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RADIATIVE CORRECTIONS IN THE STRONGLY INTERACTING LIMIT OF THE STANDARD ELECTROWEAK MODEL *†

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Abstract

Radiative corrections to the parameters of the standard electroweak model are considered in case that there is no light Higgs particle.

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INTRODUCTION

It is well known^{1,2} that in the standard model the Higgs and longitudinally polarized W, Z sector is strongly interacting if the Higgs mass is sufficiently large. To see this, recall the basic formulae of the standard model:

$$m_H^2 = 2\lambda v^2 \approx \frac{\lambda}{8}(TeV)^2, \quad \Gamma_H = \frac{3m_H^2}{32\pi v^2} \approx \frac{1}{2}m_H \left(\frac{m_H}{TeV} \right)^2. \quad (1)$$

Here m_H is the Higgs mass, λ is the self-coupling constant for the Higgs sector and v is the vacuum expectation value of the Higgs field. That is, the lagrangian for the Higgs sector is

$$\mathcal{L}_H = \mathcal{D}_\mu \varphi \mathcal{D}^\mu \bar{\varphi} - \lambda \left(|\varphi|^2 - \frac{v^2}{2} \right)^2. \quad (2)$$

Eq. (1) tells us that if $m_H > TeV$, $\lambda \sim O(1)$ and $\Gamma_H/m_H \sim O(1)$. The first statement implies strong self coupling and the second implies that in this limit the Higgs “particle” will be at best a broad resonance. Under these conditions it is worth reconsidering^{3,4} the perturbative calculations⁵ of radiative corrections within the context of the standard model.

The reason that the longitudinal components of the W and Z particles are strongly interacting in the large Higgs mass limit is that they are remnants of the Higgs sector. Before electroweak gauge symmetry breaking the spectrum includes massless Yang-Mills fields and four real scalars that can be represented by

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} i\pi^+ \\ \frac{1}{\sqrt{2}}(H - i\pi^0 + v) \end{pmatrix}, \quad (3)$$

where H is the physical Higgs field, and $\pi^{\pm 0}$ are the would-be Goldstone bosons that are “eaten” to become the longitudinal components of the $W^\pm$ and Z particles.

The coupling of ordinary quarks and leptons to longitudinally polarized W, Z and to the Higgs particle is suppressed by a factor $m_{q,\ell}/m_W$. However

strong rescattering corrections to low energy weak transitions arise through corrections to the propagators of transversely polarized W, Z . These can be described in terms of two parameters⁶: T and S . The T -parameter measures the deviation from the tree-level relation $m_W = \cos\theta_W m_Z$ and is related to the more familiar ρ -parameter by $\alpha T = \rho - 1$. The S -parameter measures corrections to the W and Z wave function renormalizations, which are equal ($U = 0$ the notation of ref. 6).

TOOLS

Three tools that can be used in the analysis of strong rescattering effects are the equivalence theorem^{2,7,8}, dispersion relations and chiral symmetry^{7,9}. The equivalence theorem states that S-matrix elements involving external longitudinally polarized W and Z particles can be evaluated, up to corrections of order $m_{W,Z}/E_{W,Z}$, by replacing $W^\pm, Z \rightarrow \pi^\pm, \pi^0$ on the external lines, that is,

$$S[n(W_L^\pm, Z_L) + X \rightarrow n'(W_L^\pm, Z_L) + X'] = S[n(\pi^\pm, \pi^0) + X \rightarrow n'(\pi^\pm, \pi^0) + X'] + O(m_{W,Z}/E_{W,Z}). \quad (4)$$

Dispersion relations express interaction amplitudes as appropriately weighted integrals over S-matrix elements, which can in turn be evaluated using (4). Provided we work in the Landau gauge, with propagators

$$\Delta^\pi = \frac{i}{p^2}, \quad \Delta_{\mu\nu}^{V_T} = \frac{-i}{p^2 - m_V^2} \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right), \quad V = W, Z,$$

the distinction between the (weakly coupled) transverse vectors V_L and (strongly coupled) scalars π is unambiguous.

