investigation.

With regard to the modified impeller-to-bowl seal, the results are inconclusive. The main conclusion that can be drawn is that pump performance can be very adversely affected by impeller-to-bowl seal abrasion and wear. This effect was evident during initial testing of the pump at which time rock particles from the side walls of the water return annulus were washed down and injected by the pump. Subsequent, further cleaning of the well eliminated most of the particles. Therefore, effects of abrasion on the modified seal were not evidenced.

The radial clearance between the lands machined on the impeller and the plastic ring was .004 inch. This appeared to be close to the minimum practical clearance for the pump being tested. Further work needs to be done with impeller-to-bowl seal arrangements and with abrasion resistant materials. Further work in this area as well as in the diffuser area is beyond the financial scope of the present program.

Installation And Removal Of Pump With On-The-Farm Tools

All installations and removals of the pump, packer, and 4 inch column pipe have been accomplished with the tripod and three ton chain hoist shown in figure 25. Removal or replacement of the pump and 95 feet of column pipe requires 12 to 16 man hours. At least a two man crew is required. Some damage to the threads on the 4 inch pipe was experienced on occasion. However, a hand applied threading die was sufficient to clean up the damage. The pump was installed six times and removed five times without replacing any shafting or pipe. As previously stated, a four inch column pipe can support a pump and 250 foot head of water in a sixteen inch diameter casing. Thus, very many well pump systems in the U.S. can be retrofitted for installation or removal with on-the-farm tools. The ability of the farmer to remove or replace a pump at his convenience and at lower cost should contribute to better pump system maintenance and higher sustained efficiency.

Summary of Experimental Results

1. With a packer both column pipe and well casing can be effectively used as water conduits from a well. The advantages gained are as follows:
 - a) A three to five point improvement in system efficiency over that

using the nominal size column pipe listed by the manufacturer for a given pump. The gain for the pump tested was three points.

- b) Significant reduction in column pipe cost. The saving for the pump tested was \$280 per 100 feet. The saving for a well that would normally use a 10 inch column pipe would be \$1050.00 per 100 feet which is more than enough to pay for the packer.

(1979 prices)

- c) Greater ease of installation due to use of smaller column pipe, allowing installation and removal with on-the-farm tools.
 - d) Straighter hanging of smaller column pipe and line shaft in crooked wells resulting in lower bearing friction as well as easier installation.
 - e) Ability to make field repairs to damaged threads on smaller column pipes.
 - f) Greater control by the farmer over repair and maintenance schedules with reduced need for hired crane or well service rigs.
2. Polishing the pump impellers by the pump manufacturer resulted in a two to three point increase in pump system efficiency. The maximum system efficiency was 81 percent at 500 gpm.
 3. Analysis of the pump impeller showed a slight negative angle of attack at the inlet to the impeller vanes at a flow rate of 550 gpm and a higher impeller work input to the fluid at the hub than at the shroud. Cutting back the vanes to equalize the work input across the impeller outlet resulted in a 1.6 point increase in system efficiency, raising the maximum system efficiency to 82.6 percent.
 4. From results of analysis and tests it is indicated that the diffuser configuration on the pump as shipped is relatively insensitive to changes in axial position of the impeller and may be the most efficient configuration to use. Further evaluation is indicated.
 5. Results from tests with the pump in various stages of modification indicate that side sealing of the impeller and bowl, which is the least sensitive to axial adjustment of the impeller, is vulnerable to abrasion of the sealing surfaces by sand and rock particles. Methods of sealing which are less sensitive to abrasion should be developed.
 6. Water can be delivered to the surface with the objective system efficiency of 82 percent.

V COST ANALYSIS

There are at least three categories of costs to be considered. The first is that associated with changes in component arrangement and material changes. The second is that associated with reduced energy consumption due to changes in pump design or manufacture which increase efficiency. The third is installation. They will be discussed in order in the following sections.

CHANGES IN COMPONENT ARRANGEMENT

The main projected change in component arrangement comes with the use of a packer which allows use of a small column pipe. The most significant saving in energy with the use of a packer may result from retrofitting of wells that are now filled with column pipes that are smaller than they should be for the flow being carried. The effect on energy loss can best be illustrated by considering the initial fitting of a well with a 14 inch casing and a water flow of 1300 gpm. With the use of a packer and a 5 inch column pipe, the hydraulic friction loss per 100 feet would be less than 0.6 ft of water. The recommended maximum diameter column pipe for a 1300 gpm pump is ten inches which results in a 1.6 ft of water head loss per 100 feet. The 1978 cost of 100 feet of 5 inch column pipe was \$760.00, and for 100 feet of 10 inch column pipe it was \$1690.00. With a packer cost of \$600, there would be a saving of \$330 for a 100 ft installation or \$1260 for a 200 ft installation plus a bonus of approximately one point in higher pumping efficiency. In addition, the pump could be removed from the well and replaced with relatively light weight equipment. With the use of heat treated alloy steel shafting, a 4 inch column pipe could be used with a saving of \$1500.00 for a 200 ft installation. The use of high strength steel shafting is discussed in the next section.

HIGH STRENGTH STEEL LINE SHAFTING

As previously stated, the friction losses associated with line shafts and their bearings increase as the square of the line shaft diameter. Therefore it behooves the irrigator to select the minimum size shaft possible for a given application. According to manufacturer's catalogs, a 1300 gpm pump operating at a 200 ft depth requires a 1 3/16 diameter AISI No. C - 1045 cold rolled steel shaft. If AISI No. 4340 heat treated steel is used, the diameter may be reduced to 1 inch. In 1978 the cost of heat treated AISI No. 4340 steel shafting was

\$60.15/ 100 lbs as opposed to \$38.85/ 100 lbs for AISI No. 1045 steel. For a 200 ft length of 1 inch diameter AISI No. 4340 shafting the cost of the basic material would have been \$321.20 in 1978 and for a 200 ft length of 1 inch diameter AISI No. 1045 shafting it would have been \$207.46, a difference of \$113.74. The cost of 200 ft of inner column and shaft assembly for a 1 inch diameter shaft would have been \$1,620 and for a 1 3/16 inch diameter shaft, it would have been \$2380, a difference of \$760. Thus, if it is assumed that the increase in basic material cost only were added to the cost of the 1 inch diameter shafting, the cost saving would have been \$646.26 on shafting plus the operating savings from 26.4 percent reduction in bearing friction loss. In this case, it would allow the use of the lower cost 4 inch column pipe with the packer.

CHANGES IN PUMP DESIGN

The major cost of changes in impeller and diffuser design result from engineering research and development. The cost of fabricating impeller or diffuser configurations with improved efficiency need not be greater than that for the unimproved configurations. There are increased costs with the purchase and use of polishing machines and improved seals. It is not possible with the meager data gathered in this program to make a cost analysis for these two items.

