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STABILITY IN STRAIGHT STELLARATORS

BY

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Stability in Straight Stellarators

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Abstract

The stability of the straight stellarator against localized interchange modes is investigated employing the Mercier-Greene-Johnson criterion. Critical values of β are obtained both numerically and analytically. The conclusion is that for classical helical stellarators the average limiting β 's are quite low of order three to four percent.

Because of the recently revived interest in stellarators produced by the favorable results on Wendelstein VII-A,¹ it is of interest to reinvestigate the various limits on β for plasmas confined in stellarators.

Since there is nominally no current in the stellarator, the principal instability limiting β should be the interchange. Further, because of transport considerations it is generally agreed that stellarators must have very large aspect ratios. Therefore, as a starting point in stability investigations, we feel it is important to first establish stability limits for straight stellarators.

Analytic stability results already exist for stellarators with infinitesimally strong helical windings. These are characterized by the small parameter δ where the bounding magnetic surface of the plasma is given by

$$r = a[1 + \delta \cos(l\theta + hs)] \quad (1)$$

in cylindrical coordinates (r, θ, s) and l characterizes the number of helical windings. The critical β for stability is an infinitesimal of order δ^2 , and in fact for $l = 3$ and $ha \ll 1$, $\beta_c = \delta^2$ where β_c is the critical value of the plasma evaluated on the magnetic axis.² From this analytic theory there was no way to correctly estimate the possible size of δ , so no real estimate of β_c could be obtained. However, a value of $1/3$,

corresponding to an equilateral triangle, for Eq. (1) had been suggested leading to a value of $1/9$ or 10% for β_c .

Using computers it is now possible to determine values of β for finite values of δ , and it will appear that $\delta < 1/3\sqrt{3}$ so $\beta_c < 1/27 \approx 4\%$. In order to carry out these finite δ corrections, we employ the Mercier-Greene-Johnson criterion for stability against localized modes in toroidal geometries to the straight stellarator with ends identified.³ This criterion can be written as

$$\begin{aligned}
 F = & \frac{\pi^2}{16\pi} \left(\oint \frac{d\ell}{B} \right)^2 + \pi' \oint \frac{d\ell}{B} \oint \frac{B^2}{|\nabla\psi|^2} [p'V'' - (\sigma - I')\chi'] \frac{d\ell}{B} \\
 & - 4\pi (p'V')^2 \oint \frac{1}{B^2} \frac{d\ell}{B} \oint \frac{B^2}{|\nabla\psi|^2} \frac{d\ell}{B} + 4\pi V'^2 \left[\left(\oint \frac{\sigma B^2}{|\nabla\psi|^2} \frac{d\ell}{B} \right)^2 \right. \\
 & \left. - \oint \frac{\sigma^2 B^2}{|\nabla\psi|^2} \frac{d\ell}{B} \oint \frac{\chi^2}{|\nabla\psi|^2} \frac{d\ell}{B} \right] > 0 .
 \end{aligned} \tag{2}$$

F is only defined for magnetic lines that close after going around the torus a finite number of times. It must be positive for an interchange instability localized about that line to be stable, and it must be positive on all such lines for the system to be totally stable. In F all primes denote derivatives with respect to ψ the toroidal flux, χ is the flux the short way. I is the toroidal current inside a magnetic surface ψ and $\sigma = \underline{j} \cdot \underline{B}/B^2$ denotes the parallel current. This criterion is derived for geometrically toroidal plasmas, but we may apply it to straight plasmas with ends identified as well. If we apply

this criteria to straight cylindrical plasmas, diffuse linear pinches, it reduces to precisely the Suydam criteria⁴ (see Appendix A).

