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DOE/PC/79935--14

DE91 000694

"CIRCULATION IN GAS-SLURRY COLUMN REACTORS"
DEPARTMENT OF ENERGY. CONTRACT NUMBER DE-FG22-87PC79935
FINAL REPORT - REPORT DOE/PC/79935-14

AUGUST 15, 1990

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SUMMARY

Circulation in bubble columns detracts from their performance in that gas is carried on average more rapidly through the column, and the residence time distribution of the gas in the column is widened. Both of these factors influence mass-transfer operations in bubble columns. Although circulation has been modeled with moderate accuracy, there is still a dearth of good data and reliable models for bubble columns. To remedy this omission, circulation prediction and measurement has been undertaken using probes, one-dimensional models, laser Doppler velocimetry, and numerical modeling.

Firstly, local void fraction was measured using resistance probes and a newly developed approach to determining air/water threshold voltage for the probe. A tall column of eight inch diameter was constructed of Plexiglas and the distributor plate was manufactured to distribute air evenly through the base of the column. Data were gathered throughout the volume at three different gas throughputs. Void distributions were quite flat near the distributor, becoming more parabolic higher in the column. Orthogonal traverses of the column with the probes verified that the circulation was symmetric. Bubble velocities proved difficult to measure using twin probes with cross-correlation because of radial bubble movement. However, data have been obtained for several central positions in the column at different flowrates of air. Measured bubble velocities agreed acceptably with predictions of a one-dimensional model, and this model has also been extended to include a drift-flux approach.

Secondly, a series of three-dimensional mean and RMS bubble and liquid velocity measurements have been obtained for a turbulent flow in a laboratory model of a bubble column. These measurements have been made using a

three-component laser Doppler velocimeter (LDV), to determine velocity distributions non-intrusively. A hexagonal bubble column cross-section was selected to allow relatively easy optical penetration by the LDV into the column. Bubble column turbulent liquid circulation for these experiments was due to injection of air into water from a single vertically oriented jet on the column centerline.

Measured distributions of mean vertical and radial liquid and bubble velocities indicated a toroidal recirculating liquid mean flow which was driven by the air jet. Mean velocity was upwards near the column centerline, radially outwards near the free surface, downwards near the column wall, and inwards in the bottom of the column. Laser light sheet flow visualization was qualitatively consistent with the quantitative LDV data. Mean circumferential (swirl) velocity was essentially zero. Measured RMS velocity fluctuations were the same order of magnitude as measured mean velocities for both phases. Thus, turbulence intensities were on the order of 100% throughout the column. The turbulence field was non-isotropic, in that the radial RMS velocity was consistently larger than either the vertical or circumferential RMS velocity. This non-intrusive set of three-component mean and turbulent liquid and bubble velocity data in a laboratory bubble column is unique and previously unavailable in the literature.

Thirdly, the gas-liquid flow inside a vertically situated circular, isothermal column reactor has been simulated numerically. The gas-liquid flow is assumed to be in the bubbly flow regime which is characterized by a suspension of discrete air bubbles in a continuous liquid phase such as a glycerol-water mixture. The mathematical formulation is based on the conservation of mass and momentum principle for the liquid phase. The gas

velocity distribution is calculated via an empirically prescribed slip velocity as a function of void fraction. The interface viscous drag forces are prescribed empirically. A profile shape is assumed for the void ratio distribution and the magnitude of it is calculated as part of the solution. The influence of various profile shapes is investigated. Results with the void ratio distribution calculated from the conservation of mass equation for the gas phase are also presented. The mathematical model has been implemented by modifying a readily available computer code for single phase Newtonian fluid flows. The numerical discretization is based on a finite volume approach.

The numerical predictions show a reasonably good agreement with measurements. The circulation pattern seems not to be so sensitive to the actual shape of the void fraction profiles, but the inlet distribution of it is important.

CIRCULATION IN GAS-SLURRY COLUMN REACTORS

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SECTION 1

INTRODUCTION

In a Fischer-Tropsch synthesis hydrogen and carbon monoxide are combined in the presence of a catalyst to produce hydrocarbons. The input gases can be generated using fluid bed gasifiers. Synthesis can be carried out in a bubble column where the gases are passed through a slurry of catalyst and high molecular weight oils produced by the synthesis itself (Stern et al., 1985). The bubble column reactor offers several advantages over conventional fixed bed reactors: lower hydrogen to monoxide ratios can be tolerated and hot-spots are controlled in the bed so that catalyst deactivation is reduced. Although gas may be introduced evenly through a distributor plate into the slurry, circulation patterns can develop in bubble column systems. The most common of these patterns involves a high concentration of gas bubbles in the column center and less near the wall. In consequence a gas-slurry mixture rises up the column center and slurry flows back down near the column wall. The circulation is known as "gulfstreaming" (Freedman and Davidson, 1969). At the bottom and top of the column there are strong radial velocity components to complete the circulation pattern.

Gulfstreaming reduces the efficiency of many unit operations by "shortcircuiting" the air bubbles, thus reducing their residence time and holdup in the vessel. To maintain an acceptable residence time of bubbles in the system, bubble columns must often be designed far taller than if there were no gulfstreaming. On the other hand, the presence of large circulation patterns leads to the formation of turbulent eddies in the column, promoting mixing and mass and heat transfer. Residence time distribution is also altered by the presence of circulation, an important consideration in predicting conversions (Chang and Smith, 1983). Therefore the degree of circulation in the column must be predicted if accurate column designs are

needed. More complex circulation patterns can also be observed, with cells side-by-side (in large diameters - see Otero et al, 1985) or above one another (reported in fluidized beds by Lin et al., 1985). A single cell reverse pattern, with slurry rising at the walls and flowing back at the center is also possible. State-of-the-art models for circulation are still not sufficient for accurate column design. This research has attacked the problem of circulation prediction and measurement through experimental work using probes, one dimensional modeling, laser doppler velocimetry measurements, and numerical modeling. These three areas are addressed in the following sections.

SECTION 2

PROBES AND ONE-DIMENSIONAL MODELS

2.1 INTRODUCTION

This section of the report presents circulation results obtained from a laboratory scale bubble column, and demonstrates that one dimensional models meet with success in predicting circulation rate in tall columns.

2.2 LITERATURE REVIEW

The vast majority of circulation research has been performed on air-water bubble columns, but this can be extended in principle to the case of Fischer-Tropsch column provided that the fluid properties of the pulp are properly considered. Several bubble column researchers have accounted for circulation by modeling the overall holdup of gas in the column (see the reviews by Lockett and Kirkpatrick, 1975 and Shah et al., 1982 and the paper by Deckwer et al., 1974) but this approach is not general and cannot be used to infer the actual fluid velocity distribution in the vessel. For one dimensional analytic models, circulation is usually quantified in terms of the velocity distribution across the diameter at half of the column height. Usually the velocity near the center is upward while the velocity near the wall is downward although reverse circulation has been documented and more complex patterns are possible in large columns. Similar distributions in gas-liquid bubble columns have been measured or proposed by many different workers for a variety of column diameters (Rietema and Ottengraf, 1970; Hills, 1974; Rietema, 1982; Freedman and Davidson, 1969; Lamont, 1958, Steinemann and Buckholz, 1984) but little theoretical analysis is given in the literature.

Rietema and Ottengraf (1970) presented an analysis for the circulation in a Newtonian viscous liquid, while Hills (1974) examined turbulent air-water flow in a column and offered a model which used a force balance and a simple eddy viscosity model. A more detailed analysis along these lines has been presented by Miyauchi et al. (1981) but also involves simplifying assumptions

about fluid shear stress which may not hold true in real column circulation. A model for circulation in Pachuca tanks (Clark, 1984) was restricted to vessels with draft tubes. Clark et al. (1987) developed a model based on simple mixing length theory, and a similar model using two mixing lengths has been offered by Michaelides and Nikitopoulos (1987). Anderson and Rice (1989) have also offered an elegant mixing length approach, while a more phenomenological analysis has been offered by Molerus and Kurtin (1986). However, there is still a need for additional accurate data on void distributions and circulation velocities in bubble columns to verify the application of these models over a broad range of column sizes and operating conditions.

2.3 EXPERIMENTAL APPARATUS AND APPROACH

A hollow cylindrical bubble column was designed to study gas-liquid circulation. It was constructed out of quarter-inch thick and eight inch internal diameter acrylic tubing. The column was composed of three attachable thirty inch sections. Each section has two, three quarter-inch flanges so that each piece could be bolted together. To prevent leaking between each section, O-rings were placed in the flange on the top section. The reason for having three sections was that it allowed for the column to be taken apart, transported, and placed together again with the least risk of possible damage to the column. Also, with three sections, the distributor plate can be changed easily. Beneath the column, a nine inch plenum of the same diameter as the column was located. To make sure that the column was perfectly horizontal, four adjustable screws are placed at the bottom of the stand. Figure 2.1 shows a schematic of the column.

Along the column walls, two resistance port holes were placed perpendicular to one another at six inch vertical intervals. The port holes are tapped with quarter-inch NPT. The two probe ports at each height allow

for readings across two orthogonal diameters so that data can be taken various locations in the column to give a good evaluation of the circulation patterns.

The air needed for the project was supplied from shop source. A pressure regulator was used to control air flowrate. The air was led through a dehumidifier and then entered a 1/2" 1-10 SCFM air flowmeter. Flowrates read were for pressures at standard conditions (14.7 lb/in^2). When the flowmeter operated at higher pressure, a correction was needed to calculate the true flowrate.

From the flowmeter, the air was introduced into the plenum. Air passed through a distributor plate into the bottom of the column. The plate used was designed to give an even distribution of air, as shown in Figure 2.2.

To acquire local data, resistance probes were used. Resistance probes have been used previously (Herringe & Davis (1974) and Nassos & Bankoff (1967)) to measure the differences in resistance from the probe tip being in a bubble or immersed in liquid. These probes were used to estimate a time-average local void fraction.

In using resistance probes, the production and maintenance of such probes are very important in receiving reliable results. The design of a resistance probe is shown in Figure 2.3: the only location on the probe that should detect a bubble or liquid is at its tip. This is an important consideration when measuring local void fraction or bubble velocities. Since the results are supposed to be local, any part of the probe except for the tip detecting bubbles or liquid would be unacceptable. To insure that the probe tip was the only place uninsulated, the probe was checked on a regular basis. If the uninsulated tip becomes too big, smaller bubbles could go by undetected and the researcher would significantly under-estimate the local void fraction. Also, the size of the needle was important. The bigger the needle, the more obtrusive the probe becomes to the overall circulation pattern and the chance

that a bubble could deflect away because of its size is increased.

The manufacturing of a good resistance probe was a delicate process. First, a thin stainless steel sewing needle was bent at an angle of approximately 45° . The eye of the needle was soldered to some fine insulated wire. Next the wire was passed through a twelve inch length of 12-gauge stainless steel tubing. The tubing has an outer diameter of 0.11 inches with a wall thickness of 0.011 inches. The needle was now spray-painted to make it insulated from the tubing. With a strong epoxy, a length of the needle was glued into the inside of the tubing leaving the tip protruding. If the needle remained insulated from the tubing, the resistance probe had been manufactured successfully. If this was not achieved, the probe would not be able to detect a bubble or liquid at its tip. This problem was easily checked with a resistance measurement.

With a successful probe made, one final step was needed to complete the process. To produce a conducting tip, the needle tip was abraded with fine sandpaper. To help limit long term electro chemical effects (Galaup, 1975), the needle was re-painted and sanded regularly.

Besides estimating local void fractions, this research was intended to estimate local bubble velocities. In order to do this, a different double resistance probe was needed. This probe contained two needles (Herringe & Davis, 1976) with one directly above the other (as shown in Figure 2.4). The same steps taken to make the single probe also applied in making the double probe.

In the collection of experimental data, the resistance probe, an intermediate circuit and the data acquisition board in a Zenith computer were the main components. To help take the data and reduce it, computer programs were written with the software package Microsoft Quickbasic 3.0. The schematic of the intermediate circuit for the data acquisition system is shown

in Figure 2.5.

The probe and circuit supplied close to zero voltage to a data acquisition board when the probe tip is in a bubble because the circuit was broken between the probe tip and tubing. When the probe tip was immersed in a liquid, the circuit was completed and a voltage was read between 2.5-3.25V. This variation of 0.75 was believed to be due to the electrochemical effects caused by the use of a direct current circuit and by variations between different probe tips. Galaup (1975) discussed the use of an alternating sinusoidal current supply, which greatly reduces the erosion to the probe tip. However, for this approach to be effective, Galaup stated that the frequency of the supply voltage had to be much greater than the frequency of the phenomenon being studied. With the constant checking of the probe tip, the use of an AC circuit did not prove necessary during the present research.

In the circuit shown in Figure 2.5, there is an extra resistor of $1M\Omega$ along the common ground line. This resistor created a greater voltage drop from when the probe was in a bubble or liquid. If this resistor was removed, the voltage of the probe in liquid read around 0.7 volts.

In theory, the signal from the probe should display a square wave with one output voltage representing a bubble and the other output voltage liquid. From viewing Figure 2.6, the trace is far from being a square wave. Due to the finite tip size and the time taken for the liquid film to drain from or reform on the tip, the signal becomes more rounded. Hence the results for a time-averaged local voltage are underestimated if a threshold voltage is not carefully chosen. A threshold voltage is a cutoff point above which the signal voltage represents a liquid and below which it represents a bubble. A newly developed protocol for identifying the best threshold voltage is discussed later in this section.

Since the probe's purpose was to detect the bubbles or liquid, a data

acquisition board was needed to enact the probe, take the data, and reduce the data so that it can be analyzed. The data acquisition board used was a RTI-815-F from Analog Devices (Norwood, MA). With the compatible software language Quickbasic, the acquisition board acted as an interface between the resistance probe and the Zenith Z286 PC. This board has various analog/digital, input/output, and time related functions. For this research, the analog inputs were multi-plexed to allow for 32 single-ended and 32 pseudo-differential input channels. The board has a 12-bit analog-to-digital converter. This constitutes a conversion resolution of 4096 counts over input signals ranging from -10V to +10V (1 count $\approx$ 4.88 mV). The board also has the ability to amplify a low-level signal and take data at a maximum speed of 71,400 MHz. The board was operated in the Direct Memory Access (DMA) mode. When the DMA is accessed, the computer's microprocessor gives up its control to the DMA controller. By skipping over the microprocessor, data can be taken rapidly and stored in Random Access Memory (RAM) until it is needed. Once all the data is taken, the microprocessor regains control of the RAM.

The process of taking data involved three subroutines from the RTI board library. They were called the Initialize, Scan, and Check routines. The Initialize routine reads the hardware configuration data from the disk. It clears all previously defined logical channel definitions and makes sure the DMA buffer allocated. If either is not achieved, an error code is returned. Before the data acquisition begins, the Initialize routine must be executed. The next routine implemented was SCAN. This routine reads a specified number of analog inputs into DMA buffer from multiple channels. It goes to the channel specified and then loops continuously over the 32 input channels until the number data points specified is reached. To allow for all 32 input channels to be used, and AC1585-1 Screw Termination Panel was added to the board. This extension is displayed in Figure 2.7. Only two input channels

are directly connected to the probes. However, the other channels are linked to the two main channels to allow for the probe to take readings on all 32 channels. The last routine used was the CHECK routine. The CHECK routine tests for the completion of the SCAN and COLLECT operations.

In estimating the global void fraction or the overall gas hold-up, the mixture expansion was measured. As Hills (1974) and Molerus (1985) discussed, this theory compares the volume of an unaerated liquid phase to a new aerated volume of a gas-liquid mixture. Since the column used has a constant cross-sectional area, the global void fraction was estimated by the comparisons of the heights before and after aeration. The equation for the global void fraction is

$$\bar{\epsilon} = \frac{H_m - H_L}{H_m}$$

where

H_m = height of two-phase gas-liquid mixture

H_L = height of single-phase liquid before aeration

$\bar{\epsilon}$ = global void fraction

A problem with the mixture expansion is estimating the height of the column once the air is introduced. As Molerus (1985) observed, at high flowrates, a foam region appears at the top of the mixture. The foam was not considered when estimating the height, H_m , in the column. From observation, there are not any circulation patterns occurring in this region. Also, the mixture density in this foam was considerably less than the average density of the gas and liquid in the bulk of the column. It was for these two reasons that the foam region was disregarded. In this research, the mixture expansion was used solely as a comparison tool to verify resistance probe data. This comparison provided a check on the reliability of the resistance probes, by

comparing global holdup measured in the column.

In estimating the local void fraction, the method used to determine the threshold voltage was very important. If the threshold voltage was not chosen wisely, the local void fraction would be underestimated or overestimated. Previous workers have generally used a Schmitt trigger to perform this function, but in the present work it was implemented with software. A program was written to collect the voltage from the probe at a large number of discrete times (generally 1000). Each voltage was then examined to see whether a bubble or liquid was present at the probe at that time, and a time-averaged gas void fraction was computed. To establish a cutoff voltage, the probe was operated at a fixed point with constant two phase flow conditions in the column. During preliminary work time-averaged void fraction was computed using tentative threshold voltages varying from 0 to 3V, and a voltage of 1.625V was selected as a good threshold voltage in the preliminary research since this lay on the "plateau" of a plot of measured void fraction against threshold voltage.

