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DYNAMIC SOIL PRESSURES ON RIGID CYLINDRICAL VAULTS

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**Under contract to
Engineering Research and Applications Division
Department of Nuclear Energy
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Upton, N.Y. 11973**

February 1993

**Prepared for
OFFICE OF ENVIRONMENTAL RESTORATION AND WASTE MANAGEMENT
DEPARTMENT OF ENERGY, WASHINGTON, D.C.**

MASTER 82

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Printed in the United States of America
Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161

NTIS price codes:
Printed Copy: A04; Microfiche Copy: A01

ABSTRACT

A critical evaluation is made of the dynamic pressures and the associated forces induced by ground shaking on an upright, circular, rigid vault that is embedded in a uniform viscoelastic stratum of constant thickness and infinite extent in the horizontal plane. Both the vault and the stratum are presumed to be supported on a non-deformable base undergoing a space-invariant, uniform horizontal motion. The effects of both harmonic and earthquake-induced excitations are examined. Simple approximate expressions for the responses of the system are formulated, and comprehensive numerical data are presented which elucidate the underlying response mechanisms and the effects and relative importance of the various parameters involved. The parameters investigated include the height to radius ratio for the vault, the conditions at the vault-medium interface, and the material properties of the stratum. In addition to valuable insights into the response of the particular system being examined, the results presented provide a conceptual framework for the analysis and interpretation of solutions for more involved systems as well.

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EXECUTIVE SUMMARY

The study reported herein is the second in a series of investigations motivated by the need to gain improved understanding of the responses to earthquakes of deeply embedded and underground cylindrical tanks storing radioactive wastes, and to develop rational but simple methods of analysis and design for such systems.

It provides a critical evaluation of the dynamic pressures and the associated forces induced by ground shaking on an upright, rigid, circular cylindrical vault that is embedded in a uniform, viscoelastic stratum of constant thickness and infinite extent in the horizontal plane. The bases of both the vault and the stratum are presumed to undergo a space-invariant, uniform horizontal motion. The effects of both harmonic and earthquake induced motions are examined, with special attention given to excitations the dominant frequencies of which are small compared to the fundamental natural frequency of the stratum. Simple, approximate expressions for the responses of the system are formulated, and comprehensive numerical data are presented which elucidate the underlying response mechanisms and the effects and relative importance of the numerous parameters involved. In addition to the characteristics of the ground motion, the parameters investigated include the ratio of the stratum height to the vault radius, the conditions at the vault-medium interface, and the material properties of the stratum. For the limiting case of a vault of infinite radius, the system considered reduces to a stratum retained by a straight wall. The response of the latter system was examined critically in the previous study.

The system is analyzed by a procedure similar to that employed previously by Tajimi. However, whereas Tajimi's solution is based on the assumption of vanishing vertical displacements, the solution presented herein is based on the assumption of vanishing vertical normal stresses. The interrelationship of the two solutions is examined.

The principal conclusions of the study may be summarized as follows:

1. With the analytical expressions presented, the dynamic pressures and forces exerted on and in the vault may be evaluated reliably and cost-effectively.
2. The comprehensive numerical data presented and the analysis of these data provide not only valuable insights into the effects and relative importance of the numerous parameters involved, but also a conceptual framework for the analysis and interpretation of the solutions for more involved systems.

3. Poisson's ratios for the medium close to $1/2$, Tajimi's analysis of the problem leads to unacceptable results. This deficiency can be eliminated, however, by an appropriate modification of the dilatational wave velocity for the medium.
4. When expressed in terms of $\pi \rho \ddot{x}_g R H^2$, where ρ = the mass density of the medium, $\ddot{x}_g$ = the peak acceleration of the ground motion, H = the depth of the stratum, and R = the radius of the vault, the static or low-frequency value of the total force exerted on the vault increases from 1.57 for broad vaults with values of $H/R \rightarrow 0$ to 4.91 for a relatively slender vault with $H/R = 3$. These values are for vaults that are fully bonded to a medium with Poisson's ratio $\nu = 1/3$. The larger relative forces for the more slender vaults are due to the lower capacity of the deep strata to transfer the forces acting on them by horizontal shearing action to the base.
5. Because the effective stiffness of vaults with a smooth interface is lower than for a rough interface, the forces exerted on such vaults are smaller. When normalized in the manner indicated above, the static value of the normalized total force increases from 1.03 for $H/R = 0$ to 4.21 for $H/R = 3$.
6. For harmonically excited systems, the peak amplification factor at resonance also increases with increasing H/R , the magnitude of the increase being an inverse function of the damping factor for the medium, δ . This trend, which is also true of systems subjected to earthquake ground motions, reflects the reduced capacity of the taller, more slender vaults to dissipate energy by radiation of waves.
7. Almost independently of the system parameters and the characteristics of the ground motion, the forces exerted on the vault increase approximately as a half-sine wave from zero at the base to a maximum at the top. As a result, the maximum base moment in the vault may be expressed as the product of the maximum base shear and approximately 60 percent of the stratum depth, H .
8. For systems with a rough interface, approximately 60 percent of the total base shear and of the associated bending moment in the vault is contributed by normal pressures, whereas the remaining 40 percent is contributed by circumferential shearing stresses. These factors may, for all practical purposes, be considered to be independent of the characteristics of the forcing function.

ACKNOWLEDGEMENT

This study was carried out at Rice University under Grant 568821 from the Brookhaven National Laboratory, Upton, New York. This support is acknowledged gratefully. Appreciation also is expressed to Drs. M. Reich and K. Bandyopadhyay for their understanding project management.

SECTION 1

INTRODUCTION

The study reported herein was motivated by the need to gain improved understanding of the response to earthquakes of deeply embedded and underground cylindrical tanks storing nuclear wastes, and to develop rational but simple methods of analysis and design for such systems.

The dynamic response of embedded structures has been of considerable interest in recent years, and a number of methods of varying degrees of accuracy, efficiency and sophistication has been developed for its evaluation. For an account of these methods and of the computer programs that have been developed for their implementation, the reader is referred to the publications listed at the end of this contribution [1-22]. Of special interest in this regard are the papers in the proceedings of the workshop on the analysis and interpretation of the seismic response data obtained from the nuclear power plant test model in Lotung, Taiwan [Ref. 19], and a subsequent report by Hadjian et al [20].

Despite the value of these contributions, however, the dynamic response of deeply embedded structures is not well understood. There is, in particular, paucity of information regarding the underlying response mechanisms and the magnitude and distribution of the pressures and associated forces induced in such structures. It is believed that there is a two-fold need at this time: (1) To use existing or suitable improvements of existing methods to critically study and interpret the responses of particular classes of systems for which relatively simple analytical solutions are possible; and (2) to assess the accuracy of the simpler available procedures which are particularly useful for preliminary design purposes and, where necessary, propose appropriate improvements.

The present paper is responsive to the first need. It provides critical evaluations of the dynamic pressures and the associated forces induced by ground shaking on an upright, rigid, circular cylindrical vault that is embedded in a uniform, viscoelastic stratum of constant thickness and infinite extent in the horizontal plane. The bases of both the vault and the stratum are presumed to undergo a space-invariant, uniform horizontal motion. The effects of both harmonic and earthquake-induced motions are examined, and special attention is given to excitations the dominant frequencies of which are small compared to the fundamental natural frequency of the stratum. Simple, approximate expressions for the responses of the system are formulated, and comprehensive numerical data are presented which elucidate the underlying response mechanisms and the effects and relative importance of the

numerous parameters involved. The parameters investigated include the ratio of the stratum height to the vault radius, the vault-medium interface conditions, and the material properties of the stratum. For the limiting case of a vault of infinite radius, the system considered reduces to a stratum retained by a straight wall, the response of which was examined critically in [22].

The system is analyzed by a procedure similar to that employed by Tajimi [1]. However, whereas Tajimi's solution is based on the assumption of vanishing vertical displacements, the solution presented herein is based on the assumption of vanishing vertical normal stresses. For the limiting case of a stratum retained by a straight wall, the assumption of vanishing vertical stresses has been shown to be superior to that of vanishing vertical displacements [22]. The same is also expected to be true of the generalized system investigated herein. The interrelationship of the solutions based on the two assumptions is demonstrated herein.

