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SIGV—A CODE TO EVALUATE PLASMA REACTION RATES TO A SPECIFIED ACCURACY

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SIGV—A CODE TO EVALUATE PLASMA REACTION RATES TO A SPECIFIED ACCURACY

ABSTRACT

A FORTRAN code to evaluate binary reaction rates $\langle \sigma v \rangle$ for a plasma to a specified accuracy is described. Distribution functions permitted are (1) two Maxwellian species at different temperatures, (2) beam-Maxwellian, (3) cold gas with Maxwellian, and (4) beam-plasma with mirror distribution of the form $f(v) \propto f(v) M(\cos \theta)$. Several functional forms are permitted for $f(v)$ and $M(\cos \theta)$. Cross-section subroutines for a number of interactions involving hydrogen, helium, and electrons are included, as is a routine allowing input of numerical data. The code is written as a subroutine to allow ready incorporation into larger plasma codes.

1. INTRODUCTION

A frequent problem in fusion research is the evaluation of the binary reaction rate $\langle \sigma v \rangle$ given by

$$\langle \sigma v \rangle \equiv \int dv_1 \int dv_2 f_1(v_1) \times f_2(v_2) v_R \sigma(v_R), \quad (1)$$

where f_1 and f_2 are the distribution functions of the interacting species and σ is the cross section at the relative speed $v_R \equiv |v_1 - v_2|$. Equation (1) has been evaluated for special distribution functions and common interactions. The work of Riviere,¹ which considered beam injection into plasmas with Maxwellian or mirror distributions, is a prominent example.

In connection with studies of beam injection into reactor plasmas, need arose for evaluation of $\langle \sigma v \rangle$ for mirror plasmas with strongly anisotropic distribution functions. Although the results of Riviere¹ have been extensively used for such studies, they were obtained under the assumption of isotropic angular distribution functions with injection normal to the field lines and plasma energy equal to beam energy. Miller and Miley² have performed similar calculations without the restrictions of isotropy and normal injection. To carry out these calculations, it is necessary to perform a three-dimensional numerical integration. Miller and

Miley² do this with a product rule involving Gauss-Legendre quadrature.³ Such a method requires a fixed number of integrand evaluations, and it is difficult (but not impossible) to check the accuracy of the numerical integration. Such accuracy checks are useful, not only to judge the final accuracy of the results, but also to reduce the number of points used in the quadrature if the desired accuracy is considerably exceeded.

To avoid the limitations just described, at Lawrence Livermore Laboratory (LLL) we have designed a code with the following features:

- Allowance for anisotropic plasma distribution function with non-normal beam injection, and
- Estimation of the accuracy of the integration, with the choice of the number of quadrature points then governed by the desired accuracy.

In addition, the code includes:

- Provision for use of a 32-point Gauss-Laguerre formula for Maxwellian distribution functions,
- A structure to facilitate modification or incorporation into a larger code, and
- A "driver" program that enables the code to be easily run from the terminal of a PDP-10.

In Sec. 2, we present the several forms for $f(v)$ treated in the code. Section 3 includes a discussion of the numerical methods used and the subroutines. Examples of code usage and some results are given in Sec. 4.

2. SPECIAL FORMS FOR DISTRIBUTION FUNCTIONS

The code treats several special forms for the distributions functions; we shall describe each case in the following paragraphs.

Two Maxwellian Components

For this case, each component has a spherically symmetric distribution function but has a Maxwellian speed distribution at different temperatures:

$$f = \left(\frac{m}{2\pi kT} \right)^{3/2} \exp \left(-\frac{mv^2}{2kT} \right), \quad (2)$$

where m is the species mass, T is its temperature, k is the Boltzmann constant, and $v \equiv |\underline{v}|$. For this form, the integral of Eq. (1) can then be reduced to

$$\langle \sigma v \rangle = 2 \left(\frac{2m_1 kT_2 + 2m_2 kT_1}{\pi m_1 m_2} \right)^{1/2} \times \int_0^\infty e^{-x} \sigma(v_R) x \, dx, \quad (3)$$

where

$$v_R^2 = \frac{2m_1 kT_2 + 2m_2 kT_1}{m_1 m_2} x. \quad (4)$$

An obvious further simplification occurs when $T_1 = T_2$.

Beam of Speed v_B and Maxwellian Plasma

In this case, one of the distribution functions of Eq. (1) is a delta function in speed and angle, and one integral may be immediately evaluated. The integral over the velocity space of the Maxwellian gas can be transformed to one over relative speed and two spherical angles. The integrations over angles may be carried out, and we find

$$\langle \sigma v \rangle = \left(\frac{2kT}{\pi m y} \right)^{1/2} e^{-y} \int_0^\infty e^{-x} \sigma(v_R) \times \sinh \left[2(xy)^{1/2} \right] x^{1/2} dx, \quad (5)$$

with

$$y \equiv mv_B^2/2kT, \quad (6)$$

and

$$v_R^2 \equiv 2kTx/m, \quad (7)$$

where m is the plasma mass.

Maxwellian Plasma Interacting with Gas at Rest

This case may be either evaluated from the general form of Eq. (1) or taken as the limit of Eq. (5) as $y \rightarrow 0$. We find

$$\langle \sigma v \rangle = 2 \left(\frac{2kT}{\pi m} \right)^{1/2} \int_0^\infty e^{-x} \sigma(v_R) x \, dx, \quad (8)$$

with v_R given by Eq. (7).

Beam of Speed v_B and Mirror-Confined Plasma

This type of distribution function is a good approximation to the ion distribution function occurring in mirror-confined plasmas. We take

$$f(\underline{v}) = f(v)M(\mu)P(\phi), \quad (9)$$

where $\mu \equiv \cos \theta$. For mirror-confined plasma, θ and ϕ are the angles of the velocity vector $\underline{v}$ relative to the $\underline{B}$ vector. Then Eq. (1) becomes

$$\begin{aligned} \langle \sigma v \rangle = & \int_0^\infty f(v) v^2 dv \int_{-1}^{+1} M(\mu) d\mu \\ & \times \int_0^{2\pi} P(\phi) \sigma(v_R) \left\{ v_B^2 + v^2 - 2v_B v \left[\mu_B \mu \right. \right. \\ & \left. \left. + \left(1 - \mu_B^2 \right)^{1/2} \left(1 - \mu^2 \right)^{1/2} \cos(\phi - \phi_B) \right] \right\}^{1/2} d\phi, \end{aligned} \quad (10)$$

with

$$v_R^2 = v_B^2 + v^2 - 2v_B v \left[\mu_B \mu + (1 - \mu_B^2)^{1/2} \right. \\ \left. \times (1 - \mu^2)^{1/2} \cos(\phi - \phi_B) \right], \quad (11)$$

and

$$\mu_B \equiv \cos \theta_B.$$

Here, θ_B and ϕ_B are the cone and polar angles of the beam direction relative to $\hat{B}$.