INSTALLATION COSTS

With the use of a packer and small column pipe, a pump can be removed and replaced with on-the-farm equipment by farm personnel, saving the \$60 to \$100 per hour cost of a crane or well drilling rig. This ability is important from standpoint of emergencies as well as pump repair and maintenance.

SUMMARY REMARKS ON COST ANALYSIS

The foregoing examples illustrate the potential of cost reductions for irrigation pumping systems that also improve efficiency of operation. It appears that if all of the approaches to pump improvement are made including engineering costs, there can easily be a large net decrease in the hardware cost to the irrigator as well as a significant reduction in energy usage.

VI INDUSTRY ACCEPTANCE

The results of the initial survey reported in reference 1 showed a generally positive attitude among pump manufacturers toward development of a simplified method of pump analysis and design. Of the turbine well pump manufacturers, two showed the greatest interest, and one of those, a leader among pump manufacturers, was asked to cooperate in the analysis and testing program. That manufacturer was very cooperative throughout the program. The engineer from that company who was assigned to work with Aerospace Research Corporation has been especially impressed with the possibility of reducing column pipe costs, as well as improving efficiency, by means of the packer.

A conference for demonstration of the pump operation, removal of the pump from the well, and discussion of the program with pump manufacturers at Aerospace Research Corporation was held. Of eleven invited pump companies that were considered to be prominent manufacturers of vertical turbine pumps, six responded and three sent representatives. Cost of travel was given as a deterrent by two California companies that asked for literature. Those in attendance agreed with the approach and exhibited considerable interest in the use of the packer. There was discussion regarding the problems of using the packer in wells with different levels of perforations and with sand. Entrained sand which is currently being pumped out with the water in some wells might settle in the column pipe and casing if the velocity were reduced. Arrangement for periodically closing off the casing to column pipe annulus at the top of the well and pumping through the column pipe only might relieve the problem should it occur. Reduction in pump installation costs appeared to be of as much interest as reducing energy consumption. There was some question on the corrosion resistance of heat treated alloy steel as opposed to cold rolled steel for open line shaft installations. Experience has shown generally that there is not much difference in corrosion rates in water. There was discussion about acceptance by the end user. It was generally agreed that the lower first costs plus the energy savings would greatly encourage acceptance. The ability of the farmer to pull his pump was considered a favorable factor for the farmer, but that it probably would be played down by service companies. All agreed that impeller to bowl seals are a major problem and that plastic seals are less subject to abrasion than metal seals, but use of plastics will require more development.

VII POTENTIAL FOR ENERGY CONSERVATION

A survey of irrigation pump usage was made by Aerospace Research Corporation in 1976 under Contract No. EY-76-C-03-1260 and the results were reported in reference 1. The survey which was confined to the contiguous United States was made to determine the energy input to irrigation pumps and the possible changes that could be made to reduce that input. Each of the agricultural extension specialists in the major irrigation states was sent a questionnaire in an effort to obtain data for an energy use evaluation. The primary data needed for the evaluation were total irrigated acreage, percent supplied by wells, percent sprinkler fed irrigation, average pumping lift, average pressure required by the sprinkler, average pressure required for movement from well to point of gravity feed, and pump operating efficiency. Irrigation specialists in Texas, Colorado, and Arizona provided field test results which showed turbine pump efficiencies ranging from 50 to 58 percent. For those states that did not provide pump efficiencies a figure of 57 percent was used although the actual figure is probably closer to 50 percent. None of the estimated pump efficiencies exceeded 60 percent. All of these efficiencies include column pipe and line shaft bearing losses. The horizontal pump efficiencies averaged approximately 60 percent. The calculations were performed in three steps. First the energy required and possible savings were calculated for lifting the water to the surface for all water supplied by wells. Second the energy required and possible energy savings were calculated for moving the sprinkler dispensed water through the sprinkler heads. Third the energy required and possible savings were calculated for moving the gravity dispensed water to the point of release. A drive efficiency of 25 percent was used to calculate equivalent oil consumption. In some states, especially those where water is supplied from a central authority, no pumping is required at the farm. This was taken into account in estimating possible savings. Using the data from the states that responded as shown in reference 1, a possible total saving of energy equivalent to approximately 21 million barrels of oil per year was projected for an overall pumping efficiency increase to 82 percent. Savings for the remaining states were estimated using the irrigated acreages. Savings for Idaho, Montana, Nevada, Utah, and Washington were based on data from neighboring states. For the remainder of the fifty states there was a total of 2,362,876 irrigated

acres (There were no figures given for New Jersey). For these states an annual average of 12 acre inches per year was assumed, the same as Florida which was used as a comparison state to project an annual possible saving of the equivalent of 564,000 bbls of oil. The estimated savings for California may be susceptible to the largest error. It is the largest user of irrigation water, but the variation in ground water conditions as well as acre inches per year used throughout the state makes calculation difficult. It was assumed that irrigation requirements in southern California would be similar to those for Arizona and for the north similar to those for Oregon. This resulted in the use of an estimated 200 ft. average lift and 56 acre inches per year which may be low. An overall total of possible savings equal to 23.3 million barrels of oil is projected. This is approximately equal to .13 quadrillion Btu. Steps have been taken by many agricultural extension agents to help reduce energy use. Some of their publications are listed in reference 1.

Subsequent to compilation of the data shown in reference 1, the results of a survey made by the U.S. Department of Agriculture were published in reference 10. Adjusting the data in that report to reflect savings that might be achieved in raising pumping efficiencies from 57 percent to 82 percent resulted in a figure of 24.6 million bbls of oil per year as compared to the figure of 23.3 million shown in reference 1. In converting the data of reference 10, a delivered efficiency of 28 percent for electric power was used to determine the equivalent amount of oil needed to power electric pumps.

The impact of possible savings may be seen if the number of irrigation wells in thirteen of the major irrigation states is considered. (from ref. 1)

STATE	NO. OF WELLS	STATE	NO. OF WELLS
Arizona	7,000	Kansas	19,000
Arkansas	7,000	Louisiana	7,000
California	50,000*	Nebraska	52,014
Colorado	20,000	New Mexico	15,000
Florida	12,000	Oklahoma	6,239
Idaho	7,000	Texas	90,500
*estimated		Wyoming	3,600

Total No. Wells 296,353

The total estimated possible savings for these states is the equivalent of 21 million bbls of oil per year or 71 barrels per well.