Aside from the fact that scalars are technically simpler to treat than vectors, the equivalence theorem allows us to exploit the chiral symmetry of the strongly

coupled scalar sector, which, at sufficiently low energies, is just a rescaled version of low energy pion physics in QCD. At center of mass energies $m_W^2 \ll s \ll m_H^2$, the Higgs lagrangian (2) reduces to the effective lagrangian

$$\begin{aligned}
\mathcal{L}_H = & m_W^2 W_{T\mu}^+ W_T^{-\mu} + \frac{1}{2} m_Z^2 Z_{T\mu} Z_T^\mu + \frac{1}{2} \partial_\mu \pi^i \partial^\mu \pi^j \left(\delta_{ij} + \frac{\pi_i \pi_j}{v^2 - \pi^2} \right) + \dots \\
& + e A^\mu J_\mu^A + \frac{g}{2} \left(W_T^{+\mu} J_\mu^- + \text{h.c.} \right) + \frac{g}{2 \cos \theta} Z_T^\mu J_\mu^Z \\
& + g \pi^0 \left(m_W W_T^2 + m_Z Z_T^2 \right) + \dots \\
& + \frac{1}{4} g^2 \frac{\partial^\mu \pi \cdot \partial_\mu \pi}{m_H^2} \left(W_T^2 + \frac{1}{2 \cos^2 \theta} Z_T^2 \right) + \dots
\end{aligned} \tag{5}$$

The first line contains the V_T mass terms and the lagrangian for the strongly self-coupled pions; the dots represent higher derivative operators. This lagrangian is invariant under parity ($\pi \rightarrow -\pi$) and “strong isospin” I under which the pions transform in the obvious way. The second line gives the pion couplings to V_T and the photon field A^μ via the currents J_μ that are appropriate combinations of axial and vector pionic currents with $I = 1, 0$; their normalization is fixed by low energy theorems. These terms induce vacuum polarization effects that contribute to the parameter S . In the third line the dots stand for different charge combination counterparts of the first two terms. In the limit $m_H \rightarrow \infty$ these are the only terms that contribute to the T -parameter in order g^2 in the standard model. In fact, the first three lines of (5) are model independent and are equally applicable to the low energy limit of technicolor theories, for example, or any model in which $\rho = 1$ at tree level due to the chiral symmetry of the gauge symmetry-breaking sector. As written, the last line is specific to the standard model; this term also contributes to T for finite Higgs mass. Its contribution is formally of order g^4 , but if scattering in the $I = J = 0$ channel is dominated by a resonance with the mass and width of the

standard model Higgs particle, the order g^2 contribution of a “light” Higgs + V_T loop is recovered. Such a term, as well as additional higher derivative and higher dimension terms, represented by dots, are expected to be present quite generally. Outside the context of the standard model m_H should be interpreted as the effective cut-off Λ of the low energy theory, that is, the scale above which the lagrangian (5) is no longer the correct description. This scale could be the technirho mass, for example, in technicolor models.

The standard model Higgs contribution to S, T is finite; the one-loop result is^{5,6,10}

$$S_{SM}^{(1)} = \frac{1}{12\pi} \ln(m_H^2/m^2), \quad T_{SM}^{(1)} = -\frac{3}{16\pi \cos^2\theta} \ln(m_H^2/m^2), \quad (6)$$

as defined by subtracting out the standard model result evaluated at some reference value m of the Higgs mass, of the order of m_W . The effective nonlinear theory defined in (5) is nonrenormalizable; the one-loop contribution must be expressed in terms of the effective cut-off Λ . The result is¹¹

$$S_{NL}^{(1)} = \frac{1}{12\pi} \ln(\Lambda_S^2/m^2), \quad T_{NL}^{(1)} = -\frac{3}{16\pi \cos^2\theta} \ln(\Lambda_T^2/m^2), \quad (7)$$

where $m \approx m_W$ is the effective infrared cut-off, and the ultraviolet cut-offs $\Lambda_S, \Lambda_T \sim \Lambda$ include an *a priori* unknown correction from finite contributions. As previously noted^{11,12}, the one-loop corrections in the effective nonlinear σ -model reproduce those found¹⁰ in the standard model, provided the cut-offs are replaced by the Higgs mass.

Conventional analyses⁵ assume $m_H \leq 1\text{TeV}$; then overall fits to the data imply a constraint on the top quark mass: $m_t \leq 200\text{GeV}$. Why not allow m_H , or, in the present context, Λ , to become arbitrarily large, thus relaxing this constraint? One loop-corrections to the standard model Higgs mass are

quadratically divergent:

$$m_H^2 = m_H^2(\text{tree}) + a \frac{g^2}{16\pi^2} \Lambda^2, \quad (8)$$

where a is a number of order unity. The infinite term when $\Lambda \rightarrow \infty$ can of course be absorbed into a renormalization. However if there are any large scales in the theory, such as a GUT scale or the Planck scale (which is indeed present), the fact that the Higgs mass is not protected by any symmetry implies that finite radiative corrections should be given by the second term in (8) with Λ replaced by the appropriate scale. This is the well known gauge hierarchy problem. If we allow the Higgs mass to become very large, physics at scales below that mass is described by the effective theory defined by (5). The Higgs particle no longer appears, but one-loop corrections within that theory renormalize the kinetic energy term with the result that the renormalized parameter v is

$$v^2 = v_{\text{tree}}^2 \left(1 - \frac{\Lambda_{eff}^2}{8\pi^2 v^2} \right) = v_{\text{tree}}^2 \left(1 - 2 \left[\frac{\Lambda_{eff}}{3TeV} \right]^2 \right) \approx \left(\frac{1}{4} TeV \right)^2, \quad (9)$$

where the right hand side is the value determined by experiment. It is obvious that (6) makes sense only if $\Lambda < 3TeV$.