In the present version of the code, $P(\phi)$ is taken as a constant:

$$P(\phi) = 1/2\pi. \quad (12)$$

For $M(\mu)$ the following forms have been implemented:

- Rensink form:

$$M(\mu) = (1/N) \begin{cases} (1 - \mu^2/\mu_0^2)^m, & \mu < \mu_0 \\ 0, & \mu \geq \mu_0 \end{cases}, \quad (13)$$

with $\mu_0^2 = (R-1)/R$, $m = 1/2$ or 1, and the normalization constant given by

$$N = \begin{cases} \pi\mu_0/2, & m = 1/2 \\ 4\mu_0/3, & m = 1 \end{cases}.$$

- Holdren form⁴:

$$M(\mu) = (1/N) \left\{ \mu_0^2 + \mu^2 + (3\mu_0^2 - 1) \right. \\ \left. \times \ln \left[(1 - \mu^2)/(1 - \mu_0^2) \right] \right\}, \quad (14)$$

with

$$N/2 = 4\mu_0^3/3 + (3\mu_0^2 - 1) \\ \times \left\{ \ln \left[(\mu_0 + 1)/(\mu_0 - 1) \right] - 2\mu_0 \right\}$$

- Ideal distribution⁵:

$$M(\mu) = i/N \left\{ \tan^{-1} \left[(\mu_0^2 - \mu^2) / (1 - \mu_0^2) \right]^{1/2} \right. \\ \left. + 3 \left[(1 - \mu_0^2)(\mu_0^2 - \mu^2) \right] / \left[5 - 3\mu^2 - 2\mu_0^2 \right] \right\}, \quad (15)$$

where N is evaluated numerically.

- Isotropic distribution:

$$M(\mu) = 1/2. \quad (16)$$

For the speed distribution, a Maxwellian speed distribution as in Eq. (2) is available, as well as a model form for mirror-confined ions:

$f(v) =$

$$(1/N) \begin{cases} 0, & v \leq v_H \\ (v - v_H) \exp \left[-(v - v_0)^2/v_W^2 \right], & v > v_H \end{cases}. \quad (17)$$

The reference speed v_0 is related to the speed v_P at which $f(v)$ is a maximum by

$$v_0 = v_P - v_W^2/2(v_P - v_H). \quad (18)$$

The three parameters of this distribution, v_H , v_W , and v_0 , can be adjusted to fit a distribution computed from numerical solutions of the Fokker-Planck equation. The normalization constant N is computed from

$$2N = v_W^4 (S^2 + 1) \exp(-S^2) \\ + v_W^3 (3v_0 - v_H) \left\{ [1 - \text{SGN}(x)] F(3/2) \right. \\ \left. + \text{SGN}(x) \Gamma(3/2, S^2) \right\} \\ + v_0 v_W (3v_0 - 2v_H) \exp(-S^2) \\ + v_0^2 v_W (v_0 - v_H) \left\{ [1 - \text{SGN}(x)] \Gamma(1/2) \right. \\ \left. + \text{SGN}(x) \Gamma(1/2, S^2) \right\}, \quad (19)$$

with $x = v_H - v_0$ and $S = x/v_W$.
 $\Gamma(a, x)$ is the incomplete gamma function defined by

$$\Gamma(a, x) = \int_x^\infty e^{-t} t^{a-1} dt. \quad (20)$$

For mirror-confined ions, $v_H < v_0$ and $x < 0$. Nonetheless, Eq. (20) allows for any sign for x .

Another quantity of interest is the mean-square-speed that is needed to compute the mean plasma energy. It is found from

$$\begin{aligned} 2N \langle v^2 \rangle = & v_W^6 (S^4 + 2S^2 + 2) \exp(-S^2) \\ & + v_W^5 (5v_0 - v_H) \left\{ [1 - \text{SGN}(x)] \right. \\ & \times \Gamma(5/2) + \text{SGN}(x) \Gamma(5/2, S^2) \left. \right\} \\ & + 2v_W^4 v_0 (5v_0 - 2v_H) (S^2 + 1) \exp(-S^2) \\ & + 2v_W^3 v_0^2 (5v_0 - 3v_H) \left\{ [1 - \text{SGN}(x)] \right. \end{aligned}$$

$$\begin{aligned} & \times \Gamma(3/2 + \text{SGN}(x) \Gamma(3/2, S^2) \left. \right\} \\ & + v_W^2 v_0^3 (5v_0 - 4v_H) \exp(-S^2) \\ & + v_W v_0^4 (v_0 - v_H) \left\{ [1 - \text{SGN}(x)] \right. \\ & \times \Gamma(1/2) + \text{SGN}(x) \Gamma(1/2, S^2) \left. \right\}. \quad (21) \end{aligned}$$

Examination of Eq. (10) shows that the integration over ϕ may be carried out analytically in three special cases:

- $\mu_B = 1$; i.e., $\theta_B = 0$.
- $v_B = 0$.
- Isotropic angular distribution [Eq. (16)].

In the last case, the angle of the beam is irrelevant, and we can set it equal to 0 to perform the integration over ϕ . In these special instances, the integration reduces to a two-fold integration over v and μ ; we use this simplification in the code.

3. DESCRIPTION OF CODE

We include here a description of the subroutines in the code. Where we have chosen a particular numerical technique, e.g., a method for numerical quadrature, it is discussed in the particular subroutine. Our discussion is directed toward those who might wish to incorporate these subroutines in another code, include other distribution functions, or transfer the code to another computer. Others desiring only to compute with this code can skip to the next section (Sec. 4) on usage. Some additional details on COMMON blocks are included in the file SIGV.DOC, which is reproduced in the Appendix. A complete listing of the source is provided on microfiche in a packet inside the back cover of this document.

The general philosophy of the code structure is to allow easy modification by breaking up the calculations into many subroutines. Many of these can be eliminated or combined for special application. In addition, all but the driver program are written in "standard" FORTRAN with many comments to enable easy conversion to another computer.

SIGV is a driver program that either calls the subroutine SGV for evaluation of $\langle \sigma v \rangle$ or, as an option, calls a subroutine to directly evaluate $\sigma(v_R)$. The NAMELIST input may be from a terminal, in which case it can be interactive, or from a disc file. Input variables are mnemonic and explained in the listing of the program. A small help package listing the options for the $f(v)$, $M^A(\mu)$, and types of cross sections may be invoked (see examples in Sec. 4).

SGV is the main subroutine that computes parameters for the actual integration and calls subroutines for evaluation of $\langle \sigma v \rangle$. Its purpose is to set up the appropriate calls for the evaluation of Eqs. (3), (5), (8) or (10) to the accuracy prescribed in the value of EPSILON. If the input variable IVELDI = 3, SGV will call the much faster subroutines SGVIMX and GLQ to evaluate Eqs. (3), (5), or (8) with a 32-point Gauss-Laguerre quadrature. Then, of course, the resulting value of $\langle \sigma v \rangle$ is of unknown accuracy, although spot checks of accuracy can be made by using the full integration option in SGV. Further explanation of input and common variables can be found in the

documentation section of SIGV.DOC in the Appendix.

