COMPUTER SIMULATION OF LOWER-HYBRID
HEATING IN TOKAMAKS

by

JOAN M. OGDEN

MASTER

There is no objection from the patent
point of view to the publication or
dissemination of the document(s)
referred to in this letter.

BROOKHAVEN PATENT GROUP

5/19/81 by Clee

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

COMPUTER SIMULATION OF LOWER-HYBRID

HEATING IN TOKAMAKS

by

Joan M. Ogden

Final Report for Contract Number: DE-AC01-80ER53106 ✓

April 29, 1981

There is no objection from the present
point of view to the publication or
dissemination of the document(s)
noted in this letter.

BROOKHAVEN PATENT GROUP

5/19/81 By CAC

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

JH8

TABLE OF CONTENTS

- I. INTRODUCTION
- II. PHYSICAL MODEL
 - A. Wave-Plasma Interaction
 - 1. Wave Propagation
 - 2. Wave Absorption
 - a. Electron and Ion Damping
 - b. Quasilinear Model of Electron Landau Damping
 - c. Comparison of Linear and Quasilinear Electron Landau Damping
 - 3. Quasilinear Current Drive
 - B. Transport Model
 - C. Including the Model in BALDUR
- III. NUMERICAL RESULTS
 - A. PLT Simulations
 - B. INTOR Reactor Simulations
- IV. SUMMARY AND CONCLUSIONS

I. INTRODUCTION

Lower hybrid heating is a promising source of auxiliary power and current for tokamaks. In lower hybrid experiments with significant heating, and particularly in proposed reactor designs, the absorption characteristics of the plasma change in time as the plasma heats. Effective heating requires an input spectrum of RF waves, which continue to penetrate and deliver power to the central plasma, as conditions change. We have studied how to maximize heating by adding a self-consistent lower hybrid model to the BALDUR 1-D tokamak transport code.^(1, 2)

A cylindrical model of wave propagation was used. An RF spectrum of about 50 "modes" was introduced at the plasma edge, and local electron and ion power absorption terms were computed for each mode as it travelled into the plasma. Linear⁽³⁾ and quasilinear⁽⁴⁾ models for electron Landau damping were included for comparison and the local quasilinear current was computed.

It was assumed that ion damping was negligibly small except near the linear mode conversion layer. A mode propagated into the plasma losing power to the electrons via linear or quasilinear Landau damping, until the linear mode conversion (LMC) layer was reached. At this radius any power remaining was given to the ions. We have not included several effects, which can be important under certain conditions, such as variation of $k_{\parallel}$, due to toroidal geometry, scattering of incident waves by density fluctuations, parametric instabilities and stochastic ion heating.⁽⁵⁻⁸⁾ Recent work^(7, 8) has shown that $k_{\parallel}$ variation has a large effect on wave penetration when the RF antenna is located on the inside or outside of the torus. However, for waves launched from the bottom of the torus, the variation in $k_{\parallel}$ is less pronounced, and the rays are

closer to the cylindrical approximation used in our model.

The code has been applied to simulate current PLT experiments ⁽⁹⁾ and the INTOR ⁽¹⁰⁾ reactor. For INTOR we have examined how to optimize the input spectrum for efficient heating of the plasma center. The predictions of linear and quasilinear theory were compared. Under certain conditions (high wave fields, very sharply peaked input spectrum), the quasilinear electron Landau damping was considerably less than the linear damping, allowing efficient direct heating of the ions and a 70%-reduction in the amount of power needed for ignition. However, for spectra which were only moderately peaked in $k_{\parallel}$, the quasilinear model predicted greater edge damping than the linear model. The quasilinear results suggest that it is necessary to have a narrow time-dependent "tuned" input spectrum for efficient ion heating. Simulations with optimized "tuned" RF input spectra indicate that ignition is possible with 25MW, using extrapolations of present transport coefficients. Our PLT simulations indicate that heating of about 2-3eV/kW of input power is likely over a range of densities $n = 1.6$ to $3 \times 10^{13} \text{ cm}^{-3}$. The quasilinear theory gives slightly more pessimistic results than the linear theory due to enhanced edge losses. In some cases, significant quasilinear currents were predicted.

