MASTER

SPECIMEN-TEST MACHINE COUPLING AND ITS IMPLICATIONS FOR PLASTIC-DEFORMATION MODELS

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ABSTRACT

The equation relating uniaxial strain rates to the crosshead speed of a test machine of arbitrary stiffness is carefully examined. It is demonstrated that an accurate formulation of the specimen/machine coupling relation requires exact additivity of elastic and plastic strain contributions and that this is only possible with a specific set of definitions for these strains. When these definitions are implemented to insure additivity, two strain-dependent terms in the coupling relation are identified that have been treated as constants in previous studies. One term is shown to account for the decreasing true total strain rate with increasing plastic strain at a fixed crosshead speed. The second term accounts for strain dependence of the combined specimen/machine modulus due to change in length and area of the specimen.

Several applications of the coupling relation are given that illustrate the importance of treating the specimen and test machine as a well-defined system when inferring information about plastic-deformation models from raw data. Topics covered include (1) effect of machine stiffness on workhardening behavior, (2) variation of total and plastic strain rate during tensile deformation, (3) effect of machine stiffness on stress relaxation behavior,

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(4) relationship between stress rates before and after a change in crosshead speed, and (5) deductions about mobile dislocation density from observed stress-time behavior.

KEY WORDS: Tensile test, stress relaxation, strain, strain rate, machine stiffness, crosshead speed.

INTRODUCTION

There is no test more widely used to study the mechanical behavior of solids than the uniaxial tensile test. In engineering applications, standardized tensile test results provide a convenient means of comparing different materials for selection or qualification purposes. It is also favored by workers studying fundamental deformation mechanisms of solids because the simple stress state involved eases interpretation of experimental data.

The term 'tensile test' of course refers to the uniaxial nature of the specimen loading. However, a number of different tests fall into this category. These can be classified according to the parameter that is controlled during the test. Examples of these are the constant-crosshead-speed extension test, the constant-strain-rate test, the constant-stress (creep) test, the constant-strain (relaxation) test, and the constant-cyclic-stress (or -strain) amplitude. Investigators seeking to formulate time-dependent elevated temperature deformation models rely heavily on ^{Test}the constant-stress (creep) where the measured parameter is time-dependent uniaxial strain. At temperatures less than about one-half of the melting temperature of the specimen, the most common methods for deformation studies include the constant-crosshead-speed extension test and the stress relaxation test.

Several examples from the literature can be cited to indicate that the deformation response of solids is dependent on the nature of the test machine used. Krausz and Craig [1], Rohde and Nordstrom [2], and Smith and White [3] have reported that results of stress relaxation tests depend on the stiffness of the test machine. Wielke [4] has demonstrated that activation volumes determined using crosshead-speed cycling depend on machine stiffness. Rohde and Nordstrom [5] have indicated that the flow stress at a given strain and crosshead-speed and the subsequent stress relaxation behavior from this flow stress are machine stiffness-dependent. Finally, Holbrook, et. al. [6] have shown that the stress-time response following crosshead-speed changes and also its measurability are partly governed by load-train stiffness.

The need to include machine characteristics in the interpretation of tensile behavior was set forth clearly in the pioneering work of Johnston and Gilman [7]. That work and subsequent studies by Hahn [8] and Gillis [9] explored the influence of machine stiffness on yield point behavior and clearly demonstrated that the specimen-test machine interaction has an important effect on observed mechanical behavior.

The purpose of the present paper is to develop carefully the specimen-test machine coupling relation and indicate its usefulness in the interpretation of deformation behavior, and to illustrate the results with a number of applications. An accurate treatment of specimen-test machine coupling has been presented previously by Hart [10] in a paper that is cited most frequently for its treatment of tensile instability. Unfortunately, the attention to detail exercised in Hart's development is seldom found in other published works. More frequently than not, specimen-machine interactions are treated in an approximate fashion in the literature without critical assessment of the significance of the approximations. The applications we present in this study are meant to illustrate the importance of viewing the specimen and test machine as well-defined systems whose properties should be known when drawing conclusions about deformation mechanisms from experimental data. Additionally, we hope to point out how the behavior in a tensile test depends on the machine used, and, given that fact, supply formulae that facilitate the analysis of experimental data.

In the following section we give an exact development of the specimen-test machine coupling relation for a machine of arbitrary stiffness. We include outlines of several techniques for the determination of machine stiffness.

Our primary concern is the accurate treatment of machine stiffness and the resultant effects on mechanical behavior. Other machine properties such as overshoot, drift, damping, etc. [11,12] are not included since they are not characteristic of all machines. In the subsequent section we give a number of examples illustrating use of the coupling relation.

SPECIMEN-TEST MACHINE COUPLING

Consider a tensile specimen of initial gage length L_0 and initial cross-sectional area A_0 that is being elongated uniformly by the crosshead of a test machine. At any time the instantaneous specimen length is L_T and its time rate of change is denoted by $\dot{L}_T$. The total extension of the gage length is ΔL_T ($= L_T - L_0$), which, in general, includes elongation due to all sources - elasticity, plasticity, anelasticity, thermal strain, etc., though in this paper we limit the discussion to only elastic and plastic contributions to ΔL_T . If the crosshead speed is given by $\dot{X}$, we can write

$$\dot{X} = \dot{L}_T + \dot{L}_M \quad (1)$$

where $\dot{L}_M$ is the composite machine deflection rate due to deflections of the crosshead, grips, linkages, load cell, specimen shoulders, etc. If all elements of L_M are linearly elastic, Equation (1) can be written

$$\dot{X} = \dot{L}_T + \frac{\dot{P}}{K} \quad (2)$$

where K is the composite spring constant and $\dot{P}$ is the time rate of change of load. Dividing through Equation (1) by L_0 , we have