APPENDIX 1

REFERENCES

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APPENDIX 2

IMPELLER ANALYSIS COMPUTATIONAL PROCEDURE

The following analysis method which was derived from reference 2 is based upon the use of stream filaments represented by streamlines drawn in between the hub and shroud of an impeller on the meridional plane as shown in figure 1 (a). Lines normal to the streamlines are drawn in as shown on figure 1 (a) with the zero point starting at the hub. All flow is assumed to be nonviscous.

The following symbols are used in the analysis:

- a parameter equal to $\frac{\cos^2 \beta}{r_c} - \frac{\sin^2 \beta}{r} \frac{dr}{dn}$
- b parameter equal to $2 \omega \sin \beta \frac{dr}{dn}$
- F force per unit mass
- F_n force exerted by the vane normal to meridional streamline.
- f factor accounting for area taken up by vane cross section.
- g acceleration due to gravity.
- n distance along normal to streamline in meridional plane, ft.
- p_a average relative static pressure between vanes lbs/ft².
- p_i total pressure upstream of pump inlet, lbs/ft².
- p_s relative static pressure on the suction face of the vane, lbs/ft².
- p_d relative static pressure on the pressure face of the vane, lbs/ft².
- Δp difference in relative static pressure between driving and suction faces of the vane.
- q average velocity between vanes relative to the impeller, ft/sec.
- q_d velocity relative to the pressure face of the vane, ft/sec.
- q_s velocity relative to the suction face of the vane, ft/sec.
- r radius of particle from axis of impeller rotation, ft.
- r_c radius of curvature of streamline in meridional plane, positive when convex side faces axis of rotation, ft.
- u tangential velocity relative to impeller, positive in the direction of rotation, ft/sec. equal to $q \sin \beta$.
- w flow rate, lb/sec.
- z distance in axial direction, ft.

- α angle between tangent to streamline in meridional plane and axis of rotation, deg.
- β angle between vane and axial-radial plane on streamline surface of revolution (negative for backward curved vanes), deg.
- λ prerotation term equal to $r(\omega r + u)$ at inlet.
- ρ mass density lb - sec 2 /ft 4
- ω angular velocity of impeller, radians/sec.

The following equation from NACA Report 1082, reference 2, is used to determine velocities in the impeller passage:

$$\frac{dq}{dn} = q \left(\frac{\cos^2 \beta}{r_c} - \frac{\sin^2 \beta}{r} \frac{dr}{dn} \right) - 2\omega \sin \beta \frac{dr}{dn} + \frac{F_n}{q} - \frac{\omega}{q} \frac{d\lambda}{dn} \quad (1)$$

For impellers which have blade elements normal to the hub and where there is no prerotation of the fluid at inlet the terms $\frac{F_n}{q}$ and $\frac{\omega}{q} \frac{d\lambda}{dn}$ drop out leaving a first-order differential equation linear in q . In order to arrive at a solution for impellers with vanes with elements that are not normal to the hub and/or with prerotation at the inlet it is usually quicker to solve for q and to neglect these terms in the first solution. The values of q thus found may then be used in the force and prerotation terms for the second solution and may be treated in the same manner as those terms dependent upon the geometry of the impeller.

The equation then becomes:

$$dq = aq - b \quad (2)$$

When equation (2) is solved for q

$$q = e^{\int_0^n a \, dn} \left(q_0 - \int_0^n b e^{-\int_0^n a \, dn} \, dn \right) \quad (3)$$

(4) The weight flow across any normal may be obtained from equation

$$w = 2\pi \int_0^n r f \rho g q \cos \beta \, dn \quad (4)$$

Once solutions for q result in satisfying continuity, the average relative static pressure, relative static pressures on the vane surfaces, and relative velocities on the vane surfaces may be computed as follows:

$$p_a = \frac{\rho \omega^2 r^2}{2} - \frac{\rho q^2}{2} + p_i - \rho \omega \lambda_0 \quad (5)$$

For viscous flow the loss in relative total pressure would have to be subtracted from the right side.

From Appendix D of reference 2

$$\Delta p = \rho q \cos \beta \left(\frac{2\pi r f}{N} \right) \left(\frac{du}{dz} \cos \alpha + \frac{q \sin \beta \sin \alpha}{r} + 2 \omega \sin \alpha \right) \quad (6)$$

From equation (5) and assuming a linear distribution of relative velocity from driving to suction face at a given radius.

$$\Delta q = \frac{\Delta p}{\rho q} \quad (7)$$

$$q_s = q + \Delta q/2 \quad (8)$$

$$q_d = q - \Delta q/2 \quad (9)$$

From equation (5)

$$p_s = \frac{\rho \omega^2 r^2}{2} - \frac{\rho q_s^2}{2} + p_i - \rho \omega \lambda_o \quad (10)$$

and

$$p_d = \frac{\rho \omega^2 r^2}{2} - \frac{\rho q_d^2}{2} + p_i - \rho \omega \lambda_o \quad (11)$$

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Suggestions for the Numerical Routine

Estimated streamlines and normal lines are drawn in on the meridional plane as shown in figure 1(a). Closely spaced r and z coordinates for the streamlines may be used to arrive at an equation for the streamline curve. It was found that a seventh order polynomial using APL programming language provided an adequately accurate curve description. The equations of the streamlines thus devised were then used to determine streamline curvatures and the angles α at points on the streamlines.

For programming purposes, using APL language, two matrices may be created with the number of rows equal to the number of streamlines and the number of columns equal to the number of normals with the numbers in one matrix corresponding to the r coordinates at points of streamline and normal intersection and numbers in the other matrix corresponding to the z coordinates at points of intersection.

With the foregoing, the values required for computing the parameters a and b can be computed if the value of the angle β is given at each point. Vane shapes vary greatly in pumps, with two styles apparently dominating. In one, the vane elements are approximately normal to the hub and shroud. In the other, the elements are approximately normal to a plane perpendicular to the axis of rotation. In all cases the vane angle must be defined at the points of intersection of the streamlines and normals in order to compute parameters a and b .

Once the parameters are known, equations (3) and (4) may be used to arrive at a solution by successively choosing values of q_0 at the hub, computing q for each stream tube and, by the trapezoidal rule, computing the overall weight flow. The iterative procedure for finding the q_0 that will produce the correct overall weight flow for each normal is as follows:

Velocities at the hub may be chosen which will encompass the velocity range, for example 0 to 100 ft/sec. Ten of these values in steps of ten may be inserted into the program to arrive at a weight flow for each value. These values are then compared to the designated weight

flow and the two velocities which encompass the designated weight flow are reinserted into the program. This routine is automatically repeated until the overall designated weight flow is matched. If the weight flows at all normals for individual stream tubes are not considered close enough, the streamlines may be redrawn and the routine repeated until the desired accuracy is achieved.