NORMALIZE carries out the evaluation of the normalization constants for the distribution functions, and also computes the mean-square speed for the mirror distribution from Eq. (21). The incomplete gamma function is evaluated with the aid of the subroutine MDGAM, which computes

$$P(a, x) = [\Gamma(a)]^{-1} \int_0^x e^{-t} t^{a-1} dt$$

$$= 1 - \Gamma(a, x)/\Gamma(a). \quad (22)$$

MDGAM is available from the International Mathematical and Statistical Libraries (IMSL) file 6 on computers at the Magnetic Fusion Energy Computer Center (MFECC).

PHINT is a routine to carry out the ϕ integration. Integration over ϕ is done with a trapezoidal rule since the integrand is periodic and little or no improvement in accuracy is expected from using a higher-order method.⁷ The number of evaluation points starts with 2 and doubles until the relative error (or change in integration values) meets a criterion that may be specified by the user, but is defaulted to EPSILON/9.

AMUINT with AMUGRAND evaluates the μ integral with the aid of the interleaving Gaussian quadrature routine QUAD of Patterson.⁸ Up to 255 points are used in the evaluation, but only enough are used at any call to satisfy the relative error criterion that may be specified by the user, and is defaulted to EPSILON/3.

VINT and VGRAND evaluate the v integral, again using the QUAD routine of Patterson.⁸ Here, the infinite upper limit of the integral must be replaced by a finite value b . For the Maxwellian distribution, this is taken as

$$b = C(3kT/2m)^{1/2}; \quad (23)$$

and for the mirror distribution as

$$b = v_p + C v_w, \quad (24)$$

where

$$C = \left\{ \text{MAX} [4, -\text{ULVCON} \times \text{LOG}_{10}(\text{EPSILON})] \right\}^{1/2}. \quad (25)$$

ULVCON can be specified by the user, and is defaulted to 2. For the mirror distribution, the lower limit is $a = v_H$, since $f(v)$ is assumed to vanish in the loss cone.

An estimate of the truncation error introduced by the above procedure is also computed. We note that the integrand behaves approximately as $\exp(-\alpha x)$ at large x , which can be integrated from b to ∞ . We estimate the truncation error by fitting this form to the last two integrand points. This estimated relative truncation error is available to the user as EPSTR. Of course, if the error is greater than EPSILON, the quantity ULVCON mentioned above should be increased to increase the upper limit b .

VDI, VD2, AMUDI, etc. are the functional forms for $f(v)$ and $M(\mu)$ listed in Sec. 2. It is clear that additional forms may be readily added.

SGNUM evaluates $\sigma(v_R)$ for tabulated values of $\sigma(10^{-16} \text{ cm})$ vs beam energy (eV). The data are stored in common block SGVNUM, which may be filled with NAMELIST input (see example in Sec. 4). The form of the NAMELIST is described in the comment in the FORTRAN source for the main program.

SGIHH, SGIEH, SGXHH, etc. are subroutines for evaluating $\sigma(v_R)$ via fitted functional forms. The comments in each subroutine list the sources of the data. Various fusion plasma ionization and charge-exchange interactions involving electrons, hydrogen atoms, ions and molecules, and helium are included.

The two cliché files SGV.CLI and SGVMX.CLI are used for COMMON blocks. MSRUN is a local subroutine for timing information and should be replaced on other computers.

4. USE OF THE SIGV CODE

The present version of the code has been compiled and is available for use on the PDP-10 in M-Division at LLL under user number [30, 317]. Files available are:

SIGV.FOR—source
SGV.CLI, SGVMX.CLI—cliché files
SIGV.REL—relocatable binary image, including IMSL routines and timing routine MSRUN

SIGV.EXE—executable code
 SIGV.TST—data file for examples below
 ELI.DAT—cross-section data for electron-lithium case in SIGV.TST.

All of the above, except for the relocatable and executable files, are also available from a LIX file on the MFE network under FILEM:

FILEM RDS 3040 .PHYSICS SIGV

The following examples illustrate some of the features available.

Example (1): We start the program and tell it that all input and output are to be with the user terminal (TTY); then we ask for a list of options.

```
.RUN SIGV
```

```
PROGRAM FOR <SIGMA-V>.
EXECUTION STARTED AT 15:09 ON 22-Aug-78
TYPE INPUT UNIT NUMBER (5=TTY, 2=DISK): 5
TYPE OUTPUT UNIT NUMBER (5=TTY, 3=DISK): 5
```

```
PLEASE NAMELIST INPUT NEW VALUES
SA IHPL=1$
```

```
POSSIBLE CROSS SECTIONS ARE(ISICHA)...
```

```
1..... IHH
2..... IEH
3..... XHH
4..... IHHE
5..... IENE
6..... XHNE
7..... XHEHE
8..... IEHE
9..... XIEH
10..... IEH
11..... IEH2
12..... XHH2
13..... IHH2
14..... NUM
```

```
POSSIBLE ANGULAR DISTRIBUTIONS ARE(IMUDIS)...
```

```
1..... HOLD
2..... RNS.5
3..... RMS1.
4..... IDEAL
5..... ISO
```

```
POSSIBLE SPEED DISTRIBUTIONS ARE(IVELDI)...
```

```
1..... MAX
2..... MIRROR
3..... ISMAX
4..... SIGMA
```

In the next five examples, we evaluate $\langle \sigma v \rangle$ for charge transfer between D and D^+ under varying assumptions regarding the distribution function. Beam energy is 100 keV and mean plasma energy is 80.1 keV.

Example (2): First, we simply compute σ at beam speed v_B and from the product $\sigma(v_B)v_B$. We also ask for extended diagnostics by setting IDIAG = 1.

```
PLEASE NAMELIST INPUT NEW VALUES
SA EBK=100., IVELDI=4, IDIAG=15
```

```
SIGMA AT BEAM ENERGY FOR YHH
BEAM MASS 2.00E+00 AMU BEAM ENERGY 1.00E+02 KEV
BEAM VELOC 3.11E+08 CM/SEC SIGMA 1.10E+16 CM^2
SIGMA V 3.40E+06 CM^3/SEC
TIME USED 0.100 SEC
```

Example (3): Try it with a Maxwellian velocity distribution function and the Gauss-Laguerre quadrature.

```
PLEASE NAMELIST INPUT NEW VALUES
SA EPLAS=80.1, IVELDI=35
```

```
ANGULAR DIST: ISO , SPEED DIST: ISMAX , SPECIES: YHH
BEAM MASS 2.00E+00 PLASMA MASS 2.00E+00 AMU
BEAM ENERGY 1.00E+02 PLASMA ENERGY 8.01E+01 KEV
PLASMA V 3.180E+08 CM^3/SEC
TIME USED 0.115 SEC
```