We now consider straight stellarators with finite helical fields. Then a vacuum magnetic field may be given in terms of the magnetic potential ϕ by

$$\begin{aligned} B &= \nabla \phi, \\ \phi &= B_0 \left[s + \frac{\epsilon}{h} I_\ell (hr) \cos(\ell\theta + hs) \right]. \end{aligned} \quad (3)$$

More general vacuum fields may be obtained by summing over ℓ and h , but we restrict ourselves to a single term. A magnetic surface function ψ is obtained by

$$\nabla \phi \cdot \nabla \psi = 0. \quad (4)$$

Noting that ψ is a function of r and $u = \ell\theta + hz$, we have

$$\frac{\partial \psi}{\partial x} I'_\ell \epsilon \cos u - I_\ell \epsilon \frac{\ell^2}{2} \sin u \frac{\partial \psi}{\partial u} + (1 - I_\ell \epsilon \sin u) \frac{\partial \psi}{\partial u} = 0 \quad (5)$$

where $x = hr$ and the solution of Eq. (5) is

$$\psi \approx \frac{1}{2} x^2 - \epsilon I'_\ell x \sin u. \quad (6)$$

There is a separatrix of the magnetic surfaces when $\partial \psi / \partial x = \partial \psi / \partial u = 0$ or at $x = x_s$, $u = \pi/2$ where x_s is given by

$$\epsilon I_\ell \left(1 + \frac{\ell^2}{x_s^2} \right) = 1 . \quad (7)$$

The parameter δ , defined only for small values of ϵ by Eq. (1), may be extended to finite values by

$$\delta \equiv \frac{\epsilon I'_\ell}{x_o} (x_o) , \quad (8)$$

where x_o is related to Ψ on the outer surface by $\Psi = (1/2)x_o^2$.

From Eqs. (6) and (7) we can express Ψ in terms of x_s and equate it with $(1/2)x_o^2$. We then have for $x_s \ll 1$, and $\ell = 3$

$$x_o = \frac{1}{3^{1/2}} x_s , \quad c = \frac{16}{3x_s} . \quad (9)$$

Thus, from Eq. (8)

$$\delta = \frac{I'_\ell(x_o)}{x_o} \epsilon = \frac{1}{16} \frac{1}{3^{1/2}} \frac{16}{3} = \frac{1}{3\sqrt{3}} , \quad (10)$$

and hence the smallness parameter δ has the limit $1/3\sqrt{3}$ as $x_o \rightarrow 0$. In fact for nonzero values of x_o , δ is still smaller.

If we attempt to extend the small δ stability results to this maximum value of δ (assuming no other changes), we get $\beta_c = 1/27 = 3.7\%$, a value considerably smaller than used to be supposed for $\ell = 3$ stellarators.

In order to substantiate this smaller result for β , we apply the criterion of Eq. (2) to magnetostatic equilibrium

based on the vacuum configuration (3). Strictly speaking for finite p it is necessary to solve the magnetostatic equilibrium equations to determine the quantities entering into Eq. (2). However, because the critical β is supposed small, we may work to lowest order in the pressure and treat all the terms in Eq. (2) as vacuum terms with the exception of p and σ . The third and fourth terms are second order in the pressure and we drop them. The first term is the shear term, which is positive definite and balances the negative contributions of the second term. Finally I' must vanish for stellarators. The stability criterion thus reduces to

$$\frac{\lambda^2}{16\pi} \oint \frac{d\ell/B}{V'} > p' V'' \oint \frac{B^2}{|\nabla\psi|^2} \frac{d\ell}{B} - \oint \frac{B^2 \sigma}{|\nabla\psi|^2} \frac{d\ell}{B} . \quad (11)$$

The first term on the right represents the normal expansional energy of destabilization related to the component of the curvature along the pressure gradient. The second term is related to curvature in the surface, and is perhaps not so familiar. It is related to p' as follows.

Expanding the magnetostatic equilibrium equations about a vacuum field we have first that p must be a function of ψ alone. Next the current is determined by

$$\underline{j} = \frac{B \times \nabla \psi}{B^2} p' + \sigma \underline{B} . \quad (12)$$

Finally, σ must be determined to make $\nabla \cdot \underline{j} = 0$ or

$$\underline{B} \cdot \nabla \sigma = -p' \frac{\underline{B} \times \nabla \psi}{B^4} \cdot \nabla B^2. \quad (13)$$

Equation (13) is a magnetic differential equation that only uniquely determines σ up to a function of ψ alone. This function is then fixed so that the total current I' vanishes. I' is made up of contributions from the perpendicular, diamagnetic current in Eq. (12) as well as from the particular solution of Eq. (13). [See Appendix A, Eqs. (A-9) and (A-10) where for the pinch the contribution of $\underline{j}_\perp$ is clear.]