II. PHYSICAL MODEL

A. Wave-Plasma Interaction

1. Wave Propagation

We have assumed that the waves propagate according to the local linear dispersion relation for electrostatic waves with $k_{\perp} \gg k_{\parallel} \neq 0$, $\omega_{ci} \ll \omega \ll \omega_{ce}$, in a warm plasma. ⁽¹¹⁾ We also take $n_{\parallel}^2 = (k_{\parallel} c / \omega)^2 > 1 + \omega_{pe}^2 / \omega_{ce}^2$ to satisfy accessibility. The linear dispersion relation was obtained by setting the real part of the dielectric function to zero.

$$K_r = (K_i - \alpha)k_i^2 + K_{ii}k_{ii}^2 = 0 \quad (1)$$

where:

$$K_i = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2}$$

$$\alpha = 3 \frac{\omega_{pi}^2}{\omega^2} \left(\frac{k_i v_{the}}{\omega} \right)^2 + \frac{3}{4} \frac{\omega_{pe}^2}{\omega_{ce}^2} \left(\frac{k_i v_{the}}{\omega_{ce}} \right)^2$$

$$K_{ii} = 1 - \frac{\omega_{pe}^2}{\omega^2} \approx - \frac{\omega_{pe}^2}{\omega^2}$$

$$v_{the,i} = \left(\frac{T_{e,i}}{m_{e,i}} \right)^{1/2}$$

The imaginary part of the dielectric function was

$$K_i = \pi \frac{\omega_{pe}^2}{k^2 v_{the}^2} \left. \frac{\partial F}{\partial w} \right|_{w = \omega/k_i v_{the}} + \text{ion damping term} \quad (2)$$

where $w = v_{ii}/v_{the}$ and F was the electron distribution function integrated over v_i . The function F was normalized so that $\int F(w)dw = 1$. The ion damping term in K_i was neglected except near the linear mode conversion (LMC) layer. According to this picture, waves travelled into the plasma until the LMC layer was reached; e.g., when

$$\omega_{pi}^2 = \frac{\omega^2}{1 - \frac{\omega^2}{\omega_{ce}\omega_{ci}} + \frac{k_{ii}v_{the}}{\omega} \left[12 T_i/T_e + 3 \left(\frac{\omega^2}{\omega_{ce}\omega_{ci}} \right)^2 \right]^{1/2}} \quad (3)$$

At this radius the lower hybrid wave coupled to a short wavelength ion mode which was quickly damped (Figure 1).

2. Wave Absorption

a. Electron and Ion Damping

The power absorbed via Landau damping from a mode with parallel wavenumber $k_{||}$ as it travels between radii a and r is given by the energy conservation equation^(12)

$$\nabla \cdot (v_{gr} W_k) = \frac{\partial W_k}{\partial t} \quad (4)$$

where:

$$W_k = \omega \frac{\partial K_r}{\partial \omega} \epsilon_k$$

$$\int \epsilon_k dk_{||} = \frac{E^2}{8\pi}$$

$$v_{gr} = \frac{\partial \omega}{\partial k_{||}}$$

If $\partial W_k / \partial t$ is due to dissipative losses, we have:

$$\frac{\partial W_k}{\partial t} = 2k_{i\perp} v_{gr} W_k \quad (5)$$

where:

$$k_{i\perp} = -K_i / \frac{\partial K_r}{\partial k_{||}} \quad (6)$$

Substituting for K_i and $\partial K_r / \partial k_i$, we obtain for the electron spatial damping factor

$$k_{i\perp} = \frac{\frac{\pi}{2} k_{\parallel} \left(\frac{m_i}{m_e} \right)^{1/2} \left(\frac{\omega}{k_{\parallel} v_{the}} \right)^2 \frac{\partial F}{\partial w}}{\left(\frac{\omega^2}{\omega_{LH}^2} - 1 \right)^{1/2}} \quad (7)$$

where:

$$\omega_{LH}^2 = \frac{\omega_{pi}^2}{1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}}$$

We have assumed that the ion contribution to K_i and $k_{i\perp}$ is zero except near the LMC point, where $k_{i\perp} \cdot l_{cm} \gg 1$. In cylindrical coordinates Equation 4 becomes

$$\frac{1}{r} \frac{\partial}{\partial r} (r v_{gr} W_k) = 2k_{i\perp} v_{gr} W_k \quad (8)$$