The responses of other classes of relatively simple embedded structures and the reliability of available simple methods of analysis will be examined in subsequent publications.

SECTION 2

SYSTEM AND METHOD OF ANALYSIS

The system investigated is shown in part (a) of Fig. 2.1. It consists of a rigid circular cylinder of radius R that is embedded in a soil layer of constant thickness H and infinite extent in the horizontal plane. Idealized as a homogeneous, viscoelastic material with frequency-independent properties, the soil layer is free at the upper surface and bonded to the base. The interface between the cylinder and surrounding soil is presumed to be either rough or smooth, as explained in greater detail in later sections. Both the cylinder and the base of the layer are excited by a space-invariant, uniform motion, the acceleration of which at any time t is $\ddot{x}_g(t)$. It is desired to evaluate the resulting normal pressures and circumferential shearing stresses along the cylinder-soil interface, and the associated shearing forces and bending moments in the cylinder.

The properties of the layer are defined by its mass density, ρ , shear modulus, G , Poisson's ratio, ν , and the material damping factor, δ . The latter factor is considered to be frequency-independent and the same for both shearing and axial deformations. The effects of both harmonic and transient excitations are examined.

The displacements relative to the moving boundary and the resulting stresses and forces for the base-excited system considered in part (a) of Fig. 2.1 can be shown to be identical to those of the force-excited system shown in part (b), for which the boundary is stationary and the layer is subjected to uniform lateral body forces of intensity $-\rho\ddot{x}_g(t)$. For nearly static conditions of loading, for which the dominant frequency of the excitation is very low compared to the fundamental natural frequency of the stratum, the action of the force-excited system may be easier to visualize than that of the base-excited system.

2.1 Governing Equations and Assumptions

When referred to the cylindrical system of coordinates, r, z, θ , shown in Fig. 2.1, the equations of motion of the medium may be written as

$$\frac{\partial \sigma_r}{\partial \xi} + \frac{1}{\xi} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{R}{H} \frac{\partial \tau_{rz}}{\partial \eta} + \frac{\sigma_r - \sigma_0}{\xi} = \rho R \frac{\partial^2 u}{\partial t^2} + \rho R \ddot{x}_g(t) \cos \theta \quad (1)$$

$$\frac{\partial \tau_{r\theta}}{\partial \xi} + \frac{1}{\xi} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{R}{H} \frac{\partial \tau_{\theta z}}{\partial \eta} + \frac{2}{\xi} \tau_{r\theta} = \rho R \frac{\partial^2 v}{\partial t^2} - \rho R \ddot{x}_g(t) \sin \theta \quad (2)$$

$$\frac{\partial \tau_{rz}}{\partial \xi} + \frac{1}{\xi} \frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{R}{H} \frac{\partial \sigma_z}{\partial \eta} + \frac{1}{\xi} \sigma_{rz} = \rho R \frac{\partial^2 w}{\partial t^2} \quad (3)$$

in which ξ and η are dimensionless position coordinates defined by

$$\xi = r/R \quad \text{and} \quad \eta = z/H \quad (4)$$

σ_r , σ_θ and σ_z are the normal stresses at an arbitrary point and time along the r , θ and z coordinates, respectively; u , v and w are the corresponding displacements relative to the moving boundary; and $\tau_{r\theta}$, τ_{rz} and $\tau_{\theta z}$ are the shearing stresses in the r - θ , r - z , and θ - z planes, respectively. The sign convention for stresses and displacements is the same as that used in theory of elasticity. Specifically, displacements are positive when directed along the positive directions of the corresponding coordinate axes; normal stresses are positive when they induce tension; and the positive directions of the shearing stresses are as indicated in the inset diagrams of Fig. 2.1(a).

The stress components are related to the displacement components by

$$\sigma_r = \frac{\lambda^* + 2G^*}{R} \frac{\partial u}{\partial \xi} + \frac{\lambda^*}{R} \left[\frac{1}{\xi} \frac{\partial v}{\partial \theta} + \frac{u}{\xi} + \frac{R}{H} \frac{\partial w}{\partial \eta} \right] \quad (5)$$

$$\sigma_\theta = \frac{\lambda^* + 2G^*}{R} \left[\frac{1}{\xi} \frac{\partial v}{\partial \theta} + \frac{u}{\xi} \right] + \frac{\lambda^*}{R} \left[\frac{\partial u}{\partial \xi} + \frac{R}{H} \frac{\partial w}{\partial \eta} \right] \quad (6)$$

$$\sigma_z = \frac{\lambda^* + 2G^*}{H} \frac{\partial w}{\partial \eta} + \frac{\lambda^*}{R} \left[\frac{\partial u}{\partial \xi} + \frac{1}{\xi} \frac{\partial v}{\partial \theta} + \frac{u}{\xi} \right] \quad (7)$$

$$\tau_{r\theta} = \frac{G^*}{R} \left[\frac{1}{\xi} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial \xi} - \frac{v}{\xi} \right] \quad (8)$$

$$\tau_{rz} = \frac{G^*}{R} \left[\frac{R}{H} \frac{\partial u}{\partial \eta} + \frac{\partial w}{\partial \xi} \right] \quad (9)$$

$$\tau_{\theta z} = \frac{G^*}{R} \left[\frac{R}{H} \frac{\partial v}{\partial \eta} + \frac{\partial w}{\partial \theta} \right] \quad (10)$$

in which G^* and λ^* are the complex-valued Lamé constants

$$G^* = G(1 + i\delta) \quad (11)$$

$$\lambda^* = \frac{2\nu}{1 - 2\nu} G^* \quad (12)$$

and $i = \sqrt{-1}$.

Fundamental to the analysis presented is the assumption that no vertical normal stresses develop anywhere in the medium. As a consequence, the following expression is obtained from equation (7)

$$\frac{\partial w}{\partial \eta} = -\frac{\lambda^*}{\lambda^* + 2G^*} \frac{H}{R} \left[\frac{\partial u}{\partial \xi} + \frac{1}{\xi} \frac{\partial v}{\partial \theta} + \frac{u}{\xi} \right] \quad (13)$$

which, when substituted into equation (5), leads to

$$\sigma_r = \psi_o^2 \frac{G^*}{R} \frac{\partial u}{\partial \xi} + (\psi_o^2 - 2) \frac{G^*}{R} \left[\frac{1}{\xi} \frac{\partial v}{\partial \theta} + \frac{u}{\xi} \right] \quad (14)$$

with

$$\psi_o = \sqrt{\frac{2}{1-\nu}} \quad (15)$$

Finally, on substituting equations (5), (6), (8), (9) and (10) into equations (1) and (2) and making use of equation (13), the equations expressing the equilibrium of forces in the radial and circumferential directions can be written as

$$\begin{aligned} \psi_e^2 \frac{\partial}{\partial \xi} \left[\frac{1}{\xi} \frac{\partial}{\partial \xi} (\xi u) + \frac{1}{\xi} \frac{\partial v}{\partial \theta} \right] - \frac{1}{\xi} \frac{\partial}{\partial \theta} \left[\frac{1}{\xi} \frac{\partial}{\partial \xi} (\xi v) - \frac{1}{\xi} \frac{\partial u}{\partial \theta} \right] + \frac{R^2 \partial^2 u}{H^2 \partial \eta^2} = \\ \frac{\rho R^2 \partial^2 u}{G^* \partial t^2} + \frac{\rho R^2}{G^*} \ddot{x}_g(t) \cos \theta \end{aligned} \quad (16)$$

$$\begin{aligned} \psi_e^2 \frac{1}{\xi} \frac{\partial}{\partial \theta} \left[\frac{1}{\xi} \frac{\partial}{\partial \xi} (\xi u) + \frac{1}{\xi} \frac{\partial v}{\partial \theta} \right] + \frac{\partial}{\partial \xi} \left[\frac{1}{\xi} \frac{\partial}{\partial \xi} (\xi v) - \frac{1}{\xi} \frac{\partial u}{\partial \theta} \right] + \frac{R^2 \partial^2 v}{H^2 \partial \eta^2} = \\ \frac{\rho R^2 \partial^2 v}{G^* \partial t^2} - \frac{\rho R^2}{G^*} \ddot{x}_g(t) \sin \theta \end{aligned} \quad (17)$$

in which

$$\psi_e = \sqrt{\frac{2-\nu}{1-\nu}} \quad (18)$$

Equations (16) and (17) are solved subject to the boundary conditions

$$u|_{\xi=1} = 0 \quad (19)$$

$$u|_{\eta=0} = 0 \quad (20)$$

$$v|_{\eta=0} = 0 \quad (21)$$

and the following additional condition along the interface of the cylinder and soil: for a rough interface,

$$v|_{\xi=1} = 0 \quad (22)$$

and for a smooth interface

$$\tau_{r\theta}|_{\xi=1} = 0 \quad (23)$$

Equation (19) expresses the continuity of the radial displacements along the cylinder-soil interface, whereas equations (20) and (21) express the continuity of the horizontal displacements at the base of the layer. Because of the approximate nature of the method of analysis employed, the additional interface condition of $w = 0$ for a rough interface and of $\tau_{rz} = 0$ for a smooth interface cannot be satisfied.