Subsequent analysis of high-speed traces of the probe voltage signal has demonstrated that this primitive thresholding was unsatisfactory. Also, comparison of increase in column height, with the local measured void fraction integrated throughout the column volume has shown that the probe with primitive thresholding may underestimate void fraction by as much as 40%. The trace in figure 2.6, showing voltage versus time for a probe in the 8-inch diameter column, demonstrates that the probe signal is certainly not a square wave. Brief intersections with bubbles lead to a small voltage drop, which may be detected by the primitive thresholding, and more significant intersections still show a finite rate of voltage drop and rise. The investigators have concluded from these traces and from cumulative frequency plots of measured voltage (see figures 2.8 and 2.9) that a better threshold is

a voltage which is very slightly lower than the typical voltage when the tip is in water. A final criterion chosen for the threshold was 0.025V less than the voltage at the 50th percentile on the cumulative frequency plot of voltage, this percentile being an excellent estimate of the probe voltage in water. Since measured void fractions are always less than 50%, this approach proves to be reliable and quite objective. The new thresholding criterion has led to an improvement in void fraction measurement: the void fraction measured by the probes is then less than 10% lower than the void fraction implied by column expansion.

Voidage profiles at various heights and flowrates could now be measured with a reliable method of estimating the threshold voltage. For the experimental analysis, voidage profiles were developed at three heights of one, two, and three feet of water each subsequently aerated with 4.6, 7.3, and 10.4 CFM of air. To show the circulation development, the probe was inserted at various heights in the column. To obtain an accurate time average over a long time period, ten runs of 1200 readings at 62.5 points per second were taken. To check the consistencies of the probes, a sample standard deviation for small sets of experimental data was calculated using the equation:

$$\sigma = \sqrt{\sum_{i=1}^N \frac{(\bar{\epsilon} - \epsilon(r))^2}{N-1}}$$

where

σ = sample standard deviation

N = number of experimental tests

$\bar{\epsilon}$ = average of experimental tests

The probe tip was placed an inch from the far column wall and moved away from the wall in one inch increments until the tip was an inch from the probe port opening in the near side wall. At the same time, a second profile was developed perpendicular to the other profile at the same height. This gives a more 3-dimensional impression of the circulation process, and it also serves

as a check on the probes. Instead of one local void fraction, there are two local void fractions (which should be quite similar) at each radius. The results should be similar because the column was level, air was introduced evenly through the distributor plate, and the circulation should be symmetrical. If the results from the two probes varied too much, it was found that one of the probes was not functioning properly. In this instance, the faulty probe was removed and replaced.

In estimating the bubble velocity in the bubble column, two different approaches were taken. With each method, the technique had to be able to estimate bubble velocities ranging from -20 to 50 in/s. This range was found by adding the expected bubble slip velocity (Serizawa et al, 1974a) to the expected circulation velocities found using one-dimensional theory and the measured void profile. The first approach used a cross-correlation technique to estimate an overall bubble velocity, while the other method measured individual bubble velocities. In both techniques, the results were not very reliable. However, in the center of the column, the results were more acceptable.

Bubble velocities were determined in a circulating system (initially two feet of water), at 6", 12", 18", and 24" above the distributor plate, and at air flowrates of 4.6, 7.3, and 10.4 CFM. From these results, a liquid velocity profile could be compared with the one dimensional force balance approach. The distance between the lower and upper probe tip was 3/8". Velocity data was taken at a sampling rate of 2000Hz (1000 Hz per probe tip) for eight seconds.

The cross-correlation technique (Herringe & Davis, 1974b and Serizawa et al, 1974) searches for a time delay between probe tips by multiplying the signal from the lower probe by the signal of the upper probe a moment of time, Δt , later, and then determining the Δt for which this product is a maximum.

The signals of the two probes are multiplied according to the following equation:

$$F(\Delta t) = \sum_{i=1}^T V_L(t) V_U(t+\Delta)$$

where $F(\Delta t)$ = sum of multiplied signals at a specified time delay

$V_L(t)$ = signal of lower probe

$V_U(t+\Delta t)$ = signal of upper probe

To illustrate this point Figure 2.10 shows an actual experimental traces of the signals from the two probe tips and Figure 2.11 displays the cross-correlation technique determining the time delay between probe tips. With the time delay known, the actual velocity can now be calculated by the equation

$$V_B = \frac{\Delta x}{\Delta t_A}$$

where V_B = bubble velocity

Δx = probe tip separation

Δt_A = actual time delay

To relate liquid and bubble velocity,

$$V_B = V_L + V_b$$

where V_B = bubble velocity in circulation relative to column

V_L = liquid velocity

V_b = rise velocity of a single bubble in quiescent fluid

The rise velocity of a single bubble can be (Harmathy, 1960) calculated by the equation:

$$V_b = 1.53 \left[\frac{\sigma g \Delta \rho}{2 \rho_L} \right]^{1/4}$$

where σ = surface tension between air and water
 g = gravity
 $\Delta\rho$ = difference in density between air and water
 ρ_L = liquid density

This equation applies for bubble sizes ranging from 2-30 mm which were the approximate size of the bubbles seen in the column. In this way liquid velocities could be inferred from probe measurements.

To test the cross-correlation approach, an attempt to measure a single bubble's velocity was made. Quarter-inch nominal diameter tubing was attached to the air supply. The tubing was placed through a probe port in the column and the end of the tubing was located at the center of the column. Next, the double resistance probe was inserted above the tubing so that bubbles from the tubing could impact the probe. Air was then introduced into the column at such a low flowrate that individual bubbles rose from the tubing periodically. As the isolated bubbles approached the probe, the data acquisition program was started and the probe took data. A trace of two bubbles impacting the dual probe is displayed in Figure 2.12. The cross-correlation program estimated a bubble velocity of 10.2 in/s. These results coincide with the single bubble theory, and it also demonstrates the accuracy and ability of the cross-correlation technique to measure the rise velocity of a single bubble.

Implementing the cross-correlation technique was accomplished through the use of computer software. The technique was broken into two programs with one taking and storing the data while the other program retrieved the data and reduced it. The first program, RAWDATA, was identical to the data taking process for the local void fraction with the only exception being the sampling rate. The CORRELATION program began with retrieving the stored data and separating the data into the lower and upper probe arrays. Another program

was also written that could measure individual bubble velocities (termed the "window" method). Serizawa et al, (1974) used a similar approach called the "Multi-channel Technique". They took down the time a bubble hit the lower and upper probe and with the separation between the probes known, the velocity of that individual bubble during circulation could be calculated. In this case the RAWDATA program was used again for the data taking process. Also, the technique used to determine a threshold voltage of each probe was implemented in the program REDUCTION to determine the precise time a bubble hit and left the two probes. This data was stored in data files and reduced later in the WINDOW program.

The WINDOW program takes the time a bubble impacts the lower probe and searches for the same bubble to hit the upper probe within a specified time frame or "window frame". This window frame was based on the expected bubble velocity found from one dimensional theory. From the predicted velocity the window frame was opened up to look at times representing 50% before and after the expected bubble velocity. After a bubble impacts the lower probe, several scenarios are possible. Serizawa et al, (1974) noted (a.) the bubble does not hit the second probe within the time span, (b.) the bubble deflects and misses the probe, and (c.) another bubble reaches the upper probe before the first bubble arrives. See figure 2.13. There are two other possibilities that could affect this technique. One possibility is if a bubble hits the lower probe away from the center, the bubble could deflect and hit the upper probe centrally. This would cause an overestimation in the bubble velocity. The other possibility was just the opposite. In this case, a bubble could hit the lower probe at its center and deflect to where the bubble would impact the upper probe at its side. This case would cause an underestimation of the bubble velocity. Figure 2.13 displays all the possibilities leading to velocity maledistimation.

In the WINDOW program when a bubble was found at the upper probe within the window frame, the following equation was executed:

$$\text{mult} = \frac{[S_L - S_U] + [E_L - E_U]}{2}$$

where mult = time delay for individual bubble
 S_L = starting time of bubble on lower probe
 S_U = starting time of bubble on upper probe
 E_L = ending time of bubble on lower probe
 E_U = ending time of bubble on upper probe

To make sure that the bubble on the upper probe was not computed with another bubble on the lower probe, a string condition was placed on it to disallow the bubble from further computations. When there was not a bubble on the upper probe within the time frame, a new starting time for a bubble on the lower probe was found and the process repeated itself. While Serizawa et al, (1974) only used the starting times, the ending times on the probes were used in this research to help compensate for the under and overestimations demonstrated in Figure 2.13. All the time delays for the individual bubbles were totaled and averaged to find the actual time delay between the probe tips.

2.4 EXPERIMENTAL RESULTS

The results found for the local void fraction were reliable. In comparison with the expansion of the mixture upon aeration; the local void fraction was underestimated by only 8-15% as shown in Table 2.1. With this small error, the threshold technique described above was taken to be reliable, since some underestimation might be expected due to deflection of bubbles from the probe.

Void fraction profiles were measured across the column diameter for the following conditions: at 1, 2 and 3 feet of unaerated water in the column, at

4.6, 7.3 and 10.4 scfm air flow, and at various heights from 6 inches above the distributor to just below the mixture surface. Results are shown in figures 2.14 to 2.40.

From observing the profiles near the distributor plate, it can be seen that the profile is somewhat flat. This type of distribution at the plate was to be expected because the air was introduced into the column evenly. However, as the probe was inserted at each height above 6", the profiles began to peak more near the center of the column. In the gulfstreaming effect, the liquid recirculates near the column wall. There is a radial inward liquid velocity above the plate forcing the bubbles inward (Freedman & Davidson, 1969). This results in a higher local void fraction near the center of the column. Again, the results back up this premise. Another relevant point about the reliability of the probes was the fact that at any location in the column, the local void fraction between the two probes transversing two diameters normal to one another was rarely over 10% in difference.

The bubble velocity results were less consistent than the void fraction results. At times, when the probe was away from the center, the time constants found from cross-correlation varied from 0.0005 to .0125 seconds which implied bubble velocities of 750 to 30.61 in/s, which was inconceivable. As seen from the experimental trace of the two probe signals in Figure 2.41, there is not a consistent pattern of bubbles impacting the lower probe then proceeding on to the upper probe. Poor results were ascribed to the bubble's radial motion which was not taken into consideration. Generally when the probe was located away from the center of the column, where a majority of the bubbles have some radial motion, the results found were poor relative to data taken at the column center. The bubble velocities found at the center of the column at 6" and 12" above the plate at three air flowrates are compared in Table 2.2 to the predicted bubble velocity found by using the one dimensional

model of Clark et al. (1987) which is described later in this section. The table shows that the cross-correlation technique and the predicted bubble velocity varied from 5-25%.

The WINDOW program results proved less reliable than the cross-correlation technique. The problem with the program was that the window frame for a bubble to pass the upper probe affected the overall results. Figure 2.42 shows a histogram of individual bubble velocities in the center of the column at 12" above the plate and an air flowrate of 4.6 CFM. From these figures, it can be seen that the majority of the bubbles were found at the high velocity. Also, expected histogram should mimic a bell shaped curve with the best choice at the peak of the curve. Figure 2.42 does not demonstrate this.

Table 2.3 shows the number of bubbles striking the probes over 8 seconds at 12" above the plate and at the three different flowrates. This table concurs with the voidage data and demonstrates that the majority of the bubbles are located near the center while bubbles are scarce at the column wall.

2.5 ONE DIMENSIONAL MODELING

This section discusses two aspects of one dimensional modeling. Firstly, a computer implementation of the model of Clark et al. (1987) is presented since this was the model used to process the results presented above. Secondly, novel research relating the drift-flux model to bubble columns is discussed.

An important aspect of this research was to model the liquid velocity from the void fraction data. Early researchers (Rietema & Ottengraf, 1970; Hills, 1974) modeled the liquid velocity by a momentum balance, but viscosity or eddy viscosity was used in the model. However, in this research, mixing length theory was used to model the liquid velocity.

In the present research, the column's four inch radius was divided into 500 evenly spaced intervals. Since in the experimental work time-averaged local void fractions in the column were measured an inch apart, the voidages were divided into four sections of 125 intervals at 0.008 inch per interval. The inner voidages were computed by finding the slope between the end points. With the voidage profile computed, a density profile for the mixture of air and water in the column was calculated based on the local void fraction by the equation

$$\rho(r) = \rho_L (1 - \epsilon(r)) + \rho_G \epsilon(r) \quad 2.1$$

where $\rho(r)$ = mixture density of radius r

ρ_L = liquid density

ρ_G = gas density

The second term of the righthand side is often neglected due to the low density of air. The average mixture density, ρ_{ave} , and the average mixture densities $\rho_i(r)$, within central cylinders of radius r were calculated by the equation:

$$\rho_i(r) = \frac{1}{r} \sum_{i=1}^r \rho(r) \quad 2.2$$

This is acceptably accurate in the case of a flattened void profile, although strictly the more exact approach used by Clark et al. (1987) is to be preferred. At the time of writing data are being reprocessed using the exact approach. The shear stress profile was computed next using the force balance equation.

$$T(r) = T_w \left[1 + Rg \left(\frac{\rho_{ave} - \rho_i(r)}{2T_w} \right) \right] \left(\frac{r}{R} \right) \quad 2.3$$

where T_w = wall shear stress (an unknown).

The shear stress may also be related to the velocity gradient in the column.

$$T(r) = T_L(r) + T_T(r) \quad 2.4$$

where $T_L(r)$ = laminar shear stress

$T_T(r)$ = turbulent shear stress

The laminar shear stress equation is:

$$T_L(r) = \mu \frac{du(r)}{dr} \quad 2.5$$

where μ = viscosity

$\frac{du(r)}{dr}$ = liquid velocity gradient

The turbulent shear stress is given by

$$T_T(r) = -\rho(r) L^2(r) \left| \frac{du(r)}{dr} \right| \frac{du(r)}{dr} \quad 2.6$$

where $L(r)$, the mixing length, is given by Clark et al., (1987),

$$L(r) = R \left[.14 - .08 \left(\frac{r}{R} \right)^2 - .06 \left(\frac{r}{R} \right)^4 \right] \quad 2.7$$

From these equations one can find $\frac{dv}{dr}$ as a function of r for an assumed value of T_w . At the column wall there is the boundary condition that liquid velocity must be zero, so that the axial liquid velocity distribution across the column is readily found.

An additional equation required before the liquid velocity profile can be computed is derived from the fact that the net flowrate of liquid across a cross-section of the column must be zero for a batch process column.

$$Q = 2\pi \int_0^R V_L(r) r (1 - \epsilon(r)) dr = 0 \quad 2.8$$

These equations were solved to yield the liquid velocity distribution in the following way. A value for wall shear was assumed, and the shear stress distribution across the column was computed. From the shear stress, the velocity gradient, and then the liquid velocity were found. If the net flowrate of liquid across the column cross-section was then not zero, a new value of wall shear stress was assumed, and the process repeated. When the net flow of liquid was zero, the correct liquid velocity profile had been found. Figure 2.42 shows the calculation scheme. Figures 2.44 through 2.46

show computed profiles of the local void fraction, mixture density, and the shear stress with one foot of water in the column, the probes were located at 6" above the distributor plate, and an air flowrate of 4.6 CFM. Figures 2.47 to 2.73 show velocity profiles computed in this way from the voidage data. Table 2.2 shows that centerline velocities found in this way agree acceptably with experimental data.

A second model was developed to predict the holdup in bubble columns with either zero or low liquid throughout using the drift-flux approach. Zuber and Findlay (1964) have presented an exhaustive derivation for the drift-flux model, which is based on the argument that holdup in a two-phase flow (typically bubble, slug or churn gas-liquid flow) is influenced by two separate phenomena. Firstly, it is acknowledged that the gas rises locally relative to the liquid due to phase density differences, a fact which may often be neglected in high velocity flows. Secondly, where a velocity distribution exists in the pipe, and where the gas is inhomogenously distributed across the pipe diameter, the gas may be concentrated in a faster or slower region of flow, thus affecting the average gas holdup. The model is usually presented in the form

$$\frac{\bar{W}_g}{\bar{\epsilon}} = C_o (\bar{W}_g + \bar{W}_l) + \frac{\overline{U_{gm}} \bar{\epsilon}}{\bar{\epsilon}} \quad 2.9$$

where $\bar{W}_g$ is gas volumetric flux (superficial velocity), $\bar{W}_l$ is liquid volumetric flux, $\bar{\epsilon}$ is void fraction, $\overline{U_{gm}}$ is relative velocity between the gas and the mixture (i.e. $\overline{U_{gm}} = \overline{U_g} - (\bar{W}_g + \bar{W}_l)$, where $\overline{U_g}$ is gas velocity) and an overscore denotes an average over the column cross-section. The profile constant C_o is given by

$$C_0 = \frac{\overline{\epsilon (W_g + W_L)}}{\overline{\epsilon} (W_g + W_L)} \quad 2.10$$

and is a measure of the interaction of the void and velocity distributions. Where gas is more concentrated in the faster region of flow, $C_0 > 1$, and C_0 is often taken as 1.2 for fast upward bubble flows, as shown in Table 2.4 below.

The term $\overline{U_{gm}} \overline{\epsilon} / \overline{\epsilon}$ is the weighted average drift velocity, accounting for local slip. Since it has been shown that relative phase (bubble rise) velocity $U_{GL} (=U_g - U_L)$ varies little over the pipe diameter (Serizawa et al., 1975), and since $U_{gm} = U_g (1-\epsilon)$, with $\epsilon < 0.25$ in bubble flow, the weighted average drift velocity is often taken as the rise velocity of a bubble in an infinite medium, U_∞ . Experimental data generally support this simplification.