The effective theory (5) has been used, in conjunction with various assumptions on the strong interaction dynamics (the dots in the first line) to predict multi- W_L, Z_L production rates at the SSC and LHC. The production mechanism in that case is a V_L, V_L fusion process that is allowed⁷ by the kinematics of the bremsstrahlung of longitudinally polarized bosons from fermions. There is an infrared divergence at small momentum transfer that gives an order E_V/m_V enhancement to the process $f \rightarrow f + V_L$ which exactly compensates the order m_V/E_V suppression implied by the equivalence theorem (4):

$$g_{ffV_L} = g_{ff\pi} + O(m_V/E_V) = g \frac{m_f}{m_V} + O(m_V/E_V). \quad (10)$$

In the following I adopt a similar approach to the calculation of S, T .

STRONG RESCATTERING CORRECTIONS TO S

The S parameter is determined as

$$\frac{g^2}{16\pi} S = \frac{g^2}{16\pi} (S_\rho - S_a) = \frac{\partial}{\partial p^2} (\Pi_\rho^W - \Pi_a^W) \Big|_{p^2=0}, \quad (11)$$

where $\Pi_{\rho,a}$ is the vacuum polarization due to the vector (V) and axial vector (A) components, respectively of the pionic currents $J^\pm = V^\pm + A^\pm$ that couple to $W^\pm$ as in the second line of (5). Assuming a once subtracted dispersion relation, one finds

$$S_i = \frac{1}{2\pi} \int_{m^2}^{\Lambda_i^2} \frac{dt}{t^2} \sum_F |\mathcal{M}_i(t^2)|^2, \quad (12)$$

where $\mathcal{M}_i(t)$ is the transition amplitude from a virtual V_T with squared momentum $p^2 = t$ to a final state state F_i that is an on-shell collection of pions with quantum numbers (as the labels suggest) $I = J = 1$, $C = P = -1$ for $i = \rho$, and $I = J = 1$, $C = P = -1$ for $i = a$, and $\sum_F$ implies the sum over all such final states. To turn this result into a number requires assumptions on the dynamics of the strongly coupled sector, as I will illustrate with two examples.

1) The lowest mass contributions to $\mathcal{M}_{\rho,a}$ are from 2π and 3π intermediate states, respectively. In the case that there are no resonances below the effective cut-off, these states may dominate the dispersion integrals, since phase space considerations together with low energy theorems imply that

$$\left| \frac{\mathcal{M}(n\pi + 1)}{\mathcal{M}(n\pi)} \right| \sim \epsilon(t) = \left(\frac{\sqrt{t}}{16\pi v} \right)^2 \approx \left(\frac{\sqrt{t}}{3TeV} \right)^2 \leq \epsilon(\Lambda).$$

If this is the case, $\mathcal{M}_i$ is determined by an Omnès equation¹³:

$$|\mathcal{M}_i(t)| = t C_i g^2 \exp \left(\frac{t}{\pi} P \int \frac{du}{u} \frac{\delta_i(u)}{u-t} \right), \quad (13)$$

where the constants C_i are determined by phase space:

$$C_\rho = \frac{1}{96\pi} = \frac{64\pi^2}{3} C_a,$$

and δ_i is the elastic scattering phase shift in the appropriate channel. The values of $\delta(u)/(u)|_{u=0}$ and

$$\mathcal{M}_i(t)/t|_{t=0} = g^2 C_i$$

are fixed by low energy theorems that follow from the chiral $SU(2)$ symmetry of the effective theory. Extrapolation of the low energy theorems up to the effective cut-off gives

$$S = S_{NL}^{(1)} \left(1 + O[\epsilon^2(\Lambda^2)] \right) = \frac{1}{12\pi} \ln(\Lambda^2/m^2) \left(1 + O \left[\frac{\Lambda}{3GeV} \right]^4 \right) \sim (.1 \text{ to } .2) \quad (14)$$

for $\Lambda = (1 \text{ to } 3TeV)$, that is, the corrections to the one-loop result (7) are less than order unity since consistency require $\Lambda < 3TeV$, $\epsilon(\Lambda) < 1$.