It is reasonably straightforward to carry out the solution for σ and the evaluation of the terms in Eq. (11) since they lead to ordinary differential equations and integrals in one variable on each magnetic surface (see Appendix B). Thus, for each surface we obtain an optimum p' and by integrating this p' we find a maximum pressure on axis represented by β_{peak} . Values of β averaged over the plasma volume $\langle \beta \rangle_2$ are given in Table 1 for various values of ϵ and x_0 , the mean radius of the outer surface. For comparison values of $\langle \beta \rangle_1$ are also listed that are obtained by including only the $p'V''$ term in Eq. (11). It is seen that the inclusion of the σ terms leads to some differences in the values of β but these differences are not very substantial for our case.

As a check on our results, it is of interest to compare them with small ϵ or δ results. Previously only the small χ ,

$l = 3$ result $\beta = \delta^2$ was available. By expanding Eq. (11) for small ϵ , more general results can be obtained. This expansion is carried out in Appendix C.

For $l = 3$ the small δ limit and the more exact numerical results $\langle \beta \rangle_2$ are compared in Fig. 1. It is seen that the agreement is perfect for small ϵ and not terribly bad over the entire range. Thus, the smaller values of β are produced essentially by the lower possible limit on δ , Eq. (10), rather than any breakdown in the small δ stability theory.

In conclusion we have done a more careful analysis of the stability of straight stellarators employing the Mercier-Greene-Johnson criteria. These results naturally extend the older known results but because of the limit on the amplitudes of the helical fields imposed by the separatrix, the values of stable β that one can obtain are disappointingly low, of order a few percent. Thus, it may be necessary to develop more exotic configurations if the stellarator is to become a practical device.

Acknowledgments

Helpful discussions with Dr. John L. Johnson are gratefully acknowledged. This work was supported by United States Department of Energy Contract No. DE-AC02-76-CHO-3073.

APPENDIX A

Reduction of the Stability Criterion for a Diffuse Linear Pinch

It is of interest to illustrate the stability criterion by applying it to a linear pinch for which all equilibrium quantities are independent of θ and z and depend only on r . We take the pinch of length L , which is the identification length. The equilibrium satisfies

$$j_{\theta} B_z - j_z B_{\theta} = \frac{dp}{dr} . \quad (A1)$$

We also have $d\psi = B_z 2\pi r dr$ so

$$v' = \frac{L}{B_z} , \quad (A2)$$

$$x' = \frac{L}{2\pi} \left(\frac{B_{\theta}}{r B_z} \right) , \quad (A3)$$

$$x'' = \frac{L}{(2\pi)^2 r B_z} \frac{d}{dr} \left(\frac{B_{\theta}}{r B_z} \right) , \quad (A4)$$

$$v'' = \frac{1}{2\pi r dr B_z} d \left(\frac{L}{B_z} \right) = - \frac{L}{2\pi r B_z^3} \frac{d}{dr} B_z , \quad (A5)$$

$$P' = \frac{dp}{dr} \left(\frac{1}{2\pi r B_z} \right). \quad (A6)$$

Now consider the evaluation of Eq. (1) on a "closed line" lying on the cylinder of radius r . Let the line close after passing the identification length N times. Then each integral $\oint dl/B$ has a constant integrand so each such factor gives a contribution

$$\oint \frac{dl}{B} = \oint \frac{dz}{B_z} = \frac{NL}{B_z} \quad (A7)$$

times its integrand.