Example (4): Repeat using the Patterson quadrature. Note that the default value EPSILON = 10^{-3} is used.

```
PLEASE NAMELIST INPUT NEW VALUES
SA IVELDI=15
```

```
ANGULAR DIST: ISO , SPEED DIST: MAX , SPECIES: YHH
BEAM MASS 2.00E+00 PLASMA MASS 2.00E+00 AMU
EPSILON 1.00E-03 PLASMA V 3.11E+08 CM/SEC
EST REL ERROR 1.70E-04 EST TPRC ERROR 1.24E-05
ALINE IS 0.00E+00 BLANK IS 0.01E+02
TOTAL CALLS 81 V FUNCTION CALLS 31
AVE MU CALLS 26.7 AVE PHI CALLS 1.0
BEAM ENERGY 1.00E+02 PLASMA ENERGY 8.01E+01 KEV
SIGMA V 3.179E+06 CM^3/SEC
TIME USED 0.093 SEC
```

Example (5): Switch to the mirror speed distribution, but maintain an isotropic angular distribution.

```
PLEASE NAMELIST INPUT NEW VALUES
SA IVELDI=2, EHCDF=13., EPEAF=65., EWIDTH=5., IMUDIS=5., LETAB=60.,
RATHIR=2.5
```

```
ANGULAR DIST: ISO , SPEED DIST: MIRROR , SPECIES: YHH
BEAM MASS 2.00E+00 PLASMA MASS 2.00E+00 AMU
BEAM THETA 60.00 MIRROR RATIO 2.00E+00
HOLE ENERGY 1.30E+01 PEAK ENERGY 6.50E+01 KEV
ENERGY WIDTH 6.50E+00 DENSITY 5.39E+01 KEV
VNDPH IS 1.18E+03 PHIFR IS RELATIVE
EPSILON 1.00E-03 ULVCM 2.00E+00
EST REL ERROR 6.09E-04 EST TPRC ERROR 2.50E-04
ALINE IS 1.30E+01 RLDF IS 2.05E+02
TOTAL CALLS 401 V FUNCTION CALLS 15
AVE MU CALLS 26.7 AVE PHI CALLS 1.0
BEAM ENERGY 1.00E+02 PLASMA ENERGY 8.01E+01 KEV
SIGMA V 3.15E+06 CM^3/SEC
TIME USED 0.552 SEC
```

Example (6): Now use a more realistic form for $M(\mu)$, namely that of Eq. (13) with $m = 1/2$, and inject the neutral beam at 60° to the magnetic field into a plasma with mirror ratio = 2. Note that the

integration over μ did not converge once: i.e., the change in the integral value in going from I27 to 255 quadrature points was greater than EPSILON/3 = $1/3 \times 10^{-3}$. Noting that only 15 points were required for the v -integration, we would suspect that overall accuracy is better than 10^{-3} . We could check this by: relaxing somewhat the criterion on the μ integral either by changing to a value less than the default value of 3 or by increasing EPSILON.

PLEASE NAME LIST INPUT NEW VALUES
5/ JMBIS*25

ANAL INPT:	2.00E+05	DEPR INPT:	WTRF	1.00E+02	YR00	
MASS	2.00E+00	PLASMA	MASS	2.00E+00	AMU	
VEAM THICK	5.00E-01	VEAM	THICK	2.00E+00		
VEAM INPT	1.00E+01	VEAM	INPT	1.00E+01		
VEAM WIDTH	6.00E-01	VEAM	WIDTH	5.00E+01		VEV
VEPR INP	1.00E+03	VEPR	INP	1.00E+03		VEV
VEPRLEN	1.00E+03	VEPR	LEN	2.00E+00		
VEPR EFF	7.50E+00	VEPR	EFF	2.00E+00		
VEPR INP	1.00E+05	VEPR	INP	2.00E+00		
TOTAL CALS	31279	VE	STRUCTURE	FACTOR	1*	
AWI M. CALS	55.3	AWI	DEI	CALS	60.0	
DEPR INPT	1.00E+02	DEPR	INPT	5.00E+01		VEV
SIGMA V	3.30022599E-08	VE	STRUCTURE	FACTOR	1	
NAME	VE	VE	VE	VE	VE	
TIME INPR:	27.00E SIK					

cross section has been measured using an electron beam, thus $X_{\text{MASS}} = M_e$; however, because the probing beam is lithium, $B_{\text{MASS}} = 7.0$. Following this example, we terminate by setting $I_{\text{QUIT}} = 1$.

```
PLEASE NAMELIST INPUT NEW VALUES
  SA=1SIGMA=14,IVEIDI=3,INUDIS=5,PMASS=7.,PLMASS=6.,GAM=11=6.
  PLMASS=6.1,EFFAM=2.5
TYPE CROSS SECTION FILE NAME, LEFT=JUSTIFIED.
  FLI.DAT
```

[illegible]

THESE SONT LES SEULES VENTES

[illegible]

Example 7: In the last example, we input tabulated cross section data for the interaction.

This reaction is the dominant one in using i_+^+ beam to experimentally determine the space-charge potential of a mirror plasma, e.g., in the plugs of the Tandem Mirror Experiment (TMX) at LLNL. This

As an example of a more extensive calculation, in Fig. 1 we show $\langle \sigma v \rangle$ vs θ_B for a D-D⁺ charge-exchange reaction. Other parameters are the same as in example (6). We note marked dependence on θ_B , reflecting the "hole" in the distribution function of the plasma velocity. Also shown in Fig. 1 is the value of $\langle \sigma v \rangle$ computed for an isotropic angular distribution function. Evidently, the error in assuming an isotropic mirror distribution function could be substantial.

Fig. 1. Binary charge-exchange reaction rate $\langle\sigma v\rangle$ vs beam angle θ_B for charge-transfer of 100-keV D^0 beam injected into 80.1-keV mirror-confined plasma. The solid line shows the value with the correct plasma angular distribution function. The dashed line shows the value computed if the angular distribution function is assumed isotropic.

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APPENDIX. SIGV. DOC LISTING

(A listing of the source is provided on microfiche in a pocket on the inside back cover of this document.)

***** SCV SUBROUTINE *****

SGV IS A PDP-10 FORTRAN SUBPROGRAM WHICH NUMERICALLY EVALUATES BINARY REACTION RATE COEFFICIENTS <SIGMA-V> FOR CERTAIN ASSUMED DISTRIBUTION FUNCTIONS.

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DOCUMENTATION LAST UPDATED ON 18 AUG. 1978

FILES AVAILABLE ON THE LLL PDP-10 FROM [30,317]:

SIGV.FOR--SOURCE FOR ALL ROUTINES.

SGV.CLI,SGVMX.CLI--CLICHE'S FOR SIGV.FOR.

SIGV.REL--RELOCATABLE BINARIES INCLUDING IMSL ROUTINES AND TIMING ROUTINE MSRUN.

SIGV.TST--A TEST DATA DECK

FLI.DAT--NUMERICAL CROSS SECTION DATA FOR LITHIUM TEST CASE IN SIGV.TST.