2) As a second example, assume that the dispersion integrals in (12), (13) are dominated by a single resonance. Then

$$S = \frac{1}{12} \left(\frac{m_\rho}{\Gamma_\rho} - \frac{3}{64\pi^2} \frac{m_a}{\Gamma_a} \right) \sim .4, \quad (15)$$

where m_i, Γ_i are the masses and widths of the resonances in the relevant channels, and the number on the right hand side of (15) corresponds to scaling the QCD values by the factor $v/f_\pi \approx 250 \text{ GeV}/90 \text{ MeV}$. This can give a value slightly larger than the result (14).

Contributions to S in (possibly?) realistic technicolor models have been studied in ref. 14. The scaling law from QCD to the electroweak scalar sector is generally more complicated than the simple ansatz used above. It depends on the number of technicolors (generally greater than 3) and the number of techniflavors (generally greater than 2). There are more pseudoscalars in addition to the “eaten” pions and the prediction for S is in general larger than the experimental result⁵, which is compatible with zero but slightly favors a negative value. A negative value for S is hard to come by in the class of models considered here, although a scenario that could give this has been found¹⁵ in the context of “walking technicolor”.

STRONG RESCATTERING CORRECTIONS TO T

The T parameter is determined as

$$\alpha T = \frac{1}{m_W^2} \left(\Pi^W - \cos^2 \theta \Pi^Z \right) \Big|_{p^2=0} = \frac{1}{m_W^2} \left(\Pi_t^W - \cos^2 \theta \Pi_t^Z \right) \Big|_{p^2=0}, \quad (16)$$

where $\Pi_t^{W,Z}$ is the vacuum polarization due to intermediate states with one transversely polarized vector boson and a system of pions. The use of Ward identities that follow from the chiral symmetry of the lagrangian (2) or (5) allows us to reexpress (16) as⁴

$$\alpha T = \frac{1}{p^2} \left(\Pi^{\pi^+} - \Pi^{\pi^0} \right) \Big|_{p^2=0}, \quad (17)$$

i.e., in terms of the difference of charged and neutral pion propagators. Note

that since the strong interaction lagrangian in (5) is isospin invariant, a nonzero value for (17) can arise only if a transversely polarized vector boson is present in the loop, so it is of order g^2 . Using (17), one only has to consider transitions permitted at lowest order in g by the couplings in the second line of (5), *i.e.* $\pi \rightarrow V_T + n\pi \rightarrow \pi$. Kinematics restrict the quantum number of the intermediate $n\pi$ states to $J = 0$, and the quantum numbers of the currents J restrict the I, C, P quantum numbers. Thus we obtain⁴

$$T = -T_\pi + T_\sigma + T_{I=2} + O(g^2). \quad (18)$$

In fact, only the first term occurs at order 1 (order g^2 in αT) in the nonlinear effective theory defined by (5). This corresponds to a $\pi + V_T$ intermediate state. If there is no resonance in the $I = 1$, $C = -P = +1$ channel, then the rescattering correction to the one loop result (7) gives a correction of order $\epsilon(\Lambda^2) < 1$. The other terms are formally of order g^2 and arise from higher dimension operators, such as the last term in (5), and are scaled accordingly by inverse powers of the effective cut-off. These contributions can be expressed by dispersion relations:

$$T_i = \frac{1}{2\pi} \int_{m^2}^{\Lambda_i^2} \frac{dt}{t} \sum_F |\mathcal{M}_i(t^2)|^2, \quad (19)$$

where now $\mathcal{M}_i(t)$ is the transition amplitude from a virtual V_T with squared momentum $p^2 = t$ to a virtual V_T + a pionic final state state F_i that is an on-shell collection of pions with the quantum numbers $J = 0$, $I = 1$, $C = -P = 1$ for $i = \pi$; $J = 0$, $I = 0$, $C = P = 1$ for $i = \sigma$, *etc.* and again $\sum_F$ implies the sum over all such final states. I will evaluate T_σ in two extreme cases analogous to the examples used above.

- 1) The lowest mass contribution to $\mathcal{M}_\sigma$ is from the 2π intermediate state.