Denote the first three terms of Eqs. (2) by I, II, III and note that the last two terms cancel. Then we have from Eqs. (A4) and (A7)

$$I = \frac{\kappa^2}{16\pi} \left(\oint \frac{dl}{B} \right)^2 = \frac{1}{4} \frac{N^2 L^4}{(2\pi r B_z)^4} \left[r \frac{d}{dr} \left(\frac{B_\theta}{r B_z} \right) \right]^2. \quad (A8)$$

In order to evaluate the second term we need $\sigma - I'$. By definition

$$B^2 \sigma = \underline{j} \cdot \underline{B} = j_\theta B_\theta + j_z B_z \quad (A9)$$

and $I' = j_z/B_z$. From this, Eqs. (A1) and (A9), we have

$$(\sigma - I') = \frac{B_\theta}{B^2 B_z} \frac{dp}{dr}. \quad (A10)$$

(Note that j_1 contributes to I' .) Then from Eqs. (A2), (A5), (A6), and (A10)

$$\begin{aligned}
 II &= v' \left(\oint \frac{d\ell}{B} \right)^2 \frac{B^2}{|\nabla\psi|^2} [p'v'' - (\sigma - I')] , \\
 &= \frac{L}{B_z} \left(\frac{NL}{B_z} \right)^2 \frac{B^2}{(2\pi r B_z)^2} \left[\frac{dp}{dr} \frac{1}{2\pi r B_z} \left(\frac{L}{B_z} \right) \frac{dB_z}{dr} \frac{1}{2\pi r B_z} \right. \\
 &\quad \left. - \frac{B_\theta}{B_z^2} \frac{dp}{dr} \frac{d}{dr} \left(\frac{B_\theta}{r B_z} \right) \frac{L}{(2\pi)^2 r B_z} \right] , \\
 &= \frac{N^2 L^4}{B_z^2} \frac{1}{(2\pi r)^4} \left[- \frac{B_\theta^2}{B_z^2} \frac{dB_z}{dr} - r B_\theta \frac{d}{dr} \left(\frac{B_\theta}{r B_z} \right) \right] \frac{dp}{dr} . \tag{A11}
 \end{aligned}$$

Making use of the equilibrium Eq. (A1) to re-express one of the p' factors in the third equation, we have

$$III = - \frac{N^2 L^4}{B_z^8 (2\pi r)^4} \left(- \frac{B_\theta}{B_z} \frac{dB_\theta}{dr} - \frac{dB_z}{dr} - \frac{B_\theta^2}{r B_z} \right) \frac{dp}{dr} . \tag{A12}$$

Combining Eqs. (A11) and (A12), we find

$$II + III = \frac{2N^2 L^4}{B_z^8 (2\pi r)^4} \frac{B_\theta^2}{r} \frac{dp}{dr} . \tag{A13}$$

Finally, from Eqs. (A8) and (A13)

$$I + II + III = \frac{N^2 L^4}{B_z^8 (2\pi r)^4} \left\{ \frac{B_z^4}{4} \left[r \frac{d}{dr} \left(\frac{B_\theta}{r B_z} \right) \right]^2 + \frac{2B_\theta^2}{r} \frac{dp}{dr} \right\} > 0 \quad (A14)$$

for stability. The first term in the bracket is positive, and the second term is negative. Thus, we have stability if

$$-\frac{dp}{dr} < \frac{1}{8} \frac{B_z^4}{B_\theta^2} r^3 \left[\frac{d}{dr} \left(\frac{B_\theta}{r B_z} \right) \right]^2, \quad (A15)$$

which is the standard Suydam criterion⁴ for stability.

This calculation is of interest for a number of reasons. First, it ties the Mercier-Greene-Johnson calculation for general stability to a well-known criterion indicating the role various stabilizing and destabilizing terms may play. For example, it throws some light on the variables σ and I' . (The diamagnetic current can contribute to the toroidal current, and if the latter is supposed to be zero, then the arbitrary constant in σ must be chosen to cancel this contribution as well as that due to j_z .)