The drift-flux model has not remained inviolate. There is a mounting body of data to show that either C_0 or $\overline{U_{gm}} \overline{\epsilon} / \overline{\epsilon}$ cannot be taken as constant over an operating range. For example, Hills (1976), using a 0.15m pipe, acquired data which led to the development of a modified drift-flux model. Also Shipley (1984), working with a "toy tower" circulation loop of 0.457m diameter, found his data best correlated by the dimensional equation.

$$\frac{\overline{W_g}}{\overline{\epsilon}} = 1.2 (W_g + W_L) + 0.24 \text{ (m/sec)} + 0.35 \left(\frac{\overline{W_g}}{\overline{W_g} + \overline{W_L}} \right)^2 (gD\overline{\epsilon})^{1/2} \quad 2.11$$

and some physical argument was provided for this approach. Clark and Flemmer (1985), using a 100mm diameter forced circulation loop in bubble flow, preferred to vary C_0 as a function of void fraction, since C_0 was found to be near 0.95 at low void fractions and near 1.2 when $\epsilon \approx 0.2$. Subsequently Clark and Flemmer (1986) fitted the same data to a drift-flux model with two profile constants

$$\frac{\bar{W}_G}{\bar{E}} = C_G \bar{W}_G + C_L \bar{W}_L + \frac{U_{GM} \bar{E}}{\bar{E}}$$

2.12

with $C_G = 1.95$ and $C_L = 0.93$. Jones (1985) acquired data in a bubble column with draft tubes ranging from 44 to 146mm in diameter. A drift-flux model overestimated the circulation in this column rate unless C_0 was set to very high values (2 to 5) (Clark and Jones, 1987). Further argument on the variation of C_0 has been presented by Lorenzi and Sotgia (1978), while careful consideration of the definition of C_0 in the light of experimentally measured void and velocity profiles presented by Galaup (1975) and Serizawa et al. (1975) also supports the argument that C_0 cannot remain constant.

The bubble column represents the extreme case of bubble flow where net liquid velocity is zero or low and the gas simply bubbles up through the liquid. Generally buoyancy effects and gas maldistribution cause circulation of liquid in the column. Even when gas is evenly distributed over the column base, circulation can occur, and it is worth taking the time to consider the mechanism governing circulation startup, since this has not been discussed elsewhere.

Let us assume that a column with even gas distribution starts to operate in an "ideal" or one-dimensional mode, where the gas void distribution is even over the cross section at all heights in the column, so that the mixture is homogeneous and has the same density throughout the column. At this stage no gross liquid circulation would occur. Since most two phase systems involve some degree of turbulence or mixing, let us propose that at some time a few bubbles move toward the center of the column, at any height, so that the concentration of bubbles is suddenly slightly higher near the center. Consider the mechanistic sequence of events which may arise. The mean mixture density in the central region of the column is now lower than density in the

outer annulus, so that the hydrostatic head is greater over the height of the annulus than over the same height of the central core. Both the annular and central regions have the same pressure at the liquid surface, so that the difference in hydrostatic head causes a radial pressure difference deeper in the column (in fact, at every height below the point where the bubbles moved inward initially). This in turn causes an inward radial movement of liquid and initiates liquid circulation.

Bubbles rising from the distributor plate in the annular region are now moved inward a little by this radial inward liquid movement: the result is that bubble concentration near the center of the column increases and the liquid circulation increases. There is a positive feedback between the circulation and the radial bubble movement. Rapidly a stronger circulation pattern develops and the radial inward currents above the distributor plate continue to sweep more of the bubbles toward the center of the column (by analogy this is an inverted classifier).

One might expect that the circulation pattern would continue to increase indefinitely, but increasing upward velocity in the central region reduces bubble holdup in that region and hence the difference in hydrostatic heads tends to some limit. Driving force arising from the difference in heads is consumed by dissipation in the fluid (energy balance) or shear at the column walls (momentum balance).

The argument may also start by assuming that a small number of bubbles move initially into the annular region, in which case the inverse pattern is set up. Such patterns have been known to occur, especially in fluidized beds (Surma, 1986, Lin et al., 1985). In fact, this argument could be applied to any pattern, even multiple patterns in large tanks (Otero et al., 1985). However, it would seem that in the case of bubble columns and fluidized beds, there are some non-stochastic grounds for the selection of a specific pattern

by initial bubble movement because at high gas velocities the circulation pattern is always upward at the center. Transition patterns have also been observed (Lin et al., 1985).

Consider a cross section of a tall bubbled column at half of the height of the mixture in the column: radial effects may be neglected. Let us pursue an analysis which commences with the same approach as the one-dimensional model derived above. The gas void distribution in the column can usually be well described by a relationship of the form

$$\epsilon(r) = \epsilon_c [1 - (r/R)^P] \quad 2.13$$

where ϵ_c is the voidage at the center of the column and R is the radius of the column. Figures 2.14 to 2.40 bear out this relationship.

The density, $\rho(r)$, as a function of position is given then given by:

$$\rho(r) = \rho_L (1 - \epsilon(r)) + \rho_G \epsilon(r) \quad 2.14$$

where ρ_L is the liquid density, and where ρ_G , the gas density, is usually neglected. Substituting the relationship for voidage into equation 2.14:

$$\rho(r) = \rho_L [1 - \epsilon_c + \epsilon_c (r/R)^P] \quad 2.15$$

Axial shear stress, $T(r)$, may be found (Clark, et al., 1987) using the same force balance as previously employed

$$T(r) = \tau_w (r/R) + 1/2 r g (\bar{\rho} - \rho_1(r)) \quad 2.16$$

where g is gravitational acceleration and τ_w is the unknown wall shear stress. $\bar{\rho} - \rho_1(r)$ is the difference between average density over the whole radius, and average density within a radius, r , given by:

$$\bar{\rho} - \rho_1(r) = \rho_L \left[\frac{2 \epsilon_c}{P+2} \right] \left[1 - \left(\frac{r}{R} \right)^P \right] \quad 2.17$$

For the fluid in the column, the rheological mixing length model which takes into account turbulent momentum transfer was again used. Shear stress was accordingly taken as:

$$\tau(r) = \mu \left(\frac{dU}{dr} \right) + L^2 \rho \left| \frac{dU}{dr} \right| \left(\frac{dU}{dr} \right) \quad 2.18$$

where μ is the viscosity, and L is the mixing length used above, in equation 2.7.

The liquid velocity profile, $U(r)$, may be solved by equating shear stress in equations 2.16 and 2.18:

$$0 = T_w \left(\frac{r}{R} \right) + \frac{rg}{2} (\bar{\rho} - \rho_l) - \mu \frac{du}{dr} - L^2 \rho \left(\frac{du}{dr} \right) \left(\frac{du}{dr} \right) \quad 2.19$$

Using an assumed wall shear stress the values for $\frac{du}{dr}$ and U can be found. Integrating $U(r) (1 - \epsilon)$ over the column cross-section then yields the liquid flux, W_L :

$$W_L = \frac{2}{R^2} \int_0^R U(r) (1 - \epsilon) r dr \quad 2.20$$

Note that local void fraction plays a role in this equation, whereas it was neglected in the simpler one dimensional analysis above. If W_L is equal to the desired liquid flux, then the assumed T_w was correct. For example, if no net liquid flow is desired (batch bubble column), T_w would have to be chosen until the integral in equation 2.20 assumed a value of zero. For the purpose of lucidity and efficiency, equation 2.19 may be made dimensionless, as follows:

$$0 = \left[N + \frac{Ga}{2} \frac{\epsilon}{p+2} \right] r' - 2 \frac{dU'}{dr'} \left(\frac{r}{R} \right)^2 (1 - \epsilon) Ga \left(\frac{dU'}{dr'} \right) \left| \frac{dU'}{dr'} \right| \quad 2.21$$

where:

$$N = \frac{T_w}{\mu} \sqrt{\frac{D}{g}} \quad 2.22$$

$$U' = \frac{U}{\sqrt{gD}} \quad 2.23$$

$$r' = \frac{r}{R} \quad 2.24$$

$$\text{And } Ga = \frac{\sqrt{gD} \cdot D \rho_L}{\mu} \quad 2.25$$

which is a Galileo number; D is column diameter.

Equation 2.20 may be rearranged to give:

$$\frac{W_L}{\sqrt{gD}} = 2 \int_0^1 U' (1 - \epsilon) r' dr' \quad 2.26$$

In order to determine the drift flux parameter, C_o , it is necessary to find the average flux of the gas, W_g . The local gas volume flux is given by:

$$W_g = \epsilon U + \frac{\epsilon}{1 - \epsilon} V_v \quad 2.27$$

Hence:

$$\begin{aligned} W_g + W_L \\ = U + \left(\frac{\epsilon}{1 - \epsilon} \right) V_v \end{aligned} \quad 2.28$$

And by definition of C_o :

$$C_o = \frac{\int_0^R \left[\epsilon U + \left(\frac{\epsilon^2}{1 - \epsilon} \right) V_v \right] r dr}{\epsilon c \left(1 - \frac{2}{p+2} \right) \int_0^R \left[U + \left(\frac{\epsilon}{1 - \epsilon} \right) V_v \right] r dr} \quad 2.29$$

Or:

$$C_o = \frac{\int_0^1 \left[\epsilon u' + \left(\frac{\epsilon^2}{1 - \epsilon} \right) Fr \right] r' dr'}{\epsilon c \left(1 - \frac{2}{p+2} \right) \int_0^1 \left[u' + \left(\frac{\epsilon}{1 - \epsilon} \right) Fr \right] r' dr'} \quad 2.30$$

where the froude number, Fr, is given by:

$$Fr = \frac{V_v}{\sqrt{gD}} \quad 2.31$$

The drift flux parameter thus depends on four variables, viz. G_a , Fr , ϵ_0 , and p .

This circulation model was used to calculate the drift-flux parameter C_0 for a wide range of column diameters, void distributions, fluid properties and local bubble rise velocities. Table 2.6 relates void distribution parameters to average voidage in the column. Figures 2.74 to 2.82 show how C_0 varies with G_a , Fr and void distribution. It is interesting to note that C_0 assumes very high values over most of the operating range, since the velocity profile is dictated entirely by buoyancy effects rather than by a net flow up the column (i.e. wall effects). This is evident in figure 2.83 which shows one of Hills' void profile with the sympathetic velocity distribution. This calculated velocity distribution agrees well with the velocity distribution measured by Hills using a Pavlov tube, which is a device that infers fluid velocity from the pressure field around a cylinder. Figure 2.84 serves to validate the model further with comparison of predicted centerline velocities measured by Hills for various gas flowrates.

The model was also used to find C_0 when there is a net flow in the column. In figure 2.85 the curve for $Fr = 0.25$ corresponds to air-water upflow in a 100 mm diameter pipe with a typical air-water bubble rise velocity of 250 mm/sec. and with Re based on net upflow of liquid in the pipe. At high velocities it can be seen that C_0 tends to a value between 1 and 1.5 which is what has been observed in practice (see table 2.4). However, at lower velocities buoyancy effects become significant with C_0 assuming values as high as 3 or 4. This explains overestimation of void fraction and hence circulation rate in some air lift circulation devices (see comments by Clark and Jones, 1987). At a larger pipe diameter ($Fr = 0.113$) buoyancy effects play an even greater role, with a very high centerline velocity existing even at average velocities of 10 m/sec. One must conclude the C_0 is highly

variable in these circumstances so that a drift-flux model should be used with caution for low velocity or large diameter systems. Figure 2.86 illustrates some of the predicted velocity profiles which led to the construction of figure 2.87. The competing buoyancy and wall effects are evident here. Figure 2.88 yields C_0 when the void fraction is a maximum at the pipe center over a broad range of liquid flow Reynolds numbers for a 500 mm diameter pipe ($Fr = 0.113$) for both upflow and downflow. A choking condition is evident when the liquid downflow counteracts the upward rise of the bubbles, causing a void fraction to be present although $(W_g + W_l) = 0$. In consequence C_0 assumes very high values.

It is well documented in the literature (Galaup, 1975; Serizawa et al., 1975; Nakoryakov et al., 1981) that saddle shape void profiles can also exist in low void fraction flows, and this is in keeping with the argument presented above for the "inverse" circulation pattern in bubble columns. Figures 2.88 show results as velocity profiles from the model for air-water upflow using the annular gas void distribution illustrated in figure 2.89. Buoyancy effects are so strong for the case of the 500mm pipe ($Fr = 0.113$) that the velocity profile remains saddle shaped (see figure 2.88) at all credible operating velocities. As a result C_0 remains above unity, despite the fact that the bubbles are concentrated near the wall. However, for a 100mm pipe ($Fr = 0.25$), where wall effects are more significant, the computed velocity profile was similar to that in a single phase flow at higher velocities, so that C_0 was driven to a value below unity.

One may conclude from this drift-flux analysis that in large diameter pipes C_0 will always be greater than unity, assuming very high values at low net flowrates. However, in small diameter pipes C_0 will still assume high values due to buoyancy effects at low velocities, but the value of C_0 will depend strongly on the gas void distribution at higher velocities. When the

void distribution is saddle-shaped, as occurs at low void fractions, C_0 will be slightly below 1. At higher void fractions, where c is a maximum at the center, C_0 will assume typical values in the range 1.3 to 1.5. It can be seen that pipe size, flowrate and void fraction can all influence C_0 , so that it is not surprising that many authors have found the necessity to modify drift-flux models to account for these problems. One must conclude that a direct drift-flux approach is not suitable unless the void distribution is known and that buoyancy effects are insignificant.

All of the velocity profiles generated above required prior knowledge of the void profile present in the column, so that a priori prediction of circulation is still not facilitated. However, by considering the arguments presented above on the origin of the circulation pattern, and by assuming all bubble motion to be causal (that is, caused only by gravity and average interphase drag) above the distributor, it should be relatively easy to develop an iterative model for the prediction of circulation in a vessel with even gas distribution, without having to assume bubble distribution across the diameter. Select, for the first iteration, an approximate circulation pattern in terms of the velocity distribution across a diameter half way up the column. Then, from continuity considerations, the average radial flux of liquid in the lower half is known at each value of the radius. Transport of bubbles towards the column center by the radial liquid flow in the entrance region can be determined quite accurately, since it is the total radial flux that determines the net bubble movement and the actual radial velocity need not be known as a function of height. The approach yields a gas flux and a voidage distribution half way up the column which can in turn be used to infer a liquid velocity distribution by applying the model described above: the calculation process is then repeated until the predicted velocity distribution agrees with the assumed velocity distribution. In other words we have two

models to relate void distribution and velocity profile. One is the force balance model, used extensively above. The other involves the "classification" of bubbles which are moved radially as they rise from the distributor to the column center. When these two models agree on the relationship between void and velocity distribution, we might assume that this is a stable circulation solution. This is discussed in detail below, and an example follows.

Consider the zone in a bubble column just above the distributor plate, and assume that the column contains one large circulation cell, with liquid upflow at the center and downflow at the walls. In this zone there must accordingly be a radially inward flow of liquid. Bubbles rising from the distributor plate will be carried a radial distance inward before they rise to the center section of the column. Let us assume that this lower zone has height h (which is shown to be an arbitrary value) and that the radial velocity components are even over this height, see figure 2.90. Fluid delivered or removed axially from above to this lower zone will dictate the radial velocity profile. Let $U(r)$ denote the axial velocity in the center zone and $V(r)$ the radial velocity within the lower zone. From conservation of mass, neglecting the volume occupied by bubbles,

$$-2\pi hrV = -\int_r^R 2\pi rU \quad dr \quad 2.32$$

which agrees with the fact that V must be zero at the column center and column wall. A bubble rising from the plate with a vertical velocity U_b will spend a time

$$t_z = h/U_b \quad 2.33$$

within this zone, and during that time will be swept inwards with a velocity $V(r)$, so that when it leaves the zone it will have moved inwards by a distance given by the integral

$$\int_{t=0}^{t_z} V(r) dt.$$

2.34

Practically (numerically!) one may trace the path of any bubble through this lower zone, assuming its lateral movement is governed causally by the liquid velocity, to find its position upon leaving the lower zone. For many bubbles, the gas flux $W_{gb}(r)$ at the base of the lower zone (at the distributor) may be translated into $W_g(r)$ in the central zone. Note that $W_g(r) = \epsilon(r) (U_L(r) + V_v)$, in other words, the gas flux is the product of void fraction and gas velocity. This suggests the technique for determining circulation in a bubble column (or a pipe containing low velocity flows) with a knowledge only of distribution performance. The technique involves an assumption of $\epsilon(r)$, followed by calculation of $U(r)$. Hence $V(r)$ may be calculated, and $W_{gb}(r)$ and $V(r)$, $W_g(r)$ and then $\epsilon(r)$ may be predicted. If this last void distribution is dissimilar from the one originally assumed, a new $\epsilon(r)$ must be assumed and the procedure repeated. Presumably a stable solution results when a given $\epsilon(r)$ (or $W(r)$) will predict the same $\epsilon(r)$ (or $W(r)$), provided that the system can reach that stable condition upon startup. Perturbation studies could also be conducted in this way to assess how robust any particular circulation pattern will be.