$$v_0 = \dot{\epsilon}_T + \frac{\dot{S}}{M_0} \quad (3)$$

where $\dot{e}_T$ is the total engineering strain rate, $\dot{S}$ is the engineering stress rate, $\dot{P}/A_0$, and we have substituted V_0 for $\dot{X}/L_0$ and M_0 for $K \frac{L_0}{A_0}$. Equation (3) is exact for a load-independent K . This is normally a good approximation in the load range where most deformation studies are conducted. For situations where the non-linearity in the load dependence of L_M cannot be neglected, K^{-1} in Equation (2) should be replaced by the Taylor series expansion

$$K^{-1} = K_i^{-1} + \left(\frac{dK^{-1}}{dP} \right)_{P_i} (P - P_i) + 1/2 \left(\frac{d^2 K^{-1}}{dP^2} \right)_{P_i} (P - P_i)^2 + \dots$$

where K_i is the composite spring constant evaluated at some reference load, P_i , and the derivatives in the expansion are evaluated at P_i . In the remainder of the paper we focus on machine behavior that can be represented by a constant spring constant, K , and therefore a constant machine modulus, M_0 . Cases are presented in the literature where the machine apparently cannot be represented by a unique value for K . Under those circumstances the coupling relations become extremely complex. We will point out ways for measuring K later in this section.

Two additional points should be made concerning Equation (3). First, it is not necessary for $\dot{X}$ to be constant for Equation (3) to hold. Secondly, implicit in the division by L_0 is the requirement that all points in the gage length are identical - in area, microstructural homogeneity, temperature, etc. Clearly, then, Equation (3) is not valid after the onset of any local phenomena such as necking or shear band formation.

To convert Equation (3) to a form useful for deformation studies, the engineering strain rate and stress rate must be converted to true strain rate and true stress rate. With the aid of the strain and strain rate definitions given in the appendix and the definition of true stress, $\sigma = S (1 + e_p)$,

Equation (A-14), where ϵ_p is the engineering plastic strain, we find

$$\frac{V_0}{g} = \dot{\epsilon}_T \left[1 - \frac{\sigma}{M} \right] + \frac{\dot{\sigma}}{M} \left[1 + \frac{\sigma}{E} \right] \quad (4)$$

where

$$M = gq M_0 \quad ,$$

$$g = 1 + e_T \quad ,$$

$$q = 1 + e_p \quad ,$$

and $\dot{\epsilon}_T$ is the true total strain rate*. Equation (4) is exact provided that M_0 is load-independent, that straining is uniform within the gage length, and that plastic deformation alters only the shape of the material in the gage length, but not its volume. Since true strain rates are additive (but not engineering strain rates, as shown in the appendix), Equation (4) can be expressed in terms of true plastic strain rate, $\dot{\epsilon}_p$, by making the substitution

$$\dot{\epsilon}_T = \dot{\epsilon}_p + \frac{\dot{\sigma}}{E} \quad .$$

Then

$$\frac{V_0}{g} = \dot{\epsilon}_p \left[1 - \frac{\sigma}{M} \right] + \frac{\dot{\sigma}}{C} \quad (5)$$

where C is the combined specimen-machine modulus given by

$$\frac{1}{C} = \frac{1}{E} + \frac{1}{M} \quad . \quad (6)$$

*Note that Equation (4) is based upon unloaded area. In practice, negligible difference is introduced by substituting q for g , since $g = (1 + e_E)q$ and $e_E \ll 1$. However, the relationship using g are algebraically simpler; no terms involving Poisson's ratio are present.

Equation (5) contains two strain-dependent terms, g and C , that are commonly treated as constants in publications involving the use of a specimen/machine coupling equation to interpret deformation behavior. The term g accounts for decreasing true strain rate due to specimen lengthening at a fixed crosshead speed for straining in tension and an increasing true strain rate for straining in compression. (Note that for uniform tension, true strain is always less than engineering; in compression, the reverse holds). The combined modulus C is strain-dependent due to the influence of strain on the effective machine modulus M . Plastic straining in tension causes an increase in length and a decrease in area of the specimen and results in an increase in the effective machine modulus M ($= gq M_0$). The reverse is true for plastic straining in compression.

The other term in Equation (5) that is routinely neglected is the quantity $(1 - \sigma/M)$ which forms a product with the true plastic strain rate. It is customarily assumed [10] that σ/M is negligible compared to unity, but this is a questionable assumption for "soft" test machines. It is easy to show that

$$\frac{\sigma}{M} = \frac{P}{KL_T}$$

which indicates also that for short gage lengths or large loads the quantity σ/M could be appreciable, depending on the value of K . In a survey of machine stiffnesses found in the literature, Hamstad and Gillis [13] reported values of K ranging from $4.56 \frac{\text{MN}}{\text{m}}$ ($26,000 \frac{\text{pounds}}{\text{inch}}$) to $29.2 \frac{\text{MN}}{\text{m}}$ ($166,500 \frac{\text{pounds}}{\text{inch}}$). As an example, for a 6.25 mm diameter, 25 mm gage length steel specimen ($E = 2.1 \times 10^5$ MPa) these values of K translate to M_0 values of $E/57$ to $E/9$. Stresses routinely experienced in tensile testing of steel can be in the order of $E/200$, a sizeabl

fraction of these M_0 values. Clearly then, neglect of the σ/M term in the coupling relation, Equation (5), can result in an error in the determination of $\dot{\epsilon}_p$ if a "soft" test machine is used.

Determination of Machine Stiffness

From the prior discussion, it is obvious that the machine stiffness must be well-known to accurately assess deformation behavior during tensile tests. Through use of the coupling relation, several techniques can be identified for determining the machine stiffness.

