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SUBJECTED TO LARGE CURRENT PULSES

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COMPUTATION OF THE CURRENT DENSITY IN NONLINEAR MATERIALS SUBJECTED TO LARGE CURRENT PULSES*

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Abstract — The finite element method and the finite difference method are used to calculate the current distribution in two nonlinear conductors. The first conductor is a small ferromagnetic wire subjected to a current pulse that rises to 10,000 Amperes in 10 microseconds. Results from the transient thermal and transient magnetic solvers of the finite element code FLUX2D are used to compute the current density in the wire. The second conductor is a metal oxide varistor. Maxwell's equations, Ohm's law and the varistor relation for the resistivity and the current density of $\rho = \alpha j^{-\beta}$ are used to derive a nonlinear differential equation. The solutions of the differential equation are obtained by a finite difference approximation and a shooting method. The behavior predicted by these calculations is in agreement with experiments.

INTRODUCTION

We computed the time history of the current density in two nonlinear conductors carrying short-current pulses. The first is a 0.075-mm-diam iron wire. This wire constitutes the sample in a pulse-heating experiment designed to measure the density and electrical resistivity of materials in the liquid state. In this experiment, the current pulse, which heats the wire, has a 60-microsecond duration and a 10-microsecond risetime. For proper results, the sample must be heated uniformly, that is, the current must diffuse to the center of the wire within the time of interest. The second nonlinear conductor is a 36-in. stack of 3-in.-diam zinc oxide varistors. The stack carries a current pulse of 10,000 Amperes that has risetime of 100 nanoseconds. We were interested in the effect of the nonlinear variation of the resistivity with current density on the overall resistance of the varistor stack.

PULSE HEATING OF IRON WIRE

Very high temperature thermophysical properties of many materials have been measured using a dynamic pulse heating technique [1]. In this technique, a thin cylindrical sample is subjected a fast axial current that heats the material to temperatures of about 2000 K. In order for this technique to yield accurate measurements of the physical properties, the sample must become heated uniformly within the time intervals of interest of the experiment. Calculations of current and heat diffusion show that this technique yields the required uniform heating for nonferromagnetic wires [2]. In the calculation reported here, we address the question of uniform heating in ferromagnetic wires.

The equation governing the linear diffusion of the current density,

$$\nabla^2 j + \frac{\rho}{\mu} \frac{\partial j}{\partial t} = 0,$$

where ρ is the resistivity and μ is the permeability, shows that, to a first approximation, the diffusion time for current density decreases with increasing permeability. The diffusion of the Ohmic heating, $j^2 \rho$, occurs at the same rate as the current density. In nonferromagnetic materials, the permeability is that of free space, μ_0 . In ferromagnetic materials, μ varies from μ_0 when the materials are saturated, to values as much as a thousand times greater than μ_0 near the coercive point.

Thus, one expects that the diffusion will be much slower in a magnetic wire than in a nonmagnetic one.

The transient magnetic solver of the finite element code FLUX2D [3] takes into account the magnetization behavior, $B(H)$, of ferromagnetic materials. Using the data base section of this code, we specified the $B(H)$ curve for the iron used in the experiments. Calculations were made on a 10° arc of the cross section of the wire.

Thermal effects were included by allowing for temperature variations in the resistivity and the permeability. In general, in ferromagnetic materials, the permeability drops to μ_0 as the temperature approaches the Curie temperature, while the resistivity increases with temperature; for example, in iron [4]

$$\rho = 8.57 + 9.135 \times 10^{-2} T \quad \mu = \Omega - m,$$

for $T < 1200$ K. Fortunately, these temperature effects are in a favorable direction for decreasing the diffusion time.

Local values of the temperature were obtained for the 10° section of wire with the Ohmic heating source $j^2 \rho$ through the use of the transient thermal solver of FLUX2D. Values for j were supplied by the solution of the transient magnetic problem. The temperature dependence of the resistivity, the specific heat and the thermal conductivity were described in external subroutines that were linked to the solvers. The internal FLUX2D model for the temperature and field variations of the permeability were used for the initial magnetization curve of iron. This combination of the transient magnetic and transient thermal solvers is not automatic in FLUX2D, but the results from each, *e.g.*, the temperature and the current density, are supplied in convenient files by the post processor.

Permeability is also highly rate dependent. In general, μ decreases as the rate of change of the applied field increases. This effect was not included. Instead, for a worst case calculation, values from the dc magnetization curve were used.

