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DEFORMATION OF THE CANDIA METALLIC
RING SEAL TEST FIXTURE*

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ABSTRACT

A fixture being used at Candia Laboratories Albuquerque for testing a metallic ring-type seal is described and analyzed. Formulas for the deformation of the fixture are derived based on a simple mathematical model. Numerical results are presented for a set of parameters typical of an actual test.

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Introduction

In the course of developing a design for a shipping cask to transport spent fuel from an LMFBR, Sandia Laboratories Albuquerque has become involved in seal testing. The particular system being investigated consists of a thin metal seal ring which is located between the main cask body and an end cap. Applying sufficiently large clamping pressures crushes the seal ring and forms the seal. The fixture being used for the testing is shown in Fig. 1. Three jack screw machines apply the clamping load to brackets which transfer the force to a massive hub or edge ring. The hub has an inner radius of 368 mm (14.5 in.), a wall thickness of 76 mm (3 in.) and its height is 279 mm (11 in.). Using jack screws rather than many bolts greatly simplifies measuring the total clamping force. Since the edge ring is loaded excentrically through the brackets, a large torque is applied at the loading points and because the loads are applied at only three locations there is a considerable span between load points. Both of these factors tend to encourage deformation in the edge ring and if the displacements in the edge ring become large enough, the seal performance can be degraded. During preliminary testing signs of deformation in the edge ring were observed and this motivated the structural analysis presented here.

Formulation

For purposes of this analysis the edge ring is considered alone. Any stiffening effect due to the curved end cap that bolts to the edge ring is ignored. The problem to be analyzed is the edge ring loaded at three locations by equal, excentrically applied downward forces. The edge ring is kept in equilibrium by a distributed pressure from the seal ring which is applied at the inner edge of the bottom face of the ring. For

simplicity the clamping pressure distribution is assumed to be uniform around the circumference. This is the pressure distribution that would result if the edge ring were rigid.

To simplify the analysis, the problem is broken up into three simpler problems. Figure 2 shows a ring cross section and the distribution of loads applied in each of the three problems. Summing the three solutions gives the solution to the original problem. The first is three concentrated excentrically applied loads that are reacted by equal and opposite concentrated loads applied at the center of the bottom face of the

FIG. 2

ring. The second problem considers the ring loaded by three forces, equal, opposite and colinear with the reacting forces in the first problem. Equilibrium is maintained by an upward line load on the center bottom face of the ring which is uniformly distributed around the circumference. Finally the third problem consists of two line loads, uniform around the circumference and applied on the bottom face of the ring. One is applied at the center bottom face of the ring and is equal and opposite to the line load of problem 2 and the other is upward on the inner face of the ring.

There are two approaches that could be used in analyzing these problems, idealizing the end ring as a short shell where the loads are applied in the direction of the shell axis, or as a very stocky ring subjected to out-of-plane loads. The second approach is simpler and is the one used here even though it is probably less accurate. In general, rings loaded out of their planes are indeterminate but the symmetry conditions of the present problems make them determinate. That is, the stress resultants in the ring can be determined from symmetry and equilibrium conditions alone. These stress resultants along with the symmetry conditions and elastic bar theory are sufficient to determine the deflections in the edge ring.

Displacements in a Curved Bar

Let bar ab be deformed and displaced. The displacement at any point in the bar ξ , can be calculated using the displacement and rotation at a and the change in shape of the bar between a and b .

FIG. 3

The notation used for displacement calculation is

$\vec{\Delta}(\xi)$ is the displacement at point ξ
 $\vec{\omega}$ is a vector representing the rotation of a
 cross section with respect to its initial position
 $\frac{d\vec{\omega}}{ds}$ is the rate of change of cross sectional rotation
 per unit of arc length. For a rigid body $\frac{d\vec{\omega}}{ds} = 0$

The direction of $\vec{\omega}$ is determined from the actual rotation by the right hand rule. For example, in the cantilevered beam shown in Fig. 4 the direction of $\vec{\omega}$ at b is into the paper.