The most straightforward method is the total deformation technique. With this method, a tensile specimen is strained for a predetermined time τ in a machine operating at a constant crosshead speed. Provided that the strain is distributed uniformly within the gage length and the machine stiffness is constant, Equation (3) can be integrated to give

$$V_0 \tau = e_T + \frac{P_\tau}{KL_0} \quad (7)$$

where P_τ is the load at time τ . If an independent determination of the total engineering strain e_T is made with an extensometer, this equation can be solved for K . It should be emphasized that if this method for determining K is used, several values of τ should be employed to identify any variation of K with load. If significant variations are noted, Equation (7) itself is inaccurate since the integration performed on Equation (3) assumed a constant K . For a non-constant K , we have from Equation (3)

$$V_0 \tau = e_T + \frac{1}{L_0} \int_0^{P_\tau} \frac{dP}{K} \quad (8)$$

If K^{-1} is expressed in a power series as before, the integral in Equation (8) can be evaluated and the load dependence of K to any desired order can be determined using a system of linear equations and measurements of e_T at various τ_p values. For this technique a specimen material with a low yield strength and high work-hardening rate is recommended since this combination will provide measurable strains over a wide range of loads.

A second technique for determining machine stiffness is by the elastic loading method. With this method, $\dot{P}$ is measured for loading situations where yielding of the specimen does not occur. Substituting in Equation (3) the identity (A-15)

$$\dot{\sigma} = (1 + e_p) \dot{S} + \sigma \dot{\epsilon}_p, \quad (9)$$

we have

$$\frac{V_0}{g} = \dot{\epsilon}_p \left[1 + \frac{\sigma}{E} \right] + \frac{q}{C} \dot{S} \quad (10)$$

where again, $q = (1 + e_p)$. For elastic loading, $\dot{\epsilon}_p = 0$, and Equation (10) can be solved for C to give

$$C = gq \frac{L_0}{A_0} \frac{\dot{P}}{\dot{X}}. \quad (11)$$

Then M and finally K are determined using Equation (6). For this technique a specimen material with a high yield strength is recommended to provide a large load range for an accurate determination of $\dot{P}$. Unlike the total deformation method, this method for determination of K requires knowledge of the elastic modulus of the specimen. However, it does not require an independent

measurement of strain. For a comparison of stiffness determinations using these the first two techniques, the reader is referred to the paper by Hockett and Gillis [14].

A third technique that has been used for K determination could be called the crosshead speed change method. With this method, the load rates $\dot{P}$ are measured immediately before and after a change in crosshead speed. The standard assumption when using this method is that the true plastic strain rate remains continuous at the instant of the crosshead speed change. Gillis and Medrano [15] and Fortes and Proença [16] have made machine stiffness determinations using this method. To use the method, Equation (10) can be solved for the quantity $\dot{\epsilon}_p[1 + \sigma/E]$ which is continuous during a crosshead speed change. This results in the relation

$$\frac{\dot{X}_1}{gL_0} - \frac{q}{C} \frac{\dot{P}_1}{A_0} = \frac{\dot{X}_2}{gL_0} - \frac{q}{C} \frac{\dot{P}_2}{A_0}$$

where $\dot{P}_1$ and $\dot{P}_2$ are the load rates immediately before and after a change in crosshead speed from $\dot{X}_1$ to $\dot{X}_2$. Solving for C gives

$$C = gq \frac{L_0}{A_0} \frac{\dot{P}_2 - \dot{P}_1}{\dot{X}_2 - \dot{X}_1}, \quad (12)$$

from which K is found using Equation (6). As with the elastic loading method, the crosshead speed change method requires no independent measurement of strain for stiffness determination but does require knowledge of the elastic modulus of the specimen.

Gillis and Medrano [15] reported finding large scatter in stiffness determinations with the crosshead speed change method using speed changes

corresponding to the start of a stress relaxation test ($\dot{\chi}_2 = 0$). This is consistent with the work of Holbrook et. al. [6] who have demonstrated that the initial load rate during stress relaxation is a difficult quantity to measure accurately. Fortes and Proença [16] used crosshead speed changes corresponding to the reloading after a stress relaxation test ($\dot{\chi}_1 = 0$) and reported significantly less scatter in stiffness determinations. In view of the findings of Fortes and Proença [16] and Holbrook et. al. [6], use of the crosshead speed change technique can be expected to yield more reliable results for machine stiffness if determinations are limited to speed changes of the type $\dot{\chi}_2 > \dot{\chi}_1$.

A final suggested technique for measuring machine stiffness can be called the stress relaxation method. If the machine stiffness is constant over the range of load reduction in a stress relaxation test ($\dot{V}_0 = 0$), Equation (3) yields

$$K = \frac{-\Delta P}{L_0 \Delta e_T} \quad (13)$$

where ΔP is the difference between the load at any time during relaxation and the load at the start of relaxation (a negative quantity for relaxation in tension) and Δe_T is the increase (for tension) in total engineering strain at that time from the value at the start of relaxation. This technique requires a sensitive, independent measure of strain, but does not require knowledge of the specimen modulus. A highly strain-rate-sensitive specimen material is recommended, however, as well as a reasonably large value of initial crosshead speed in order to maximize load reduction and, hence, strain increase. This technique differs from the first three methods in that the crosshead is immobile for this determination of stiffness. Depending on the machine, it is possible that the static machine stiffness is dif-

ferent than the effective stiffness when the crosshead is moving [17].