With the relative displacements u and v determined, the stresses σ_r and $\tau_{r\theta}$ are computed from equations (14) and (8), and the vertical displacements, w , are determined by integration of equation (13) subject to the boundary condition

$$w|_{\eta=0} = 0 \quad (24)$$

If needed, the remaining stress components may be determined from equations (6), (9) and (10).

It should be noted that equation (3), which expresses the equilibrium of the vertical forces, is not satisfied in this approach, neither is the condition of vanishing shears at the free surface. For the limiting case of a straight wall, it has been shown [22] that these violations have a minor influence on the magnitudes of the wall pressures and forces, which are the quantities of primary interest in this study. The same is also expected true of the cylindrical boundary considered herein.

Mention has already been made of the fact that Tajimi's solution for the problem is based on the assumption of vanishing vertical displacements, w . His expressions for the various response quantities can be deduced from those presented herein by replacing both the factor ψ_o in equation (14) and the factor ψ_e in equations (16) and (17) by the factor

$$\chi_o = \sqrt{\frac{2(1-\nu)}{1-2\nu}} \quad (25)$$

The latter factor represents the ratio of the dilatational and shear wave velocities for the medium.

The interrelationship of the solutions based on the $\sigma_z = 0$ and $w = 0$ approximations is examined in a later section. In the interim, it is worth observing from equations (25) and (14) that, as $\nu \rightarrow 0.5$, the radial normal stresses in the solution for the $w = 0$ approximation tend to infinity, a result which is clearly unacceptable. Except where specifically otherwise indicated, the solutions presented herein are for the assumption of vanishing vertical normal stresses.

2.2 Harmonic Response

For a harmonic input motion of acceleration

$$\ddot{x}_g(t) = \ddot{X}_g e^{i\omega t} \quad (26)$$

the radial and tangential displacements, $u = u(\xi, \theta, \eta, t)$ and $v = v(\xi, \theta, \eta, t)$, may be expressed in the form

$$u = U(\xi, \theta, \eta) e^{i\omega t} \quad (27)$$

and

$$v = V(\xi, \theta, \eta) e^{i\omega t} \quad (28)$$

in which $\ddot{X}_g$ is the acceleration amplitude of the input motion; U and V are functions of ξ , θ and η representing the amplitudes of the resulting displacements relative to the moving boundary; and ω is the circular frequency of the excitation and of the resulting response.

The functions $U(\xi, \theta, \eta)$ and $V(\xi, \theta, \eta)$ may be expressed as linear combinations of the natural modes of vibration of the soil stratum computed on the assumption that it acts as a series of vertical cantilever shear-beams, as

$$U(\xi, \theta, \eta) = \sum_{n=1,3}^{\infty} U_n \sin \frac{n\pi}{2} \eta \cos \theta \quad (29)$$

$$V(\xi, \theta, \eta) = \sum_{n=1,3}^{\infty} V_n \sin \frac{n\pi}{2} \eta \sin \theta \quad (30)$$

in which $n = \text{an odd integer}$, and U_n and V_n are functions of the radial position coordinate, ξ . Note that these expressions satisfy automatically the boundary conditions defined by equations (20) and (21).

On substituting equations (26) through (28) into equations (16) and (17), making use of equations (29) and (30), and expanding $\ddot{X}_g$ in the form

$$\ddot{X}_g = \frac{4}{\pi} \ddot{X}_g \sum_{n=1,3}^{\infty} \frac{1}{n} \sin \frac{n\pi}{2} \eta \quad (31)$$

one finds that U_n and V_n must satisfy the system of ordinary differential equations

$$\begin{aligned} \psi_e^2 \frac{d}{d\xi} \left[\frac{1}{\xi} \frac{d}{d\xi} (\xi U_n) + \frac{1}{\xi} V_n \right] - \frac{1}{\xi} \left[\frac{1}{\xi} \frac{d}{d\xi} (\xi V_n) + \frac{1}{\xi} U_n \right] \\ - \left[\left(\frac{n\pi R}{2H} \right)^2 - \frac{\rho \omega^2 R^2}{G^*} \right] U_n = \frac{4}{n\pi} \frac{\rho R^2 \ddot{X}_g}{G^*} \end{aligned} \quad (32)$$

$$-\psi_e^2 \frac{1}{\xi} \left[\frac{1}{\xi} \frac{d}{d\xi} (\xi U_n) + \frac{1}{\xi} V_n \right] + \frac{d}{d\xi} \left[\frac{1}{\xi} \frac{d}{d\xi} (\xi V_n) + \frac{1}{\xi} U_n \right] - \left[\left[\frac{n\pi R}{2H} \right]^2 - \frac{\rho \omega^2 R^2}{G^*} \right] V_n = -\frac{4}{n\pi} \frac{\rho R^2 \ddot{X}_g}{G^*} \quad (33)$$

Equations (32) and (33) are solved by the approach followed by Tajimi [1], by expressing U_n and V_n in terms of the potential functions $P_n(\xi)$ and $S_n(\xi)$ as

$$U_n = \frac{dP_n}{d\xi} + \frac{S_n}{\xi} \quad (34)$$

and

$$V_n = -\frac{P_n}{\xi} - \frac{dS_n}{d\xi} \quad (35)$$

On substituting the latter expressions into equations (32) and (33), one obtains the system of equations

$$\psi_e^2 \frac{d}{d\xi} L_1(P_n) + \frac{1}{\xi} L_2(S_n) = \frac{4}{n\pi} \frac{\rho R^2 \ddot{X}_g}{G^*} \quad (36)$$

$$\psi_e^2 \frac{1}{\xi} L_1(P_n) + \frac{d}{d\xi} L_2(S_n) = \frac{4}{n\pi} \frac{\rho R^2 \ddot{X}_g}{G^*} \quad (37)$$

where L_1 and L_2 are Bessel operators given by

$$L_1 = \frac{d^2}{d\xi^2} + \frac{1}{\xi} \frac{d}{d\xi} - \left[\alpha_n^2 + \frac{1}{\xi^2} \right] \quad (38)$$

and

$$L_2 = \frac{d^2}{d\xi^2} + \frac{1}{\xi} \frac{d}{d\xi} - \left[\beta_n^2 + \frac{1}{\xi^2} \right] \quad (39)$$

in which

$$\alpha_n = \beta_n / \psi_e \quad (40)$$

$$\beta_n = \frac{n\pi R}{2H} \sqrt{1 - \frac{\phi_n^2}{1 + i\delta}} \quad (41)$$

$$\phi_n = \frac{1}{n} \frac{\omega}{\omega_1} \quad (42)$$

ω_1 = the fundamental circular natural frequency of the stratum, defined by

$$\omega_1 = \frac{\pi v_s}{2 H} \quad (43)$$

and $v_s = \sqrt{G/\rho}$ = the shear wave velocity for the stratum.