FIG. 4

In this notation the displacement at any point ξ on the bar can be written

$$\vec{\Delta}(\xi) = \vec{\Delta}(a) + \vec{\omega}(a) \times \vec{\rho}_{a\xi} + \int_a^\xi \frac{d\vec{\omega}}{ds} \times \vec{\rho}_{s\xi} ds \quad (1)$$

Equation (1) includes the effects of flexure and torsion but neglects the contribution of shortening of the bar axis. Thus, for example, Eq. (1) will not give the displacement of an axially loaded bar prior to buckling. In the case of a ring loaded by forces normal to its plane the radial component of $\frac{d\vec{\omega}}{ds}$ is proportional to the bending moment and the circumferential component is proportional to the torque.

Solution

For the analysis that follows the vertical or out-of-plane deflection at the load points is taken as the datum, i.e., set to 0, and the deflections are calculated relative to it. To calculate displacements all the terms in Eq. (1) must be specified. Using coordinates defined in Fig. 5, the $\vec{\rho}$ vectors can be written

FIG. 5

$$\vec{\rho}_{a\xi} = (\cos \theta - 1) \vec{i} + \sin \theta \vec{j} \quad (2a)$$

and

$$\vec{\rho}_{s\xi} = (\cos \theta - \cos \psi) \vec{i} + (\sin \theta - \sin \psi) \vec{j} \quad (2b)$$

where $\vec{i}$ and $\vec{j}$ are unit vectors in the $\theta = 0$ and $\theta = \frac{\pi}{2}$ directions. From simple elastic bar theory $\frac{d\omega}{ds}$ can be written in terms of the bending moment, m , and torque, t , at point s .

$$\frac{d\omega}{ds} = \frac{-m}{EI} \vec{e}_r + \frac{t}{JG} \vec{e}_\theta \quad (3)$$

where $\vec{e}_r$ and $\vec{e}_\theta$ are unit vectors in the radial and circumferential directions at s and EI and JG are the standard flexural and torsional rigidity terms from elastic bar theory. The minus sign in Eq. (3) is due to the sign convention adopted for m .

The positive senses for m and t are as shown below.

FIG. 6

If expressions for m , t and $\vec{\omega}_a$ in terms of the applied forces can be found, then these along with Eqs. (2) and (3) can be substituted into Eq. (1) and the displacements calculated.

As an example of how m and t are calculated, consider Problem 1, and in particular, a free body of one-third of the ring, $0 - \epsilon < \theta < \frac{2\pi}{3} - \epsilon$, where ϵ is a very small angle.

FIG. 7

The couples of the applied forces of Problem 1 are replaced by their resultant torques, $T = Fd$, applied at the load points. From symmetry the moments and torques are the same at a and b . Summing moments on the free body and setting the result in the x and y directions equal to 0 gives two equations for the unknowns m and t . The solution is

$$m_a = m_b = T/2\sqrt{3} \quad (4a)$$

$$t_a = t_b = T/2 \quad (4b)$$

To calculate $m(\theta)$ and $t(\theta)$ consider a free body from point a to some intermediate point c (see Fig. 7). Again summing moments and setting the radial and circumferential components at c equal to 0 gives the desired result;

$$m(\theta) = T \left[(\cos \theta)/\sqrt{3} + \sin \theta \right] / 2 \quad (5a)$$

$$t(\theta) = T \left[(\sin \theta)/\sqrt{3} - \cos \theta \right] / 2 \quad (5b)$$

If $\vec{\omega}_a$ can be found everything needed to use Eq. (1) will be known. From symmetry $\vec{\omega}_b$ is equal to $\vec{\omega}_a$ except for a rotation of $2\pi/3$ (120°). The difference between the rotations at a and b is given by

$$\vec{\omega}_b - \vec{\omega}_a = \int_a^b \frac{d\vec{\omega}}{ds} ds = \int_0^{2\pi/3} \left(\frac{-m}{EI} \vec{e}_r + \frac{t}{JG} \vec{e}_\theta \right) R d\theta \quad (6)$$