In a calculation for a 0.0075-mm-diam wire, and a current pulse that rises linearly to 10000 Amperes in 10 microseconds, we found that the current and flux densities, and hence the heating, diffusion to the center of the wire in about 11 microseconds. The diffusion time was adequate for the experiment. Experiments using the dynamic pulse heating technique for both iron and nickel wires also showed no signs of nonuniform heating [5].

CURRENT DENSITY IN A Zn-O VARISTOR

The zinc oxide varistor is a ceramic semiconducting device with a characteristic trait of a highly nonlinear relation between current and voltage. For very low currents and for very high currents, the voltage across a varistor is approximately linear, but for currents in the range of about 10^{-9} Amps to approximately 100 Amps, the voltage varies inversely with the power of current as $V = k_1 I^{-k_2}$, where k_1 and k_2 are positive real numbers. The electrical behavior of the varistor, in particular, the value of k_2 , is highly dependent upon its composition, which includes ZnO grains, an intergranular medium of mainly Bi_2O_3 , spinel particles and a variety of doping elements, and on the fabrication processes. Values of k_2 between 4 and 50 have been reported [6,7].

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Typically, varistors are used to protect circuits from surges from lightening and power line transients. In a new application, a stack of varistors is used to regulate the voltage in the accelerators of the Dual Axis Radiographic Hydrotest (DAHRT) Facility at Los Alamos National Laboratory [8]. In all these applications, the varistors are subjected to very short, large amplitude current and voltage pulses. Analysis and design of varistor circuits depend upon the transient behavior of the varistor under these pulses.

Previous calculations of the current and voltage responses of varistors were based on an equivalent circuit model derived from considerations of the microscopic structure of the Zn-O grains surrounded by the insulating materials. The varistor is represented as a capacitor in parallel with a voltage dependent resistor [6]. In this paper, we discuss a different approach in which a differential equation for the flux density throughout the varistor is derived from the current-voltage relation and Maxwell's equations. The solution of this differential equation gives the local, time-dependent values of the flux density from which the current density, the resistivity, and the circuit quantities of resistance, voltage, and current can be calculated.

To derive the differential equation, we express the current-voltage relation as a relation between the resistivity ρ and the current density j :

$$\rho = \alpha j^{-\beta} \quad (1)$$

for current densities between, approximately, 10^{-8} A/cm² to 100 A/cm². For current densities out of this range, ρ is constant and the current-voltage characteristics are linear. In Eq. (1), α and β are positive real constants. The values of α and β , and the values of the current density that specify the nonlinear region depend on the composition of the varistor and its fabrication history. The $\rho - j$ characteristics of the commercially available material [9] used in the initial varistor test at the DAHRT Facility are shown in Fig. 1. For this material, $\alpha = 1.1376 \times 10^5$ and $\beta = 0.968$ for j between 1.0×10^{-5} and 8.0×10^6 A/m² and ρ in $\Omega \cdot m$. For j less than 1.0×10^{-5} , we hold ρ constant at 7.87×10^9 ; for j greater than 8.0×10^6 , ρ is $2.36 \times 10^2 \Omega \cdot m$. These

Fig. 1. Varistor resistivity versus current density. For j between 1.0×10^{-5} and 8.0×10^6 A/m², the resistivity is given by Eq. (1) with $\alpha = 1.1376 \times 10^5$ and $\beta = 0.968$. For $j > 1.0 \times 10^{-5}$, $\rho = 7.87 \times 10^9$; for $j > 8.0 \times 10^6$, $\rho = 2.36 \times 10^2 \Omega \cdot m$.

constant values of ρ correspond to resistances over a 36-in. stack of varistors of 1.5-in. radii of $1.57 \times 10^{12} \Omega$ for the lower current density region and 4.74Ω in the upper current density region.

From Ohm's law, $j = E/\rho$, and (1) we write that, in the nonlinear region,

$$E = \alpha j^{1+\beta} \quad (2)$$

For the case of a cylindrical varistor that carries an axial, time-varying current $I(t)$, the substitution of (2) into Maxwell's equation $\nabla \times E = -\dot{B}$ and performing the differentiation in cylindrical coordinates yields

$$\frac{\partial j}{\partial r} = \frac{-1}{\alpha(1-\beta)} j^{-\beta} \dot{B} \quad (3)$$

where j is the axial component of the current density and B is the azimuthal flux density. Taking the curl of the equation $\nabla \times B = \mu j$ of Maxwell and the subsequent substitution from (3) gives

$$B''(r, t) = \frac{\mu^{1-\beta}}{\alpha(1-\beta)} \dot{B} \left(B' + \frac{B}{r} \right)^{-\beta} - \frac{B'}{r} + \frac{B}{r^2} \quad (4a)$$