The homogeneous solution of equations (36) and (37), P_n^h and S_n^h , is obtained by requiring that

$$L_1(P_n^h) = L_2(S_n^h) = 0 \quad (44)$$

This leads to

$$P_n^h(\xi) = A_n' K_1(\alpha_n \xi) + C_n' I_1(\alpha_n \xi) \quad (45)$$

and

$$S_n^h(\xi) = B_n' K_1(\beta_n \xi) + D_n' I_1(\alpha_n \xi) \quad (46)$$

where I_1 and K_1 are modified Bessel functions of the first order and first and second kind, respectively, and A_n' , B_n' , C_n' and D_n' are integration constants that remain to be determined.

The particular solutions of equations (36) and (37), P_n^p and S_n^p , can be written as

$$P_n^p = S_n^p = E_n \xi \quad (47)$$

where E_n is a constant which, on substituting equation (47) into either equation (36) or (37), is found to be

$$E_n = -\frac{8 \rho \ddot{X}_g H^2}{\pi^3 G} \frac{1}{n^3} \frac{1}{1 - \phi_n^2 + i\delta} \quad (48)$$

On deleting from equations (45) and (46) the terms with the function I_1 which increases without bound as $\xi \rightarrow \infty$, and substituting the expressions for $P_n = P_n^h + P_n^p$ and $S_n = S_n^h + S_n^p$ into equations (34) and (35), one obtains

$$U_n = -\frac{16 \rho \ddot{X}_g H^2}{\pi^3 G} \frac{1}{n^3} \frac{1}{1 - \phi_n^2 + i\delta} \left[1 + \left\{ \frac{1}{\xi} K_1(\alpha_n \xi) + \alpha_n K_0(\alpha_n \xi) \right\} A_n - \frac{1}{\xi} K_1(\beta_n \xi) B_n \right] \quad (49)$$

and

$$V_n = \frac{16 \rho \ddot{X}_g H^2}{\pi^3 G} \frac{1}{n^3} \frac{1}{1 - \phi_n^2 + i\delta} \left[1 - \frac{1}{\xi} K_1(\alpha_n \xi) A_n + \left\{ \frac{1}{\xi} K_1(\beta_n \xi) + \beta_n K_0(\beta_n \xi) \right\} B_n \right] \quad (50)$$

in which A_n and B_n are dimensionless integration constants related to A'_n and B'_n by

$$\frac{A'_n}{A_n} = \frac{B'_n}{B_n} = \frac{16 \rho \ddot{X}_g H^2}{\pi^3 G} \frac{1}{n^3} \frac{1}{1 - \phi_n^2 + i\delta} \quad (51)$$

The constants A_n and B_n may now be determined by satisfying the boundary conditions defined by equations (19) and (22) or (23). For a rough interface defined by equations (19) and (22),

$$A_n = -\frac{1}{\Delta_n} [2K_1(\beta_n) + \beta_n K_0(\beta_n)] \quad (52a)$$

and

$$B_n = -\frac{1}{\Delta_n} [2K_1(\alpha_n) + \alpha_n K_0(\alpha_n)] \quad (52b)$$

where

$$\Delta_n = [K_1(\alpha_n) + \alpha_n K_0(\alpha_n)] [K_1(\beta_n) + \beta_n K_0(\beta_n)] - K_1(\alpha_n) K_1(\beta_n) \quad (52c)$$

whereas for a smooth interface defined by equations (19) and (23),

$$A_n = -\frac{1}{\Delta_n} [4K_1(\beta_n) + 2\beta_n K_0(\beta_n) + \beta_n^2 K_1(\beta_n)] \quad (53a)$$

$$B_n = -\frac{1}{\Delta_n} [4K_1(\alpha_n) + 2\alpha_n K_0(\alpha_n)] \quad (53b)$$

and

$$\Delta_n = [K_1(\alpha_n) + \alpha_n K_0(\alpha_n)] [3K_1(\beta_n) + 2\beta_n K_0(\beta_n) + \beta_n^2 K_1(\beta_n)] - K_1(\beta_n) [3K_1(\alpha_n) + \alpha_n K_0(\alpha_n)] \quad (53c)$$

2.2.1 Far-Field Response. As $\xi \rightarrow \infty$, the terms involving the Bessel functions in equations (49) and (50) vanish, and equations (27) and (28) reduce to their far-field or free-field values

$$u|_{\xi \rightarrow \infty} = U_f(\eta) \cos \theta e^{i\omega t} \quad (54)$$

$$v|_{\xi \rightarrow \infty} = -U_f(\eta) \sin \theta e^{i\omega t} \quad (55)$$

in which

$$U_r(\eta) = -\frac{16\rho\ddot{X}_g H^2}{\pi^3 G} \sum_{n=1,3}^{\infty} \frac{1}{n^3} \frac{1}{1-\phi_n^2+i\delta} \sin \frac{n\pi}{2} \eta \quad (56)$$

defines, as it should, the displacement amplitudes of a base-excited, vertical cantilever shear-beam of the same height and material properties as the stratum.

2.2.2 Wall Pressures and Forces. With the displacements u and v established, the radial pressures and circumferential shearing stresses induced on the vault-medium interface may be determined from equations (14) and (8) by setting $\xi = 1$. These may be expressed as

$$\sigma_r|_{\xi=1} = \sigma(\eta) \cos\theta e^{i\omega t} \quad (57)$$

$$\tau_{r\theta}|_{\xi=1} = \tau(\eta) \sin\theta e^{i\omega t} \quad (58)$$

where $\sigma(\eta)$ and $\tau(\eta)$ are complex-valued amplitudes that are functions of η and are given by

$$\sigma(\eta) = -\frac{8\psi_\sigma}{\pi^2} \rho \ddot{X}_g H \sqrt{1+i\delta} \sum_{n=1,3}^{\infty} \frac{1}{n^2} \frac{g_n(\alpha_n, \beta_n)}{\sqrt{1-\phi_n^2+i\delta}} \sin \frac{n\pi}{2} \eta \quad (59)$$

and

$$\tau(\eta) = \frac{8}{\pi^2} \rho \ddot{X}_g H \sqrt{1+i\delta} \sum_{n=1,3}^{\infty} \frac{1}{n^2} \frac{h_n(\alpha_n, \beta_n)}{\sqrt{1-\phi_n^2+i\delta}} \sin \frac{n\pi}{2} \eta \quad (60)$$

In the latter expressions

$$\psi_\sigma = \frac{2}{\sqrt{(1-\nu)(2-\nu)}} \quad (61)$$

and g_n and h_n are dimensionless factors defined by the following. For a rough interface,

$$g_n(\alpha_n, \beta_n) = \frac{2 + \beta_n K_r(\beta_n)}{K_r(\alpha_n) + \psi_e K_r(\beta_n) + \beta_n K_r(\alpha_n) K_r(\beta_n)} \quad (62a)$$

and

$$h_n(\alpha_n, \beta_n) = \frac{2\psi_e + \beta_n K_r(\alpha_n)}{K_r(\alpha_n) + \psi_e K_r(\beta_n) + \beta_n K_r(\alpha_n) K_r(\beta_n)} \quad (62b)$$

where

$$K_r(x) = K_0(x) / K_1(x) \quad (63)$$

and x is a complex-valued quantity representing either α_n or β_n . For a smooth interface,

$$g_n(\alpha_n, \beta_n) = \frac{4 + 2\beta_n K_r(\beta_n) + \beta_n^2 + 2\psi_c^2 [2 + \alpha_n K_r(\alpha_n)]}{[\beta_n + 2K_r(\beta_n)] [\psi_c + \beta_n K_r(\alpha_n)] + 2K_r(\alpha_n)} \quad (64a)$$

and

$$h_n(\alpha_n, \beta_n) = 0 \quad (64b)$$

The real and imaginary parts of $K_r(x)$ are plotted in Fig. 2.2 as a function of the real-valued amplitude of the independent variable, $|x|$, and its phase angle, ϕ . Note that, with increasing $|x|$, the real part of $K_r(x)$ tends to unity, whereas the imaginary part tends to zero.