The right hand side of Eq. (6) can be evaluated since m and t are known and

$$\vec{e}_r = \cos \theta \vec{i} + \sin \theta \vec{j} \quad (7a)$$

$$\vec{e}_\theta = -\sin \theta \vec{i} + \cos \theta \vec{j} \quad (7b)$$

In graphical form Eq. (6) is

FIG. 8

With $\vec{\omega}_b - \vec{\omega}_a$ known, $|\vec{\omega}_a|$, which is $|\vec{\omega}_b - \vec{\omega}_a|/\sqrt{3}$, can be calculated. Also the direction of $\vec{\omega}_a$ is a $-2\pi/3$ rotation from the direction of $\vec{\omega}_b - \vec{\omega}_a$. For problem 1 the result is

$$\vec{\omega}_a = \text{TR}^2 \left[\frac{1}{EI} \left(\frac{\pi}{9} + \frac{1}{4\sqrt{3}} \right) + \frac{1}{JG} \left(\frac{\pi}{9} - \frac{1}{4\sqrt{3}} \right) \right] \vec{j} \quad (8)$$

That is, the rotation at the load point "a" is a twist only.

Substituting Eqs. (2), (3) and (8) into Eq. (1) and evaluating the result yields an out-of-plane displacement given by

$$\begin{aligned} \Delta_1(\theta) = & \frac{-\text{TR}^2}{2EI} \left\{ \sin \theta \left[\frac{-\theta}{2\sqrt{3}} + \frac{\sin 2\theta}{4\sqrt{3}} + \frac{1-\cos 2\theta}{4} \right] \right. \\ & + \cos \theta \left[\frac{-1 + \cos 2\theta}{4\sqrt{3}} - \frac{\theta}{2} + \frac{\sin 2\theta}{4} \right] \Bigg\} \\ & + \frac{\text{TR}^2}{2JG} \left\{ \sin \theta \left[\frac{-\theta}{2\sqrt{3}} + \frac{\sin 2\theta}{4\sqrt{3}} + \frac{1-\cos 2\theta}{4} - 1 \right] \right. \\ & + \cos \theta \left[\frac{-1 + \cos 2\theta}{4\sqrt{3}} - \frac{1}{\sqrt{3}} + \frac{\theta}{2} + \frac{\sin 2\theta}{4} \right] + \frac{1}{\sqrt{3}} \Bigg\} \end{aligned} \quad (9)$$

$$+ \frac{TR^2}{EI} \left(1 - \cos \theta \right) \left[\frac{\pi}{9} + \frac{1}{4\sqrt{3}} + \frac{EI}{JG} \left(\frac{\pi}{9} - \frac{1}{4\sqrt{3}} \right) \right]$$

and in particular at midspan

$$\Delta_1 \left(\frac{\pi}{3} \right) = \frac{TR^2}{2} \left(\frac{1}{EI} + \frac{1}{JG} \right) \left(\frac{\pi}{9} - \frac{1}{2\sqrt{3}} \right) \quad (10)$$