As can be seen from Equations (3) - (6), significant simplifications in the coupling relation result if the machine modulus is very large. In particular, for $M_0 \rightarrow \infty$ we have

$$V_0 = \dot{e}_T = g \dot{e}_T = g (\dot{e}_p - \dot{\sigma}/E) \quad (14)$$

From the definition of M_0 , it can be seen that the machine modulus can be made arbitrarily large by selecting the appropriate value of L_0/A_0 . Extremely large values of L_0/A_0 are not practical, however. Decreasing A_0 decreases load values in a test and results in load measurement difficulties. Increasing L_0 increases the possibility of non-uniform deformation in the gage length. The effect of an infinite machine modulus is obtainable, however, by using a servo-controlled test machine in strain-control mode. The controlled parameter in this situation is L_T (hence e_T), not crosshead position. The feedback signal for the closed-loop strain control is obtained from an extensometer measurement of L_T made directly on the specimen.

In the case of closed-loop strain control, V_0 is not a consideration (although V_0 actually varies according to Equation (3) to maintain a prescribed $\dot{e}_T$), and all that is required to determine true strains and strain rates are the definitions given in the Appendix relating these quantities to e_T and $\dot{e}_T$. It can be seen that these are the same as those in Equation (14) using the coupling relations with $M_0 \rightarrow \infty$, hence, the concept of infinite machine modulus for servo-controlled machines in strain-control follows. Before proceeding to the next section, it should be mentioned that servo-controlled machines in either load- or stroke-control do not simulate infinite stiffness. The characteristic stiffness of the machine must be known in these cases to use the coupling relation.

APPLICATIONS

Several examples will now be given that illustrate the use of the exact form of the coupling relation in analysis of tensile test data.

Interpretation of Work Hardening Behavior

A realistic treatment of the work hardening capacity of a material is essential for a successful deformation model. In this section we outline the technique for accurate determination of work hardening behavior from tensile tests. We distinguish between two types of work hardening behavior, observed and actual. Briefly, observed work hardening is the measured increase in the flow stress of the material for a measured increase in strain. As shown below, this measured behavior is influenced by machine stiffness and loading conditions, and these effects must be accounted for to deduce the actual work hardening behavior.

Consider a tensile specimen that is being loaded by a tensile machine and experiences an increment in total true strain $d\epsilon_T$. We may divide the differential expression

$$d\epsilon_T = d\epsilon_E + d\epsilon_p$$

by $d\sigma$ to arrive at

$$\frac{1}{H_T} = \frac{1}{H_E} + \frac{1}{H_p} \quad (15)$$

where H_T is the observed total work hardening coefficient, $\frac{d\sigma}{d\epsilon_T}$, and H_E and H_p are similarly defined. H_T is the slope of a true stress-true strain curve at any point. By definition, $\epsilon_E = \sigma/E$, thus H_E is equal to the elastic

modulus E . Since H_T is a measured quantity and E is known, Equation (15) provides us with a method for calculating H_p , the increase in flow stress for an increase in plastic strain. We note in passing that for loading of a tensile specimen near the yield stress (where $H_T \approx E$), Equation (15) indicates that $H_p \gg E$. We feel this behavior simply suggests that the quantity $\frac{d\sigma}{d\epsilon_p}$ is not well-defined for near-elastic loading. The reciprocal, $\frac{d\epsilon_p}{d\sigma}$, is well-defined, however. Representation of stress-strain test results by plotting H_p vs. σ has become popular of late [18]. It would appear, in light of the near-yield peculiarity with H_p , that an alternate graphical representation of stress-strain behavior using $1/H_p$ vs. σ is advisable.

Next, consider substituting the identity

$$\dot{\epsilon}_p = \frac{\dot{\sigma}}{H_p}$$

into the coupling relation, Equation (5), and solving for $1/H_p$:

$$\frac{1}{H_p} = \frac{\frac{V_0}{g\dot{\sigma}} - \frac{1}{C}}{(1 - \frac{\sigma}{M})} \quad (16)$$

This equation demonstrates that the quantity H_p can be determined from measurement of $\dot{\sigma}$ and g ($= 1 + e_T$), and also, that H_p is dependent on a prescribed parameter, V_0 , and on machine stiffness (through M and C). Equation (16) also clearly indicates that H_p is not a fundamental material parameter, but is dependent on the experimental loading conditions. For example, letting $V_0 = 0$ in Equation (16), corresponding to a stress relaxation test, we find that

$$H_p = - (1 - \frac{\sigma}{M}) C$$

which is a negative quantity. This is not an indication of work softening during stress relaxation, but a demonstration that H_p is an observed work hardening coefficient and should not be equated with the actual work hardening behavior.

The correspondence between H_p and the actual work hardening coefficient θ_p is, however, easy to develop. If the specimen flow stress depends only on plastic strain and plastic strain rate, we may write

$$d\sigma = \left(\frac{\partial \sigma}{\partial \epsilon_p} \right)_{\dot{\epsilon}_p} d\epsilon_p + \left(\frac{\partial \sigma}{\partial \ln \dot{\epsilon}_p} \right)_{\epsilon_p} \frac{d\dot{\epsilon}_p}{\dot{\epsilon}_p} \quad (17)$$

The partials in this expression correspond to the work hardening rate at constant true plastic strain rate and the strain rate sensitivity of the flow stress, respectively. Renaming these partials θ_p and γ , Equation (17) can be rewritten:

$$H_p = \theta_p + \gamma \frac{\ddot{\epsilon}_p}{\dot{\epsilon}_p^2} \quad (18)$$

Provided the strain rate sensitivity of the material, γ , is known, Equation (18) enables us to calculate θ_p from a measured value of H_p and knowledge of the true plastic strain rate and the true plastic strain acceleration $\ddot{\epsilon}_p$. It is obvious from this equation that for a constant true plastic strain rate test [19], where $\ddot{\epsilon}_p = 0$, θ_p is determined directly, since it is equal to H_p . In other loading situations (e.g. constant V_0), θ_p must be found from Equation (18) using experimental determinations for H_p , $\dot{\epsilon}_p$ and $\ddot{\epsilon}_p$. If the experimental data is available in the form of true stress versus time, and an independent measurement of strain is available, H_p , $\dot{\epsilon}_p$, and $\ddot{\epsilon}_p$ can be determined from

Equation (16), Equation (5), and the time derivative of Equation (5), respectively.