2.2.3 Limiting Solutions for Straight Wall. As H/R tends to zero, α_n and β_n tend to infinity, $K_r(\alpha_n)$ and $K_r(\beta_n)$ tend to unity, g_n for both the rough and smooth interface conditions tends to unity, and equation (59) reduces, as it should, to the corresponding expression for a straight wall presented as equation (39) in [22]. Similarly, for the rough interface condition, h_n tends to unity, and equation (60) reduces to

$$\tau(\eta)|_{H/R \rightarrow 0} = \frac{8}{\pi^2} \rho \ddot{X}_g H \sqrt{1+i\delta} \sum_{n=1,3}^{\infty} \frac{1}{n^2} \frac{1}{\sqrt{1-\phi_n^2+i\delta}} \sin \frac{n\pi}{2} \eta \quad (65)$$

This expression defines the shearing stresses induced on a straight wall when the base of the system is excited parallel to, rather than normal to, the wall.

2.2.4 Shear and Moment in Cylinder. With the stress amplitudes along the cylinder-soil interface established, the force amplitude per unit of cylinder height, $F(\eta)$, is determined by integration to be

$$F(\eta) = \int_0^{2\pi} [\sigma(\eta) \cos^2 \theta - \tau(\eta) \sin^2 \theta] R d\theta = \pi R [\sigma(\eta) - \tau(\eta)] \quad (66)$$

in which the positive direction of $F(\eta)$ is considered to coincide with that of the base motion. Equation (66) will appear as a series of the form

$$F(\eta) = \sum_{n=1,3}^{\infty} F_n \sin \frac{n\pi}{2} \eta \quad (67)$$

in which the factors F_n represent the amplitudes of the component terms.

The expressions for the shearing force, $Q(\eta)$, and the bending moment, $M(\eta)$, across a normal section are then determined by integration to be

$$Q(\eta) = \int_{\eta}^1 F(\eta_o) d\eta_o H = \frac{2H}{\pi} \sum_{n=1,3}^{\infty} \frac{1}{n} F_n \cos \frac{n\pi}{2} \eta \quad (68)$$

and

$$M(\eta) = \int_{\eta}^1 F(\eta_o) (\eta_o - \eta) d\eta_o H^2 = \frac{4H^2}{\pi^2} \sum_{n=1,3}^{\infty} \frac{1}{n^2} F_n \left[\sin \frac{n\pi}{2} - \sin \frac{n\pi}{2} \eta \right] \quad (69)$$

2.3 Transient Response

With the harmonic response of the system established, the response to an arbitrary transient excitation is evaluated by the Discrete Fourier Transform (DFT) technique in combination with the Fast Fourier Transform (FFT) algorithm [23]. In the application of this procedure, the duration of the forcing function should be increased by the addition of a sufficiently long band of zeros [24] to eliminate the aliasing errors that may be introduced. For the solutions presented herein, the duration of this band was taken equal to 10 times the fundamental natural period of the layer idealized as a cantilever shear-beam

(a) Ground-Excited

Fixed Base
(b) Force-Excited

Figure 2.1 System Considered

Figure 2.2 Real and imaginary parts of complex-valued function $K_r(x)$.

SECTION 3

NUMERICAL DATA FOR STATIC EFFECTS

It is desirable to begin by examining the responses obtained for harmonic excitations the frequencies of which are small compared to the fundamental natural frequency of the stratum (i.e., for values of $\phi_1 \rightarrow 0$). Such excitations and the resulting effects will be referred to as static, a term which should not be confused with that normally used to represent the effects of gravity forces. In the equivalent, force-excited version of the problem shown in Fig. 2.1(b), the static excitation is represented by a set of horizontal body forces of constant intensity $-\rho \ddot{X}_g$.

3.1 Magnitude and Distribution of Wall Pressures and Forces

The maximum values of the static normal wall pressure, σ_{st} , and of the corresponding circumferential shearing stress, τ_{st} , are attained at the top. Determined from equations (59) and (60), these values, normalized with respect to $\rho \ddot{X}_g H$, are plotted in Fig. 3.1 as a function of the ratio of stratum height to cylinder radius, H/R . These plots are for a medium with Poisson's ratio $\nu = 1/3$, a value which, except where otherwise indicated, is also used for all other solutions presented in subsequent sections. The solid lines in the figure define the stresses for a rough interface, whereas the dashed line defines the normal pressure for a smooth interface.

It is observed that: (1) Both components of stress increase with increasing H/R ; and (2) the normal pressure for the smooth interface is generally higher than for the rough, the difference becoming larger with increasing H/R . The first trend reflects the decreasing capacity of the deeper layers to transmit a part of the forces acting on the medium by horizontal shearing action to the base. The second trend is a consequence of the ability of the rough interface to resist by circumferential shearing action a part of the force that gets transmitted to it. For a straight wall ($H/R \rightarrow 0$), the normal stress is naturally independent of the interface condition, and the shearing stress is given by equation (65).

Unlike the normal wall pressures which have been shown to be generally smaller for the rough interface than for the smooth, the total force per unit of cylinder height should be larger for the rough interface. Being the "stiffer" of the two, the rough interface would be expected to attract a higher proportion of the forces acting on the medium. That this is indeed the case is demonstrated in Fig. 3.2, in which the top value of the total force per unit of cylinder height, $F_{st}(1)$, is plotted as a function of the slenderness ratio, H/R . The results in this case are normalized with respect to $\pi \rho \ddot{X}_g R H$. Note

that, almost independently of the value of H/R , the force for the rough interface exceeds that for the smooth by about $0.741\pi\rho\ddot{X}_gRH$. For broad vaults with $H/R \rightarrow 0$, the latter value represents exactly the contribution of the circumferential shearing stresses defined by equation (65). For reasons noted in the following paragraphs, similar relationships should also hold approximately for the resulting shearing forces and bending moments in the vault.

The heightwise variations of the amplitudes of the normal pressures and circumferential shearing stresses exerted on the vault are shown in Fig. 3.3 for systems with a rough interface and values of $H/R = 0$ and 3. These plots are normalized to unit values at the top. The variations for intermediate values of H/R fall between those displayed and are not presented. The same is also approximately true of the variations of the normal pressures for a smooth interface.

If the response of the system were evaluated considering the contribution of only the fundamental mode of vibration of the stratum, the interface stresses and forces would have been proportional to $\sin(\pi\eta/2)$, the shearing forces in the wall would have been proportional to $\cos(\pi\eta/2)$, and the distribution of the bending moments would have been given by

$$\frac{M(\eta)}{M_b} = 1 - \sin\frac{\pi}{2}\eta \quad (70)$$

Furthermore, the base shear would have been $(2/\pi)H$ times the peak interface force, and the base moment would have been $(2/\pi)H$ times the base shear. While the indicated distributions are somewhat different from the exact, the maximum values of the base shears and moments are reasonable approximations of the exact values. The exact distributions of the shearing forces and bending moments in the vault of the systems considered in Fig. 3.3 are shown in Fig. 3.4.

The exact values of the base shear and base moment for systems with different values of H/R are listed in Table 3.1 for both the rough and smooth interface conditions. Also listed for the rough condition are the components of the base shear contributed by the radial and the circumferential components of the interface stresses. Incidentally, for the limiting case of a medium retained by a straight wall, $H/R = 0$, the values of the base forces per unit of wall width are determined by dividing the tabulated values for the smooth interface by πR .

3.2 Effect of Poisson's Ratio

The numerical data and plots presented so far are for media with Poisson's ratio $\nu = 1/3$. As an indication of the sensitivity of the various response quantities to the value of ν involved, the peak values of the normal and circumferential shearing stresses at the top of the cylinder are plotted in Fig. 3.5 as a function of ν for systems with rough interface and several different values of the slenderness ratio H/R . The normal stress in these plots is normalized with respect to $\psi_o\rho\ddot{X}_gH$, the common multiplier of equation (59), whereas the shearing stress is normalized with respect to $\rho\ddot{X}_gH$, the corre-

sponding multiplier of equation (60).

It is observed that, for the limiting case of $H/R = 0$, the quantities in Fig. 3.5 are independent of ν , a fact noted previously in [22], and that, even for the remaining values of H/R , they are insensitive to variations in ν . It follows that, to a reasonable degree of approximation, the normal stresses may be considered to be proportional to ψ_σ , whereas the shearing stresses may be considered to be independent of ν .