Defining the power flux due to a mode with $n_{\parallel}$ through a cylindrical surface $P_r(n_{\parallel}, r) = 2\pi R \cdot 2\pi r v_{gr} W_k$, we have

$P_r(n_{\parallel}, r) = P_r(n_{\parallel}, a) \exp \int_a^r 2k_{i\perp}(n_{\parallel}, r) dr$ where $P_r(n_{\parallel}, a)$ is the power flux from mode $n_{\parallel}$ at the edge of the tokamak. The power absorbed between the edge and radius r is

$$P_r(n_{\parallel}, a) \left[1 - \exp \int_a^r 2k_{i\perp}(n_{\parallel}, r) dr \right] \quad (9)$$

If P is the total input power at the edge integrated over $n_{||}$, define the input spectrum by a shape function $S(n_{||})$ such that $\int S(n_{||}) dn_{||} = 1$ and

$$P = \int P S(n_{||}) dn_{||} = \int P_r(n_{||}, a) dn_{||} \quad (10).$$

Equation (10) provides the boundary condition for P_r .

b. Quasilinear Model of Electron Landau Damping

When the wave fields are sufficiently large, the electron velocity space distribution function $F(w)$ can be modified near the resonant region of velocity space; e.g., when $w_1 \leq w \leq w_2$, where $w_1 = \omega/k_{||2} v_{the}$, $w_2 = \omega/k_{||1} v_{the}$, and $k_{||1}$ and $k_{||2}$ are the minimum and maximum parallel wavenumbers in the input spectrum (Figure 2). Collisions tend to remove any non-Maxwellian features. Fisch (4) has solved the Fokker-Planck equation for $F(w)$.

$$\frac{\partial F}{\partial \tau} = \frac{\partial}{\partial w} \left[D(w) \frac{\partial F}{\partial w} \right] + \frac{\partial F}{\partial \tau} \Big|_{coll} \quad (11)$$

In steady state, when collisions balance wave absorption,

$$F(w) = C \exp \left[- \int_0^w \frac{w' dw'}{I + w'^3 D(w')} \right] \quad (12)$$

where: $\tau = v_o t$

$w = v_{||}/v_{the}$

$D = D_{QL}/v_o v_{the}^2$

$$D_{QL} = \frac{8\pi^2 e^2}{m_e} \frac{\epsilon_k}{v_{the}^w} \left(\frac{k_{||}}{k} \right)^2, \quad w_1 \leq w \leq w_2$$

= 0, otherwise

$$v_0 = \frac{\omega_{pe}^4 (2 + Z_i) \ln \Lambda}{n_e v_{the}^3 4\pi}$$

and C is a normalization constant such that $\int_{-\infty}^{\infty} F(w)dw = 1$. We note that $F(w)$ is a Maxwellian when $D = 0$ (negligible wave fields) and flat in the limit of large fields, $w^3 D \gg 1$. When $w^3 D \gg 1$, which is satisfied for most cases of interest,

$F(w)$ is given by

$$\begin{aligned} F(w) &= C \exp\left(\frac{-w^2}{2}\right) & , w \leq w_1 \\ &C \exp\left(\frac{-w_1^2}{2}\right) & , w_1 \leq w \leq w_2 \\ &C \exp\left(\frac{-w_1^2}{2} + \frac{w_2^2}{2} - \frac{w^2}{2}\right) & , w_2 \leq w \end{aligned} \quad (13)$$

Integrating $F(w)$ over w gives an approximate value for C.

$$C = \frac{1}{(2\pi)^{1/2} + \exp\left(\frac{-w_1^2}{2}\right)(w_2 - w_1 + 1)} \quad (14)$$

The derivative is

$$\begin{aligned} \frac{\partial F(w)}{\partial w} &= -C w \exp\left(\frac{-w^2}{2}\right) & , w \leq w_1 \\ &-C \frac{1}{1 + w D(w)} \exp\left(\frac{-w_1^2}{2}\right) & , w_1 \leq w \leq w_2 \\ &-C w \exp\left(\frac{-w_1^2 + w_2^2 - w^2}{2}\right) & , w_2 \leq w \end{aligned} \quad (15)$$