To compute Δp a seventh order polynomial representing a plot of u versus z may be derived for use in the equation (6) and an appropriate program written for the solution to the equation. For most pump impellers the blade force F_n may be neglected. Extreme care must be taken to assign the correct sign to values of β , r_c , and du/dz .

APPENDIX 3

A MATHEMATICAL MODEL FOR TWO-DIMENSIONAL, AXISYMMETRIC, INCOMPRESSIBLE, TURBULENT, VISCOUS FLOW IN ANNULAR DUCTS

by: John D. Stanitz

ABSTRACT

It has been established that variable prewhirl, produced by adjustable inlet guide vanes upstream from a pump, is an efficiency-effective means of controlling pump head under conditions of varying flow rate and/or rotational speed. However, many pumps cannot tolerate the blockage resulting from guide vanes in the inlet eye. It then becomes necessary to locate the vanes well upstream from the pump inlet, where an enlarged annular area can accommodate the guide vane blockage. This arrangement is attractive provided the prewhirl created by the vanes does not degenerate or distort in an undesired or unexpected manner, due to friction on the annular duct walls and turbulent mixing within the flow. An analysis has been made and a computer program REALAD developed to investigate the effect of viscous skin friction and turbulent mixing on the decay and/or distortion of the swirl flow in annular ducts, where the meridional configuration is such that the meridional streamline radii of curvature and slopes can be neglected. From the numerical results, it is concluded that for practical ratios of annular channel length to channel height, such as might be required if pump inlet guide vanes were located far enough upstream to avoid choking the pump inlet, the decay and/or distortion of the swirl flow is probably not serious, and pump inlets with variable inlet guide vanes can be developed for good pump performance. However, it is recommended that the analysis and computer program be extended to include the effects of meridional streamline curvature and slope, and that the approximation regarding wall shear stress be removed.

INTRODUCTION

It has been established that variable prewhirl, produced by adjustable inlet guide vanes upstream from the pump, is an efficiency-effective means of controlling pump head under conditions of varying flow rate and/or rotational speed. However, many pump designs cannot tolerate the blockage resulting from guide vanes in the inlet eye. It then becomes necessary to locate such vanes well upstream from the pump inlet, where an enlarged annular area can accommodate the guide vane blockage. This arrangement is attractive provided the prewhirl created by the vanes does not degenerate or distort in an undesired or unexpected manner, due to friction on the annular duct walls and turbulent mixing within the flow. The object of this study is to create a mathematical model for predicting at arbitrarily specified stations along the annular flow path the meridional and tangential velocity distributions for axisymmetric, turbulent, viscous, swirl flow.

ANALYSIS

In this analysis, the symmetrical, annular duct flow is treated as consisting of a finite number of contiguous annular filaments, within each of which flow conditions may vary in the direction of flow but are constant normal to that direction. These filaments are acted upon by "body forces" generated by viscous friction at the walls and by turbulent mixing between adjacent filaments within the flow.

FLOW GEOMETRY

The axisymmetric flow field is described in the meridional (z, r) plane of FIGURE 1. The duct, as shown, is perfectly general. However, as the result of a subsequent assumption, the geometry will become more restricted in this particular study.

ASSUMPTIONS

As already stated, flow conditions in any given filament are considered constant normal to the direction of flow. These conditions are, in addition

to the static pressure p , the meridional flow direction u and the velocity components c_m , c_r and c_z (shown in Figure 1), and the tangential velocity component c_θ (normal to the plane of Figure 1).

FIGURE 1. Axisymmetric Flow Field in the Meridional Plane.

The filament adjacent to each wall is assumed to be acted upon by a viscous skin friction force, which becomes part of the body force F acting on that filament. Also acting on each filament are forces due to the turbulent mixing between adjacent filaments. These momentum mixing forces are likewise part of the body force acting on each filament. In order to maintain the same flow rate within each filament, the exchange flow due to turbulence between adjacent filaments is assumed equal at any given location in the flow field.

This study has been further simplified by assuming negligible effect of meridional streamline radius of curvature and slope on the equation of motion, and by limiting the application of the study to cases in which the meridional velocity c_m is everywhere positive. The former assumption effectively restricts

the meridional configuration of the annular ducts to straight, or near-by straight, lines of constant, or approximately constant radius lines for both the hub and shroud. However, this assumption of negligible meridional streamline radius of curvature and slope is not crucial to a solution, and the curvature and slope terms are carried along in the general analysis. The fluid is incompressible.

CONTINUITY

Consider the flow rate ΔW_i in any fluid filament i at any point along its path. From FIGURE 2, the continuity condition gives

$$\Delta W_i = \rho c_{mi} \pi (r_{2i} + r_{1i}) \left(\frac{r_{2i} - r_{1i}}{\cos \mu_i} \right) \quad (1)$$

where ρ is the weight density of the incompressible fluid.

FIGURE 2. Flow Conditions in Any Fluid Filament i At Any Point Along Its Mean Path in the Meridional Plane.

If there are N filaments from hub to shroud, then from equation (1), the total flow rate W in the annular duct becomes

$$W = \sum_{i=1}^N \Delta W_i = 2\pi (r_s - r_H) \rho \sum_{i=1}^N \frac{c_{mi} r_{mi} k_i}{\cos \mu_i} \quad (2)$$

where $r_{mi} = \frac{1}{2} (r_{2i} + r_{1i})$ (2a)

and $k_i = \frac{r_{2i} - r_{1i}}{r_s - r_H}$ (2b)

in which r_s and r_H are the shroud and hub radii, respectively, along the normal.

Finally, dividing equation (2) by W gives

$$1.0 = \sum_{i=1}^N \frac{\Delta W_i}{W} = \sum_{i=1}^N \Delta \psi_i = \frac{2\pi (r_s - r_H)}{W/\rho} \sum_{i=1}^N \frac{c_{mi} r_{mi} k_{mi}}{\cos \mu_i} \quad (3a)$$

where

$$\Delta \psi_i = \frac{c_{mi} r_{mi} k_i}{\cos \mu_i} \cdot \frac{2\pi (r_s - r_H)}{W/\rho} \quad (3b)$$

where the stream function ψ is constant along streamlines.

EQUATIONS OF MOTION

The equations of motion for incompressible, axisymmetric flow are

$$\frac{-g}{\rho} \frac{\partial p}{\partial r} = -g F_r + c_z \frac{\partial c_r}{\partial z} + c_r \frac{\partial c_r}{\partial r} - \frac{c_\theta^2}{r} \quad (4a)$$

$$0 = -g F_\theta + c_z \frac{\partial c_\theta}{\partial z} + c_r \frac{\partial c_\theta}{\partial r} + \frac{c_r c_\theta}{r} \quad (4b)$$

$$\frac{-g}{\rho} \frac{\partial p}{\partial z} = -g F_z + c_z \frac{\partial c_z}{\partial z} + c_r \frac{\partial c_z}{\partial r} \quad (4c)$$

where subscripts r , θ and z indicate components in the r , θ and z directions, respectively, and F is the dimensionless, distributed body force (lb/lb)

resulting from skin friction at the walls and turbulent, momentum mixing within the flow.