In practice, however, the experimental record from a tensile test is ordinarily in the form of engineering stress versus time or engineering stress versus total engineering strain. In this case, Equation (5) is inconvenient for calculation of $\dot{\epsilon}_p$ and ϵ_p to use in Equation (18). By differentiating Equation (3) and using (A3) and (A13) we find that the coupling relations for $\dot{\epsilon}_p$ and ϵ_p in terms of engineering quantities are

$$\dot{\epsilon}_p = \left(\frac{V_0}{g} - \frac{\dot{S}q}{C} \right) / \left(1 + \frac{qS}{E} \right) \quad (19)$$

and

$$\ddot{\epsilon}_p = \left[\frac{V_0}{g} - \frac{\ddot{S}q}{C} - \left(\frac{\dot{\epsilon}_T}{g} \right)^2 - \frac{q}{E} \left(2 \dot{S}\dot{\epsilon}_p + S \dot{\epsilon}_p^2 \right) \right] / \left(1 + \frac{qS}{E} \right) \quad (20)$$

The values for $\dot{\epsilon}_T$ and $\dot{\epsilon}_p$ in Equation (20) are found from Equations (3) and (19), respectively. Also, to a very good approximation in Equations (19) and (20), the denominators can be equated to one since $qS \ll E$.

For tensile test data in the ^{form of} ~~form of~~ S versus e_T , stress rate, $\dot{S}$, is not available but is easily found from the slope of the engineering stress-strain trace, $h_T (= \frac{dS}{de_T})$. Using Equation (3):

$$V_0 = \dot{S} \left(\frac{1}{h_T} + \frac{1}{M_0} \right) \quad (21)$$

The expression relating engineering work-hardening rates analogous to Equation (15) for true work-hardening rates is easily shown to be

$$\frac{1}{h_T} = \frac{1 + e_p}{h_E} + \frac{1 + e_E}{h_p} . \quad (22)$$

Equation (22) is not particularly useful since h_E does not, in general, equal E ($h_E = E$ only for $e_p = 0$). Determination of h_T from Equation (21) or directly from the slope of the engineering stress-strain curve is useful, however, since h_T can be related to H_T . The correspondence is

$$H_T = \frac{q(gh_T + S)}{1 + \frac{qS}{E}} \approx q(gh_T + S) , \quad (23)$$

where, as before,

$$\begin{aligned} g &= 1 + e_T \approx 1 + e_p \\ q &= 1 + e_p . \end{aligned}$$

For completeness, we include the relation between H_p and h_p :

$$H_p = q (q h_p + S) \quad (24)$$

Finally, simple expression relating h_E to $H_E (= E)$ does not exist since h_E is defined in terms of engineering stress, which does not account for the current value of strain. However, it can be shown that

$$\frac{h_E}{h_T} = \frac{E(1 + \frac{qS}{E})}{g h_T + S} = \frac{qE}{H_T} \quad (25)$$

Variation of Strain Rate in a Tensile Test

A plastically deforming specimen appears to the moving crosshead as a nonlinear element in series with the spring which represents the machine respons

If Equation (3) is used to solve for the partitioning of crosshead displacement between specimen and machine, we obtain

$$\dot{L}_T = \frac{\dot{X}}{1 + \frac{h_T}{M_0}} \quad (26)$$

where h_T is the slope of the engineering stress-strain curve. This equation illustrates that the total engineering strain rate $\dot{L}_T/L_0$ is, in general, variable for a constant crosshead speed test, depending on the instantaneous engineering work-hardening coefficient and the machine stiffness. This behavior has been noted in a number of studies [13, 20, 21]. It can be seen from Equation (26) that only for an infinitely stiff machine (servo-controlled in strain mode) is the total engineering strain rate identically constant during a tensile test.

Identification of the true strain rate behavior can be accomplished by dividing Equation (26) by $L_T = L_0 (1 + e_T)$, giving

$$\dot{\epsilon}_T = v_0 / \left(\left(1 + e_T \right) \left(1 + \frac{h_T}{M_0} \right) \right) \quad (27)$$

If the experimental record is in terms of engineering stress versus time, Equation (21) is useful for relating h_T to $\dot{S}$. True elastic strain rate is most easily determined by using Equation (23) to solve for H_T . Then (23) yields

$$\dot{\epsilon}_E = \dot{\epsilon}_T \left(\frac{H_T}{E} \right) \quad (28)$$

True plastic strain rate is then determined from

$$\dot{\epsilon}_p = \dot{\epsilon}_T - \dot{\epsilon}_E$$

Shown in Figure 1 is an example of strain rate variation during a tensile test. Chosen for illustration is the strain-rate-insensitive constitutive behavior

$$\epsilon_p = \epsilon_0 \left(\frac{\sigma}{\sigma_0} \right)^N, \quad (29)$$

where ϵ_0 , σ_0 , and N are constants.