An example of this new technique follows, based on the data of Hills. In his paper, data is presented for void and velocity distributions in a 138mm diameter bubble column operating with "Plate B", which by description should introduce gas evenly over the base of the column. From Hills' Figure 11, liquid velocity profiles were obtained for gas fluxes of 169 mm/sec and 38 mm/sec. From Hills' Figure 6, data on void fraction were obtained for the same gas throughputs. A computer program was written in Fortran 77 to translate the discrete values of velocity $U(r)$ into radially inward

velocities, $V(r)$, in the lower zone above the distributor. This zone was assumed to be one column radius high, but it can be shown readily that this height is arbitrary (a "dummy variable") in this simplified analysis. The program then examined a flux of bubbles rising through this radial flow, being carried inward as they rose. In this way $W_g(r)$ was predicted from $W_{gb}(r)$, where $W_{gb}(r)$ was assumed to be an even flux with Hills' plate B.

Figure 2.91 shows the axial velocities in the center zone and radial velocities in the lower zone for $h=R$ and for the gas flowrate of 38 mm/sec. Figure 2.92 shows the computed values of $W(r)$ compared with the values of $W(r) (= \epsilon (U_L + V_V))$ found from Hills' paper. While agreement is not excellent, the trends are correct. Errors are ascribed to (i) inaccuracy in the data, (ii) the discrete and crude approach to finding $V(r)$, (iii) the fact that the volume occupied by bubbles in the lower zone was not considered fully in this simplified model, (iv) the fact that bubble motion might not have injected air evenly and (v) the fact that the distributor plate might not have injected air evenly over the bed cross section. Considerable disagreement toward the column center also arises because errors in volumetric flows are mapped into very small area increments, yielding high errors in computed velocity in this region. Data for the 169 mm/sec gas flowrate were in greater error, most likely due to cause (iii) above.

Improving this approach will simply lead the researcher to use of a more rigorous fluid computational analysis, as discussed in section 4 of this report.

DISTANCE FROM DISTRIBUTOR PLATE	AIR FLOWRATE(CFM)		
	46	73	104
6	12.17	16.13	19.15
12	13.09	16.40	19.26
GLOBAL MEASUREMENTS	14.28	18.64	21.95
	PERCENT VOIDAGE		

a.)

DISTANCE FROM DISTRIBUTOR PLATE	AIR FLOWRATE(CFM)		
	46	73	104
6	11.58	14.98	17.90
12	12.28	15.43	18.09
18	12.12	15.67	17.66
24	13.06	14.94	18.90
GLOBAL MEASUREMENTS	13.52	17.24	20.00
	PERCENT VOIDAGE		

b.)

DISTANCE FROM DISTRIBUTOR PLATE	AIR FLOWRATE(CFM)		
	46	73	104
18	12.50	15.47	17.41
24	12.27	14.90	17.22
36	12.70	15.38	17.12
GLOBAL MEASUREMENTS	13.23	16.78	19.10
	PERCENT VOIDAGE		

c.)

Table 2.1 Comparison of Global Measurement and Resistance Probe Technique in Determining Overall Gas Hold-up

- a. 1 ft. of water
- b. 2 ft. of water
- c. 3 ft. of water

	Flowrate (CFM)	Predicted (in/s)	Experimental (in/s)
6"	4.6	30.13	32.05
	7.3	31.99	35.71
	10.4	34.54	44.12
12"	4.6	25.37	30.99
	7.3	28.37	34.4
	10.4	30.56	35.71

Table 2.2

Comparison of Predicted and Experimental Bubble Velocities
in the center of the column

Flowrate (CFM)	Distance from Wall				
	1	1.75	2.5	3.25	4
4.6	97.6	136.4	144.2	139.4	146.2
7.3	105	104.2	181.4	197	180
10.4	160.6	181	208	219.8	228

Table 2.3

Bubble Distribution over 8 seconds with 2 ft of water, Probe
12" above Distributor Plate, and $Q_g = 4.6$ CFM

<u>Author</u>	<u>Comment</u>	<u>Value of C_0</u>
Wallis	Recommendation	1.2
Govier and Aziz	Recommendation	1.2 - 1.3
Ishii and Grolmes	Recommendation	$1.2 - 0.2 (\rho_G/\rho_L)^{0.5}$
Nassos and Bankoff	69 mm pipe	1.1
Best Fit/Data of Borishanskiy et al.	11 mm pipe	1.187
Hewitt	Recommendation	1.2
Zuber and Findlay	50 mm pipe (includes slug flow)	1.2
Zuber et al.	Recommendation	1.2 - 1.3
Galaup	42 mm pipe	1.13
Clark and Flemmer	50 mm pipe	1.16

Table 2.4: VALUES DETERMINED FOR C_0 IN CHURN-TURBULENT BUBBLE FLOW

Center void fraction ϵ_c	Exponent ρ	Average void fraction $\bar{\epsilon}$
0.1	2	0.050
0.2	2	0.100
0.3	2	0.150
0.1	3	0.060
0.2	3	0.120
0.3	3	0.180
0.1	7	0.078
0.2	7	0.156
0.3	7	0.233

Table 2.5. Void fractions used in modeling.

Figure 2.1 - Design of Bubble Column

Figure 2.2 - Design of Distributor Plate

Figure 2.3 - Design of Resistance Probe

Figure 2.4 - Double Resistance Probe for Bubble Velocity Measurements

Figure 2.5 - Data Acquisition System Schematic

Figure 2.6 - Voltage Trace showing Interaction between Bubble and Probe

Figure 2.7 - AC1585-1 Screw Termination Panel with 50 Pin Cable

Figure 2.8 - Local Void Fraction versus Probe Tip Voltage

Figure 2.9 - Cumulative Percentage of Reading Voltage versus Voltage

Figure 2.10 - Experimental Trace form Lower and Upper Probe

Figure 2.11 - Cross-Correlation of Experimental Traces

CROSS-CORRELATION

Figure 2.12 - Double Resistance Probe Detecting Individual Bubbles

a.)

b.)

c.)

Figure 2.13 - Possible Errors in Dual Probe Performance

d.)

e.)

- a. Window closed before bubble hit upper probe
- b. Bubble Misses Upper Probe
- c. Another bubble hit upper probe
- d. Overestimation of Velocity
- e. Underestimation of Velocity

Figure 2.13 - Possible Errors in Dual Probe Performance (continued)

Figure 2.14 Voidage Profile with 1 ft. of Water, Probes - 6" Above Distributor Plate, and $Q_g = 4.6$ CFM.

Figure 2.15 - Voidage Profile with 1 ft. of Water, Probes 12" Above Distributor Plate, and $Q_g = 4.6$ CFM.

Figure 2.16 - Voidage Profile with 1 ft. of Water, Probes - 6" Above Distributor Plate, and $Q = 7.3$ CFM.

Figure 2.17 Voidage Profile with 1 ft. of Water, Probes - 12" Above Distributor Plate, and $Q_g = 7.3$ CFM.

Figure 2.18 Voidage Profile with 1 ft. of Water, Probes - 6" Above Distributor Plate, and $Q_g = 10.4$ CFM.

Figure 2.19 Voidage Profile with 1 ft. of Water, Probes - 12" Above Distributor Plate, and $Q_y = 10.4$ CFM.

Figure 2.20 Voidage Profile with 2 ft. of Water, Probes - 6" Above
Distributor Plate, and $Q_g = 4.6$ CFM.

Figure 2.21 Voidage Profile with 2 ft. of Water, Probes 12" Above
Distributor Plate, and $Q_g = 4.6$ CFM.

Figure 2.22 Voidage Profile with 2 ft. of Water, Probes - 18" Above Distributor Plate, and $Q_g = 4.6$ CFM.

Figure 2.23 Voidage Profile with 2 ft. of Water, Probes - 24" Above Distributor Plate, and $Q_g = 4.6$ CFM.

Figure 2.24 Voidage Profile with 2 ft. of Water, Probes - 6" Above Distributor Plate, and $Q_y = 7.3$ CFM.

Figure 2.25 Voidage Profile with 2 ft. of Water, Probes - 12" Above
Distributor Plate, and $Q_g = 7.3$ CFM.

Figure 2.26 Volume Profile with 2 ft. of Water, Probes 18" Above Distributor Plate, and $Q_g = 7.3$ CFM.

Figure 2.27 Voidage Profile with 2 ft. of Water, Probes = 24" Above Distributor Plate, and $Q_y = 7.3$ CFM.

Figure 2.28 Voidage Profile with 2 ft. of Water, Probes - 6" Above Distributor Plate, and $Q_g = 10.4$ CFM.

Figure 2.29 Voidage Profile with 2 ft. of Water, Probes - 12" Above Distributor Plate, and $Q_g = 10.4$ CFM.

Figure 2.30 Voidage Profile with 2 ft. of Water, Probes - 18" Above Distributor Plate, and $Q = 10.4$ CFM.

Figure 2.31 Void Profile with 2 ft. of Water, Probes - 24" Above Distributor Plate, and $Q_g = 10.4$ CFM.

Figure 2.32 Voidage Profile with 3 ft. of Water, Probes - 18" Above Distributor Plate, and $Q_s = 4.6$ CFM.

Figure 2.33 Voidage Profile with 3 ft. of Water, Probes - 24" Above Distributor Plate, and $Q_y = 4.6$ CFM.

Figure 2.34 Voidage Profile with 3 ft. of Water, Probes - 36" Above Distributor Plate, and $Q_g = 4.6$ CFM.

Figure 2.35 Voidage Profile with 3 ft. of Water, Probes - 18" Above Distributor Plate, and $Q_g = 7.3$ CFM.

Figure 2.36 Voidage profile with 3 ft. of water, Probes - 24" Above Distributor Plate, and $Q_{d, \text{max}} = 7.3 \text{ CFM}$.

Figure 2.37 Voidage Profile with 3 ft. of Water, Probes - 36" Above Distributor Plate, and $Q_y = 7.3$ CFM.

Figure 2.38 Voidage Profile with 3 ft. of Water, Probes - 18" Above Distributor Plate, and $Q_g = 10.4$ CFM.

Figure 2.39 Voidage Profile with 3 ft. of Water, Probes - 24" Above Distributor Plate, and $Q_g = 10.4$ CFM.

Figure 2.40 Voidage Profile with 3 ft. of Water, Probes - 36" Above
Distributor Plate, and $Q = 10.4$ CFM.

Figure 2.41 - Experimental Trace of Double Resistance Probe with Inconsistent Data

Figure 2.42 - Histogram of Bubble Velocities with 2 ft. of water
 Probes 12 in. from Distributor Plate @ center of column.
 $Q(g) = 4.6$ CFM, Window Frame, $V_{max} = 1.4$ Vpred.
 $V = 0.6$ volts.

FLOW CHART DIAGRAM

Figure 2.43 Liquid Velocity Model Flowchart

Figure 2.44 - Calculated Local Void Fraction Profile with 1 ft. of water
Probes 6 in. from Distributor Plate, $Q(g) = 4.6$ CFM

Figure 2.45 - Calculated Mixture Density Profile with 1 ft. of water
Probes 6 in. from Distributor Plate, $Q(g) = 4.6$ CFM

Figure 2.46 - Calculated Shear Stress Profile with 1 ft. of water.
Probes 6 in. from Distributor Plate, $Q(g) = 4.6$ CFM

Figure 2.47 - Calculated Liquid Velocity Profile with 1 ft. of Water,
Probes - 6" above Distributor Plate, $T_w = -1.932 \times 10^5$ psi,
and $Q_y = 4.6$ CFM (Figure 4.1 is Corresponding Voidage
Profile)

L1114

Figure 2.48- Calculated Liquid Velocity Profile with 1 ft. of Water,
Probes - 12" above Distributor Plate, $T_w = -2.06 \times 10^3$ psi,
and $Q_u = 4.6$ CFM (Figure 4.2 is Corresponding Voidage
Profile)

Figure 2.49- Calculated Liquid Velocity Profile with 1 ft. of Water,
 Probes - 6" above Distributor Plate, $T_w = -2.36 \times 10^3$ psi,
 and $Q_y = 7.3$ CFM (Figure 4.3 is Corresponding Voidage
 Profile)

L116

Figure 2.50 Calculated Liquid Velocity Profile with 1 ft. of Water, Probes - 12" above Distributor Plate, $T_w = -2.43 \times 10^{-4}$ psi, and $Q_g = 7.3$ CFM (Figure 4.4 is Corresponding Voidage Profile)

Figure 2.51 - Calculated Liquid Velocity Profile with 1 ft. of Water, Probes - 6" above Distributor Plate, $T_w = -2.73 \times 10^{-4}$ psi, and $Q_g = 10.4$ CFM (Figure 4.5 is Corresponding Voidage Profile)

L118

Figure 2.52- Calculated Liquid Velocity Profile with 1 ft. of Water, Probes - 12" above Distributor Plate, $T_w = -2.76 \times 10^4$ psi, and $Q_y = 10.4$ CFM (Figure 4.6 is Corresponding Voidage Profile)

L2054

Figure 2.53- Calculated Liquid Velocity Profile with 1 ft. of Water, Probes - 6" above Distributor Plate, $T_w = -1.86 \times 10^{-4}$ psi, and $Q_y = 4.6$ CFM (Figure 4.7 is Corresponding Voidage Profile)

L214

Figure 2.54 - Calculated Liquid Velocity Profile with 2 ft. of water, Probes - 12" above Distributor Plate, $T_w = -2.14 \times 10^{-3}$ psi, and $Q_y = 4.6$ CFM (Figure 4.8 is Corresponding Voidage Profile)

L2154

Figure 2.55 Calculated Liquid Velocity Profile with 2 ft. of Water, Probes - 18" above Distributor Plate, $T_v = -2.16 \times 10^{-4}$ psi, and $Q_y = 4.6$ CFM (Figure 4.9 is Corresponding Voidage Profile)

1224

Figure 2.56 Calculated Liquid Velocity Profile with 2 ft. of water, Probes - 24" above Distributor Plate, $T_v = -2.30 \times 10^{-4}$ psi, and $Q_g = 4.6$ CFM (Figure 4.10 is Corresponding Voidage Profile)

L2056

Figure 2.57 Calculated Liquid Velocity Profile with 2 ft. of water, Probes - 6" above Distributor Plate, $T_w = -2.27 \times 10^{-4}$ psi, and $Q_g = 7.3$ CFM (Figure 4.11 is Corresponding Voidage Profile)

L216

Figure 2.58 Calculated Liquid Velocity Profile with 2 ft. of Water.
Probes - 12" above Distributor Plate, $T_w = -2.48 \times 10^{-4}$ psi,
and $Q_y = 7.3$ CFM (Figure 4.12 is Corresponding Voidage
Profile)

L2156

Figure 2.59 Calculated Liquid Velocity Profile with 2 ft. of Water,
Probes - 18" above Distributor Plate, $T_v = -2.66 \times 10^{-4}$ psi,
and $Q_g = 7.3$ CFM (Figure 4.13 is Corresponding Voidage
Profile)

L226

Figure 2.60 Calculated Liquid Velocity Profile with 2 ft. of water, Probes - 24" above Distributor Plate, $T_w = -2.78 \times 10^3$ psi, and $Q_y = 7.3$ CFM (Figure 4.14 is Corresponding Voidage Profile)

L2058

Figure 2.61 Calculated Liquid Velocity Profile with 2 ft. of Water, Probes - 6" above Distributor Plate, $T_w = -2.6 \times 10^{-3}$ psi, and $Q_g = 10.4$ CFM (Figure 4.15 is Corresponding Voidage Profile)

Figure 2.62 Calculated Liquid Velocity Profile with 2 ft. of water, Probes - 12" above Distributor Plate, $T_w = -2.79 \times 10^{-3}$ psi, and $Q_g = 10.4$ CFM (Figure 4.16 is Corresponding Voidage Profile)

L2158

Figure 2.63 Calculated Liquid Velocity Profile with 2 ft. of Water,
Probes - 18" above Distributor Plate, $T_w = -2.92 \times 10^{-4}$ psi,
and $Q_g = 10.4$ CFM (Figure 4.17 is Corresponding Voidage
Profile)

L228

Figure 2.64 Calculated Liquid Velocity Profile with 2 ft. of Water, Probes - 24" above Distributor Plate, $T_w = -3.06 \times 10^4$ psi, and $Q_y = 10.4$ CFM (Figure 4.18 is Corresponding Voidage Profile)

L3154

Figure 2.65 Calculated Liquid Velocity Profile with 3 ft. of water, Probes - 18" above Distributor Plate, $T_w = -2.22 \times 10^4$ psi, and $Q_y = 4.6$ CFM (Figure 4.19 is Corresponding Voidage Profile)

L324

Figure 2.66 Calculated Liquid Velocity Profile with 3 ft. of Water, Probes - 24" above Distributor Plate, $T_v = -2.26 \times 10^{-4}$ psi, and $Q_y = 4.6$ CFM (Figure 4.20 is Corresponding Voidage Profile)

L334

Figure 2.67 Calculated Liquid Velocity Profile with 3 ft. of Water, Probes - 36" above Distributor Plate, $T_w = -2.33 \times 10^{-4}$ psi, and $Q_y = 4.6$ CFM (Figure 4.21 is Corresponding Voidage Profile)

L3156

Figure 2.68 Calculated Liquid Velocity Profile with 3 ft. of Water,
Probes - 18" above Distributor Plate, $T_v = -2.61 \times 10^{-4}$ psi,
and $Q_y = 7.3$ CFM (Figure 4.22 is Corresponding Voidage
Profile)

L326

Figure 2.69 Calculated Liquid Velocity Profile with 3 ft. of water,
Probes - 24" above Distributor Plate, $T_w = -2.63 \times 10^4$ psi,
and $Q_y = 7.3$ CFM (Figure 4.23 is Corresponding Voidage
Profile)

L336

Figure 2.70 Calculated Liquid Velocity Profile with 3 ft. of Water,
Probes - 36" above Distributor Plate, $T_w = -2.69 \times 10^{-4}$ psi,
and $Q_y = 7.3$ CFM (Figure 4.24 is Corresponding Voidage
Profile)

L3158

Figure 2.71 - Calculated Liquid Velocity Profile with 3 ft. of Water, Probes - 18" above Distributor Plate, $\tau_w = -2.87 \times 10^{-4}$ psi, and $Q_g = 10.4$ CFM (Figure 4.25 is Corresponding Voidage Profile)

L328

Figure 2.72 Calculated Liquid Velocity Profile with 3 ft. of Water,
Probes - 24" above Distributor Plate, $T_w = -2.92 \times 10^{-4}$ psi,
and $Q_y = 10.4$ CFM (Figure 4.26 is Corresponding Voidage
Profile)

L338

Figure 2.73 Calculated Liquid Velocity Profile with 3 ft. of water, Probes - 36" above Distributor Plate, $T_w = -2.92 \times 10^{-3}$ psi, and $Q_y = 10.4$ CFM (Figure 4.27 is Corresponding Voidage Profile)

Figure 2.74. Profile constant C_o for various Fr and Ga for void distributions given by [1] with $\epsilon_c = 0.1$ and $\rho = 2$. $Fr = 0.25$ would correspond to a typical bubble rising in a 100 mm dia column of water. $Fr = 0.05$ would correspond to a 2.5 m dia column of water. Laboratory scale air-water bubble columns would have $Ga = 10^4$ to 10^6 and the industrial scale would extend to $Ga = 10^8$. $Ga < 10^3$ would occur for viscous systems. The void fraction used here is lower than in an industrial column.