Second it may be used to resolve certain paradoxes involving the relation of V'' to stability. For example, we may construct an unstable system with zero V'' as follows. Imagine that we have a long straight conductor along the z -axis carrying a current I_0 . Impose on this a small uniform B_z -field. Then from Eq. (A5) V'' is zero. But for this case Eq. (A15) becomes

$$-\frac{dp}{dr B_z^2} < \frac{1}{8} \frac{r^3}{B_\theta^2} \frac{d}{dr} \left(\frac{B_\theta}{r} \right). \quad (A16)$$

If we allow the pressure gradient (assumed small) to be balanced by modifying B_θ , then by choosing B_z small, Eq. (A16) gives instability. (Of course, the volume per flux the short way is not constant.) Then in the Mercier-Greene-Johnson criterion the σ and I' terms are responsible for the instability.

APPENDIX B

Numerical Methods

In solving $\psi(r,u) = \text{constant}$ numerically, we solved the partial differential equation directly using the Runge-Kutta method, rather than solving r versus u using the exact solution with Newton's method. In the numerical solution, the coordinates $x = hr$ and T (which is proportional to u) are used.

Once the parameters l and c are fixed, x and u are calculated typically at 21 different ψ 's labeled by x_0 . The quantity x_0 is for the equivalent circular radius. To determine the gradient, (x,u) are also calculated at one tenth of the interval of the magnetic surface, ddx_0 .

The differential area of the magnetic surface is calculated by simply summing $x \cdot ddx$. The step length of the angle u is kept constant and the total area is multiplied by 2π to obtain the differential area ds in the (r,θ) plane. If one multiplies this area ds by B_z , it becomes $d\psi$. Thus $ds/d\psi$ is proportional to $dV/d\psi$. This is calculated and $d^2V/d\psi^2$ is estimated by taking the difference in different magnetic surfaces. The quantity $B^2/|\nabla\psi|^2$ is then averaged in the $\oint dl/B$ sense. The $|\nabla\psi|^2$ is easily calculated because $\psi(r,u)$ is calculated at x_0 and $x_0 + ddx_0$. The rotational transform is calculated from the

exact expression ψ [Eq. (1)] and by knowing x could be expressed also as a function of u from the known expression (see Eqs. 5.39 and 5.41 of Ref. 5). Once the rotational transform is known, the shear can be calculated as the difference in rotational transform at neighboring magnetic surfaces. By balancing shear with the $V''p'$ term so that the energy integral becomes zero, we can calculate the critical $dp/d\psi$ or dp/dx_0 . By integrating dp/dx_0 , it is possible to obtain the pressure profile, hence average p , $\langle p \rangle$.

The correction of the parallel and perpendicular currents of Johnson-Greene (see Eq. 13) amounts to the calculation of $\nabla B^2 \cdot (\underline{B} \times \nabla \psi)$ and $\nabla \psi \times \underline{B}$. Since the numerical method is used to calculate the neighboring magnetic surfaces at the distance ddx_0 , $\nabla \psi$ can be calculated almost without any effort. Similarly the magnetic field is expressed analytically, and the gradient of magnetic field squared can be easily obtained numerically. Combining those terms, the correction to the β -limit is obtained without undue numerical difficulty. The length of the computer instruction is about 500-600 lines.

APPENDIX C

The Stability Criterion for Small ϵ , Arbitrary $h a$, and l

Since the equilibrium magnetic field of Eq. (3) is very near a straight field, V' and χ' can be evaluated by the method of averages as in the review article of Morozov and Solov'ev.⁵ First we observe that because the single stabilizing term of Eq. (2) involves χ'' which is proportional to ϵ^2 all the terms must be of order ϵ^4 in order to derive a stability criterion. This means p' must be of order ϵ^2 and all other factors such as V' , V'' , etc., may be evaluated for vacuum fields. Further, from Eq. (13) r can be shown to be at least of order ϵ^3 . Thus we need only keep the first term and the $p'V''$ part of the second term.