It follows, then, that

$$\epsilon_T = \frac{\sigma}{E} + \epsilon_0 \left(\frac{\sigma}{\sigma_0} \right)^N \quad (30)$$

and

$$\frac{1}{H_T} = \frac{1}{E} + N \frac{\epsilon_0}{\sigma_0} \left(\frac{\sigma}{\sigma_0} \right)^{N-1}. \quad (31)$$

For purposes of this illustration, Equation (4) has been solved for $\dot{\epsilon}_T$ (instead of using Equation (27) since H_T can be inserted directly in the result:

$$\dot{\epsilon}_T = v_0 / \left((1 + e_T) \left(1 - \frac{\sigma}{M} + \frac{H_T}{M} \left(1 + \frac{\sigma}{E} \right) \right) \right) \quad (32)$$

The curves in Figure 1 have been calculated for a condition where both ϵ_E and ϵ_p equal 0.002 when $\sigma = \sigma_0$. Thus, σ_0 represents the 0.2 percent

offset yield strength, occurring at $\sigma_0 = 0.002E$ (60 ksi for steel). A value of $N = 5$ has been chosen which corresponds to a work hardening exponent of 0.2. Curves are given in Figure 1 for two values of machine stiffness, $E/10$ and infinitely stiff. It can be seen from this figure that for the "soft" machine ($E/10$), all strain rates are variable during a constant V_0 tensile test of this model material. However, for an infinite stiffness ($M_0 = \infty$), $\dot{e}_T$ is constant throughout the test and as can be seen from either Equation (27) or (32), true total strain rate is simply $V_0/(1 + e_T)$. Clearly, true plastic strain rate is zero at the beginning of the test for either stiffness and is virtually equal to $\dot{e}_T$ after yielding. There is a rapid variation of $\dot{e}_p$ near the yield stress, the details of which are dependent on machine stiffness. It should be obvious that a true-plastic-strain-rate tensile test is, in principle, not possible over the entire strain range. However, as described by Hartley and Jenkins [19], with a computer-controlled infinite-stiffness machine the test can be closely approximated after yielding has occurred.

Stress Relaxation Tests

The stress relaxation test is commonly used to examine the strain rate dependence of the flow stress for a given material. The test consists of stopping the crosshead after a predetermined amount of strain and measuring the subsequent decrease in load with time. Using the coupling relation, the measured load rate can be converted to a plastic strain rate, and this strain rate can be plotted against stress to ascertain the strain rate sensitivity of the flow stress.

From Equation (5), it follows immediately that during a stress relaxation test ($V_0 = 0$), the stress rate and strain rate are related by

$$\dot{\epsilon}_p = - \dot{\sigma} / C(1 - \frac{\sigma}{M}) \quad . \quad (33)$$

An unusual feature of Equation (33) is the presence of the term $1 - \sigma/M$. To the authors' knowledge, this term has never been included in published studies on stress relaxation behavior. The significance of the term depends on the stiffness of the machine used for testing. For servohydraulic machines in strain-control, M is effectively infinite, and Equation (33) reduces to

$$\dot{\epsilon}_p = - \dot{\sigma} / E \quad .$$

For "soft" machines, the effect of the term depends on the stress level. Neglect of this term results in an underestimate of the magnitude of the plastic strain rate during relaxation.

Since raw data for a stress relaxation test is normally in the form of load versus time, it is useful to express the right-hand side of Equation (33) in engineering terms. Substituting the identity (A-15)

$$\dot{\sigma} = q (\dot{S} + S \dot{\epsilon}_p)$$

into Equation (33) we arrive at

$$\dot{\epsilon}_p = q \dot{S} / C(1 + \frac{qS}{E}) \approx \frac{q\dot{S}}{C} \quad . \quad (34)$$

Often an empirical constitutive law for plastic strain rate as a function of stress is substituted for $\dot{\epsilon}_p$ in Equation (33) and integrated with the assumption of constant microstructure to yield the stress-time behav-

ior [22]. This is then fitted to the experimental stress-time data to identify the parameters in the empirical law. The assumption of constant microstructure is questionable, however, for a stress relaxation test [2, 23-26]. Clearly from Equation (33), plastic strain accrues during relaxation, the magnitude of which depends on machine stiffness for a given stress drop.

If the flow stress at a constant temperature depends only on microstructure (denoted by Γ) and plastic strain rate, we may write as a differential constitutive law

$$\dot{\sigma} = \left(\frac{\partial \sigma}{\partial \Gamma} \right)_{\dot{\epsilon}_p} \dot{\Gamma} + \left(\frac{\partial \sigma}{\partial \ln \dot{\epsilon}_p} \right)_{\Gamma} \frac{\ddot{\epsilon}_p}{\dot{\epsilon}_p} \quad (35)$$

which incorporates the influence of microstructural changes on the flow stress. This is essentially Equation (17) with Γ instead of ϵ_p as an internal state variable. The coupling relation for stress relaxation, Equation (33), can then be substituted into Equation (35) to give

$$- C \left(1 - \frac{\sigma}{M} \right) \dot{\epsilon}_p = \left(\frac{\partial \sigma}{\partial \Gamma} \right)_{\dot{\epsilon}_p} \dot{\Gamma} + \left(\frac{\partial \sigma}{\partial \ln \dot{\epsilon}_p} \right)_{\Gamma} \frac{\ddot{\epsilon}_p}{\dot{\epsilon}_p} \quad (36)$$

as a differential equation governing stress relaxation. As an example, consider the case where microstructure depends only on plastic strain (but without recovery or loss of mobile dislocations). Then Equation (36) can be written

$$- C \left(1 - \frac{\sigma}{M} \right) \dot{\epsilon}_p = \theta_p \dot{\epsilon}_p + \frac{\gamma \ddot{\epsilon}_p}{\dot{\epsilon}_p}, \quad (37)$$

which is readily integrated if the functional forms of the properties θ_p and γ are known.