Figure 2.75. Profile constant C_O for various Fr and Ga for void distributions given by [1] with $\epsilon_c = 0.2$ and $\rho = 2$. The comments on Fr and Ga in the legend to figure 2.74 apply.

Figure 2.76. Profile constant C_O for various Fr and Ga for void distributions given by [1] with $\epsilon_c = 0.3$ and $p = 2$. This would be a realistic void fraction for an industrial column. The comments on Fr and Ga in the legend to figure 2.74 apply.

Figure 2.77. Profile constant C_o for various Fr and Ga for void distributions given by [1] with $\epsilon_c = 0.1$ and $\rho = 3$. The comments on Fr and Ga in the legend to figure 2.74 apply.

Figure 2.78. Profile constant C_o for various Fr and Ga for void distributions given by [1] with $\epsilon_c = 0.2$ and $\rho = 3$. The comments on Fr and Ga in the legend to figure 2.74 apply.

Figure 2.79. Profile constant C_O for various Fr and Ga for void distributions given by [1] with $\epsilon_c = 0.3$ and $\rho = 3$. The comments on Fr and Ga in the legend to figure 2.74 apply.

Figure 2.80. Profile constant C_o for various Fr and Ga for void distributions given by [1] with $\epsilon_c = 0.1$ and $\rho = 7$. The comments on Fr and Ga in the legend to figure 2.74 apply.

Figure 2.81. Profile constant C_O for various Fr and Ga for void distributions given by [1] with $\epsilon_c = 0.2$ and $p = 7$.

Figure 2.82. Profile constant C_O for various Fr and Ga for void distributions given by [1] with $\epsilon_c = 0.3$ and $\rho = 7$.

Figure 2.83. Void profile from the data of Hills (15.2 cm dia column, air superficial velocity 169 mm/s) and the sympathetic liquid velocity profile, found using the method of Clark *et al.* (1987). It is evident that the flow is buoyancy-driven.

Figure 2.84. The model was used to predict the centerline liquid velocity in the column when there was no net liquid flow for various gas flow rates and values of the void distribution parameter, p . (Note that ϵ_c can be found given p , W_g and the velocity across the column.) Hills' measured values of centerline velocity are close to the predicted lines for values of $p = 7$ to 10. At very high gas flow rates the mixture properties and behavior will deviate significantly from those of the liquid phase, causing error in the predictions using the present model.

Figure 2.85. The profile constant, C_o , was calculated for the case where there was a net upflow in the column (represented by the liquid flow $Re = \rho U D / \mu$) but when buoyancy effects were still significant. $Fr = 0.25$ corresponds to a bubble rise velocity of 250 mm/s in a 100 mm dia pipe, and $Fr = 0.113$ for a 500 mm dia pipe.

Figure 2.86. Axial liquid velocity profiles for the case of a 500 mm dia bubble column ($Re = 0$) and pipe carrying a two-phase upflow ($Re > 0$), when the void fraction is greatest at the center.

Figure 2.87. A plot yielding values of C_O , for upflow and downflow, in a 500 mm pipe with the void fraction having a maximum at the pipe center. Note the choking condition for the downflow case, where C_O assumes very high values; this is when the bubbles cannot rise upward through the downflow, and when the downflow velocity is not strong enough to carry them downward.

Figure 2.88. When the bubbles are concentrated in an annular region, buoyancy forces and wall effects are in competition during two-phase upflow. For this case in a 500 mm dia pipe, liquid flow is downward at the center for net upflow $Re < 500,000$. The void profile used to generate this plot is given in figure 2.89.

Figure 2.89. This annular void distribution, intended to imitate saddle-shaped distributions observed by some previous researchers, was used to generate the velocity profiles shown in figure 2.88.

Figure 2.90. To model radial bubble motion, a two-zone, one-dimensional approach was used. In the central zone, all fluid motion was considered axial. In the lower zone, all liquid motion was considered radial. Bubbles rising vertically through the radial field were moved inward. In this way an uneven void distribution can develop although gas introduction through the distributor may be even.

Figure 2.91. Distribution of radial velocity in the lower zone (v) and axial velocity in the center zone (U), corresponding to Hill's air-water column, with an air superficial velocity of 38 mm/s.

Figure 2.92. The gas flux, $W(r)$, computed from bubble motion in the lower zone, is compared here with the $W(r)$ found from the product of $\epsilon(r)$ and $(U_L)(r)$ are given in Hills' paper.

SECTION 3

LASER DOPPLER VELOCIMETRY

3.1 INTRODUCTION

As part of a combined numerical (Celik and Wang, 1990), analytical (Clark, Atkinson, and Flemmer, 1987), and experimental (Gross and Kuhlman, 1990) study of bubble column hydrodynamics, measurements have been made of mean and fluctuating bubble and liquid velocities in a laboratory model bubble column. The three component laser Doppler velocimeter (LDV) described by Kuhlman and Gross (1989) has been used to obtain the bubble and liquid velocity data non-intrusively. Use of a non-intrusive flow measurement technique is essential for accurate measurements in a recirculating flow such as the circulation in a bubble column.

Many of the previous experimental studies of bubble column circulation have used intrusive probes to measure velocities, and generally only vertical velocity has been measured. For example, Hills (1974) utilized resistance and pressure probes to measure radial distributions of voidage and mean vertical velocity for turbulent flow, over a range of gas flow rates. The liquid circulation was observed to be upwards on the column centerline and downwards at large radius, near the column wall. A similar circulation pattern has been measured by photographic means for a laminar flow in a bubble column with a very small void fraction by Reitma and Ottengraf (1970).

More recently, Durst, et al., (1984) have used a non-intrusive single channel LDV to determine the mean liquid velocity distribution for an axisymmetric, laminar flow in a cylindrical column. The liquid flow was driven by a single column of discrete bubbles injected at the centerline. Again, a toroidal recirculating liquid flow pattern was measured. Flow visualization photographs clearly indicated that the presence of a probe in their bubble column could alter this liquid circulation from a single

recirculation cell, to multiple cells. This points clearly to the need for non-intrusive velocity measurements in the recirculating bubble column flow.

Studies of the bubbly, two-phase flow in a vertical circular pipe have been conducted by Serizawa, et al., (1975), and by Currie and Brankovic (1987). Serizawa, et al., (1975) used a series of intrusive probes to measure both mean and turbulence quantities for both the bubble and liquid flows. There was no flow reversal in their vertical pipe, and the flow was driven by and applied pressure difference across the pipe. Radial distributions of axial bubble and liquid velocity and voidage were measured. More recently Currie and Brankovic (1987) obtained axial and radial liquid and bubble velocity measurements for an air-liquid two-phase flow in a vertical pipe with a sudden expansion, using a non-intrusive two-component LDV system. However, again the flow studied differs from the circulation in a bubble column, in that the liquid flow was driven by an applied pressure gradient.

3.2 LDV EXPERIMENTAL APPARATUS AND PROCEDURE

A schematic of the hexagonal cross-section bubble column is shown in Fig. 3.2.1. The column is made of plexiglas and is 18 cm across the flats or 21 cm across the diagonal at the top, and can accommodate water depths up to 18 cm measured from the bubble injection manifold. For the present experiments in air bubble injection manifold made from a PVC end cap (Fig. 3.2.1) with a single, central hole of 1 mm diameter has been used. Air flow rate was $7.87 \times 10^{-5} \text{ m}^3/\text{s}$ (10 SCFH). Calculated jet exit air velocity was 90 m/s, so that superficial gas velocity was only 0.3 cm/s. This resulted in an initial air jet which broke into nominal 1 cm diameter or less bubbles on the centerline near the top of the column. This air jet drove a relatively strong water circulation which was observed by flow visualization to be upwards near the central column of air bubbles, and downwards near the outer walls of the

column. Flow visualization video has indicated the upwards velocity of the air jet to be between 30 and 40 cm/sec. This configuration has been studied because it greatly simplifies the measurement of bubble and liquid velocity since there is very little interference with the laser beams due to bubbles crossing through the beam paths away from where the laser beams cross in the measurement volume to form interference fringes. This is because only the smaller bubbles (less than 1 mm diameter) are re-entrained by the water circulation to recirculate in the column. Note, then, that all bubble velocity data are actually for these small diameter bubbles. Also, the hexagonal cross-section has been selected because it eliminates the problems associated with optically penetrating a curved interface (Broadway and Karahan, 1981; Bicen, 1982; Durrett et al., 1986), while still remaining nearly cylindrical in shape.

The present velocity measurements have been obtained using a three-component, 5-beam laser Doppler velocimeter described by Buchave (1984). This system has previously been used by Kuhlman and Gross (1989) to measure the three-dimensional mean and turbulent velocity fields in an axisymmetric air jet. Since simultaneous, coincident data were measured by Kuhlman and Gross (1989), then it was possible to determine Reynolds stresses. These jet data were consistent with previous axisymmetric jet data. The LDV system consists of DANTEC 55X modular optics and a series 2000 5W Spectra Physics argon ion laser, which are mounted to a 3-D, computer-controlled traversing system, as shown in Fig. 3.2.2. Front lenses with a focal length of 0.6 m are used, so the angle between the two separate optical trains is equal to 60° . This large angle helps greatly to improve accuracy of the 3-D velocity measurements, as discussed by Meyers (1985).

Three separate LDV channels are formed, by use of color separation. The

488 nm and 514.5 nm wavelength beams form orthogonal fringes by means of a standard two-channel optical train, shown on the bottom of Fig. 3.2.2. The vertical velocity component is measured using the 488 nm beam, while the 514.5 nm beam measures a velocity component at a right angle to the optical axis, in a horizontal plane (i.e., inclined 30° right of the Y axis defined in Fig. 3.2.2). The third LDV channel uses the 476 nm wavelength beam from the laser, which is sent to the single-channel optical train indicated in Fig. 3.2.2. This LDV channel measures a velocity component in a horizontal plane at an angle 30° left of the Y axis in Fig. 3.2.2. Orthogonal horizontal ($r-\theta$) velocity components are computed by vector transformation from the non-orthogonal to the orthogonal coordinate system. Frequency shifting allows measurements in reversing flows, while the beam expanders increase signal-to-noise ratio. Probe volumes for all three LDV channels are 0.16 mm in diameter, by 3.3 mm long. Fringe spacing is nominally 5.84 μm , yielding approximately 27 fringe in the probe volume.

Backscattered light generates the output of the photomultiplier tubes which is sent to three counter processors, operated in the combined mode. Output from the three counter processors goes to a DANTEC buffer interface and coincidence filter, which accept validated data from each channel, check that the velocity measurements from each channel all were measured within a user-selectable time window which defines coincidence of the data, and measure the time between each set of measurements of the velocity components. Validated, coincident data and the measured sample interval time are sent to a PDP 11/23 microcomputer for storage. Data is reduced using techniques discussed by Edwards (1987), Meyers (1988), Yanta and Smith (1973), and Gould, et al. (1989).

Air bubble velocity data have been obtained for a matrix of twenty

different vertical depths, by eleven different radial locations. Liquid velocity data have been obtained at the same 20 vertical locations, but at 13 radial locations. These results have been obtained by simultaneously measuring the vertical velocity and one horizontal velocity component perpendicular to one of the column faces, using the 2-channel LDV, and then measuring the third velocity component, perpendicular to a neighboring face, at the same location at a later time. Thus, no Reynolds stress results have been obtained. Velocity data obtained from the two non-orthogonal horizontal LDV channels have been post-processed to obtain orthogonal radial and circumferential velocity results. At each location, a time history of 3754 validated LDV velocity measurements has been acquired for each of the three velocity components. Since the counter processors were operated in the "combined" mode, only one doppler signal was acquired for each bubble or seed particle that crossed through the probe volume. Also note that for this single doppler signal, the total number of fringes crossed by the bubble or seed particle was recorded. The sample interval time between each subsequent validated velocity measurement has also been measured. These data have been reduced by sorting the time series for each velocity component into a 100 bin velocity histogram (i.e., number of velocity measurements in each velocity range), and deleting the tails of the resulting histograms for each channels. Between 0.5-8% of the data have been rejected as spurious in this fashion, where for the majority of data files between 1-3% of the data have been omitted by this technique. Average and RMS bubble and liquid velocities have been computed by four different averaging methods: statistical averaging (sum of the velocities divided by number of data values), both with and without rejection of spurious data in the tails of the velocity histograms, and sample interval time weighting, again with and without rejection of spurious data.

Results presented herein have all been calculated using statistical averaging, and with the rejection of spurious "outlier" data. One exception to this is for the 2-D air bubble velocity data for the vertical traverse at the third from largest r location, where no data rejection was used, since these data files contained fewer samples (from 150-2300) due to a poor signal-to-noise ratio caused by a laser beam reflection off the column wall at this radius. Plotted mean and RMS velocity distributions changed by no more than 1% independent of which of these four data reduction methods was used, indicating that statistical bias is not a significant problem with the current data. Each data record at a point took between 30 and 800 seconds to acquire, with most files taking 200-300 seconds. These validated data rates of about 10 samples per second yielded calculated data densities (validated data rate times Taylor microscale; see Edwards, 1987) of between 0.06-0.6. Data densities tended to be higher and fewer spurious points were rejected for the 1-D data measurements. For the present results no velocity bias corrections have been applied to the calculated statistical averages (Edwards, 1987; Meyers, 1988; Yanta and Smith, 1973; Gould and Stevenson, 1989). This is not generally believed to be a significant problem, because of the low average validated data rates.

All bubble velocity measurements were obtained using distilled water in the column, with no added seed particles, and without any thresholding of the Doppler signals to reject large signals. Liquid velocity data reported herein have been obtained by adding 4 μ m diameter silver coated plastic sphere seed particles to the bubble column, and then by rejecting the largest Doppler signals due to the large air bubbles, through use of a 24 dB threshold setting on each of the counter processors. This level of thresholding was observed to typically reject approximately 90-95% of the Doppler signals obtained without

any seed particles added to the flow. The seed particles are expected to adequately track the present flow up to frequencies of 1-2 kHz, based on the work of Hjelmfelt and Mockros (1966). It is expected that other non-intrusive methods, such as phase Doppler techniques (Bauckhage, 1985; Drain and Phil, 1985; Foster and King, 1985) or fluorescent particles (Goldman and Seasholtz, 1988) could lead to better liquid velocity data, but these methods were not available for the present work.

3.3 LDV RESULTS - AVERAGE BUBBLE VELOCITIES

The average bubble radial, vertical and circumferential velocities are presented in Figs. 3.3.1-12. The data is plotted versus non-dimensional effective column radius r/R , where R is one half the hydraulic diameter of the column. Each figure plots the data for five vertical locations (Z , in centimeters) measured relative to the location of the air injection port at the bottom of the column. Water depth, as measured from the air injection port, was held constant at 18 cm. The air flow rate through the injection port was kept constant at $7.87 \times 10^{-5} \text{ m}^3/\text{sec}$ (10 SCFH).

No bubble or liquid velocity measurements have been obtained for $r < 0.17R$, because of the presence of the air jet for smaller R . When the probe volume was positioned at the edge of the jet (approximately for $r \approx 0.17 R$, or less) the photodetector signals would saturate, or appear extremely "noisy". Thus it was judged that no reliable velocity data could be obtained too close to, or inside, the jet itself. Also, no measurements have been obtained for $r > 0.82R$, since laser light which is reflected off the column walls (water-plastic and plastic-air interfaces) saturates the photodetectors at larger r values. It was observed that the signal quality was degraded gradually as the column wall was approached (due to reflections) and as the air jet was approached. Increased signal noise near the air jet was due both

to reflections off the jet interface, as well as the presence of smaller bubbles in the laser beam paths away from the probe volume. These smaller bubbles reduced signal quality, and eventually would destroy the doppler signal by destroying the fringe pattern formed in the probe volume (Foster and King, 1985). Figs. 3.3.1 through 3.3.4 display the average bubble radial velocity versus r/R . The radial velocity bubble profiles are negative in all but the top four vertical locations. This indicates that the entrained bubbles are moving in towards the column centerline for all but the top fifth portion of the vessel and that in the top fifth region, the bubbles are moving out toward the column wall. On average, the bubbles appear to be moving inwards at roughly 5 to 7 cm/s and outwards at a velocity on the order of 10 to 20 cm/s. In all cases, the radial bubble velocities tend toward zero, or decrease significantly, at the beginning and ending radial positions. This indicates that a solid boundary is impeding the flow. For $r < 0.17R$, the initial air jet is acting as the solid boundary, while at $r > 0.82R$ the plexiglas interface is being approached.