In (4), the primes denote differentiation with respect to the radius r ; r lies between 0 and the edge of the varistor, i.e., $r \in (0, a]$. In the linear region, the flux density is governed by the linear diffusion equation

$$B''(r, t) = \frac{\mu}{\rho_1} \dot{B} - \frac{B'}{r} + \frac{B}{r^2} \quad (4b)$$

where ρ_1 is the constant resistivity of the low current region and ρ_2 is the constant resistivity of the high-current region. From the Biot-Savart law, at the center of the varistor, that is, at $r = 0$, the flux density is zero:

$$B(0, t) = 0 \quad (5a)$$

and at the edge, $r = a$, the flux density is

$$B(a, t) = \frac{\mu_0 I(t)}{2\pi a} \quad (5b)$$

Equations (4) and (5) constitute a two-point boundary value problem. The solution is obtained by the finite difference method.

Under a finite difference approximation, (4a) becomes

$$B_{i+1} = \Delta^2 \left[k \left(\frac{B_i - B_{i-1}}{\delta} \right) \left(\frac{B_i - B_{i-1}}{\Delta} + \frac{B_i}{r_i} \right)^{-\beta} - \left(\frac{B_i - B_{i-1}}{r_i \Delta} \right) + \frac{B_i}{r_i^2} \right] + 2B_i - B_{i-1} \quad (6)$$

where $k = -\mu^{1-\beta}/\alpha(1-\beta)$; δ is the time step, $t_j - t_{j-1}$; and the present and previous values of flux density at r_i are $B_i = B(r_i, t_j)$ and $B_{i-1} = B(r_i, t_{j-1})$. For ease of notation, Eq. (6) is written with equal spatial steps, $\Delta = |r_i - r_{i-1}|$, although in practice a geometric variation in the spacing with the smallest steps located in the eddy current region near the edge is used. A similar form is obtained for Eq. (4b).

Let us set $r_1 = a$ and $r_{i_{\max}} = \Delta$. Then the boundary conditions are

$$B_{imax+1} = B(0, t_j) = 0, \quad \text{for all } j, \quad (7a)$$

and

$$B_1 = B(r_1, t_j) = \frac{\mu_0 I(t_j)}{2\pi a}, \quad \text{for all } j. \quad (7b)$$

Attempts to solve this system using the relaxation method were complicated by the presence of poles and complex-valued results in the term $(B' + B/r)^{-\beta}$. Hence, a shooting method was adopted. In this method, the value for B_1 at $r = a$ for time t_j is given in (7b) from a specification of the history of the current, $I(t)$. The selection of a value for B_2 allows for the values of the successive B_i , $i \geq 3$ to be computed. The acceptable solution satisfies condition (7a) that $B = 0$ at $r = 0$. In order to decrease the number of iterations required to calculate $B(0, t) = 0$ from (7b), we first found intervals of values of B_2 that yielded reasonable values of B_3 , B_4 , and B_5 , with reasonable values defined as those that lie between a number slightly greater than the linear solution $B_i = r_i B_1/a$ and a number slightly less than the previous solution B_i^- .

Results obtained by this method are shown in Figs. 2-6 for the material used in the DAHRT tests and for a current pulse that rises to 10,000 Amps in 100 nanoseconds, i.e.,

$$I(t) = \begin{cases} \frac{I_m}{\tau} t, & \text{for } t < \tau; \\ I_m, & \text{for } t \geq \tau, \end{cases}$$

with $I_m = 1 \times 10^4$ and $\tau = 100 \times 10^{-9}$.

Fig. 2. Current $I(t)$ in the stack of varistors for the computations described in Eqs. (6)-(7). The solid dots show the computation times t_j .

Fig. 3. The flux density calculated from Eqs. (6)-(7) using the current pulse shown in Fig. 2 and the $\rho-j$ characteristics shown in Fig. 1. The open circles on the 12-ns line are the computation points r_i .

Fig. 4. The current density computed as $\nabla \times B/\mu$ from the flux density shown in Fig. 3. The line styles shown here correspond to the times given in the legend of Fig. 3.

Fig. 5. The resistivity variations across the varistor during the times specified in Fig. 3.

Fig. 6. Resistance and voltage of a 36-in. stack of varistors of 1.5-in. radius for the calculation shown in Figs. 1-5.

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