ALL OF THESE FILES EXCEPT SIGV.REL ARE ALSO AVAILABLE IN LIX FILE

SIGV AVAILABLE FROM FILEM 3040 .PHYSICS

***** EXPLANATION OF SUBROUTINE *****

THIS SUBROUTINE WAS WRITTEN TO EVALUATE <SIGMA-V> FOR A NEUTRAL BEAM INJECTED INTO A MIPROP PLASMA WITH DISTRIBUTION FUNCTION $F(V, \mu) = \cos(\theta) \Phi(\mu)$. WE ASSUME F IS INDEPENDANT OF Φ & SEPARABLE IN V & μ : $F = F(V) * M(\mu) / N$ WHERE N IS A NORMALIZATION CONSTANT. FORMS FOR $F(V)$ & $M(\mu)$ ARE LISTED BELOW. THIS SUBROUTINE WILL ALSO COMPUTE <SIGMA-V> FOR OTHER DISTRIBUTION FUNCTIONS:

- (1) COLD GAS(ZERO SPEED) INTERACTING WITH HOT PLASMA(SET FREEM = 0.)
- (2) TWO GASES OF DIFFERENT MASSES AND TEMPERATURES, BOTH WITH MAXWELLIAN VELOCITY DISTRIBUTIONS.(SET EBEAM=0., PLMASS = REDUCED MASS AND $EPLASM = (MASS(1)*E(2) + MASS(2)*E(1)) / (MASS(1) + MASS(2))$).
- (3) 32-POINT GAUSS LAGUERRE QUADRATURE FOR PLASMA WITH MAXWELLIAN VELOCITY DISTRIBUTION--ALL CASES WITH FREEM >= 0. ARE PERMITTED.

THE CALLING SEQUENCE FOR SCV IS

SGV(NSIGMA, NMU1DIST, NVELDIST, ABMASS, APLMASS, EBEAM, THETAREAM, EPLASMA, EPSILON, PATMIRROR)

THE ARGUMENTS ARE AS FOLLOWS:

NSIGMA	CHOOSSES CROSS SECTION
1	IONIZATION OF NEUTRAL HYDROGEN BY HYDROGEN IONS.
2	IONIZATION OF NEUTRAL HYDROGEN BY ELECTRONS.
3	CHARGE EXCHANGE BETWEEN NEUTRAL HYDROGEN AND HYDROGEN IONS.

4 IONIZATION OF NEUTRAL HELIUM BY HYDROGEN IONS.
 5 IONIZATION OF NEUTRAL HELIUM BY ELECTRONS.
 6 CHARGE EXCHANGE BETWEEN NEUTRAL HELIUM AND HYDROGEN IONS.
 7 CHARGE EXCHANGE BETWEEN NEUTRAL HELIUM AND HELIUM IONS.
 8 IONIZATION OF NEUTRAL HELIUM BY HELIUM IONS.
 9 CHARGE EXCHANGE BETWEEN NEUTRAL HYDROGEN AND HELIUM IONS.
 10 IONIZATION OF NEUTRAL HYDROGEN BY HELIUM IONS.
 11 IONIZATION OF MOLECULAR HYDROGEN BY ELECTRON.
 12 CHARGE EXCHANGE BETWEEN MOLECULAR HYDROGEN AND HYDROGEN IONS.
 13 IONIZATION OF MOLECULAR HYDROGEN BY HYDROGEN IONS.
 14 CROSS SECTION INTERPOLATED IN INPUT TABLES (SEE COMMON BLOCK /SCVNUM/ EXPLAINED BELOW).
 FMUDIST PLASMA MU DISTRIBUTION
 1 HOLDREFF DISTRIBUTION (SEE MUCL. FUSION 12,267,1972).
 2 RENSINK DISTRIBUTION, $P=.5$
 $M(MU) = \text{SORT} (1. - (MU/MUZERO) ** 2)$
 3 RENSINK DISTRIBUTION, $P=1.$
 $M(MU) = 1. - (MU/MUZERO) ** 2$
 4 IDEAL DISTRIBUTION (SEE MUCL. FUSION 15,875,1975).
 5 ISOTROPIC DISTRIBUTION $M(MU) = \text{CONSTANT}$.
 NVELDIST CHOOSES SPEED DISTRIBUTION [= F(V)]
 1 MAXWELLIAN SPEED DISTRIBUTION
 2 MIRROR DISTRIBUTION. THIS OPTION NEEDS SOME NUMBERS PASSED IN COMMON /SCV/MIRROR/
 3 ISOTROPIC MAXWELLIAN. ONLY NEEDS TO DO ONE INTEGRAL USES FUNCTION SGVIMX WITH 32-POINT CAUSS-LAGUERRE.
 ABMASS BEAM SPECIES MASS (AMU)
 APLMASS PLASMA SPECIES MASS (AMU)
 EBEAM BEAM ENERGY (KEV)
 THETABEAM INJECTION ANGLE OF THE BEAM WITH RESPECT TO THE B FIELD (DEGPES)
 EPLASMA MEAN PLASMA ENERGY (KEV)
 EPSILON RELATIVE ERROR CRITERION FOR INTEGRATION
 RATMIRROR MIRROR RATIO

SCV IS RETURNED IN CM(3) SEC(-1). NONE OF THE ARGUMENT VALUES THEMSELVES ARE CHANGED BY SCV.

* * * * INTEGRATION * * * *

A TRIPLE INTEGRATION CAN INVOLVE A GREAT MANY FUNCTION EVALUATIONS. IF AN N-POINT RULE IS USED FOR EACH QUADRATURE, THEN N^3 FUNCTION EVALUATIONS ARE NEEDED. THE RELATIVE ERROR CRITERION ARGUMENT (EPSILON) IN SCV ALLOWS THE USER SOME CONTROL OVER THE ACCURACY VS. TIME TRADE OFF. HOWEVER, THE ERROR CRITERION IS ONLY A ROUGH ESTIMATE OF THE ACTUAL ACCURACY OF THE INTEGRATION. DECREASING THE ERROR CRITERION WILL INCREASE THE ACCURACY OF THE INTEGRATION, AND INCREASE THE NUMBER OF FUNCTION EVALUATIONS, AND CONVERSELY. THIS IS ABOUT ALL THAT CAN BE SAID WITH ANY DEGREE OF CONFIDENCE. THERE IS A MINIMUM OF 9 FUNCTION EVALUATIONS, AND A MAXIMUM OF ABOUT 66.6 MILLION FUNCTION EVALUATIONS.

THE RELATIVE SPEED IS A PERIODIC FUNCTION OF THE PLASMA PHI, AND HENCE THE PHI INTEGRAND IS PERIODIC, AND INTEGRATED OVER ONE PERIOD. BECAUSE OF THIS PERIODICITY, THE PHI INTEGRATION USES A

TRAPEZOIDAL RUL", AS NOTHING IS GAINED BY USING A MORE SOPHISTICATED QUADRATURE RULE. (SEE DAVIS AND RABINOWITZ, "METHODS OF NUMERICAL INTEGRATION", ACADEMIC PRESS, 1975. SECTION 2.9.1, PP 106-114.)