From equation (4b),

$$g F_{\theta} = \frac{c_z}{r} \frac{\partial}{\partial z} (r c_{\theta}) + \frac{c_r}{r} \frac{\partial}{\partial r} (r c_{\theta}) \quad (5a)$$

where from Figure 1,

$$c_z = c_m \cos \mu = c_m \frac{dz}{dm} \quad (5b)$$

and

$$c_r = c_m \sin \mu = c_m \frac{dr}{dm} \quad (5c)$$

where m is meridional distance in the direction of flow. From equations (5a) through (5c),

$$g F_{\theta} = \frac{c_m}{r} \frac{d(rc_{\theta})}{dm} = \frac{1}{r} \frac{d(rc_{\theta})}{dt} \quad (5d)$$

Thus, the tangential component F_{θ} of the distributed body force results in the time rate of change of moment of tangential momentum (i.e., the time rate of change of swirl $r c_{\theta}$).

Next, in Figure 3, consider the properties of a streamline in the meridional plane.

FIGURE 3. Properties of a Streamline in the Meridional Plane.

The meridional component c_m of the velocity c is tangent to the streamline ψ , and thus to the elemental distance dm along the streamline. The meridional streamline radius of curvature r_c is normal to the streamline and thus also to the elemental distance dm ; it is defined by

$$r_c = \frac{dm}{d\mu} \quad (6)$$

Now, along a streamline, in the meridional plane

$$\frac{dc_r}{dm} = \frac{\partial c_r}{\partial z} \frac{dz}{dm} + \frac{\partial c_r}{\partial r} \frac{dr}{dm}$$

or, from equations (5b) and (5c),

$$c_m \frac{dc_r}{dm} = c_z \frac{\partial c_r}{\partial z} + c_r \frac{\partial c_r}{\partial r} \quad (7a)$$

Also, from equation (5c),

$$c_m \frac{dc_r}{dm} = c_m \frac{d(c_m \sin \mu)}{d\mu} \frac{d\mu}{dm}$$

or, from equation (6),

$$c_m \frac{dc_r}{dm} = \frac{c_m^2}{r_c} \cos \mu + c_m \sin \mu \frac{dc_m}{dm} \quad (7b)$$

Thus, from equations (4a), (7a) and (7b),

$$-\frac{g}{\rho} \frac{\partial p}{\partial r} = -gF_r + \frac{c_m^2}{r_c} \cos \mu + c_m \sin \mu \frac{dc_m}{dm} - \frac{c\theta^2}{r} \quad (7c)$$

In a similar fashion, it can be shown that

$$-\frac{g}{\rho} \frac{\partial p}{\partial z} = -gF_z - \frac{c_m^2}{r_c} \sin \mu + c_m \cos \mu \frac{dc_m}{dm} \quad (7d)$$

Along a meridional streamline the pressure gradient is given by

$$\frac{-g}{\rho} \frac{\partial p}{\partial m} = \frac{-g}{\rho} \frac{\partial p}{\partial z} \frac{dz}{dm} - \frac{g}{\rho} \frac{\partial p}{\partial r} \frac{dr}{dm}$$

or, from equations (5b) and (5c) and from (7c) and (7d),

$$\frac{-g}{\rho} \frac{\partial p}{\partial m} = -g (F_z \cos \mu + F_r \sin \mu) + c_m \frac{dc_m}{dm} - \frac{c_\theta^2}{r} \sin \mu \quad (7e)$$

As will be considered later, the meridional component F_m of the distributed body force F , due to skin friction and momentum mixing, is tangent to the meridional direction of flow. Thus,

$$F_z = F_m \cos \mu \quad (7f)$$

and

$$F_r = F_m \sin \mu \quad (7g)$$

so that, from equations (7e) through (7g),

$$\frac{-g}{\rho} \frac{\partial p}{\partial m} = -g F_m + c_m \frac{dc_m}{dm} - \frac{c_\theta^2}{r} \sin \mu \quad (7h)$$

Equation (7h) gives the pressure gradient in the direction of flow in the meridional plane.

Finally, consider the pressure gradient along any elemental distance dn in the general direction ζ , as shown in Figure 4, where

$$\frac{dz}{dn} = \cos \zeta \quad (8a)$$

$$\text{and } \frac{dr}{dn} = \sin \zeta \quad (8b)$$

The pressure gradient along dn is given by

$$\frac{-g}{\rho} \frac{\partial p}{\partial n} = \frac{-g}{\rho} \frac{\partial p}{\partial z} \frac{dz}{dn} - \frac{g}{\rho} \frac{\partial p}{\partial r} \frac{dr}{dn}$$

or, from equations (7c) and (7d), (7f) and (7g), and (8a) and (8b),

$$-\frac{g}{\rho} \frac{\partial p}{\partial n} = -g F_m \cos (\zeta - \mu) + \frac{c_m^2}{r_c} \sin (\zeta - \mu) + c_m \frac{dc_m}{dm} \cos (\zeta - \mu) - \frac{c_\theta^2}{r} \sin \zeta \quad (8c)$$

FIGURE 4. The Elemental Distance dm at Angle μ Along a Meridional Streamline, and an Elemental Distance dn at a General Angle ζ .

Provided F_m is tangent to the meridional streamline (equations (7f) and (7g), equation (8c) is perfectly general and embodies all possible pressure gradients in the meridional plane. For example, if $\zeta = \mu$, equation (8c) becomes equation (7h). Also, if ζ equals $\mu + 90^\circ$, the pressure gradient normal to the meridional streamline becomes

$$-\frac{g}{\rho} \frac{\partial p}{\partial n} = \frac{c_m^2}{r_c} - \frac{c_\theta^2}{r} \cos \mu \quad (8d)$$

For purposes of this study, which examines the decay of swirl rc_θ in an annular duct, it is assumed that

$$\mu \approx 0^\circ$$

$$r_c \longrightarrow \infty$$

and

$$(1) \quad \zeta \approx \mu \quad \text{so that } dn \approx dz$$

$$(2) \quad \zeta \approx \mu + 90^\circ \quad \text{so that } dn \approx dr$$

For these conditions, equation (8c), gives

$$(1) \quad -\frac{g}{\rho} \frac{\partial p}{\partial z} = -gF_m + c_m \frac{dc_m}{dz} \quad (8e)$$

$$(2) \quad -\frac{g}{\rho} \frac{\partial p}{\partial r} = -\frac{c_\theta^2}{r} \quad (8f)$$

Equations (8e) and (8f) are the two equations of motion used in this study.