Stress Rates Following a Crosshead Speed Change

Abrupt crosshead speed changes are frequently used in various types of tensile tests. Examples of these are strain-rate-cycling tests, fatigue tests, and stress relaxation tests. Often the stress rate immediately following a crosshead speed change is required as a test of deformation model [6] or for characterization of the test machine [15,16].

In using the coupling relation to solve for the stress rate following a crosshead speed change, one must make the usual assumption that true plastic strain rate is continuous during a speed change. Then we have from Equation (5) for a crosshead speed change from V_{01} to V_{02} :

$$\dot{\sigma}_2 = \dot{\sigma}_1 + \frac{C}{g} (V_{02} - V_{01}) \quad , \quad (38)$$

which relates the stress rate after the speed change, $\dot{\sigma}_2$, to the prior stress rate. Again, using Equation (16) to substitute for $\dot{\sigma}_1$ in Equation (38) we also have

$$\dot{\sigma}_2 = \frac{C}{g} \left(V_{02} - \frac{V_{01}}{1 + \frac{H_{p1}}{C(1 - \frac{\sigma}{M})}} \right) \quad , \quad (39)$$

where σ is the stress at the crosshead speed change and H_{p1} is the measured plastic work-hardening rate immediately before the change.

For a stress relaxation test V_{02} is zero in Equation (39). We see then that the initial stress rate in a stress relaxation test depends strongly

on machine stiffness and, through H_{p1} , on the location on the stress strain curve where relaxation is started.

Thermally Activated Dislocation Motion

For crystalline materials deforming by dislocation motion, the plastic strain rate is related to the mobile dislocation density ρ and their average velocity v by Orowan's Equation [27]

$$\dot{\epsilon}_p = \alpha b \rho v \quad (40)$$

where α is a constant and b is the Burgers vector of the mobile dislocations. It is possible to substitute for $\dot{\epsilon}_p$ in this equation from the coupling relation, Equation (5). This would relate the variation of the product ρv to the crosshead speed, stress rate, and machine stiffness. An alternate representation which separates the variables ρ and v is found by forming the ratio $\ddot{\epsilon}_p / \dot{\epsilon}_p$:

$$\frac{\ddot{\epsilon}_p}{\dot{\epsilon}_p} = \frac{\dot{\rho}}{\rho} + \frac{\dot{v}}{v} \quad (41)$$

For substitution into the left-hand side of this expression, Equation (5) and its time derivative (we note that g , σ , M , and C in Equation (5) are variables and should not be treated as constants in the differentiation) as Equations (19) and (20) may be used.

Use of Equation (41) to study the variation of ρ in a tensile test requires knowledge of the behavior of v and vice-versa. The dislocation velocity is commonly treated as a function of an effective stress σ^* which is the difference between the applied stress σ and an internal stress σ_i .

Equation (41) can then be written

$$\frac{\ddot{\epsilon}_p}{\dot{\epsilon}_p} = \frac{\dot{\rho}}{\rho} + \frac{d \ln v}{d \sigma^*} \dot{\sigma}^* \quad (42)$$

If the internal stress is due to the long range stress fields characteristic of the microstructure, it can be assumed that the internal stress varies with plastic strain according to

$$\dot{\sigma}_i = \theta_p \dot{\epsilon}_p \quad ,$$

where θ_p is the true work hardening coefficient discussed in an earlier section. Substituting this relation into Equation (42) we have

$$\frac{\ddot{\epsilon}_p}{\dot{\epsilon}_p} = \frac{\dot{\rho}}{\rho} + \frac{d \ln v}{d \sigma^*} (\dot{\sigma} - \theta_p \dot{\epsilon}_p) \quad (43)$$

Provided the dislocation velocity dependence on σ^* and the work-hardening behavior are known, Equation (43) enables us to study $\dot{\rho}$ from observations of the stress-time or stress-strain behavior during tensile deformation.

Figure 2 illustrates a typical experimental measurement for a particular load-time history, namely the load-transient behavior seen in strain-rate cycling tests. Some special cases are labeled with letters in Figure 2. At positions A, C, and D the engineering stress rate is zero. The terms $\dot{\sigma}$, $\dot{\epsilon}_p$, and $\dot{\epsilon}_p$ in Equation (43) are defined for this situation by Equations (9), (19), and (20). Thus, $\frac{\dot{\rho}}{\rho}$ can be quantitatively evaluated if θ_p and $\frac{d \ln v}{d \sigma^*}$ are known.

SUMMARY AND CONCLUSIONS

We have developed the specimen-machine coupling relation in its exact form for engineering total strain rate, true total strain rate, and true plastic strain rate in Equations (3) - (5). With a number of applications we have shown how the coupling relation can be used to accurately interpret observed deformation behavior. We have also attempted to illustrate several cases where a machine's stiffness characteristics actually influence the deformation response of a material during tensile testing.

Several methods have been described for determining machine stiffness. In a number of cases we have indicated the benefits of performing tensile tests in a servo-controlled machine in strain mode for which the machine stiffness is effectively infinite. The benefits include a strain rate behavior that is not dependent on the current work-hardening behavior of the specimen, minimal plastic strain occurring during stress relaxation tests, and a much simpler coupling relation for use in interpreting data.