Performing a similar analysis for problem 2 gives

$$\begin{aligned} \Delta_2(\theta) = & FR^3 \left\{ \left(\frac{1}{EI} + \frac{1}{JG} \right) \left(\frac{1}{4\sqrt{3}} + \frac{\pi}{9} - \frac{3}{2\pi} \right) (1 - \cos \theta) \right. \\ & - \frac{1}{2EI} \left[\sin \theta \left(\frac{\theta}{2\sqrt{3}} + \frac{\sin 2\theta}{4\sqrt{3}} + \frac{1 - \cos 2\theta}{4} - \frac{3}{\pi} \sin \theta \right) \right. \\ & \left. \left. - \cos \theta \left(\frac{1 - \cos 2\theta}{4\sqrt{3}} + \frac{\theta}{2} - \frac{\sin 2\theta}{4} - \frac{3}{\pi} + \frac{3}{\pi} \cos \theta \right) \right] \right\} \\ & + \frac{1}{2JG} \left[-\sin \theta \left(\frac{\theta}{2\sqrt{3}} - \frac{\sin 2\theta}{4\sqrt{3}} + \frac{3 + \cos 2\theta}{4} \right. \right. \\ & \left. \left. - \cos \theta - \frac{3}{\pi} \sin \theta + \frac{3\theta}{\pi} \cos \theta \right) - \cos \theta \left(\frac{1 - \cos 2\theta}{4\sqrt{3}} - \frac{\theta}{2} \right. \right. \\ & \left. \left. - \frac{\sin 2\theta}{4} + \sin \theta + \frac{3}{\pi} - \frac{3}{\pi} \cos \theta - \frac{3\theta}{\pi} \sin \theta \right) \right. \\ & \left. \left. + \frac{1 - \cos \theta}{\sqrt{3}} - \sin \theta + \theta - \frac{3\theta^2}{2\pi} \right] \right\} \quad (11) \end{aligned}$$

and at midspan

$$\begin{aligned} \Delta_2(\pi/3) = & \frac{FR^3}{2} \left\{ \left(\frac{1}{EI} + \frac{1}{JG} \right) \left(\frac{1}{4\sqrt{3}} + \frac{\pi}{9} - \frac{3}{2\pi} \right) + \frac{1}{EI} \left(\frac{3}{\pi} - \frac{\sqrt{3}}{4} \right) \right. \\ & \left. + \frac{1}{JG} \left(\frac{3}{2\pi} + \frac{\pi}{6} - \frac{7\sqrt{3}}{12} \right) \right\} \quad (12) \end{aligned}$$

Problem 3 is equivalent to a uniformly distributed torque applied to the ring and from symmetry this loading gives no out-of-plane deflection.

The details of the computations giving the above results are quite tedious but not at all difficult. Some additional details of the analysis are given in the appendix. The appropriate parameters for the Sandia seal test fixture are

$EI = 72.5 \times 10^8 \text{ lb-in}^2$	$5.0 \times 10^{13} \text{ Pa}$
$JG = 8.16 \times 10^8 \text{ lb-in}^2$	$5.6 \times 10^{12} \text{ Pa}$
$F = 18,000 \text{ lb}$	$80,000 \text{ N}$
$R = 16.25 \text{ in}$	0.413 m
$T = F \times d = 126,000 \text{ lb-in}$	$14,200 \text{ N-m}$

and $\Delta(\pi/3) = \Delta_1(\pi/3) + \Delta_2(\pi/3)$

$$= 0.035 \text{ mm } (0.00137 \text{ in}) + 0.031 \text{ mm } (0.00124 \text{ in})$$

$$= 0.066 \text{ mm } (2.61 \times 10^{-3} \text{ in})$$

This displacement is an overestimate of the actual displacement for several reasons. First, the added stiffness of the end cap has been neglected. Also the assumed uniform distribution of pressure in problems 2 and 3 is not accurate and the actual pressure is greatest close to the load points. Finally, the load brackets apply the torque over a wide area and this reduces the length of span between the load points.

APPENDIX

Problem 1; three torques on a ring.

Because there is no net out-of-plane force applied to the ring, the shear is zero everywhere. Consider the free body shown in Fig. 7. Summing moments on the free body "ab" and taking the component in the x direction gives

$$m - \sqrt{3}t/2 + m/2 = 0 \quad (A1)$$

and in the y direction gives

$$T - t - t/2 - \sqrt{3}m/2 = 0 \quad (A2)$$

Solving for m and t gives the results shown in Eq. (4).

The bending moment and torque as functions of θ are derived by considering the moment equilibrium of free body "ac" in Fig. 7.

$$T/2 \hat{j} + \frac{T}{2\sqrt{3}} \hat{i} - m(\theta) \vec{e}_r + t(\theta) \vec{e}_\theta = 0 \quad (A3)$$

where $\vec{e}_r$ and $\vec{e}_\theta$ are defined by Eq. (7). Taking dot products of (A3) with $\vec{e}_r$ and $\vec{e}_\theta$ gives the results shown in Eq. (5). The remainder of the solution is straightforward and described in the text. The calculations are tedious but not complicated and the end result is given in Eq. (9).