MERIDIONAL BODY FORCE F_m

Figure 5 shows the meridional view of three contiguous annular filaments, one of which is adjacent to the inner duct wall

FIGURE 5. Flow Filaments in an Annular Duct.

Two types of body force act on the filament adjacent to the wall. One, the skin friction force acts opposite to the direction of flow and, assuming for convenience that the meridional component of c^2 , is given approximately by c_{m1}^2 , is given by

$$-c_f^* \frac{\rho c_{m1}^2}{2g} 2\pi r_H dz \quad (9a)$$

where c_f^* is considered a universal constant for the skin friction coefficient

based on the filament velocity c_{m1} . (Note, however, that c_f^* is a moderate function of the Reynolds number.)

The other force, the momentum mixing force, acts in the direction of flow, and results from the greater momentum of that fluid in the adjacent filament which is introduced into the wall filament by the turbulent mixing. The mixing rate per unit of area between the filaments is postulated to be

$$0.5 (\Delta\psi_1 + \Delta\psi_2) \frac{W k_T}{\pi (r_s^2 - r_H^2)} \quad (9b)$$

where k_T is considered a universal constant for turbulent mixing, and where $0.5 (\Delta\psi_1 + \Delta\psi_2) W$ is the average flow rate in the two filaments. (For further discussion on the universality of the constants k_T and c_f^* , see page 18.)

$$0.5 (\Delta\psi_1 + \Delta\psi_2) \frac{W k_T}{g} \frac{(c_{m2} - c_{m1})}{\pi (r_s^2 - r_H^2)} 2\pi (r_{m1} + 0.5(r_s - r_H) k_1) dz \quad (9c)$$

where, from equations (2a) and (2b), $2\pi(r_{m1} + 0.5(r_s - r_H)k_1)dz$ is the cylindrical area between the two adjacent filaments.

By definition, the distributed body force F_m is equal to the sum of the two forces (9a) and (9c) divided by the weight of fluid $\rho 2\pi r_{m1} k_1 (r_s - r_H) dz$, where k_1 is defined by equation (2b). Thus,

$$gF_{m1} = \frac{-c_f^* c_{m1}^2 r_H + (\Delta\psi_1 + \Delta\psi_2) k_T (c_{m2} - c_{m1}) (r_{m1} + 0.5(r_s - r_H) k_1) \frac{W}{\rho \pi (r_s^2 - r_H^2)}}{2r_{m1} k_1 (r_s - r_H)} \quad (9d)$$

Equation (9d) gives the distributed body force F_m for the flow filament adjacent to the inner wall. Similarly, for the flow filament N adjacent to the outer wall, the meridional component $F_{m,n}$ of the body force becomes

$$gF_{mn} = \frac{-c_f^* c_{m,n}^2 r_s - (\Delta\psi_{n-1} + \Delta\psi_n) k_T (c_{m,n} - c_{m,n-1} - 0.5(r_s - r_H) k_n) \frac{W}{\rho \pi (r_s^2 - r_H^2)}}{2 r_{m,n} k_n (r_s - r_H)} \quad (9e)$$

For filaments away from the walls, two body forces arise from turbulent

mixing with the two adjacent filaments. These forces, which act parallel to the filament flow in the meridional plane, are positive or negative, if the meridional velocity of the adjacent filament is greater or less, respectively. Thus, for any interior filament i , the meridional component F_{mi} of the body force becomes, from equation 9(c), etc.,

$$gF_{mi} = \frac{k_T W}{2r_{mi} k_i (r_s - r_H) \rho \pi (r_s^2 - r_H^2)} \left[(\Delta\psi_i + \Delta\psi_{i+1})(c_{m,i+1} - c_{mi})(r_{mi} + 0.5(r_s - r_H)k_i) - (\Delta\psi_i + \Delta\psi_{i-1})(c_{mi} - c_{m,i-1})(r_{mi} - 0.5(r_s - r_H)k_i) \right] \quad (9f)$$

Equations (9d), (9e) and (9f) determine the meridional component F_m of the distributed body force, which component is required for the solution of equation (8e) in finite difference form.

TANGENTIAL BODY FORCE F_θ .

In the $r\theta$ -plane, the same types of body force exist except that formulations are based on the tangential velocities c_θ , as opposed to the meridional velocities c_m used in the meridional plane. (Note here that, as was similarly done for the meridional component of the wall shear stress, the tangential component of c_1^2 is assumed, for convenience, to be given approximately by $c_{\theta 1}^2$.) Thus along the inner wall,

$$gF_{\theta 1} = \frac{-c_f^* c_{\theta 1}^2 r_H + (\Delta\psi_1 + \Delta\psi_2) k_T (c_{\theta 2} - c_{\theta 1})(r_m + 0.5(r_s - r_H)k_1) \frac{W}{\rho \pi (r_s^2 - r_H^2)}}{2r_{m1} k_1 (r_s - r_H)} \quad (10a)$$

Along the outer wall,

$$gF_{\theta n} = \frac{-c_f^* c_{\theta n}^2 r_s - (\Delta\psi_{n-1} + \Delta\psi_n) k_T (c_{\theta n} - c_{\theta n-1})(r_{mn} - 0.5(r_s - r_H)k_n) \frac{W}{\rho \pi (r_s^2 - r_H^2)}}{2r_{mn} k_n (r_s - r_H)} \quad (10b)$$

And, for any interior filament i ,

$$gF_{\theta i} = \frac{k_T W}{2r_{mi} k_i (r_s - r_H) \rho \pi (r_s^2 - r_H^2)} \left[(\Delta\psi_i + \Delta\psi_{i+1})(c_{\theta,i+1} - c_{\theta i})(r_{mi} + 0.5(r_s - r_H)k_i) \right]$$

$$-(\Delta\psi_i + \Delta\psi_{i-1})(c_{\theta i} - c_{\theta, i-1})(r_{mi} - 0.5(r_s - r_H)k_i) \quad (10c)$$

Equations (10a), (10b) and (10c) determine the tangential component F_θ of the distributed body force, which component is required for the solution of equation (5d) in finite difference form.

COMPUTER PROGRAM

A computer program REALAD, based on the foregoing analysis, has been written in FORTRAN V. It is described herein briefly, and a listing is given at the end of this appendix.