THE MU INTEGRATION IS DONE WITH PATTERSON'S QUAD SUBROUTINE. (SEE ALGORITHM 468, "ALGORITHM FOR AUTOMATIC NUMERICAL INTEGRATION OVER A FINITE INTERVAL", COLLECTED ALGORITHMS FROM CACM.) QUAD USES AN INTERLACING FAMILY OF WHOLE INTERVAL QUADRATURE FORMULAS BASED ON EXTENSIONS OF A THREE POINT GAUSS-LEGENDRE RULE. ESPECIALLY NOTEWORTHY IS THAT FUNCTION EVALUATIONS ARE NOT WASTED IN PROCEEDING FROM ONE INTEGRAL ESTIMATE OF THE NEXT, SINCE A FUNCTION EVALUATION FOR THE MU INTEGRAL INVOLVES A PHI INTEGRATION. (SEE PATTERSON, T.N.L., "THE OPTIMUM ADDITION OF POINTS TO QUADRATURE FORMULAE". MATH. COMP. 22 (1966) PP 847-856.)

A MODIFIED VERSION OF PATTERSON'S QUAD SUBROUTINE (QUAD1) IS USED TO PERFORM THE V INTEGRATION. THE V INTEGRAL RANGES FROM ZERO TO INFINITY, BUT QUAD ONLY INTEGRATES OVER A FINITE INTERVAL. THE FINITE UPPER LIMIT IS CHOSEN FROM THE PEAK OF THE SPEED DISTRIBUTION, ITS WIDTH, THE ERROR TOLERANCE AND A FACTOR ULVCON. THE LATTER IS DEFAULTED TO 2. WHICH USUALLY GIVES A RELATIVE TRUNCATION ERROR ABOUT THE SAME AS THE RELATIVE ERROR TOLERANCE. IT CAN BE INCREASED TO IMPROVE THE TRUNCATION ERROR. THE ERROR IN TRUNCATING AT A FINITE UPPER LIMIT IS ESTIMATED BY LINEARLY EXTRAPOLATING THE LAST TWO INTEGRAND VALUES ON A LOG-LOG PLOT. THE EXTRAPOLATION CAN BE INTEGRATED ANALYTICALLY FROM THE UPPER LIMIT TO INFINITY TO ESTIMATE THE ERROR. THE RELATIVE TRUNCATION ERROR ESTIMATE IS STORED IN EPTRUNC.

THE PLASMA VELOCITY DISTRIBUTION FUNCTION MUST BE NORMALIZED I.E. THE INTEGRAL OVER ALL VELOCITY SPACE OF THE DISTRIBUTION FUNCTION MUST BE 1. THE NORMALIZATION CONSTANTS ARE EVALUATED IN SUBROUTINE NORMALIZE. MOST OF THE NECESSARY INTEGRALS CAN BE PERFORMED ANALYTICALLY. THE NORMALIZATION FOR THE IDEAL MU DISTRIBUTION FUNCTION IS PERFORMED NUMERICALLY, USING THE QUAD INTEGRATION ROUTINE. THE MIRROR DISTRIBUTION NORMALIZATION, ALTHOUGH ANALYTIC, IS QUITE COMPLICATED, AND USES THE IMSL SUBROUTINE MDGAM TO EVALUATE THE INCOMPLETE GAMMA FUNCTION. THE MEAN SQUARE VELOCITY OF THE MIRROR SPEED DISTRIBUTION IS ALSO EVALUATED ANALYTICALLY.

THE ENERGY UNIT FOR THE INPUT ARGUMENTS IS KEV, AND THE MASS UNIT FOR THE INPUT ARGUMENTS IS AMU. WITHIN THE CODE, VELOCITIES ARE EXPRESSED IN KEV(.5) AMU(-.5), RATHER THAN M SEC(-1) OR CM SEC(-1). THIS WAS DONE FOR THREE REASONS. FIRST, IT CUT DOWN ON THE NUMBER OF CONVERSION FACTORS NEEDED WITHIN THE CODE. SECOND, TYPICAL VELOCITIES WOULD BE APPROXIMATELY 1. RATHER THAN 1.E+7. THIS MINIMIZES THE POSSIBILITY OF OVERFLOW ON THE PDP-10. (THE PDP-10 OVERFLOWS AT ABOUT 1.E38) . THIRD, IT MADE IT EASIER TO CHECK THE ANALYTIC NORMALIZATION OF THE MIRROR VELOCITY DISTRIBUTION FUNCTION. THE CROSS SECTION FUNCTIONS TAKE AS INPUT A SQUARE VELOCITY IN KEV AMU(-1), AND YIELD A CROSS SECTION IN CM(2). THE INTEGRATED RATE COEFFICIENT IS THEN IN UNITS OF CM(2) KEV(.5) AMU(-.5)/ THIS IS CONVERTED TO CM(3) SEC(-1) JUST BEFORE THE VALUE IS RETURNED TO THE USER.

***** SIMPLIFICATIONS *****

IF THE MU DISTRIBUTION IS ISOTROPIC AND THE VELOCITY DISTRIBUTION IS MAXWELLIAN, THEN TWO OF THE INTEGRALS CAN BE DONE ANALYTICALLY. (THE TRICK IS TO TRANSFORM TO SPHERICAL RELATIVE VELOCITY COORDINATES). IN THIS CASE, FUNCTION SCVIMX IS CALLED, AND IT IN TURN CALLS FUNCTION CLQ, WHICH USES A 32 POINT GAUSS-LAGUERRE QUADRATURE FORMULA TO EVALUATE THE SINGLE INTEGRAL. UNFORTUNATELY, THERE IS NO RELATIVE ERROR ESTIMATE AVAILABLE IN THE GAUSS-LAGUERRE QUADRATURE. YOU WILL JUST HAVE TO TAKE THE ANSWER ON FAITH OR RERUN WITH MUDIST = 5 (ISOTROPIC) AND NVELDI=1 (MAXWELLIAN) AND A SMALL EPSILON AND COMPARE ANSWERS.

IF THE THETARFAM ARGUMENT IS ZERO, OR THE FBAM ARGUMENT IS ZERO, THEN THE PHI INTEGRAL BECOMES INDEPENDENT OF PHI, AND CAN BE DONE ANALYTICALLY. IN THIS CASE THE TRIPLE INTEGRAL EVALUATION REDUCES TO A DOUBLE INTEGRAL.

IF THE ISOTROPIC MU DISTRIBUTION IS CHOSEN, THEN THE BEAM ANGLE MAKES NO DIFFERENCE. IF WE SET THE THETARFAM TO ZERO, WE CAN REDUCE THE TRIPLE INTEGRAL TO A DOUBLE INTEGRAL. (SEE PREVIOUS PARAGRAPH.)

WHEN THE BEAM SPEED IS MUCH GREATER THAN THE AVERAGE PLASMA SPEED, THE RELATIVE SPEED APPROACHES THE BEAM SPEED, AND THE RATE COEFFICIENT APPROACHES SIGMA(VBEAM) * VBFAM AND THIS INTEGRATION PROGRAM SHOULD NOT BE USED.