Substituting Eq.(15) into Eq.(7), we find the quasilinear damping factor,

$$k_{i\perp QL} = \frac{\pi C \left(\frac{m_i}{m_e} \right)^{\frac{1}{2}} k_{\parallel}}{2 \left(\frac{\omega^2}{\omega_{LH}^2} - 1 \right)^{\frac{1}{2}}} \frac{w^3 \exp \left(\frac{-w_1^2}{2} \right)}{1 + w^3 D(w)} \bigg|_{w = \frac{\omega}{k_{\parallel} v_{the}}} \quad (16)$$

An expression for the dissipative losses can be written, using Eqs. (1), (5) and (16).

$$\frac{\partial W_k}{\partial t} = 2k_{i\perp} v_{gr} W_k = \frac{-\pi C \exp \left(\frac{-w_1^2}{2} \right)}{k_{\parallel}} n_e T_e v_o \quad (17)$$

The power absorbed is independent of the wave field strength in the limit of high fields, $w^3 D \gg 1$. Integrating Eq.(17) over $k_{\parallel}$ gives the total power absorbed.

$$\frac{\partial W_{TOT}}{\partial t} = \int \frac{\partial W_k}{\partial t} dk_{\parallel} = \pi C \exp \left(\frac{-w_1^2}{2} \right) \ln \left(\frac{w_2}{w_1} \right) n_e T_e v_o \quad (18)$$

which is Fisch's result (4).

c. Comparison of Linear and Quasilinear Electron Landau Damping

Let us compare the electron Landau damping of a wave according to linear and quasilinear theory. The linear attenuation factor for a Maxwellian velocity distribution is

$$k_{i\perp L} = \frac{\left(\frac{\pi}{8} \right)^{\frac{1}{2}} \left(\frac{m_i}{m_e} \right)^{\frac{1}{2}} k_{\parallel} w^3 \exp \left(\frac{-w^2}{2} \right)}{\left(\frac{\omega^2}{\omega_{LH}^2} - 1 \right)^{\frac{1}{2}}} \quad (19)$$

The ratio of linear and quasilinear attenuation is

$$k_{i\perp QL} = k_{i\perp L} \frac{\exp\left(\frac{-w_1^2 + w_2^2}{2}\right)}{1 + \bar{w}^3 D(\bar{w})} \frac{C}{\frac{1}{(2\pi)^{1/2}}} \quad (20)$$

where the normalization factor C of the quasilinear electron distribution function is approximately equal to the linear normalization factor $1/(2\pi)^{1/2}$ for typical ranges of w_1, w_2 . From Eq.(20) it can be shown that linear and quasilinear attenuation are about equal, when the spread in the input spectrum, $\Delta w = w_2 - w_1$, satisfies

$$\frac{\Delta w}{\bar{w}} \approx \frac{\ln [1 + \bar{w}^3 D(\bar{w})]}{\bar{w}^2} \quad (21)$$

where $\bar{w} = (w_2 + w_1)/2$ is the central value of w .

For PLT or INTOR parameters linear and quasilinear damping are comparable, when $\Delta w/\bar{w} \approx 20-40\%$. Quasilinear theory predicts considerably less damping than linear theory, when $\Delta w/\bar{w} \ll \ln [1 + \bar{w}^3 D(\bar{w})]/\bar{w}^2$; e.g., when the spectrum is very narrow. Conversely, for a sufficiently broad spectrum, $k_{i\perp QL}$ is greater than $k_{i\perp L}$. Quasilinear theory implies that a narrow intense input spectrum penetrates into the plasma more easily than a broad spectrum.

3. Quasilinear Current Drive

The quasilinear current is given by

$$J_{QL} = -ne v_{the} \int_{-\infty}^{\infty} w F(w) dw.$$

Substituting for $F(w)$ gives,

$$J_{QL} = -C \exp\left(\frac{-w_1^2}{2}\right) \left(\frac{w_2^2 - w_1^2}{2}\right) n_e^{ev} \text{the} \quad (22)$$

in the large field limit.