REFERENCES

- (1) A. S. Krausz and G. B. Craig, Acta Met. 14 1807 (1966).
- (2) R. W. Rohde and T. V. Nordstrom, Scripta Met. 7 317 (1973).
- (3) I. O. Smith and M. G. White, Mat. Science and Eng. 21 185 (1975).
- (4) B. Wielke, Scripta Met., 11 87 (1977).
- (5) R. W. Rohde and T. V. Nordstrom, Scripta Met. 7 1227 (1973).
- (6) J. H. Holbrook, R. W. Rohde, and J. C. Swearengen, Acta Metallurgica, in press.
- (7) W. G. Johnston and J. J. Gilman, J. Appl. Physics 30 129 (1959).
- (8) G. T. Hahn, Acta Met. 10 727 (1962).
- (9) P. P. Gillis, 5th Int. Symp. on High Speed Testing, Boston, March, 1965.
- (10) E. W. Hart, Acta Met. 15 351 (1967).
- (11) F. Guiu and P. C. Pratt, Phys. Stat. Sol. 6 111 (1964).
- (12) F. Guiu, Scripta Met. 3 489 (1969).
- (13) M. A. Hamstad and P. P. Gillis, Materials Res. Standards 6 569 (1966).
- (14) J. E. Hockett and P. P. Gillis, Int. J. Mech. Sci. 13 251 (1971).
- (15) P. P. Gillis and R. E. Medrano, J. Materials 6 514 (1971).
- (16) M. A. Fortes and J. G. Proença, J. Testing and Eval. 4 248 (1976).
- (17) J. Preston, Instron Corporation, private communication, 1980.
- (18) See for example, U. F. Kocks, this volume.
- (19) C. S. Hartley and D. A. Jenkins, J. Metals 32 23 (1980).
- (20) I. M. Bernstein, Acta Met., 17 249 (1969).
- (21) H. R. Voorhees, ASTM Data Series, DS44, ASTM, 1969.
- (22) See for example, I. Gupta and J.C.M. Li, Met Trans. 1 2323 (1970).
- (23) G. A. Sargent, Acta Met. 13 663 (1965).

REFERENCES
(Continued)

- (24) G. Sargent, G. Jones, and H. Conrad, Scripta Met. 3 481 (1969).
- (25) K. Okazaki, Y. Aono, and M. Kagawa, Acta Met. 24 1121 (1976).
- (26) M. A. Meyers, J.R.C. Guimaraes, and R. R. Avillez, Met. Trans. 10A
33 (1979).
- (27) E. Orowan, Proc. Phys. Soc. 52 8 (1940).

FIGURES

- Fig. 1 Stress-strain and strain rate-strain behavior for the strain-rate-insensitive model material described by Equation (30). Note stress-strain is represented on a log-log plot. True plastic strain rate is the difference between the $\dot{\epsilon}_T$ and $\dot{\epsilon}_E$ curves. The vertical lines through the curves represents the strain at 0.2% offset yield strength.
- Fig. 2 Schematic load-time behavior in a strain-rate-cycling (or crosshead-speed - cycling) experiment with $\dot{X}_2 > \dot{X}_1$. At locations A, C, and D $\dot{S}$ is zero. Locations B and E are inflection points, thus $\ddot{S} = 0$.
- Fig. A1 Schematic true stress-strain curve indicating specimen gage length definitions for use in strain formulae.

APPENDIX

Development of the specimen-machine coupling relation in terms of true plastic strain rate, Equation (5), requires the exact additivity of true elastic and plastic strain components. Referring to the specimen gage length at selected points on the engineering stress-strain curve shown in Figure A-1, it can be seen that when the true elastic and plastic strains are

$$\epsilon_E = \ln \frac{L_T}{L_p} = \frac{\sigma}{E} \quad (A-1)$$

and

$$\epsilon_p = \ln \frac{L_p}{L_0} \quad , \quad (A-2)$$

additivity is satisfied because

$$\epsilon_T = \epsilon_E + \epsilon_p = \ln \frac{L_T}{L_0} \quad . \quad (A-3)$$

True strain rates are found by differentiating Equations (A-1)-(A-3) with respect to time:

$$\dot{\epsilon}_T = \frac{\dot{L}_T}{L_T} \quad (A-4)$$

$$\dot{\epsilon}_p = \frac{\dot{L}_p}{L_p} \quad (A-5)$$

and

$$\dot{\epsilon}_E = \frac{\dot{L}_T}{L_T} - \frac{\dot{L}_p}{L_p} = \frac{\dot{\sigma}}{E} \quad . \quad (A-6)$$

Engineering strain components are given by the formulae

$$e_E = \frac{L_T}{L_p} - 1 \equiv j - 1 \quad (A-7)$$

$$e_p = \frac{L_p}{L_0} - 1 \equiv q - 1 \quad (A-8)$$

STRAIN DEFINITIONS

and

$$e_T = \frac{L_T}{L_0} - 1 \equiv g - 1 \quad (A-9)$$

From comparison of Equations (A-1)-(A-3) to Equations (A-7)-(A-9) respectively, it can be seen that the conventional rules for connecting true to engineering strains are satisfied

$$\epsilon_E = \ln (1 + e_E)$$

$$\epsilon_P = \ln (1 + e_P)$$

$$\epsilon_T = \ln (1 + e_T)$$

Using Equation (A-3) it is evident that

$$1 + e_T = (1 + e_E)(1 + e_P) \quad (A-10)$$

where the quantities $1 + e_T$, etc. are often called stretch ratios [14].

Equation (A-10) indicates that engineering strains are not additive, but in fact

$$e_T = e_E + e_P + e_E e_P \quad (A-11)$$

Further

$$\dot{e}_T = (1 + e_P) \dot{e}_E + (1 + e_E) \dot{e}_P \quad (A-12)$$

i.e., engineering strain rates are not additive. True strain rates are related to engineering strain rates by

$$\dot{\epsilon}_E = \frac{\dot{e}_E}{1 + e_E} = \frac{\dot{g}}{g}$$

$$\dot{\epsilon}_P = \frac{\dot{e}_P}{1 + e_P} \quad (A-13)$$

$$\dot{\epsilon}_T = \frac{\dot{e}_T}{1 + e_T}$$