THERE IS SOME REGULARITY IN THE NAMING OF THE CROSS SECTIONS. THE FIRST LETTER REFERS TO THE ATOMIC PROCESS TAKING PLACE: "X" FOR CHARGE EXCHANGE, "I" FOR IONIZATION. THE SECOND GROUP OF ONE OR TWO LETTERS REFERS TO THE ION SPECIES: "E" FOR ELECTRONS, "H" FOR HYDROGEN (PROTON OR DEUTERON), "HE" FOR HELIUM (ALPHA PARTICLES). THE THIRD GROUP OF ONE OR TWO LETTERS REFERS TO THE NEUTRAL SPECIES: "H" FOR ATOMIC HYDROGEN, "HE" FOR ATOMIC HELIUM, AND "H2" FOR MOLECULAR HYDROGEN. FOR EXAMPLE, "IEH2" IS THE CROSS SECTION LABEL FOR IONIZATION OF MOLECULAR HYDROGEN BY ELECTRONS. THE CROSS SECTION FUNCTION NAMES CONSIST OF "SV" FOLLOWED BY THE CROSS SECTION LABEL, FOR EXAMPLE SVIEH2. NOTE, HOWEVER, THAT WHEN THE CROSS SECTION LABEL HAS FIVE CHARACTERS (LIKE IHEHE) THE CROSS SECTION FUNCTION NAME IS "S" FOLLOWED BY THE LABEL (LIKE SIHEHE).

***** COMMON BLOCK VARIABLES *****

COMMON BLOCK /SCVCONS/
THIS BLOCK CONTAINS CONSTANTS NEEDED BY SCV

PI	PI
SQRTPI	SQUARE ROOT OF PI
EMASS	ELECTRON MASS (AMU)
PROMASS	PROTON MASS (AMU)
CONVFAC	CONVERSION FACTOR FROM KEV(.5) AMU(-.5) TO CM SEC(-1).
EPSMACH	MACHINE EPSILON. HIGHEST RELATIVE ACCURACY OBTAINABLE ON THE COMPUTER

COMMON BLOCK /SCVBEAMV/

THIS BLOCK CONTAINS THE BEAM VELOCITY VARIABLES

VBEAM BEAM VELOCITY (KEV(.5) AMU(-.5))
 VBMSQ SQUARE OF BEAM VELOCITY (KEV AMU(-1))
 AMUBEAM BEAM MU. (COSINE OF THETABEAM)
 AMUBSQ SQUARE OF AMUBEAM

COMMON BLOCK /SCVPLASMAV/

THIS BLOCK CONTAINS THE PLASMA SPEED VARIABLES

THESE ARE NOT RELATED TO THE MEAN PLASMA SPEED

VPLASMA PLASMA SPEED (KEV(.5) AMU(-.5))
 VPLSQ SQUARE OF PLASMA SPEED (KEV AMU(-1))
 AMUPLASMA PLASMA MU
 AMUPLSQ SQUARE OF AMUPLASMA

COMMON BLOCK /SCVEPS/

THIS COMMON BLOCK CONTAINS THE RELATIVE ERROR CRITERIA FOR THE INTEGRATIONS

EPSPHI EPSILON FOR THE PHI INTEGRAL
 EPSMU EPSILON FOR THE MU INTEGRAL
 EPSV EPSILON FOR THE V INTEGRAL
 KMIN MINIMUM NUMBER OF SUBDIVISIONS FOR MU-INTEGRAL

COMMON /SCVEPERROR/

THIS COMMON BLOCK CONTAINS ERROR INFORMATION.

NUMERR(2C) ARRAY OF ERROR SENTINELS. (ZERO FOR NO ERROR).

INDEX ERROR
 1 NUMBER OF TIMES THE PHI INTEGRATION DID NOT CONVERGE
 2 NUMBER OF TIMES THE MU INTEGRATION DID NOT CONVERGE
 3 V INTEGRATION DID NOT CONVERGE
 4 NORMALIZATION OF THE IDEAL DISTRIBUTION DID NOT CONVERGE
 5 ERROR IN NORMALIZATION OF MIRROR DISTRIBUTION FUNCTION DUE TO CALL TO IMSL ROUTINE MDCAM.
 6 ERROR IN MEAN SQUARE VELOCITY CALCULATION FOR MIRROR DISTRIBUTION FUNCTION.
 7 INCORRECT NSIGMA ARGUMENT. DEFAULTS TO HYDROGEN HYDROGEN CHARGE EXCHANGE.
 8 INCORRECT NMUINDIST ARGUMENT. DEFAULTS TO ISOTROPIC.
 9 INCORRECT NVELDIST ARGUMENT. DEFAULTS TO MAXWELLIAN.
 10 INCORRECT ABMASS ARGUMENT. DEFAULTS TO 1.
 11 INCORRECT APLMASS ARGUMENT. DEFAULTS TO 1.
 12 INCORRECT EBEAM ARGUMENT. DEFAULTS TO 0.
 13 INCORRECT FPLASMA ARGUMENT. DEFAULTS TO 1.
 14 INCORRECT EPSILON ARGUMENT. DEFAULT DEPENDS ON IF TOO LARGE OR TOO SMALL.
 15 INCORRECT RATHIRFOR ARGUMENT. DEFAULTS TO 1.1.
 16 SEE MAXERROR(1)
 17 SEE MAXERROR(2)
 18-20 NOT USED
 NERRTOT TOTAL NUMBER OF ERRORS. (SUM OF NUMERROR ARRAY).
 EPSTR ESTIMATE OF RELATIVE TRUNCATION IN V INTEGRATION.
 EPSNOR ESTIMATE OF ERROR IN NORMALIZATION OF IDEAL DIST.

COMMON BLOCK /SCVSTATS/
THIS COMMON BLOCK CONTAINS STATISTICS CONCERNING THE FUNCTION
EVALUATIONS DONE IN SCV.

LPETOT TOTAL NUMBER OF FUNCTION (SIGMA) EVALUATIONS DONE IN
PERFORMING THE PHI INTEGRALS
LPEMAX MAXIMUM NUMBER OF FUNCTION (SIGMA) EVALUATIONS IN A
SINGLE PHI INTEGRAL
LPEMIN MINIMUM NUMBER OF FUNCTION (SIGMA) EVALUATIONS IN A SINGLE
PHI INTEGRAL
EPSPMX MAXIMUM COMPUTED RELATIVE ERROR FOR A SINGLE PHI INTEGRAL.
IF ALL THE PHI INTEGRALS CONVERGED, EPSPMX IS ZERO.
LNETOT TOTAL NUMBER OF FUNCTION (AMUGRAND) EVALUATIONS FOR THE
MU INTEGRALS.
LMEIMAX MAXIMUM NUMBER OF FUNCTION (AMUGRAND) EVALUATIONS FOR A
SINGLE MU INTEGRAL
LMEIMIN MINIMUM NUMBER OF FUNCTION EVALUATIONS (AMUGRAND) FOR A
SINGLE MU INTEGRAL
EPSPMX MAXIMUM COMPUTED RELATIVE ERROR FOR A SINGLE MU INTEGRAL.
IF ALL THE MU INTEGRALS CONVERGED, EPSPMX IS ZERO.
LVETOT NUMBER OF FUNCTION (AMUGRAND) EVALUATIONS FOR THE V INTEGRAL
EPSVMX COMPUTED RELATIVE ERROR FOR THE V INTEGRAL.
AVEPE AVERAGE NUMBER OF FUNCTION (SIGMA) EVALUATIONS FOR
THE PHI INTEGRAL
AVIME AVERAGE NUMBER OF FUNCTION (AMUGRAND) EVALUATIONS FOR
THE MU INTEGRAL.