Problem 2; out-of-plane concentrated and line loads.

From symmetry considerations the bending moment, torque, and the magnitude of rotation are the same at all concentrated load points. Also, the magnitude of the shear is 1/2 that of the concentrated force at these points.

FIG. 9

The bending moment and torque are found by considering the free body acb ($0 + \epsilon < \theta < \frac{2\pi}{3} - \epsilon$)

FIG. 10

Summing moments about b gives

$$\begin{aligned}
 & M \left[\left(1 + \frac{1}{2}\right) \vec{i} - \frac{\sqrt{3}}{2} \vec{j} \right] + T \left[-\frac{\sqrt{3}}{2} \vec{i} - \left(1 + \frac{1}{2}\right) \vec{j} \right] \\
 & + \vec{p}_{ba} \times \frac{P}{2} \vec{k} + \int_0^{\frac{2\pi}{3}} \left[\vec{p}_{b\theta} \times p \vec{k} \right] R d\theta = 0
 \end{aligned} \tag{A4}$$

where $\vec{k}$ is a unit vector pointing out of the paper and

$$\vec{p}_{b\theta} = \left(\cos \theta + \frac{1}{2}\right)\vec{i} + \left(\sin \theta - \frac{\sqrt{3}}{2}\right)\vec{j} \quad (A5)$$

Equilibrium of forces in the out-of-plane direction gives

$$F - 2\pi R p/3 = 0 \quad \text{or } p = 3F/2\pi R \quad (A6)$$

Substituting Eqs. (A5) and (A6) into (A4) gives a vector equation that decomposes into two scalar equations for M and T

$$3M/2 - \sqrt{3}T/2 = FR(\sqrt{3} - 9/\pi)/4 \quad \text{and,} \quad (A7)$$

$$-\sqrt{3}M/2 - 3T/2 = FR(3\sqrt{3}/\pi - 1)/4 \quad (A8)$$

The solutions are

$$M = FR(1/\sqrt{3} - 3/\pi)/2 \quad (A9a)$$

$$T = 0 \quad (A9b)$$

To find bending moment and torque as a function of θ the free body ac is used and taking moments about c gives

$$M\vec{i} - T\vec{j} - m(\theta)\vec{e}_r + t(\theta)\vec{e}_\theta + \vec{p}_{ca} \times F/2 \vec{k} + \int_0^\theta (\vec{p}_{c\psi} \times p\vec{k})Rd\psi = 0 \quad (A10)$$

$$\text{where } \rho_{c\psi} = R[\cos \psi \vec{i} + \sin \psi \vec{j} - \vec{e}_r(\theta)] \quad (A11)$$

Substituting (A9) and (A11) into (A10) and evaluating gives a vector moment equation. Taking the dot product of this equation with $\vec{e}_r$ and $\vec{e}_\theta$ yields the following results

$$m(\theta) = FR \left[\cos \theta / \sqrt{3} + \sin \theta - 3/\pi \right] / 2 \quad (A12)$$

$$t(\theta) = FR \left[\sin \theta / \sqrt{3} - \cos \theta + 1 - 3\theta/\pi \right] / 2 \quad (A13)$$

The remainder of the analysis follows the procedure used for Problem (1). The change in curvature $\vec{\omega}_b - \vec{\omega}_a$ is calculated and $\vec{\omega}_a$ is

$$\vec{\omega}_a = FR^2 \left(\frac{1}{EI} + \frac{1}{JG} \right) \left[\frac{1}{4\sqrt{3}} + \frac{\pi}{9} - \frac{3}{2\sqrt{3}} \right] \vec{j} \quad (A14)$$

This result for $\vec{\omega}_a$ along with Eqs. (A11), (A12) and (A13) are substituted into Eq. (1) and the result given in Eq. (11) of the text is obtained.