Let us write the vacuum field as

$$\underline{B} = \underline{B}_0 + \underline{\tilde{B}} . \quad (C1)$$

Then from Morozov and Solov'ev⁵ we have

$$\gamma' = \frac{L}{B_0} \left(1 + \frac{\overline{\tilde{B}^2}}{B_0^2} \right) \quad (C2)$$

where the bar denotes averages over one helical period. But for a single winding Eq. (3) gives

$$\underline{\tilde{B}}_r = B_0 h a I'_l \sin u , \quad (C3)$$

$$\tilde{B}_\theta = B_0 \frac{a}{r} \epsilon I_\ell(hr) \cos u, \quad (C4)$$

$$\tilde{B}_z = -B_0 h \epsilon I_\ell(hr) \cos u, \quad (C5)$$

so from Eqs. (C2) through (C5)

$$V'' = \frac{L}{B_0} + \frac{L}{2B_0} \frac{\delta^2 X^2}{[I'_\ell(X)]^2} \left[I'_\ell(x)^2 + \left(\frac{\ell^2}{x^2} + 1 \right) I_\ell^2 \right], \quad (C6)$$

$$\frac{dV'}{dr} = - \frac{L}{B_0} \frac{\delta^2 X^2}{I_\ell^2(X)^2} h \left[\frac{I'_\ell^2}{x} - 2 \left(\frac{\ell^2}{x^2} + 1 \right) I_\ell I'_\ell + \frac{\ell^2}{x^3} I_\ell^2 \right] \quad (C7)$$

where $X = ha$ and $x = hr$. Further, $\chi' = 1/2\pi$ where the rotational transform angle χ is given by⁵

$$\chi = \frac{L}{r} \frac{d\psi}{dr}, \quad (C8)$$

$$\psi = \frac{1}{hB_0^2} \frac{\tilde{B}_r \tilde{B}_\theta}{2} = \frac{\delta^2 a^2}{2} \frac{I_\ell(x) I'_\ell(x)}{r I'_\ell(X)}, \quad (C9)$$

so

$$\chi = \frac{\delta^2 L}{2} \frac{na^2}{r} \frac{\ell}{[I'_\ell(X)]^2} \left[I'_\ell^2 - \frac{2I_\ell I'_\ell}{x} + \left(\frac{\ell^2}{x^2} + 1 \right) I_\ell^2 \right]. \quad (C10)$$

Now reduction of F for a closed line on a surface of lowest order radius r gives to lowest order

$$F = \frac{N^2 L^2}{2\pi r B_0^4} \left[\frac{1}{16\pi} \left(\frac{d(1/2\pi)}{dr} \right)^2 + \frac{B_0}{2\pi r B_0^2} \frac{dp}{dr} \frac{dV'}{dr} L \right] \quad (C11)$$

where N is the number of identification lengths the line possess before it closes. Substituting Eqs. (C7) and (C10) into (C11) and carrying out the appropriate differentiation, we can determine the critical pressure gradient on each rational surface and, thus by continuity on every surface. This actual evaluation leads to a rather cumbersome analytic result which we do not give. It is easier to break it into steps. However, for $hX \ll 1$, $l = 3$ it is easy to see that the critical pressure gradient is given by

$$\frac{8\pi}{B_0^2} \frac{dp}{dr} = 2\delta^2 \frac{r}{a^2} \quad (C12)$$

so for stability the limit of the pressure at $r = 0$

$$\beta < \beta_{\max} \equiv \frac{8\pi p_{\max}}{B_0^2} = \delta^2 \quad (C13)$$

as quoted in the text.

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Table 1. Numerical Results for β

l	ϵ	x_0	$\langle \beta \rangle_1$	$\langle \beta \rangle_2$	$\left\langle \frac{1}{2\pi} \right\rangle_{\max}$
3	2.0	1.05	2.28	1.99	0.075
3	1.6	1.05	1.133		0.03
3	1.2	1.05	0.556		0.016
3	2.1	1.05	2.81	2.41	0.095*
3	0.3	2.1	0.380		0.008
3	0.59	2.1	2.91		0.042*
3	1.0	0.525	0.060	0.0618	0.0021
3	3.0	0.525	0.6285		0.0021
2	0.3	2.1	1.319	1.297	0.07
2	0.4	1.05	0.0624	0.0621	0.018
2	0.8	1.05	1.232	1.18	0.155

*Close to the separatrix.

Fig. 1 Plot of peak S versus ϵ ($i = 3$).