NUMERICAL PROCEDURE

The annular duct is described by the hub radius $RH(J)$, the shroud radius $RS(J)$ and the axial location $Z(J)$, where J goes from 1 at the inlet station to M at the exit station (maximum value of M is 50). The annulus is divided into N filaments (maximum value of N is 20). At the inlet station 1, the flow is described by the meridional velocity ratio $VR(I)$, the maximum value of which must be 1.0, the swirl angle $BD(I)$ and the percent $DPSI(I)$ of total flow for each filament, where I goes from 1 for the hub filament to N for the shroud filament. Other inputs are the flow rate W , the fluid density $RH\phi$, the universal turbulent mixing coefficient AKT , the universal skin friction coefficient $CFSTR$, the iteration accuracy EPS and the inlet mass-averaged static pressure $PSAV$.

Assuming all flow conditions are known at station $J-1$, the flow conditions at the next station J are determined as follows:

- (1) Assume values for the meridional and tangential components of the body force, $FZ(I, J)$ and $FT(I, J)$, respectively, at all values of I from 1 to N . For example, use $FZ(I, J-1)$ and $FT(I, J-1)$.
- (2) Assume a value for the meridional velocity $V(1, J)$ of the hub filament. For example, use $V(1, J-1)$.
- (3) Compute $R2(1, J)$ from equation (1), where μ_i is 0.0, and ΔW_i is equal to $W * DPSI(1)$.
- (4) Compute $RM(1, J)$ and $AK(1, J)$ from equations (2a) and (2b), respectively.

- (5) Compute the tangential velocity $U(1,J)$ for the hub filament from equation (5d) in finite difference form.
- (6) Compute the static pressure $PS(1,J)$ for the hub filament from equation (8e) in finite difference form.
- (7) Assume a value for the meridional velocity $V(2,J)$ for the next (adjacent) filament. For example, use $V(2,J-1)$.
- (8) Compute $R2(2,J)$ from equation (1), where μ_j is 0.0, and ΔW_j is equal to $W^* \text{ DPSI}(2)$.
- (9) Compute $RM(2,J)$ and $AK(2,J)$ from equations (2a) and (2b), respectively.
- (10) Compute the tangential velocity $U(2,J)$ from equation (5d) in finite difference form.
- (11) Compute the static pressure $PS(2,J)$ from equation (8f) in finite form.
- (12) Compute the meridional velocity $V(2,J)$ from equation (8e) in finite difference form.
- (13) Repeat (8) through (12) until the value of $V(2,J)$ converges.
- (14) Repeat (7) through (13) for each successive filament until a value for the radius $R2(N,J)$ of the shroud filament has been established.
- (15) Estimate a new value for the meridional velocity $V(1,J)$ of the hub filament, and repeat (3) through (14) until the value of $R2(N,J)$ equals $RS(J)$.
- (16) Compute new values of $FZ(I,J)$ and $FT(I,J)$ at all values of I from 1 to N , using equations 9(d), (e) and (f), and 10 (a), (b) and (c), respectively.
- (17) Repeat (2) through (16) until the value of $V(1,J)$ in (15) repeats for two successive cycles.

Flow conditions have now been established at axial station J . The entire cycle (1) through (17) is repeated at successive axial stations until the complete flow field in the annular duct has been established.

INPUT

The various INPUT required for Program REALAD have been generally described at the beginning of this section on Computer Program. However, nothing has been stated in regard to the numerical values for: (1) the universal turbulent mixing coefficient AKT, (2) the universal skin friction coefficient CFSTR, and (3) the universal distribution of the N values of incremental stream function DPSI (I). (Here the word "universal" has been emphasized, because it is the concept of this analysis that the values of these quantities, once established, remain constant for all geometries and flows. Thus, for example, when flow decelerates in a diffusing annulus, the local skin friction coefficient c_f , based on the local average duct velocity, will decrease. The universal skin friction coefficient CFSTR, however, is constant, because, although the viscous shear stress at the wall decreases, this decrease is accounted for by the decreased meridional velocity V (1) in the stream filament adjacent to the wall; and it is this decreased velocity to which CFSTR applies. Of course, CFSTR must vary moderately with the Reynolds number, but it is assumed not to vary with various flow geometries at the same Reynolds number.)

To establish appropriate values for AKT, CFSTR and DPSI (I), a series of solutions using Program REALAD were obtained for an annular geometry in which the hub radii R_H (J) were held constant at 100 inches, and the shroud radii R_S (J) were held constant at 101 inches. In effect, the problem was reduced to pressure flow between parallel flat plates, for which geometry, experimental results are well known. A fixed arbitrary set of flow conditions were prescribed at the channel inlet, and solutions, using REALAD, were obtained at each of 36 axial stations (J equals 2 through 37) located downstream from the inlet (J=1) and spaced 0.5 inches apart. At the exit station, flow conditions were essentially stabilized at their steady state values. The inputs AKT, CFSTR and DPSI (I) were changed from solution to solution until reasonably good agreement was obtained with known experimental results for turbulent pressure flow between parallel planes. The final results are shown in Figure 6, where the velocity distribution calculated by Program REALAD is compared with a velocity distribution based on the 1/7th power law. It is seen that the comparison is reasonably good, considering the approximate (but convenient) nature of the mathematical model. (Also, in this final solution, the

value of the skin friction coefficient CF (as usually defined) was found by equation (12) to be 0.0040.) For this solution

$$AKT = 0.500 \quad (11a)$$

$$CFSTR = 0.010 \quad (11b)$$

and the following 20 values of DPSI (I) were used:

I	DPSI (I)
1*	0.010
2	0.040
3	0.045
4	0.050
5	0.055
6	0.060
7	↓
8	
9	
10	
11	
12	
13	
14	
15	0.060
16	0.055
17	0.050
18	0.045
19	0.040
20**	<u>0.010</u>
TOTAL	1.000

(11c)

* flow filament at inner wall of annulus

** flow filament at outer wall of annulus

It is noted that the DPSI (I) values are symmetrical about the mean radius of the annulus, and that the sum of these values for the 20 filaments is 1.0. Although AKT, CFSTR and DPSI (I) have been left as inputs to the

program, it is recommended that they not be changed from the values given by (11a) through (11c) without first investigating the effect of different values on the solution for pressure flow between "parallel" flat plates, as described just previously. (There is a balance among values of AKT, CFSTR and DPSI (I) for which realistic solutions result.)

The INPUT to the program is described as follows:

CARD NO. 1

W, AN, AM, $RH\phi$, AKT, CFSTR, EPS, PSAV FORMAT (F10.3, 2F10.1, F10.4, F10.3, F10.4, F10.6, F10.2)

W	total flow rate in annulus (lb/sec)
AN	number of stream filaments (maximum value is 20.0), recommended value is 20.0 (to match (11c))
AM	number of axial stations (maximum value is 50.0)
$RH\phi$	fluid weight density (lb/ft ³)
AKT	universal turbulent mixing coefficient (see (11a), which is the recommended value)
CFSTR	universal skin friction coefficient (see (11b), which is the recommended value)
EPS	iteration accuracy (recommended value is 0.000005)
PSAV	mass-weighted average static pressure at the inlet station (psig)

CARD TYPE NO. 2

RH (J), J=1, M where M is integer value of AM
FORMAT (8F10.3)

RH (J) hub radius (ins) of annulus (see Figure 5). The variation (if any) in RH (J) with Z (J) should be such that the meridional radius of curvature of the hub is large enough that the meridional curvature term in the equations of motion can be neglected.