Since the heightwise variations of the interface stresses may also be considered to be independent of ν , the base shear for systems with values of ν different from $1/3$ may be determined from the values listed in Table 3.1 merely by modifying the component contributed by the normal stresses so as to account for the change in the value of ψ_σ and leaving the component contributed by the circumferential shearing stresses unaltered. The tabulated data for a smooth interface may be adjusted similarly by modifying the factor ψ_σ in the expression for the normal stress. The corresponding base moments may then be determined by multiplying the tabulated moments by the ratio of the adjusted to the tabulated values of base shear.

3.3 Comparison with Tajimi's Solution

Reference has already been made to Tajimi's solution [1] which is based on the assumption of vanishing vertical displacements. As an indication of the interrelationship of this solution to the one presented herein, a comparison is made in Fig. 3.6 of the normalized values of the base shear computed by the two approaches for a medium with values of Poisson's ratio ν in the range between 0 and 0.5.

The data displayed are for a vault with $H/R = 1$ that is fully bonded to the surrounding medium, but the indicated trends are typical of those obtained for other conditions as well. The solid line represents the results obtained on the $\sigma_z = 0$ assumption, whereas the dashed line represents those deduced from Tajimi's solution. The latter values were computed from equation (68) by replacing the factor ψ_σ by the factor χ_σ defined by equation (25). It is observed that, for values of $\nu \leq 1/3$, the two sets of results are indeed in excellent agreement. For the larger values of ν , however, the agreement deteriorates, and Tajimi's solution ceases to be acceptable.

Also shown in Fig. 3.6 as a dotted line are the results computed by replacing the factor χ_σ in Tajimi's solution by the factor

$$\chi_\sigma = \sqrt{\frac{2}{1-\nu}} \quad (71)$$

This modification, which follows a suggestion made by Gazetas and Dobry [25] in other studies of foundation dynamics, is tantamount to replacing the dilatational or compressional velocity for the

medium,

$$v_c = \sqrt{\frac{2(1-\nu)}{1-2\nu}} v_s \quad (72a)$$

by

$$v'_c = \sqrt{\frac{E}{(1-\nu^2)\rho}} = \sqrt{\frac{2}{1-\nu}} v_s \quad (72b)$$

in which $E = 2(1+\nu)G$ is Young's modulus of elasticity. It can be seen that, subject to this modification, Tajimi's solution can be used as a reasonable approximation over the entire range of ν values. Incidentally, the alternative suggestion in [25] of replacing the dilatational wave velocity by the so-called Lysmer velocity [25] did not yield satisfactory results.

Table 3.1: Static values of base shears and moments for systems with different interface conditions and slenderness ratios; $\nu = 1/3$

H/R	$-\frac{(Q_b)_{st}}{\pi \rho \ddot{X}_g R H^2}$			$-\frac{(M_b)_{st}}{\pi \rho \ddot{X}_g R H^3}$		
	Rough Interface			Smooth Interface	Rough Interface	Smooth Interface
	Component due to σ_r	Component due to $\tau_{r\theta}$	Total			
0.00	1.029	0.542	1.571	1.029	0.941	0.616
0.30	1.071	0.640	1.711	1.113	1.029	0.669
0.40	1.086	0.669	1.755	1.150	1.056	0.693
0.50	1.101	0.698	1.799	1.190	1.083	0.717
0.60	1.118	0.724	1.842	1.231	1.110	0.743
0.70	1.134	0.750	1.884	1.272	1.136	0.769
0.80	1.151	0.774	1.925	1.314	1.162	0.795
0.90	1.169	0.798	1.967	1.356	1.188	0.821
1.00	1.186	0.821	2.007	1.397	1.213	0.847
1.25	1.231	0.876	2.107	1.500	1.276	0.912
1.50	1.276	0.929	2.205	1.600	1.336	0.974
1.75	1.322	0.978	2.300	1.697	1.396	1.035
2.00	1.367	1.027	2.394	1.793	1.454	1.094
2.50	1.459	1.118	2.577	1.976	1.568	1.209
3.00	1.549	1.205	2.754	2.152	1.679	1.318
5.00	1.897	1.523	3.420	2.799	2.094	1.721
10.00	2.695	2.214	4.909	4.213	3.020	2.600

Figure 3.1 Effect of slenderness ratio, H/R , on maximum static values of normal and circumferential stresses induced at top of vault; $\nu = 1/3$.

Figure 3.2 Effect of slenderness ratio, H/R , on static value of force per unit of height induced at top of vault; $\nu = 1/3$.

Figure 3.3 Heightwise variations of static values of normal and circumferential stresses induced on a vault with rough interface; $\nu = 1/3$.

Figure 3.4 Heightwise variations of static shear and bending moment for vaults with a rough interface; $\nu = 1/3$.

Figure 3.5 Effect of Poisson's ratio on static values of normal and circumferential stresses induced at top of vault with rough interface

Figure 3.6 Effect of different approximations on static value of base shear computed for vaults with a rough interface; $H/R = 1$.

SECTION 4

NUMERICAL DATA FOR HARMONIC RESPONSE

4.1 Magnitude and Distribution of Forces on Cylinder

Figure 4.1 shows the variation with the frequency ratio ω/ω_1 of the force amplitude per unit height, F , induced at the top of the vault for a harmonically excited system. Normalized with respect to $\pi\rho\ddot{X}_gRH$, the results are for a system with $H/R = 1$, $\nu = 1/3$, and material damping factor $\delta = 0.1$. The vault-medium interface in these solutions is considered to be rough. Part (a) of the figure shows the real and imaginary parts of the force, whereas part (b) shows the real-valued amplitude, given by the square root of the sum of the squares of the component parts. The real part of the force is in phase with the exciting motion and represents the restraining effect of a displacement-proportional, spring-like action of the medium, whereas the imaginary part is 90° out of phase and represents the effect of a damping mechanism analogous to that of a viscous damper. These plots are similar to those presented in [22] for the corresponding pressure amplitude induced on a similarly excited straight wall.

The following trends are worth noting in Fig. 4.1:

1. For $\omega = 0$, the imaginary part of the force vanishes, and the real part reduces, as it should, to the value of $-2.8\pi\rho\ddot{X}_gRH$ shown in Fig. 3.2.
2. For values of $\omega < \omega_1$, the force amplitude is dominated by the real part, whereas for $\omega \geq \omega_1$, it is dominated by the imaginary part. Note, in particular, the very rapid and large increase of the imaginary part in the neighborhood of $\omega = \omega_1$. The relative unimportance of the imaginary part for $\omega < \omega_1$ is due to the fact that no energy gets dissipated by radiation of waves within this frequency range.
3. The absolute maximum value of the force amplitude is attained at $\omega = \omega_1$, and it is associated with an amplification factor of $12.8/2.8 = 4.57$. This is a relatively low value, indicating large overall damping for the system. This matter is considered further in later sections.

The heightwise variations of the real-valued force amplitude for the system considered in Fig. 4.1 are shown in Fig. 4.2 for several different values of the frequency ratio, ω/ω_1 . All plots are normalized

with respect to $-2.8\pi\ddot{X}_gRH$, the static value of the top force. It is observed that, for ω/ω_1 of the order of 2 or less, the force intensity increases approximately as a half-sine wave from zero at the base to a maximum at the top. It follows that the fundamental mode of vibration of the stratum is the dominant contributor to the response of the system in this case. For larger values of ω/ω_1 , the distribution is wavy because of the increased participation of the higher modes of vibration, but even in these cases, the contribution of the fundamental mode is substantial.

4.2 Base Shear

The amplification factor, AF, for the base shear in the vault of the system examined in the preceding two figures is plotted in Fig. 4.3 as a function of ω/ω_1 for three different values of δ . This factor represents the ratio of the real-valued amplitude of the base shear, $(Q_b)_r$, to the numerical value of the corresponding static shear, $(Q_b)_{st}$. The latter is determined from Table 3.1 to be $2.01\pi\rho\ddot{X}_gRH^2$.