Figs. 3.3.5 through 3.3.8 present the average bubble vertical velocity versus r/R . Vertical mean bubble velocity is largest at a fixed depth for small radius. The presence of the air jet can be directly attributed to this phenomenon. Vertical bubble velocity is zero at $r \approx 0.7R$ at all depths, where R is the effective radius of the hexagonal bubble column. Velocity magnitude increases with increasing vertical position for all but the last two traverses. For these two traverses ($Z = 15.4, 16.1$ cm) the water-air interface at the column top acts to decrease bubble velocities as the free surface is approached. The magnitude of the outwards bubble velocity near the free surface is about the same order of magnitude as the upwards bubble velocity near the air jet.

The circumferential mean bubble velocity (Figs. 3.3.9 through 3.3.12) are very nearly zero everywhere in the bubble column.

3.4 LDV RESULTS - RMS BUBBLE VELOCITIES

The RMS bubble, radial, vertical and circumferential velocities are presented in Figs. 3.4.1 through 3.4.12. The data is again plotted versus nondimensional effective column radius r/R . In all cases, the RMS velocities are smaller in the bottom half of the column, and are largest near the jet (small r) and near the free surface of the column.

In the bottom half of the column, the RMS radial and vertical velocities are essentially the same size as the corresponding mean velocity components. Radial RMS velocities are consistently larger than both RMS vertical and circumferential velocities, which are generally about the same size. In the top portion of the bubble column, both RMS velocities are more nearly equal to the magnitude of the larger of the mean vertical and radial velocities. The RMS velocities all decrease as radius is increased, similar to trends for the mean bubble velocity magnitude. Turbulence intensities are everywhere on the order of 100%, indicating very good mixing.

3.5 LDV RESULTS - AVERAGE LIQUID VELOCITIES

The average liquid radial, vertical and circumferential velocities are presented in Figs. 3.5.1 through 3.5.12. The liquid velocities follow trends which are similar to observations for the bubble velocities, except that the liquid velocities are generally smaller than corresponding bubble velocities near the top of the column and near the column centerline. This velocity difference is significant in that it clearly shows that the liquid circulation is being driven by the bubble flow.

Figs. 3.5.1 through 3.5.4 display the average liquid radial velocity versus r/R . As is the case for the average bubble radial velocities, the

liquid radial velocities are negative for Z values up to about 80% of the column height. The liquid is moving radially outwards for the remaining 20% of the column height. On average, the liquid flows radially back towards the column centerline with a velocity of roughly 2 to 5 cm/s. Liquid radial flow velocity to the column walls is roughly 8 to 10 cm/s. Zero velocity, or a significant velocity decrease at the beginning and ending radial positions, representing the air jet or wall region, is also noted.

Average liquid vertical velocity plotted versus non-dimensional effective column radius is presented in Figs. 3.5.5 through 3.5.8. Measured vertical liquid mean velocity is a smaller positive value as compared to vertical bubble mean velocity near the central air jet, and is consistently downwards beyond $r/R \approx 0.5 - 0.7$. These results further indicate that the liquid flow lags behind the bubble flow, as expected. As is the case with the vertical bubble mean velocity, vertical liquid mean velocity decreases ($Z = 14.6, 15.4, 16.1$ cm) as the free surface is approached.

The circumferential mean liquid velocity, Figs. 3.5.9 through 3.5.12, are very nearly zero everywhere in the bubble column. This is consistent with the circumferential mean bubble velocity data.

3.6 LDV Results - RMS Liquid Velocities

The RMS liquid radial, vertical and circumferential velocities are presented in Figs. 3.6.1 through 3.6.12. As expected, RMS liquid velocities are generally smaller than corresponding RMS bubble velocities near the top of the column and near the column centerline. RMS radial and vertical velocities are essentially the same size as the corresponding mean velocity components in the bottom half of the column. Again this behavior is similar to that of the bubble phase data. Radial RMS velocities are consistently larger than both

RMS vertical and circumferential velocities, which are generally about the same size. Thus, it can be said that non-isotropic turbulence exists in the bubble column.

In the top portion of the bubble column, RMS velocities are more nearly equal to the magnitude of the larger of the mean vertical and radial velocities. The RMS velocities all decrease as radius is increased, similar to trends for the mean liquid velocity magnitude. Liquid turbulence intensities are everywhere on the order of 100%, again indicating very good mixing.

3.7 FLOW VISUALIZATION AND VELOCITY COMPARISONS

Figs. 3.7.1 and 3.7.2 are examples of flow visualization photography. Flow visualization was accomplished by passing a 5 watt argon-ion laser beam through a cylindrical glass rod which turned the beam into a two-dimensional light sheet. The light sheet was then passed in a vertical plane through the bubble column along the minor diameter. This had the effect of illuminating the flow field in a two-dimensional plane. Photographs and video tape were used to further investigate the flow.

Fig. 3.7.1 is a view along the major diameter of the column. It should be noted that because of the plexiglas intersection, there appear to be two air jets when in fact there is only one. Fig. 3.7.2 is a view along the minor diameter of the bubble column. Both photographs provide additional information that is consistent with mean flow velocity information. Bubbles can be observed to rise rapidly near the central air jet, with most bubbles leaving the bubble column at the free surface. Smaller bubbles are observed to move radially outwards near the top of the column, downwards at large r , and inwards at the bottom of the column. A relatively strong toroidal recirculating vortex flow is observed in the top half of the column. The

central air jet velocity was observed from video tape to be on the order of 30 to 40 cm/s.

Figs. 3.7.3 and 3.7.4 allow side by side comparisons between radial and vertical bubble/liquid mean velocities. Both figures clearly indicate that the liquid phase lags the bubble phase near the air jet and near the free surface of the bubble column.

Figure 3.2.1 Schematic of hexagonal cross-section bubble column for laser Doppler velocimeter (LDV) measurements.

Figure 3.2.2 Top view of three-component laser doppler velocimeter (LDV).

Figure 3.3.1 Average radial bubble velocity measurements in the bottom quarter of the bubble column.

Figure 3.3.2 Average radial bubble velocity measurements in the second quarter of the bubble column.

Figure 3.3.3 Average radial bubble velocity measurements in the third quarter of the bubble column.

Figure 3.3.4 Average radial bubble velocity measurements in the top quarter of the bubble column.

Figure 3.3.5 Average vertical bubble velocity measurements in the bottom quarter of the bubble column.

Figure 3.3.6 Average vertical bubble velocity measurements in the second quarter of the bubble column.

Figure 3.3.7 Average vertical bubble velocity measurements in the third quarter of the bubble column.

Figure 3.3.8 Average vertical bubble velocity measurements in the top quarter of the bubble column.

Figure 3.3.9 Average circumferential bubble velocity measurements in the bottom quarter of the bubble column.

Figure 3.3.10 Average circumferential bubble velocity measurements in the second quarter of the bubble column.

Figure 3.3.11 Average circumferential bubble velocity measurements in the third quarter of the bubble column.

Figure 3.3.12 Average circumferential bubble velocity measurements in the top quarter of the bubble column.

Figure 3.4.1 RMS radial bubble velocity measurements in the bottom quarter of the bubble column.

Figure 3.4.2 RMS radial bubble velocity measurements in the second quarter of the bubble column.

Figure 3.4.3 RMS radial bubble velocity measurements in the third quarter of the bubble column.

Figure 3.4.4 RMS radial bubble velocity measurements in the top quarter of the bubble column.

Figure 3.4.5 RMS vertical bubble velocity measurements in the bottom quarter of the bubble column.

Figure 3.4.6 RMS vertical bubble velocity measurements in the second quarter of the bubble column.

Figure 3.4.7 RMS vertical bubble velocity measurements in the third quarter of the bubble column.

Figure 3.4.8 RMS vertical bubble velocity measurements in the top quarter of the bubble column.

Figure 3.4.9 RMS circumferential bubble velocity measurements in the bottom quarter of the bubble column.

Figure 3.4.10 RMS circumferential bubble velocity measurements in the second quarter of the bubble column.

Figure 3.4.11 RMS circumferential bubble velocity measurements in the third quarter of the bubble column.

Figure 3.4.12 RMS circumferential bubble velocity measurements in the top quarter of the bubble column.

Figure 3.5.1 Average radial liquid velocity measurements in the bottom quarter of the bubble column.

Figure 3.5.2 Average radial liquid velocity measurements in the second quarter of the bubble column.

Figure 3.5.3 Average radial liquid velocity measurements in the third quarter of the bubble column.

Figure 3.5.4 Average radial liquid velocity measurements in the top quarter of the bubble column.

Figure 3.5.5 Average vertical liquid velocity measurements in the bottom quarter of the bubble column.

Figure 3.5.6 Average vertical liquid velocity measurements in the second quarter of the bubble column.

Figure 3.5.7 Average vertical liquid velocity measurements in the third quarter of the bubble column.

Figure 3.5.8 Average vertical liquid velocity measurements in the top quarter of the bubble column.

Figure 3.5.9 Average circumferential liquid velocity measurements in the bottom quarter of the bubble column.

Figure 3.5.10 Average circumferential liquid velocity measurements in the second quarter of the bubble column.

Figure 3.5.11 Average circumferential liquid velocity measurements in the third quarter of the bubble column.

Figure 3.5.12 Average circumferential liquid velocity measurements in the top quarter of the bubble column.

Figure 3.6.1 RMS radial liquid velocity measurements in the bottom quarter of the bubble column.

Figure 3.6.2 RMS radial liquid velocity measurements in the second quarter of the bubble column.

Figure 3.6.3 RMS radial liquid velocity measurements in the third quarter of the bubble column.

Figure 3.6.4 RMS radial liquid velocity measurements in the top quarter of the bubble column.

Figure 3.6.5 RMS vertical liquid velocity measurements in the bottom quarter of the bubble column.

Figure 3.6.6 RMS vertical liquid velocity measurements in the second quarter of the bubble column.

Figure 3.6.7 RMS vertical liquid velocity measurements in the third quarter of the bubble column.

Figure 3.6.8 RMS vertical liquid velocity measurements in the top quarter of the bubble column.

Figure 3.6.9 RMS circumferential liquid velocity measurements in the bottom quarter of the bubble column.

Figure 3.6.10 RMS circumferential liquid velocity measurements in the second quarter of the bubble column.

Figure 3.6.11 RMS circumferential liquid velocity measurements in the third quarter of the bubble column.

Figure 3.6.12 RMS circumferential liquid velocity measurements in the top quarter of the bubble column.

Figure 3.7.1 Flow visualization along the major diameter of the bubble column.

Figure 3.7.2 Flow visualization along the minor diameter of the bubble column.

Average radial liquid velocity.

Average radial bubble velocity.

Figure 3.7.3 Comparison of radial velocity (bubble/liquid) in top quarter of bubble column.

Average vertical bubble velocity.

Average vertical liquid velocity.

Figure 3.7.4 Comparison of vertical velocity (bubble/liquid) in top quarter of bubble column.

4.1 INTRODUCTION

The study of the circulation patterns inside a column reactor is of theoretical and practical significance, because of the wide use of column reactors in industry as chemical or bio-chemical reactors. The bubble column reactor offers several advantages over conventional fixed bed reactors: e.g., lower hydrogen to monoxide ratios can be tolerated and hot-spots are controlled in the bed so that catalyst deactivation is reduced. The gas may be introduced evenly through a distributor plate into the slurry, so the circulation patterns of the continuous liquid phase can be controlled.

Numerous mathematical models are available in the literature for calculation of relevant parameters such as the gas hold-up (i.e. volume fraction of the gas phase), the liquid velocity, and the size and strength of the liquid circulation cells. Freedman and Davidson (1969), Rietema and Ottengraf (1970) and Hills (1974) have reported on approximate models based on pressure balance to calculate liquid circulation velocity. Whalley and Davidson (1974) have shown that their model based on the energy balance predicted liquid circulation velocities which are in better agreement with experiments than those obtained from the models based on pressure balance. Joshi and Sharma (1979) have adapted the energy balance method and reported good agreement between their calculations and experiments for the values of the liquid velocity and void fraction. Clark, et al. (1987, 1989) have reviewed the so called "drift flux models" and presented their improved version. These models are mainly empirical relations which relate the superficial gas and liquid velocities to the average void fraction. Clark et al. suggested an approximate model to predict the empirical constant in one of these models, and found that it varied significantly as a function of Froude number and Galileo number. Their conclusion was that "a direct drift-flux approach is not suitable unless

the void distribution is known and that buoyancy effects are insignificant." A further restriction in these models is that the size and the location of recirculation zone must be assumed *a priori*. For example, in the model proposed by Clark et al. (1989) two-zones are identified, namely a central zone where only axial flow is assumed, and a lower zone where only radial flow is assumed. Such assumptions, which are inherent to most approximate models, neglect possibility of multiple circulation cells. To eliminate such restrictive assumptions, it is necessary to develop more rigorous computational fluid flow analysis with which the circulation and the void fraction distribution can be predicted *a priori*. However such comprehensive numerical simulations of gas/liquid two-phase systems are not given much attention in the literature. This is not surprising in view of the fact that the mathematical formulation for multi-phase flows is still in a stage of development. There is no definite form of the governing equations which is generally accepted (for a review, see Stewart and Wendroff, 1984). Other difficulties such as handling of interaction terms (interface conditions), the boundary and initial conditions, and interphase instabilities, make comprehensive numerical modeling a challenging research area, and it calls for more attention from the computational fluid dynamicists.

Here, it is reported on the results of a study where the flow and the circulation patterns inside an isothermal column reactor model has been simulated numerically using a finite volume approach. As a first step, only the equations of motion and continuity for the continuous phase, and the conservation of mass equation for the discrete phase have been solved. The gas velocity field is prescribed empirically. Only the laminar, bubbly flow regime is considered. This is done to avoid the complexities of a turbulence closure model at higher superficial gas velocities. These results constitute the

where μ is the liquid viscosity. The dilatation term $\underline{z} \cdot \underline{u}$ will be neglected in the momentum equations, since the effect of this term is minimal even for a wide range of compressible fluid flows provided that the Mach number is less than 0.30. It should be noted, however, that $\underline{z} \cdot \underline{u}$ will be kept in the equation of continuity and the resulting equations would be valid for many compressible fluid flow problems unless this term becomes very large under unusual circumstances, i.e. Mach numbers much larger than 0.3. In two-phase flows the

The term $F_{12}(\underline{u}_2 - \underline{u}_1)$ in Eq.(4.3) represents the momentum exchange for liquid-phase equations, likewise, $F_{21}(\underline{u}_2 - \underline{u}_1)$ is the momentum exchange term for gas-phase equations, hence $F_{12} = -F_{21}$. If the dilatation term $\underline{z} \cdot \underline{u}_1$ is neglected, the three components of Eq.4.3 in x-, r-, θ - directions, respectively, can be written in conservative form as follows

x - Momentum

$$\begin{aligned} & \frac{\partial \rho_1 u_1}{\partial t} + \frac{\partial}{\partial x}(\rho_1 u_1 u_1) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho_1 u_1 v_1) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho_1 u_1 w_1) \\ &= \frac{\partial}{\partial x}(\tau_{xx}) + \frac{1}{r} \frac{\partial}{\partial r}(r \tau_{rx}) + \frac{1}{r} \frac{\partial}{\partial \theta}(\tau_{\theta x}) - (1-\alpha) \frac{\partial P}{\partial x} + \rho_1 g_x + F_{12}(u_2 - u_1) \quad (4.5) \end{aligned}$$

r - Momentum

$$\begin{aligned} & \frac{\partial \rho_1 v_1}{\partial t} + \frac{\partial}{\partial x}(\rho_1 u_1 v_1) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho_1 v_1 v_1) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho_1 v_1 w_1) - \rho_1 \frac{w_1^2}{r} \\ &= \frac{\partial}{\partial x}(\tau_{xr}) + \frac{1}{r} \frac{\partial}{\partial r}(r \tau_{rr}) + \frac{1}{r} \frac{\partial}{\partial \theta}(\tau_{\theta r}) - \frac{\tau_{\theta\theta}}{r} - (1-\alpha) \frac{\partial P}{\partial r} + \rho_1 g_r + F_{12}(v_2 - v_1) \quad (4.6) \end{aligned}$$

θ - Momentum

$$\frac{\partial \rho_1 w_1}{\partial t} + \frac{\partial}{\partial x}(\rho_1 u_1 w_1) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho_1 v_1 w_1) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho_1 w_1 w_1) + \frac{\rho_1 v_1 w_1}{r}$$

$$= \frac{\partial}{\partial x}(\tau_{x\theta}) + \frac{1}{r} \frac{\partial}{\partial r}(r\tau_{\theta r}) + \frac{1}{r} \frac{\partial}{\partial \theta}(\tau_{\theta\theta}) + \frac{\tau_{r\theta}}{r} - (1-\alpha)\frac{1}{r} \frac{\partial P}{\partial \theta} + \rho_1 g_\theta + F_{12}(w_2 - w_1) \quad (4.7)$$

For the axisymmetric, non-swirling (i.e. $w = 0$) flow case with the typical geometry shown in Fig.4.1, Eqs.4.5-4.7 are greatly simplified. The final form of the liquid phase equations which will be solved numerically are:

Continuity

$$\frac{\partial}{\partial x}(\rho_1 u_1) + \frac{1}{r} \frac{\partial}{\partial r}(\rho_1 r v_1) = 0 \quad (4.8)$$

x - Momentum

$$\begin{aligned} \frac{\partial}{\partial x}(\rho_1 u_1 u_1) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho_1 u_1 v_1) = & -(1-\alpha) \frac{\partial P}{\partial x} - \rho_1 g + F_{12}(u_2 - u_1) \\ & + \frac{\partial}{\partial x}(\mu_l \frac{\partial u_1}{\partial x}) + \frac{1}{r} \frac{\partial}{\partial r}(r \mu_l \frac{\partial u_1}{\partial r}) + \frac{\partial}{\partial x}(\mu_l \frac{\partial u_1}{\partial x}) + \frac{1}{r} \frac{\partial}{\partial r}(r \mu_l \frac{\partial v_1}{\partial x}) \end{aligned} \quad (4.9)$$

r - Momentum

$$\begin{aligned} \frac{\partial}{\partial x}(\rho_1 u_1 v_1) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho_1 v_1 v_1) = & -(1-\alpha) \frac{\partial P}{\partial r} + F_{12}(v_2 - v_1) \\ & + \frac{\partial}{\partial x}(\mu_l \frac{\partial v_1}{\partial x}) + \frac{1}{r} \frac{\partial}{\partial r}(r \mu_l \frac{\partial v_1}{\partial r}) + \frac{1}{r} \frac{\partial}{\partial r}(r \mu_l \frac{\partial v_1}{\partial r}) + \frac{\partial}{\partial x}(\mu_l \frac{\partial u_1}{\partial r}) - \mu_l \frac{2v_1}{r^2} \end{aligned} \quad (4.10)$$

The equations for the gas phase can be obtained from Eqs. (4.8) to (4.10) by interchanging the subscripts "1" to "2", as well as, "l" and "g".