CARD TYPE NO. 3

RS (J), J=1, M where M is integer value of AM
FORMAT (8F10.3)

RS (J) shroud radius (ins) of annulus (see Figure 5). The variation (if any) in RS (J) with Z (J) should be such that the meridional

radius of curvature of the shroud is large enough (see RH (J), above). The input values of RH (J) and RS (J), taken together, should be such that their mean values as a function of Z (J) are roughly constant.

CARD TYPE NO. 4

Z (J), J=1,M where M is integer value of AM

FORMAT (8F10.3)

Z (J) axial location (ins) of stations. (It is recommended that the increase in Z from station to station be no greater than approximately half the difference between RS (J) and RH (J).)

CARD TYPE NO. 5

DPSI (I), I=1,N where N is integer value of AN

FORMAT (8F10.3)

DPSI (I) incremental values of stream function for flow filaments (see (11c), which are the recommended values). The sum of the N values of DPSI (I) must equal 1.0.

CARD TYPE NO. 6

BD (I), I=1,N where N is integer value of AN

FORMAT (8F10.3)

BD (I) swirl angle (degrees), measured from the axial direction, at the inlet station J=1.

CARD TYPE NO. 7

VR (I), I=1,N where N is integer value of AN

FORMAT (8F10.3)

VR (I) ratio of meridional velocity V(I, 1) to the maximum value of the meridional velocity at the inlet station J=1.

This completes the input data.

OUTPUT

First, all input data, as described just previously, is printed out. Then for each of M axial stations (i.e., for J=1, M), the following output information is given: The station number J, the axial location Z, the hub radius RH, the shroud radius RS, the average static pressure PS-AV, the average total pressure PT-AV, and the skin friction coefficient CF at the

inner wall. This skin friction coefficient is related to the universal skin friction coefficient CFSTR by

$$CF \times V_{av}^2 = CFSTR \times V(1,J)^2 \quad (12)$$

where V_{av}^2 is the one-dimensional, average meridional velocity at station J, and $V(1,J)$ is the meridional velocity of the flow filament adjacent to the inner wall.

Also, at each station J for each flow filament I, the following output information is given: the filament number I, the ratio K (equation 2b), the incremental stream function DPSI(I) for the filament (input), the mean radius RM of the filament, the inner radius R1 of the filament (figure 5), the outer radius R2 of the filament (figure 5), the distance Y of the mean radius of the filament from the inner wall, the meridional velocity V, the tangential swirl velocity U, the static pressure, the total pressure, the ratio of Y to the channel width (RS-RH), and the ratio of the meridional velocity V(I,J) to the maximum meridional velocity V-MAX at axial station J.

This completes the description of the OUTPUT information. Several sample pages of printout are given following the program listing at the end of this Appendix.

RESULTS

Program REALAD has been used to study the decay and/or distortion due to viscous skin friction and turbulent mixing of the swirl flow in an annular duct of constant inner wall radius (10 ins) and constant outer wall radius (15 ins). Flow conditions were determined every 2.5 inches in the axial direction starting from 0.0 at the inlet to 40.0 inches at the exit. Two types of inlet swirl were investigated: (1) a symmetrical swirl, in which the inlet tangential velocity made a constant angle (45 degrees) with the meridional velocity for all 20 stream filaments, and (2) a skewed swirl, in which the inlet swirl angle varied linearly with filament number (I) from 20.0 degrees at the inner filament (I=1) to 67.5 degrees at the outer filament (I=20).

The results are plotted in figures 7 and 8, which compare the exit tangential velocity distribution at Z equals 40.4 inches with the inlet tangential velocity distribution at Z equals 0.0 inches. For symmetrical inlet swirl, figure 7, the decay and/or distortion of the tangential velocity is minimal. For skewed inlet swirl, figure 8, there is a moderate amount of distortion, but it is not serious considering the 8:1 ratio of channel length (40 ins) to annulus height (5 ins).

It is concluded that for practical ratios of annular channel length to channel height, such as might be required if pump inlet guide vanes were located far enough upstream to avoid choking the pump inlet, the decay and/or distortion of the swirl flow is probably not serious, and pump inlets with variable inlet guide can be developed for good pump performance. In this regard, however, it should be noted that the approximations whereby C_{m1}^2 and $C_{\theta 1}^2$ are the meridional and tangential components, respectively, of C_1^2 in the wall shear stress formulations (made in equation (9a), for example) understate the effects of skin friction when swirl exists, and therefore, the decay and/or distortion of the swirl flow is somewhat greater than indicated in figures 7 and 8.

It is probable that significant streamline curvature and slope in the meridional plane will have additional detrimental influence on the decay and/or distortion of the swirl flow, and it is therefore recommended that the analysis presented here be extended to include the effects of meridional streamline curvature and slope, as well as exact formulations for the meridional and tangential components of the wall shear stress.

SUMMARY AND CONCLUSIONS

An analysis has been made and a computer program REALAD developed to investigate the effect of viscous skin friction and turbulent mixing on the decay and/or distortion of the swirl flow in annular ducts for which the meridional configuration is such that the meridional streamline radii of curvature and slopes can be neglected. From the numerical results, it is concluded that for practical ratios of annular channel length to channel height, such as might be required if pump inlet guide vanes were located far enough upstream to avoid choking the pump inlet, the decay and/or distortion of the swirl flow is probably not serious, and pump inlets with variable in-

let guide vanes can be developed for good pump performance.

It is probable that significant streamline curvature and slope in the meridional plane will have additional detrimental influence on the decay and/or distortion of the swirl flow, and it is therefore recommended that the analysis presented here be extended to include the effects of meridional streamline curvature and slope, as well as exact formulations for the meridional and tangential components of the wall shear stress, which have been understated.

FIGURE 6. Comparison of Stabilized Meridional Velocity Ratio with 1/7th Power Law as Functions of Distance From Wall to Channel Height.

FIGURE 7. Tangential Velocity Distribution as Function of Percent of Channel Width for Symmetrical Inlet Swirl as Determined by Program REALAD.

FIGURE 8. Tangential Velocity Distribution as Function of Percent of Channel Width for Skewed Inlet Swirl as Determined by Program REALAD.