As would be expected, all curves start from unity at the left, and the one for the undamped system attains infinite values at the natural frequencies of the stratum. Soil material damping reduces significantly the resonant peaks, particularly those corresponding to the higher modes of vibration. The very significant effect of damping on the higher resonant peaks is associated with the fact that, even at high values of the frequency ratio ω/ω_1 , the fundamental mode of vibration is the dominant contributor to the response. This can more clearly be seen in Fig. 4.4, in which the exact frequency response curve for $\delta = 0.1$ is compared with that computed considering the contribution of the fundamental mode of vibration only. The excellent agreement between the two sets of results indicates that the one-mode approximation should indeed yield accurate results for transient excitations as well. These trends are similar to those reported in [22] for a stratum retained by a straight wall.

The effect of the slenderness ratio H/R on the frequency response curves for base shear is illustrated in Fig. 4.5. Shown are the results for values of H/R in the range between 0 and 3. As before, systems with a rough interface, and values of $\nu = 1/3$ and $\delta = 0.1$ are considered.

Note that increasing H/R increases the amplification factor both at resonance and throughout the lower range of ω/ω_1 values, but decreases this factor at the higher values of ω/ω_1 . This trend is similar to the effect of decreasing damping on the transmissibility factor of a viscously damped single-degree-of-freedom system, and may be explained by visualizing the action of systems having the same stratum depth, H , and decreasing vault radii, R . As the radius of the vault and hence its lateral surface decrease, the capacity of the structure to radiate and dissipate waves from its surface decreases, and this reduction leads to reduced overall damping for the system.

The resonant or absolute maximum value of the amplification factor for base shear is plotted in Fig. 4.6 as a function of H/R for systems with three different values of the material damping factor, δ .

The solid lines in this figure are for systems with a rough interface, whereas the dashed lines are for systems with a smooth interface. It is observed that the amplification factors increase more rapidly with H/R for the smooth interface than for the rough. This is attributed to the fact that, whereas the smooth interface can radiate only compressional waves, the rough interface can radiate both compressional and horizontally polarized shear waves. Accordingly, systems with a rough interface have higher damping capacities and lower resonant peaks than those with a smooth interface. Incidentally, a comparison of the frequency response curves for systems with the two interface conditions reveals that the curves crossover at values of ω/ω_1 of 1.27 in a manner entirely analogous to that indicated in Fig. 4.5. For the limiting case of $H/R = 0$, the amplification factor at resonance is given by $1/\sqrt{\delta}$.

Although the peak amplification factors in Fig. 4.6 are higher for a smooth interface than a rough, it should be recalled that the opposite is true of the magnitude of the static effects. As a result, the total force induced on a smooth interface is normally smaller than that for a rough interface.

4.3 Base Moment

The amplitude of the base moment in the vault may conveniently be expressed as the product of the base-shear amplitude and an appropriate height, h . If the response were contributed solely by the fundamental mode of vibration, considering that this mode varies as a half-sine from zero at the base to a maximum value at the free surface, the height h would have been $(2/\pi)H = 0.637H$. The exact values, identified in Fig. 4.7 for different values of H/R , are not materially different, except for a totally undamped system at exciting frequencies close to the second natural frequency of the layer.

4.4 Relative Effects of Normal and Shearing Stresses

Excepting the data shown by the dashed lines in Fig. 4.6, the dynamic response data presented in the preceding sections are for systems with a rough interface. Accordingly, they represent the combined effects of the interfacial normal pressures, σ_r , and of the corresponding circumferential shearing stresses, $\tau_{r\theta}$.

In Fig. 4.8, the component of the base shear amplitude induced by the normal pressures, $|(Q_b)_{\sigma}|$, is plotted as a fraction of the corresponding amplitude of the total base shear, $|Q_b|$, over a wide range of H/R values and three critical values of the exciting frequency. It is observed that the ratio of the two amplitudes is insensitive to variations in either H/R or ω/ω_1 , and that it may be taken approximately as 0.60. In other words, irrespective of the frequency of the excitation and the characteristics of the system, about 60 percent of the base shear and moment in the vault may be considered to be caused by the normal pressures, with the remaining 40 percent caused by the circumferential stresses. This simple approximate relationship applies for transient excitations as well.

Figure 4.1 Frequency response curves for amplitude of force per unit of height induced at top of a harmonically excited vault; rough interface, $H/R = 1$, $\nu = 1/3$, $\delta = 0.1$.

Figure 4.2 Heightwise variations of real-valued amplitude of force per unit of height induced in a harmonically excited vault; rough interface, $H/R = 1$, $\nu = 1/3$, $\delta = 0.1$

Figure 4.3 Effect of material damping on frequency response curve for base shear in vaults with a rough interface; $H/R = 1$, $\nu = 1/3$.

Figure 4.4 Comparison of frequency response curves for base shear in a vault with rough interface computed by considering one and all modes of vibration; $H/R = 1$, $\nu = 1/3$, $\delta = 0.1$.

Figure 4.5 Effect of slenderness ratio, H/R , on frequency response curve for base shear in vaults with a rough interface; $\nu = 1/3$, $\delta = 0.1$.

Figure 4.6 Effect of slenderness ratio and material damping factor on resonant amplification factor for base shear in vault of harmonically excited systems with rough and smooth interface; $\nu = 1/3$.

Figure 4.7 Height of resultant of total force induced on a vault of a harmonically excited system; rough interface, $H/R = 1$, $\nu = 1/3$.

Figure 4.8 Portion of total base shear induced by normal pressures in a fully bonded vault; $\nu = 1/3$, $\delta = 0.1$.

SECTION 5

SOLUTIONS FOR TRANSIENT RESPONSE

The response of the medium was also evaluated for the first 6.3 sec of the N-S component of the ground motion recorded during the 1940 El Centro, California earthquake. The acceleration, velocity and displacement traces of this record have been presented before in [26]. The peak value of the ground acceleration is $\ddot{x}_g = 0.312$ g, and the corresponding values of the velocity and displacement are $\dot{x}_g = 14.02$ in/sec and $x_g = 8.29$ in.

5.1 Results for Rough Interface

The solid lines in Fig. 5.1 define the amplification factor for the absolute maximum base shear induced by the El Centro ground motion in a fully bonded vault with values of H/R in the range between 0 and 3, and medium properties represented by $\nu = 1/3$ and $\delta = 0.1$. The static values of the base shear involved in the definition of this factor are listed in Table 3.1, in which $\ddot{X}_g$ must now be interpreted as the maximum acceleration of the transient ground motion, $\ddot{x}_g$. The results are plotted as a function of the fundamental cyclic natural frequency of the stratum, defined by

$$f_1 = \frac{v_s}{4H} \quad (73)$$

It should be noted that the right part of this figure, which refers to high-frequency, stiff strata, corresponds to the left parts of Figs. 4.1, 4.3, 4.4 and 4.5, whereas its left part corresponds to the right parts of the previous figures.

As a measure of the values of f_1 that may be encountered in practice, it is noted that for values of v_s in the range between 400 and 1600 ft/sec and for values of H in the range between 20 and 50 ft, the value of f_1 falls between 2 cps and 20 cps.

For very stiff strata with values of f_1 of the order of 30 cps or more, the amplification factors in Fig. 5.1 are effectively unity, and the maximum base shears reduce to the values listed in Table 3.1. As the system frequency decreases, the amplification factors generally increase, the increase being more or less proportional to the value of the slenderness ratio, H/R . These trends are consistent with those noted for harmonically excited systems in Figs. 4.5 and 4.6. Furthermore, the peak amplification factors in Fig. 5.1 are consistent with the damping values for the system implied by the plots presented in

Fig. 4.6. For highly compliant systems with very low natural frequencies, the amplification factors are naturally less than unity.

The dashed line in Fig. 5.1 defines the amplification factors computed for a system with $H/R = 1$ considering the contribution of only the fundamental mode of vibration of the medium. As anticipated from the responses of the harmonically excited systems examined earlier, the agreement with the more nearly exact solution is indeed excellent. It follows that even for the fairly complex ground shaking considered, the maximum values of the wall forces may, for all practical purposes, be considered to increase as half-sine waves from zero at the base to a maximum value at the top, and the maximum base moment may be computed as the product of the base shear and the height $h = 0.637H$.