COMMON BLOCK /SCVPAR/
THIS COMMON BLOCK CONTAINS PARAMETERS FOR ADJUSTING THE INTEGRATION
PERFORMANCE OF THE FUNCTION.

IMXPHI THE MAXIMUM NUMBER OF FUNCTION EVALUATIONS ALLOWABLE
IN THE PHIINT ROUTINE IS 2 ** (IMXPHI + 1). IMXPHI IS
SET IN DATA TO BE 9, FOR A MAXIMUM OF 1024 FUNCTION
EVALUATIONS IN PHIINT.
ULVCON CONSTANT USED FOR CHOOSING THE UPPER LIMIT OF INTEGRATION
FOR THE V INTEGRAL. INCREASING ULVCON INCREASES THE
UPPER LIMIT OF INTEGRATION. SEE THE CODING FOR DETAILS.
DATA LOADED VALUE OF 2.
EPSFMMU EPSILON FACTOR FOR THE MU INTEGRAL. SINCE VALUES OF THE
MU INTEGRAL ARE USED IN THE V INTEGRAL, THE MU
INTEGRAL SHOULD BE DONE MORE ACCURATELY THAN THE
V INTEGRAL. EPSFMMU = EPSV / EPSFMMU. DATA LOADED
VALUE OF 3..
EPSFPH EPSILON FACTOR FOR THE PHI INTEGRAL. EPSFPH = EPSFMMU/EPSFPH.
DATA LOADED VALUE OF 3.
PHINMX MAXIMUM VALUE OF THE PHI INTEGRAL ENCOUNTERED DURING THE
CALCULATION.
PHIFLG FLAG TO USE ABSOLUTE ERROR CRITERION IN PHI INTEGRAL (IF = 1)
INSTEAD OF RELATIVE ERROR CRITERION (IF = 0) (DEFAULT = 0).

COMMON BLOCK /SGVLM/
COMMON BLOCK FOR THE LIMITS OF INTEGRATION.

ALIMMU LOWER LIMIT OF INTEGRATION FOR THE MU INTEGRAL.
BLIMMU UPPER LIMIT OF INTEGRATION FOR THE MU INTEGRAL.
ALIMV LOWER LIMIT OF INTEGRATION FOR THE V INTEGRAL.
BLIMV UPPER LIMIT OF INTEGRATION FOR THE V INTEGRAL.

COMMON BLOCK /SGVSWITCH/

THIS BLOCK CONTAINS SWITCHES THAT INDICATE VELOCITY DISTRIBUTION, MU DISTRIBUTION, AND CROSS SECTION.

ISWIVD VELOCITY DISTRIBUTION SWITCH(= NVELDI).
 ISWIMD MU DISTRIBUTION SWITCH(= NMUDIS).
 ISWISG CROSS SECTION SWITCH(= PSIGMA).

COMMON BLOCK /SGVDISTPAR/

THIS COMMON BLOCK CONTAINS DISTRIBUTION FUNCTION PARAMETERS

ALPHA V CONSTANT FOR THE MAXWELLIAN SPEED DISTRIBUTION,
 $EXP(-ALPHA V * (V ** 2))$
 AMUZERO MU NAUGHT.
 AMUSQ MU NAUGHT SQUARED.

COMMON BLOCK /SGVNORM/

THIS COMMON BLOCK CONTAINS NORMALIZATION CONSTANTS.

VNORM VELOCITY INTEGRAL NORMALIZATION CONSTANT.
 AMUNORM MU INTEGRAL NORMALIZATION CONSTANT.
 PHINORM PHI INTEGRAL NORMALIZATION CONSTANT.

COMMON BLOCK /SGVISOMAX/

THIS COMMON BLOCK CONTAINS PARAMETERS NEEDED FOR THE ISOTROPIC MAXWELLIAN INTEGRATION. (32 POINT GAUSS-LAGUERRE)

BETAPSQ SQUARE OF BETAP. BETAP IS RELATED TO THE PLASMA ENERGY.

TWOBPVB $2. * BETAP * VBEM.$

MAXERR(20) ARRAY OF ERROR SENTINELS(0 FOR NO ERROR)

INDEX FRPOP

1 EBAM TOO HIGH FOR SGVIMX. USED SIGMA(VBEM)*VBEM INSTEAD.

2 ELECTRON CROSS SECTION ROUTINE SPECIFIED BUT NEITHER MASS WAS ELECTRON MASS. USED EMASS FOR PLMASS.

7,10,11,13 SAME AS NUMERR INDICES.

VBEM BEAM SPEED (KEV $^{-.5}$ AMU $^{-.5}$)

COMMON BLOCK /SGVMIRROR/

THIS COMMON BLOCK CONTAINS VARIABLES NEEDED FOR USING THE MIRROR VELOCITY DISTRIBUTION.

FHOLE HOLE ENERGY. (KEV) INPUT

EPEAK PEAK ENERGY. (KEV) INPUT

FWIDTH ENERGY WIDTH. (KEV) INPUT

FMEAN MEAN ENERGY. (KEV) OUTPUT

VHOLE HOLE VELOCITY. (KEV(.5) AMU(-.5)) OUTPUT

VPEAK PEAK VELOCITY. (KEV(.5) AMU(-.5)) OUTPUT

WIDITH VELOCITY WIDTH. (KEV(.5) AMU(-.5)) OUTPUT

VNAUGHT V NAUGHT VELOCITY. (KEV(.5) AMU(-.5)) OUTPUT

VMEAN MEAN VELOCITY. (KEV(.5) AMU(-.5)) OUTPUT

COMMON BLOCK /SGVNUM/

THIS COMMON BLOCK CONTAINS THE NUMERICAL CROSS-SECTION DATA NEEDED FOR FUNCTION SGNUM. INTERPOLATION IS DONE WITH A PROGRESSIVE SCHEME DESCRIBED IN LLL UCRL-7681(1964).

FMIN MINIMUM ENERGY FOR WHICH DATA ARE AVAILABLE(EV)

EMAX MAXIMUM ENERGY FOR WHICH DATA ARE AVAILABLE(EV)

ZMASS MASS OF BEAM SPECIES IN TAKING DATA(AMU)

NCROSS NUMBER OF DATA PAIRS(<=30)

ENC(30) ENERGY FOR CROSS(EV)

CROSS(30) CROSS SECTION(10 $^{-16}$ CM 2)