Assuming this state dominates the dispersion integral, the amplitude is determined by an Omnès equation similar to (13), and one obtains⁴

$$T_\sigma \approx C_\sigma M^{-4} \int_{m^2}^{\Lambda^2} dt t^2 \ln(\Lambda^2/t) \exp\left(\frac{t}{\pi} P \int \frac{du \delta_\sigma(u)}{u(u-t)}\right), \quad (20)$$

where M is the mass parameter appearing in the last term in (5), the constant

$$C_\sigma = \frac{9}{512\pi^3 v^2 \cos^2 \theta}$$

is determined by phase space, and δ_σ is the elastic scattering phase shift in the $I = J = 0$ channel with $\delta_\sigma(u)/(u)|_{u=0}$ fixed by low energy theorems. Assuming there is no resonance, extrapolation of the low energy theorems up to the effective cut-off gives a correction of order $\epsilon(\Lambda^2)$ to the result obtained for $\delta_\sigma \rightarrow 0$, namely

$$T_\sigma \rightarrow \left(\frac{\Lambda}{M}\right)^4 \left(\frac{\Lambda}{m_H}\right)^2 T_{SM}^{(2)}. \quad (21)$$

Here $T_{SM}^{(2)}$ is the two-loop Higgs contribution¹⁰ in the standard model, which grows quadratically with the Higgs mass and becomes comparable to the one-loop contribution (6) for $m_H \approx 10 TeV$. We see that the nonlinear formulation approximately recovers the standard model two-loop result if M and Λ are identified with the Higgs mass m_H .

2) If the integral over $\delta_\sigma(u)$ in (20) is dominated by a single resonance, we obtain⁴

$$T_\sigma = \frac{3}{16\cos^2 \theta_w} C'_\sigma \ln(\Lambda^2/m_\sigma^2), \quad C'_\sigma = \left(\frac{m_\sigma}{M}\right)^4 \left(\frac{m_\sigma}{m_H}\right)^3 \left(\frac{\Gamma_H}{\Gamma_\sigma}\right), \quad (22)$$

that is, we recover the one loop result of (6) if we identify σ with the Higgs particle, and set $M = m_H$ as in (5). The difference between cases 1) and 2) in

the context of the standard model is simply whether the resonance occurs above or below the scale at which the effective pion theory ceases to be valid. If there are new degrees of freedom that can contribute to S, T (and U) above that scale, their high energy contributions must be evaluated separately. If this is not the case, but the true theory is finite and renormalizable (*e.g.*, the standard model, albeit with a strongly coupled Higgs sector), then the full result for T must be finite. Neglecting the $I = 2$ channel and $O(\epsilon^2)$ corrections, this requires $C' = 1$, or more generally that the coefficient of $\ln \Lambda^2$ in (20) be equal and opposite to that in T_π , Eq.(7). In this case we get simply

$$T = -\frac{3}{\cos^2 \theta} \ln(\langle t \rangle / m^2), \quad (23)$$

that is, the modification of the standard model result amounts to replacing m_H^2 in (6) by the weighted average of t in the integral (20). Since $\sqrt{\langle t \rangle} < \Lambda < 3TeV$, the conclusions of the standard analyses⁵ remain essentially unchanged, since it is the ρ parameter that most severely constrains¹⁶ the top quark mass.

CONCLUSIONS

The examples considered show that the effects of strong rescattering on the analysis of precision measurements of electroweak parameters are not large enough to alter present constraints on the top quark mass and are within present experimental errors. On the other hand the deviation from the standard model prediction

$$S + \frac{4\cos^2 \theta}{9} T = 0 \quad (24)$$

could be as large as .2-.3, *i.e.* of the same order as the values of these parameters. This is expected, for example, in technicolor models¹⁴. To establish the presence

of such a deviation would require a measurement of the top quark mass to within a few GeV and a factor 10 improvement in the precision measurements of low energy parameters. Should such a discrepancy show up, it could be attributed either to effects of the type considered here, or to new high energy degrees of freedom (or both). This ambiguity could (in principle, at least) be resolved by measuring V_L, V_L scattering cross sections at the SSC (or some super SSC) via the fusion process

$$pp \rightarrow pp + X + V_L V_L. \quad (25)$$

Measurements of scattering phase shifts and/or resonance parameters would determine the integrands in the dispersion integrals up to some energy. This would allow an inference as to whether the measured parameters S, T are consistent with scattering data.

I have considered only the so-called “oblique” class of radiative corrections, that is, corrections to the propagators, or vacuum polarization. Strong rescattering effects on “non-oblique” corrections are expected to be suppressed by additional powers of g^2 or m_f/m_W , where f is an external fermion. This is because to lowest order these corrections entail the coupling of V_L to an external fermion. This expectation could be incorrect if the Feynman integral contains an infrared divergence that results in a kinematic factor $|p_V|/m_V$ that overcomes the suppression (10) as in the case of the process (25). This question is under study¹⁷.

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