The global assumptions involved in deriving Eqs. (4.8) to (4.10) are: isothermal steady, axisymmetric, incompressible flow without swirl and without

chemical reactions.

It should be noted that the way the pressure gradient terms should be handled in Eqs.(4.8 - 4.10) is a controversial issue. There is considerable debate in the literature (see for example Stewart and Wendroff, 1984) whether $\nabla[(1-\alpha)P]$ or $(1-\alpha)\nabla P$ should be used in these equations. Both forms satisfy the condition that when the corresponding momentum equations for the two phases are added, the resulting pressure gradient term must be ∇P . The equal pressure model (Stewart and Wendroff, 1984) is adopted, i.e. $P_1=P_2=P$; this pressure is distributed as $(1-\alpha)P$ and αP between the liquid and gas phases, respectively. In this regard, the pressure gradient terms are being treated as part of the interfacial momentum exchange. The surface tension effects can be included in the interfacial momentum exchange function F_{12} ; for different surface tension, σ , the drag force and hence the terminal velocity of a bubble are different. It should be noted, however, if we include a pressure difference $\Delta p = P_1 - P_2 = 4\sigma/d_b$ due to surface tension, this does not alter the form of the above equations. This is because the bubble diameter d_b and the surface tension, σ , are fixed for a given flow regime and hence $P_2 = P_1 + 4\sigma/d_b = P + 4\sigma/d_b$ and $\nabla P_2 = \nabla P$ again. In this regard, the pressure gradient terms are being treated as part of the interfacial momentum exchange. The surface tension effects are neglected in the present study.

The momentum exchange function F_{12} will be prescribed empirically in the functional form

$$F_{12} = F_{12}(\alpha, Re_b) \quad (4.11)$$

where Re_b is the bubble Reynolds number which is defined as $Re_b = \rho_L |u_2 - u_1| d_b / \mu_L$, with d_b being the bubble diameter. The explicit form of Eq.4.11 is discussed in the next subsection.

Once $F_{12} = F_{12}(\alpha, Re_b)$ is prescribed, Eqs.(4.8) through (4.10) written for

both phases constitute a closed set of 6 differential equations for the 6 unknowns, namely α , P , u_1 , v_1 , u_2 and v_2 . For the present numerical simulations these equations are reduced to 4 equations and 4 unknowns by assuming a slip velocity relation of the form

$$u_s = u_2 - u_1 = f(\alpha, Re_b) \quad (4.12)$$

This explicit form of Eq.4.12 is discussed in the following sections.

Interfacial Momentum Exchange

The momentum transfer between two phases takes place via several mechanisms, the most important of which being the viscous drag force resulting from the shear stress at the interface and the form drag due to the pressure distribution on the surface of individual bubbles. Other possible mechanisms for momentum transfer include added mass effect, magnus effect (due to rotation), pressure gradient, and shear rate effects of the surrounding fluid (see for example Hinze, 1972). For brevity these forces will not be considered in the present analysis. Instead, all these effects will be lumped into the drag function Eq. (4.11).

In the bubbly flow regime, the total drag force can be related to that of a single bubble. Hirt (1982) used the following relation for water droplets in steam (for our problem, ρ_g in the original expression is replaced by ρ_t).

$$F_{12} = \frac{3}{4} \alpha^2 (1-\alpha) \rho_t \frac{|u_1 - u_2|}{d_b} C_D \quad (4.13)$$

Syamlal and O'Brien (1988) suggested the following empirical relation for dispersed solid particles in a continuous liquid or gas phase

$$F_{12} = \frac{3}{4} \frac{\alpha^2 (1-\alpha) \rho_t |u_1 - u_2|}{d_b} C_D^* \quad (4.14)$$

where C_D^* is another empirical drag function given by

$$C_D^* = \frac{C_D (Re_b \leq Re_b/V_r)}{V_r^2} \quad (4.15)$$

In Eq. (4.15) $C_D = C_D(Re_b)$ is the drag coefficient for an isolated particle and V_r is the ratio of terminal velocity of a group of particles to that of an isolated particle. Neither of the equations (4.13) or (4.15) is strictly applicable to the bubbly flow regime. However, to a first approximation the simpler relation used by Hirt (1982) should be sufficient for our purposes, provided that C_D is replaced by an empirical relation for bubbles in water.

Such a C_D relation can be derived by curve fitting to the experimental data presented by Clift et al. (1978). For bubbles in pure liquid systems, the following function is suggested by us,

$$C_D = a Re_b^{-b} \quad (4.16)$$

The coefficients a and b are given in Table 4.1 for different Reynolds number ranges. This particular form is adapted because it simplifies the calculation of slip velocities considerably.

Table 4.1 Coefficients for Eq.(4.16) for Air Bubbles in Pure Liquids

Re_b range	a	b
0 - 2	24	1.000
2 - 10	23.66	0.981
10 - 100	14.9	0.780
100 - 1000	6.9	0.613

Simplification for the Gas Phase

Instead of solving for the gas phase momentum equations, the gas velocities are determined from a slip velocity relation of the form of Eq.(4.12).

For small void ratios (i.e. dilute flow with a dispersed gas phase) the gas velocities can be calculated in the radial and axial-direction, respectively, as

$$v_s = 0 \quad \text{or} \quad v_t = v_g \quad (4.17)$$

$$u_g = u_s + u_t \quad (4.18)$$

where the slip velocity is given by

$$u_s = U_{b\infty}(1-\alpha) \quad (4.19)$$

$U_{b\infty}$ is the terminal velocity of an isolated bubble in an infinite liquid medium. The effect of void ratio, α , on the slip velocity as given in Eq.(4.19) is suggested by Wallis (1962). $U_{b\infty}$ can be calculated by equating the drag force to the difference of the buoyancy force and the weight of the bubble. With the drag relation, Eq.(4.16), this force balance results in

$$U_{b\infty} = \left[\frac{4}{3a} \frac{(\rho_l - \rho_g)}{\rho_l} g d_b \left(\frac{\rho_l d_b}{\mu_l} \right)^b \right]^{1/(2-b)} \quad (4.20)$$

For example with $b = 1$ and $a = 24$, (i.e., Stokes range) Eq.(4.20) reduces to

$$U_{b\infty} = \frac{1}{18} \frac{(\rho_l - \rho_g)}{\mu_l} g d_b^2 \quad (4.21)$$

If the water (or liquid) is not pure, the degree of contamination may have significant influence on $U_{b\infty}$. For such cases, the empirical data presented by Clift et al. (1978) can be used. Another alternative is to use the terminal velocity relations presented by Hewitt (1982, chapter 2) where the terminal velocity of bubbles in clean fluids is expressed as a function of Re_b and the Galileo number $Ga = g\mu_l/\rho_l\sigma^3$.

4.3 DESCRIPTION OF CASES STUDIED

The numerical study simulated, as closely as possible, the experiments performed by Rietema and Ottengraf (1970) where a laminar liquid circulation

and bubble street formation were investigated in a Quickfit glass column. The geometric configuration for the glass column is shown in Fig. 4.1. The experimental conditions for the numerically simulated case were: liquid (glycerol water solution) density $\rho_l = 1153 \text{ kg/m}^3$, liquid viscosity $\mu_l = 0.35 \text{ kg/m-s}$, air flow rate $Q_a = 11.4 \text{ cm}^3/\text{s}$, gas hold-up $\epsilon_g = 74 \text{ cm}^3$, bubble diameter $d_b = 0.54 \text{ cm}$ and bubble street radius $r_s = 5.0 \text{ cm}$. The glass column had a diameter of 22 cm and a height of 122 cm. Initially the column was filled with the liquid solution up to a depth of 82 cm. If the gas hold-up of 74 cm^3 is added to the liquid volume, the total mixture volume requires a column height of approximately 82.2 cm. This value was used in the simulations. Air bubbles were formed by means of injection needles. According to experiments, vertical baffles were placed along the wall, so that a reasonably symmetrical bubble street could be created. The effect of the baffles is not considered in the present study. The liquid velocity profile from the experimental work (Rietema and Ottengraf, 1970) is shown in Fig. 4.2.

For the set of calculations where $\alpha(x,r)$ is obtained from the solution of Eq.(4.28), a step function $\alpha = \alpha_0$ for $r \leq r_{s0}$; $\alpha_0 = 0$ for $r > r_{s0}$ was assumed at the inlet (see Fig. 4.12b). α_0 was approximated as the area fraction of injection holes to the total area. A hole diameter of $0.5 d_b$ was assumed. r_{s0}/R values were varied between 0.3 and 0.9 to study the influence of distribution at the inlet on the overall circulation patterns.

Profiles for α - distribution

As a further simplification in the present model, the void fraction is assumed to vary as $\alpha(x,r) = \alpha_c(x) \alpha(r)$, and the shape of the profiles $\alpha = \alpha(r)$ are prescribed empirically. Among various shapes investigated are cosine, linear and parabolic profiles.

The cosine profile is given by

$$\alpha(r) = \begin{cases} 0.5 \alpha_c [1 + \cos(\pi r/r_s)] & r \leq r_s \\ 0 & r_s < r \leq R \end{cases} \quad (4.22)$$

where R is the column radius and r_s is the radius of the bubble street as observed from experiments (Rietema and Ottengraf, 1970). Eq.(4.22) represents a smooth function which satisfies the zero derivative conditions at $r=0$ and $r=r_s$. The linear profile is given by

$$\alpha(r) = \alpha_c (1 - r/r_s) \quad (4.23)$$

$$\begin{aligned} \text{where } r/R &= -\alpha_c(1-x/L) + 1 && \text{for } 3/4 < x/L \leq 1 \\ r_s/R &= 0.45 && \text{for } 1/4 < x/L \leq 3/4 \\ r/R &= -\alpha_c x/L + 1 && \text{for } 0 < x/L \leq 1/4 \end{aligned} \quad (4.24)$$

The parabolic profile is given by

$$\alpha(r) = \alpha_c \left(1 - \frac{r^2}{r_s^2} \right) \quad (4.25)$$

$$\begin{aligned} \text{where } r/R &= 1 - ((L-x)/L)^{0.437} && \text{for } 3/4 < x/L \leq 1 \\ r_s/R &= 0.45 && \text{for } 1/4 < x/L \leq 3/4 \\ r/R &= 1 - (x/L)^{0.437} && \text{for } 0 < x/L \leq 1/4 \end{aligned} \quad (4.26)$$

Examples of these profiles are shown in Figures 4.5b, 4.6a and 4.7a, respectively.

The center line value α_c for the three cases is determined from continuity, i. e.

$$Q_a = 2\pi \int_0^R \alpha_c \alpha(r) u_g(x, r) r dr \quad (4.27)$$

Equation for α -distribution

Calculations are also performed where $\alpha(x,r)$ is calculated from conservation of mass equation for the gas phase:

$$\frac{\partial}{\partial x} (\alpha u_2) + \frac{1}{r} \frac{\partial}{\partial r} (r \alpha u_2) = 0 \quad (4.28)$$

The advantage of prescribing the α -profiles instead of determining them from Eq.(4.28) is that all the computational uncertainties in solving Eq.(4.28), such as numerical diffusion are eliminated. Further, the former makes it possible to investigate the influence of various α -profiles on the circulation patterns.

The boundary condition for $\alpha(0,r)$ at the inlet of the bubble column is determined by the constant gas volume flow rate Q_a given by Eq.(4.27), where $\alpha(0,r)$ is prescribed as a step function, such as

$$\begin{aligned} \alpha(0,r) &= \alpha_c & \text{for } 0 < r/R \leq r_{so}/R \\ \alpha(0,r) &= 0 & \text{for } r_{so}/R < r/R \leq 1 \end{aligned} \quad (4.29)$$

where r_{so} represents the location of the last row of holes on the distributor plate. The boundary condition for $\alpha(L,r)$ at free surface is also determined by Eq.(4.27) to satisfy the continuity.

4.4 NUMERICAL METHOD

The form of equations for the continuous liquid phase (Eqs.4.9 and 4.10) is amenable for using the finite volume technique (e.g. Patankar, 1980) which has been successfully used for solution of steady, incompressible, single phase, recirculating flow problems. This formulation takes into account density variation (say due to buoyancy effects) in space which is suitable for the present problem where the microscopic density is constant but the macroscopic density varies according to $\rho_1 = \rho_0(1-\alpha)$. The so called "SIMPLE" algorithm of Patankar (1980) is employed to calculate the pressure field iteratively. However, modifications have been made to account for the modified

gravity term, pressure gradients $(1-\alpha)\partial P/\partial x$ and $(1-\alpha)\partial P/\partial r$, and additional momentum source terms, namely $F_{12}(u_2-u_1)$ and $F_{12}(v_2-v_1)$. These modifications have been incorporated in a readily available computer code, TEACH (Gosman and Ideriah, 1976; Durst and Loy, 1984) and the modified version has been used for the present calculations.

The "hybrid" difference scheme is used for discretization of the equations. This scheme has the property that it switches from central to upwind differencing for high Peclet numbers ($Pe=\rho_1 \Delta x u_1/\mu_1$) for the convective terms; for the diffusive terms, the central differencing is employed at all times. For the present application the maximum Pe had an order of magnitude of 1.0. That is, the flow was mainly dominated by viscous forces, and the central differencing is used for most of the flow region. This is an important feature of the method especially for calculating the void fraction distribution from Eq.(4.28) which has no physical diffusion. Since the central differencing is a second order accurate scheme, it minimizes the truncation errors and hence the numerical diffusion.

Boundary Conditions

No slip condition was enforced for the liquid velocities at the walls and at the air inlet (distributor plate); at the centerline ($r=0$) symmetry conditions were imposed. The free surface was assumed to be undisturbed at which the axial-velocity, u_1 , was set equal zero (i.e. no liquid flux through the surface). The radial velocity, v_1 , at the free surface was calculated from the condition $\partial v_1/\partial x=0$; this is a somewhat arbitrary condition used as a first approximation since no other information is available on v_1 .

Boundary conditions are needed for α when it is calculated from Eq.(4.28). Since there is no mass flux through the walls $\partial \alpha/\partial r=0$ was used at the side

walls; the same condition applies at the centerline due to symmetry. At the inlet, a uniform distribution $\alpha=\alpha_0$ was assumed, and α_0 was estimated from the number of holes on the distributor plate, i.e., $\alpha_0=\Sigma A_j/A$, $j=1,2,\dots,N$, A_j is the area of an individual hole, A is the total cross-sectional area of the inlet, and N is the number of the holes. Then from Eq.(4.27) the gas velocity is calculated for a given air flow rate, Q_a .

The α values at the free surface are not needed for the calculations, $\alpha=1$ was imposed at the grid points just outside the free surface. The present method uses a staggered grid arrangement in which the velocities are staggered and the scalar quantities are stored at centers of main grid cells. The gas velocity at the free surface is calculated from Eq.(4.27) to satisfy continuity once α was determined there.