5.2 Results for Smooth Interface

The response spectra for systems with rough interface presented in Fig. 5.1 may also be used to approximate the corresponding spectra for systems with a smooth interface. It is only necessary to multiply the results presented by a factor which, strictly speaking, should be a function of the natural frequency of the stratum. For high-frequency, stiff strata, for which the amplification factor for the fully bonded condition is equal to or close to unity, the multiplying factor should also be unity. For lower-frequency, more flexible strata, it is recommended that the multiplying factor be taken as the mean of unity and the ratio of the resonant amplification factors for smooth and rough interfaces presented in Fig. 4.6. Justifying this approach is the fact that for broad-banded ground motions of the type considered herein, the maximum response of the system is dominated by those harmonics of the excitation having frequencies close to, but not exclusively equal to, the fundamental natural frequency of the stratum.

5.3 A Final Comment

The shears and bending moments for the vault referred to in this paper represent exclusively the effects of the normal pressures and circumferential shearing stresses induced by the surrounding medium. To these effects, must also be added the effects of the vault inertia. For the rigid vault considered, the latter effects may be evaluated readily.

Figure 5.1 Response spectra for amplification factor for base shear in vault of systems subjected to El Centro record; rough interface, $\nu = 1/3$, $\delta = 0.1$.

SECTION 6

CONCLUSIONS

The principal conclusions of this study may be summarized as follows;

1. With the analytical expressions presented, the dynamic pressures and associated forces induced by ground shaking on a cylindrical rigid vault embedded in a stratum of uniform viscoelastic material may be evaluated reliably and cost-effectively.
2. The comprehensive numerical data that have been presented and the analysis of these data provide not only valuable insights into the effects and relative importance of the numerous parameters involved, but also a conceptual framework for the analysis and interpretation of the solutions for more involved systems.
3. Tajimi's analysis of the problem, which is based on the assumption of vanishing vertical displacements, leads to unacceptable results for values of Poisson's ratio for the medium close to $1/2$. However, this deficiency may be eliminated by an appropriate modification of the dilatational wave velocity for the medium.
4. When expressed in terms of $\pi \rho \ddot{x}_g R H^2$, the static or low-frequency value of the total force exerted on the vault increases from 1.57 for broad vaults with values of $H/R \rightarrow 0$ to 4.91 for a relatively slender vault with $H/R = 3$. These values are for vaults that are fully bonded to a medium with Poisson's ratio $\nu = 1/3$. The larger relative forces for the more slender systems are due to the lower capacity of the deep strata to transfer the forces acting on them by horizontal shearing action to the base.
5. Because the effective stiffness of vaults with a smooth interface is lower than for a rough interface, the forces exerted on such vaults, are smaller. When normalized in the manner indicated above, the static value of the normalized total force increases from 1.03 for $H/R = 0$ to 4.21 for $H/R = 3$.
6. For harmonically excited systems, the peak amplification factor at resonance also increases with increasing H/R , the magnitude of the increase being an inverse function of the damping factor for the medium, δ . This trend, which is also true of systems subjected to earthquake ground motions, reflects the reduced capacity of the taller, more slender vaults to dissipate energy by radiation of waves.

7. Almost independently of the system parameters and the characteristics of the ground motion, the forces exerted on the vault increase approximately as a half-sine wave from zero at the base to a maximum at the top. As a result, the maximum base moment in the vault may be expressed as the product of the maximum base shear and approximately 60 percent of the stratum depth, H .
8. For systems with a rough interface, approximately 60 percent of the total base shear and of the associated bending moment in the vault is contributed by normal pressures, whereas the remaining 40 percent is contributed by circumferential shearing stresses. These factors may, for all practical purposes, be considered to be independent of the characteristics of the forcing function.

SECTION 7

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SECTION 8

NOTATION

The following symbols are used in this paper:

A_n, A'_n	constants of integration
B_n, B'_n	constants of integration
E	Young' modulus of elasticity for the medium
E_n	constant of integration
F	peak value of force per unit of vault height
F_n	n^{th} term in an expansion of F in terms of natural modes of vibration of stratum
$F_{st}(\eta)$	static value of F at a distance defined by η
f_1	fundamental cyclic natural frequency of stratum
G	shear modulus of elasticity for medium
G^*	complex-valued shear modulus for medium, defined by equation (11)
g_n	dimensionless factor defined by equations (62a) and (64a)
H	depth of soil stratum or height of vault
h_n	dimensionless factor defined by equations (62b) and (64b)
i	$\sqrt{-1}$
I_1	modified Bessel function of the first kind and first order
K_0, K_1	modified Bessel functions of the second kind and zero and first order, respectively
K_r	ratio of K_0 to K_1
L_1, L_2	Bessel operators defined by equations (38) and (39), respectively
M	bending moment across a normal section of vault
M_b	bending moment at vault base
$(M_b)_{st}$	static value of base moment
n	odd integer
P_n, P_n^h, P_n^p	potential function, along with its homogeneous and particular parts
$Q(\eta)$	peak value of shearing force induced across a section of the vault defined by the dimensionless distance η
Q_b	shear at vault base
$(Q_b)_{st}$	static value of base shear
$(Q_b)_\sigma$	part of base shear contributed by normal pressures
R	radius of vault

S_n, S_n^h, S_n^p	potential function, along with its homogeneous and particular parts
t	time
U	complex-valued amplitude of radial displacement relative to moving boundary for an arbitrary point of the medium
U_f	displacement amplitude of base-excited, cantilever shear-beam representing far-field action of the medium
U_n	n^{th} term in an expansion of U in terms of the natural modes of vibration of stratum
u	radial component of displacement relative to moving boundary for an arbitrary point and time
V	complex-valued amplitude of circumferential displacement relative to moving boundary for an arbitrary point of the medium
V_n	n^{th} term in an expansion of V in terms of the natural modes of vibration of stratum
v	circumferential component of displacement relative to moving boundary for an arbitrary point and time
v_c	velocity of dilatational wave propagation in medium
v'_c	modified value of v_c , defined by equation (72b)
v_s	velocity of shear-wave propagation in medium
w	vertical displacement for an arbitrary point and time
$\ddot{X}_g$	amplitude of harmonic ground acceleration
$x_g, \dot{x}_g, \ddot{x}_g$	maximum values of displacement, velocity and acceleration of ground for a transient ground shaking
$\ddot{x}_g(t)$	instantaneous value of ground acceleration
z	vertical position coordinate
α_n, β_n	complex-valued dimensionless factors defined by equations (40) and (41)
δ	material damping factor for constant hysteretic characterization of soil damping
Δ_n	complex-valued dimensionless factor
η	z/H = dimensionless vertical position coordinate
θ	angle, measured counterclockwise from direction of ground motion
λ^*	complex-valued Lamé constant defined by equation (12)
ν	Poisson's ratio for medium
ξ	r/R = dimensionless radial position coordinate
ρ	mass density for medium
σ	amplitude of radial pressure induced on vault
σ_r	radial stress at an arbitrary point and time
$\sigma_{st}(\eta)$	maximum value of static pressure induced on vault at dimensionless distance η
σ_z	vertical stress at an arbitrary point and time
σ_θ	circumferential stress at an arbitrary point and time

τ	amplitude of circumferential shearing stresses induced on vault
τ_{rz}	shearing stress in r-z plane at an arbitrary point and time
$\tau_{r\theta}$	shearing stress in r- θ plane at an arbitrary point and time
$\tau_{st}(\eta)$	maximum value of static shearing stresses induced on vault at dimensionless distance η
$\tau_{\theta z}$	shearing stress in θ -z plane at an arbitrary point and time
ϕ_n	frequency ratio defined by equation (42), where n is an odd integer
$\chi_o, \bar{\chi}_o$	functions of ν defined by equations (25) and (71), respectively
$\psi_e, \psi_o, \psi_\sigma$	functions of ν defined by equations (18), (15) and (61), respectively
ω	circular frequency of excitation and resulting steady-state response
ω_1	fundamental circular natural frequency of stratum considered to act as a series of vertical cantilever shear-beams.

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