Convergence and Grid Independence

To ensure properly converged numerical solutions, not only the total residues of difference equations were checked, but also the net liquid circulation, Q_{net} and the maximum relative change in the axial velocity, $\Delta u_{max}/u_{max}$. The net liquid volume circulated should be zero for the present problem. This is normalized by the recirculated liquid volume, Q_{cir} , given by

$$Q_{cir} = 2\pi \int_0^{r_0} (1-\alpha) u_z r dr = -2\pi \int_{r_0}^R (1-\alpha) u_z r dr \quad (4.30)$$

where r_0 is the zero cross point for u_z . After about 300 iterations, both $\Delta u_{max}/u_{max}$ and Q_{net}/Q_{cir} values were less than 10^{-5} which are shown in Fig.4.3 and Fig.4.4, respectively. Calculations were continued another 200 iterations to ensure complete convergence.

Two uniform grid distributions were used, namely a coarse grid of 23×12 ($\Delta x=4\text{cm}$, $\Delta r=1\text{cm}$), and a fine grid of 42×23 ($\Delta x=2\text{cm}$ and $\Delta r=0.5\text{cm}$). The

difference in liquid velocity from the coarse grid and fine grid solutions was less than 1%. The fine grid solutions are presented in this paper unless stated otherwise.

4.5 RESULT AND DISCUSSION

Results with Prescribed α - Profiles

The results of the calculations using a cosine profile for the shape of $\alpha(r)$ distribution are depicted in Figs. 4.5a through 4.5d. The predicted streamlines plotted in Fig. 4.5a show the commonly observed circulation pattern with a downward flow near the wall and an upward flow near the center of the reactor. The stream function is calculated from

$$\psi = 2\pi \int_0^R (1 - \alpha) \rho_L u_L r dr \quad (4.31)$$

The total volume of the circulated liquid can be read off from Fig. 4.5a as $Q_{cir} \approx 0.16 \text{ m}^3/\text{s}$. The center of the circulation zone is predicted to be close to the free surface. As it will be discussed later, the circulation pattern is a direct result of the void-fraction distribution. The corresponding α distribution is shown in Fig. 4.5b. Though the shape of the α -distribution is assumed, its magnitude is calculated as part of the solution. Fig. 4.5b shows that α decreases with the axial distance. This occurs as a result of the increase in the gas velocity towards the free surface; to satisfy continuity Eq.(4.27) as the mean value of u_g increases, that of α should decrease. The predicted liquid velocity profiles are shown in Fig. 4.5c. The predicted value of centerline velocity, $u_{LC} \approx 22 \text{ cm/s}$, at $x/L=0.5$ is considerably higher than the measured value of $u_{LC} \approx 10.5 \text{ cm}$. The boundary of the reverse flow where $u_L=0$ is predicted as $r/R=0.50$ which is in good agreement with experiments. The agreement is also good for the maximum reverse flow velocities which are seen to be 3-4 and 3-5 cm/s for experiments and predictions, respectively. Of

course, these quantities do change with axial distance and with the prescribed bubble-street radius, r_s .

The influence of different void fraction-profile shapes on the results were also investigated where the gas flow rate and the bubble-street diameter were fixed. Fig. 4.6b shows the resulting of liquid velocity distribution with prescribed linear α distribution and Fig. 4.7b shows the resulting of liquid velocity distribution with prescribed parabolic α distribution. The linear and parabolic α -profiles does not influence the results very much. The reverse flow boundary and the maximum reverse flow velocities are affected the least. The centerline liquid velocity is somewhat higher for the parabolic profile compared to the others. These results show that it is the magnitude of the α -distribution and the bubble-street diameter which primarily affect the flow pattern in a column reactor.

Results with Varing Street Diameters

Comparisons of the results were made by varying the bubble street radii as $r_{s0}/R=0.27$, 0.45, and 0.65. The results for $r_s/R=0.27$ and 0.65 are shown in Fig. 4.8 and Fig. 4.9. These figures show that a decrease in r_{s0} causes an increase in α values, thus an increase in the liquid velocity and vice versa. This also affects the boundary of the reverse flow zone. For example, for $r_s/R=0.65$, the predicted centerline velocity is about 18cm/s and for $r_{s0}/R=0.27$ it is about 26 cm/s at the mid-height of the column. Consequently, the uncertainties in measuring the region where $\alpha \approx 0$ (i.e. no air bubbles) will result in differences between the experiments and predictions.

Results with Predicted α - Distribution

Further results of the predictions are depicted in Figs. 10-13 where

closer agreement between the measured and predicted velocity profile at $x/L=0.5$ as shown in Fig. 4.10c. The location of the measurements is reported (Rietema and Ottengraf, 1970) to be near the mid-height of the column. The magnitude of the predicted u_z values decreases because the overall α values are lower and hence less drag force is impacted on the liquid by the gas flow.

The values of the cut-off point r_{so}/R for the α -step function at $x=0$ were chosen arbitrarily. This brings the question of what exactly this value should be? Strictly speaking, this should be the location of the last row of injection holes on the distributor plate, but this information was not available from the experiments considered here. A further increase of this parameter to $r_{so}/R=0.7$ resulted in much closer agreement between the measured and predicted liquid velocities (Fig. 4.12c), but the center of the recirculation zone moved further down towards the inlet (Fig. 4.12a). The results show that, increasing the value of r_{so}/R , decreases the liquid velocity at the centerline. As shown in Fig. 4.13c, liquid velocity has a small value when the air flow is distributed over 90% of the inlet area. For the same case, Fig. 4.13b shows very little variation in α distribution in the axial direction from that prescribed at the inlet. The results depicted in Figures 4.14a and 4.14b show that the circulation is negligibly small when α is uniformly distributed over the whole cross-section of the inlet area.

$$\begin{aligned}
 q_d &= 0.3834 \times 10^{-6} & \alpha &= 0.0118 \\
 \phi &= 0.4108 \times 10^{-4} & \delta &= 0.46
 \end{aligned}$$

Fig. 4.2 Experimentally determined velocity profile, from Rietema and Ottengraf (1970).

- q_d DIMENSIONLESS AIR FLOW-RATE
- ϕ DIMENSIONLESS SLIP VELOCITY
- δ DIMENSIONLESS STREET DIAMETER

Fig. 4.3 Convergence of numerical solution with respect to the maximum relative changes in axial velocity.

Fig. 4.4 Convergence of numerical solution with respect to the net recirculated liquid mass Q_{net} .

Fig. 4.5 (a) Calculated circulation pattern
(Result from a cosine κ distribution function)

Fig. 4.5 (b) Void ratio distribution as a cosine function.

Fig. 4.5 (c) Liquid velocity profiles
 □ (experiments by Rietema and Ottengraf, 1970)

Fig. 4.5 (d) Gas velocity profile

a) Void ratio distribution

b) Liquid velocity profile

Fig. 4.6 Results with linear void ratio distribution.

a) Void ratio distribution

b) Liquid velocity profile

Fig. 4.7 Results with parabolic void ratio distribution.

Fig. 4.8 Results with $r_p/R = 0.27$

Fig. 4.9 Results with $r_p/R = 0.65$

Fig. 4.10 (a) Calculated circulation pattern
 (α is calculated from transport equation (4.28) with $r_p/R = 0.45$).

Fig. 4.10 (b) Predicted void ratio distribution.

Fig. 4.10 (c) Liquid velocity profile
 □ (experiments by Rietema and Ottengraf, 1970)

Fig. 4.10 (d) Gas velocity profile.

Fig. 4.10 (e) α -contours

Fig. 4.10 (f) Liquid velocity, gas velocity and α distribution along the centerline of the column with $r_0/R = 0.45$.

Fig. 4.11 (a) Calculated circulation pattern
 (α is calculated from transport equation (4.28) with $r_j/R = 0$).

Figure 4.11 (b) α - distribution; $r_s / R = 0.60$

Fig. 4.11 (c) Liquid velocity profile
 □ (experiments by Rietema and Ottengraf, 1970)

Fig. 4.11 (d) Gas velocity profile

Fig. 4.12 (a) Calculated circulation pattern
 (α is calculated from transport equation (4.28) with $r_s/R=0.7$)

Fig. 4.12 (b) Void ratio distribution

Fig. 4.12 (d) Gas velocity profile

Fig. 4.12 (c) Liquid velocity profile
 $\square$ (experiments by Rietema and Ottengraf, 1970)

Fig. 4.12 (a) α contours

Fig. 4.13 (a) Calculated circulation pattern
 (α is calculated from transport equation (4.28) with $r_{so}/R=0.9$)

Fig. 4.13 (b) Void ratio distribution

Fig. 4.13 (c) Liquid velocity profile
 □ (experiments by Rietema and Ottengraf, 1970)

Fig. 4.13 (d) Gas velocity profile

Fig. 4.13 (e) α contours

Fig. 4.14 (a) Calculated circulation pattern
 (α is calculated from transport equation (4.28) with $r_s/R=0.95$)

Fig. 4.14 (b) Void ratio distribution

SECTION 5

Conclusions

To improve the prediction of circulation in bubble columns, a theoretical and experimental effort has been undertaken at West Virginia University. The problem has been addressed using both probe studies and laser doppler velocimetry, and both one-dimensional and full numerical models have been proposed.

For the probe work a tall 8 inch diameter bubble column was constructed, and void distributions were measured with resistance probes. A distributor plate introduced air evenly over the column base. To acquire data from the probes, a new thresholding protocol was developed and verified. Data gathered at various column heights and gas flowrates showed that the void distribution was flattened near the distributor plate and became more parabolic higher in the column. Bubble velocity data proved more difficult to acquire when two probes were used, and their signals cross-correlated. Bubbles had too strong a radial velocity component to permit reliable vertical velocity measurement. Nevertheless, at the column center, some reliable bubble velocity data was obtained, and the results compared favorably with velocity calculated from the void profile with the one-dimensional model of Clark et al. (1987).

One-dimensional modeling was also extended to incorporate a drift-flux model, and it was shown that C_0 , the distribution parameter, could be obtained from the void profile. However, a priori prediction of holdup was not possible using this approach.

During the laser Doppler velocimeter (LDV) work, a series of 3-D mean and fluctuating mean and turbulent velocity measurements have been obtained

in a laboratory model bubble column. These data have been acquired non-intrusively through use of a three-component LDV, for vertical air injection from a single port on the centerline at the bottom of the column. Mean velocity data confirm the existence of a strong, toroidal vortex in the top half of the column with a much weaker inwards motion in the bottom half of the column. These quantitative data are confirmed by qualitative flow visualization. Circumferential mean velocity is essentially zero. The turbulence field is non-isotropic, as expected, in that radial RMS velocities are generally larger than vertical and circumferential RMS velocities. Turbulence intensities are on the order of 100% throughout the column.

The present data contribute to the understanding of the development of liquid circulation in a bubble column. Also, data are extremely useful in the development of reliable numerical models for the turbulent multiphase flow in a bubble column. However, it would be extremely useful to obtain a similar set of velocity data for a bubble injection distributor plate which created a uniform distribution of bubbles at the bottom of the column. The present measurements could also be significantly improved through the development of LDV alignment procedures to enable simultaneous measurement of all three velocity components, so that Reynolds stresses could also be determined, as an aid in turbulence model development. While it is certain that optical penetration into the bubble column by the LDV would place limits on the magnitudes of void fraction for which such data could be obtained, still it is clear that significant improvements could be made.

Numerical calculation of the liquid circulation inside an isothermal column reactor was performed in two ways: (a) with a prescribed α -profile (shape); (b) with α -distribution calculated from the transport equation for

it.

The results indicate that the actual shape of the α -profile is not that critical with respect to the circulation patterns and the liquid velocity. The overall magnitude of the void fraction as well as the bubble-street diameter seems to be more important.

The predictions including direct solution for α -distribution lead to a more realistic circulation pattern compared to experimental observations. The boundary condition for α at the inlet (i.e. the distributor plate) seems to play a dominant role in determining the overall circulation pattern. At this point numerical investigations need more input from experimental investigations.

The present mathematical model based on the continuum approach and the slip velocity relation seems to predict reasonable well the overall characteristics of the circulation in the bubbly flow regime. However, the full numerical solution of both the liquid and gas phases needs to be further investigated as well as the flow regimes other than the bubbly flow regime.

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SECTION 7

PUBLICATIONS ARISING FROM CONTRACT

Clark, N.N., Seiss, R.A. and Turton, R., "Inferring Bubble Size Distributions from Resistance Probe Measurements in Gas-Liquid, Gas-Slurry, and Three Phase Systems", 3rd International Symposium on Liquid-Solid Flows, Chicago, 1988, ASME Winter Annual Meeting, ASME Pub. FED-Vol. 75, pp. 181-188.

Clark, N.N., Van Egmond, J.W. and Nebiolo, E., "The Drift-Flux Model Applied to Bubble Columns and Low Velocity Flows", International Journal of Multiphase Flow, Vol. 16, No. 2 (1990) pp. 261-279.

Clark, N.N., and Nebiolo, E., "Circulation Results from an 8 inch Diameter Tall Bubble Column", Fine Particle Soc. Meeting, Boston, Aug. 1989.

Kuhlman, J.M. and Gross, R.W., "Three-Component Velocity Measurements In A Laboratory Bubble Column", ASME Int'l Symposium on Gas-Liquid 2-Phase Flows, Dallas, TX, 1990.

Celik, I., and Wang, Y.-Z., "Numerical Simulation of Liquid Circulation In A Bubbly Column", Symposium on Numerical Methods for Multiphase Flows", ASME/FED Spring Conference, Toronto, Ontario, Canada, 1990.

first stage of an ongoing research where the numerical solution of the continuum equations of motion for both phases are aimed at. In such a procedure, the only empiricism should come through the prescription of interfacial momentum exchange terms.

4.2 MATHEMATICAL MODEL

Consider an unsteady, gas-liquid flow inside a vertically situated circular reactor which is assumed to be isothermal and non-reacting. The gas-liquid flow is assumed to be in the bubbly flow regime which is characterized by a suspension of discrete air bubbles in a continuous liquid such as water. The mathematical model is based upon the conservation of mass and momentum for the liquid and gas phases including appropriate interface momentum exchange terms. Invoking continuum assumptions and performing a space or time averaging over a macroscopic control volume, (see e.g., Anderson and Jackson, 1967; Homsy, 1983; Drew, 1983), these equations can be written in vector form analogous to the well known Navier-Stokes equations. Let the subscripts "1" and "2" or "l" and "g" denote phase-1 (liquid) and phase-2 (gas), respectively; α be the void fraction (i.e. volume concentration of gas), $\rho_1 = (1-\alpha)\rho_l$, $\rho_2 = \alpha\rho_g$ be the macroscopic densities, ρ_l and ρ_g being the microscopic liquid and gas densities, respectively; $\underline{u}_1$ be the liquid velocity vector (u_1, v_1, w_1) and $\underline{u}_2$ the gas velocity vector (u_2, v_2, w_2) in the axial x -, radial r -, and tangential, θ - directions of cylindrical coordinates, respectively. The continuity equation representing the conservation of mass for the liquid phase is

$$\frac{D\rho_1}{Dt} + \rho_1(\underline{z} \cdot \underline{u}_1) = \frac{\partial \rho_1}{\partial t} + \underline{z} \cdot (\rho_1 \underline{u}_1) = 0 \quad (4.1)$$

where $D(\)/Dt$ is the substantial derivative and $\underline{z}$ is the divergence operator. The term $\underline{z} \cdot \underline{u}_1$ is zero for an incompressible flow. There are no source/sink

terms in Eq.4.1 because there is no phase change due to chemical or thermal reactions. For numerical treatment Eq.4.1 is rewritten in the conservative form as

$$\frac{\partial \rho_1}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho_1 r v_1) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho_1 w_1) + \frac{\partial}{\partial x} (\rho_1 u_1) = 0 \quad (4.2)$$

The momentum equations for liquid phase, in vector form (see for example Celik, 1986), are

$$\rho_1 \frac{D \underline{u}_1}{Dt} = -(1-\alpha) \nabla P - \nabla \cdot \underline{\tau} + F_{12} (\underline{u}_2 - \underline{u}_1) + \rho_1 \underline{g} + \underline{f}_c - w_1 \underline{e}_x \times \underline{u}_1 / r \quad (4.3)$$

In Eqs.(4.3), $\underline{\tau}$ is the stress tensor, $\underline{g}$ is the acceleration of gravity, P is the pressure, F_{12} is the momentum exchange function between phase-1 and phase-2, $\underline{e}_x$ is the unit vector in x - direction, $\times$ denotes a vector cross product and $\underline{f}_c$ is the additional stress vector $(0, -\tau_{\theta\theta}/r, \tau_{r\theta}/r)$ in x -, r -, and θ - directions, respectively. The components of the symmetric stress tensor $\underline{\tau}$ for Newtonian fluids in cylindrical coordinates are:

$$\begin{aligned} \tau_{xx} &= 2\mu \frac{\partial u}{\partial x} - \frac{2}{3} \mu \nabla \cdot \underline{u} \\ \tau_{rr} &= 2\mu \frac{\partial v}{\partial r} - \frac{2}{3} \mu \nabla \cdot \underline{u} \\ \tau_{\theta\theta} &= 2\mu \left(\frac{1}{r} \frac{\partial w}{\partial \theta} + \frac{v}{r} \right) - \frac{2}{3} \mu \nabla \cdot \underline{u} \\ \tau_{rx} &= \tau_{xr} = \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial r} \right) \\ \tau_{r\theta} &= \tau_{\theta r} = \mu \left(\frac{\partial w}{\partial r} + \frac{1}{r} \frac{\partial v}{\partial \theta} - \frac{w}{r} \right) \\ \tau_{x\theta} &= \tau_{\theta x} = \mu \left(\frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial w}{\partial x} \right) \end{aligned} \quad (4.4)$$

END

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