B. Transport Model

The BALDUR 1-D tokamak transport code ⁽²⁾ solves the radial equations

$$\frac{\partial n}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r D_i \frac{\partial n}{\partial r} \right) \quad (23)$$

$$\begin{aligned} \frac{3}{2} \frac{\partial (n T_e)}{\partial t} = & \frac{1}{r} \frac{\partial}{\partial r} r \left(K_e \frac{\partial T_e}{\partial r} + \frac{3}{2} T_e D_i \frac{\partial n}{\partial r} \right) \\ & + \frac{n c^2}{(4\pi r)^2} \left[\frac{\partial}{\partial r} (r B_\theta) \right]^2 + \text{Sources} - \text{Sinks} + W_{LHe} \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{3}{2} \frac{\partial (n T_i)}{\partial t} = & \frac{1}{r} \frac{\partial}{\partial r} r \left(K_i \frac{\partial T_i}{\partial r} + \frac{3}{2} T_i D_i \frac{\partial n}{\partial r} \right) \\ & + \text{Sources} - \text{Sinks} + W_{LHi} \end{aligned} \quad (25)$$

$$\frac{\partial B_\theta}{\partial t} = \frac{c^2}{4\pi} \frac{\partial}{\partial r} \left[\frac{n}{r} \frac{\partial}{\partial r} (r B_\theta) \right] - c \frac{\partial}{\partial r} (n J_{QL}) \quad (26)$$

where W_{LHe} and W_{LHi} are the electron and ion power density source terms from lower hybrid heating and J_{QL} is the quasilinear current.

Semi-empirical transport coefficients were used. The density diffusion

coefficient D_i and electron heat conductivity K_e were proportional to $1/n_e T_e^\alpha$, where α varied from 0 to 1 depending on the simulation. The ion heat conductivity K_i was the neoclassical value. In cases where $q(0) < 1$, we modelled MHD turbulence by setting the transport coefficient to 1% of the Bohm values. In some PLT simulations, a Carbon impurity was included. In these cases, the impurity transport was assumed to be neoclassical.

C. Including the Model in BALDUR

A spectrum of waves of frequency ω is launched from an array of waveguides at the plasma edge. The total power flux into the tokamak is P , and the shape of the power spectrum is defined by a function $S(n_{||})$, with $\int S(n_{||}) dn_{||} = 1$. In our simulations the spectrum is broken up into about 40 modes, and the power flux at $r = a$ for a mode with parallel index of refraction $n_{||}$ is

$$P_r(n_{||}, a) = P S(n_{||}) \Delta n_{||} \quad (27)$$

The function $S(n_{||})$ is input to the BALDUR code as a series of magnitudes. Alternatively, a time dependent input spectrum can be used, in which $S(n_{||}, t)$ is calculated at each time step from an approximation to Brambilla's theory (13). In this case, we take

$$S(n_{||}, t) = \frac{1}{\pi} \frac{N}{\left[n_{||c}(t) - n_{||} \right]^2 N^2 + 1} \quad (28)$$

where the number of waveguides N determines the width of the spectrum, and $n_{||c}(t)$ is the central value of the parallel index of refraction.

We either specify $n_{||c}(t)$ as input to the code or calculate $n_{||c}(t)$ using a feedback subroutine so that $n_{||c}(t)$ has its linear mode conversion point

at the plasma center at all times. We define the minimum and maximum values of $n_{||}$, where the power has dropped to 1% of its central value. Setting $S(n_{||,1,2})/S(n_{||,c}) = .01$, yields $n_{||,1,2} = n_{||,c} \pm \Delta n_{||}$, where $\Delta n_{||} = (99)^{1/2}/N$. In addition, we require $n_{||,1}^2 > 1 + \omega_{pe}^2/\omega_{ce}^2$, in order to satisfy the accessibility condition.

The power flux at radius r from mode $n_{||}$ is calculated by advancing Eq.(8) radially inward from $r = a$, using Eq.(27) as a boundary condition

$$P_r(n_{||}, r - \Delta r) = P_r(n_{||}, r) \exp \left[-2k_{i\perp}(n_{||}, r) \Delta r \right] \quad (29)$$

From Eqs.(12) and (16), we see that $k_{i\perp QL}$ depends upon the field strength ϵ_k through D and also upon the normalization constant C for the electron distribution function. We calculate $\epsilon_k(r)$ as a function of radius from Eqs. (4) and (29)

$$\begin{aligned} P_r(n_{||}, r) &= 2\pi R \cdot 2\pi r v_{gr} W_k \frac{\partial K}{\partial \omega} \frac{r}{\omega} \epsilon_k(r) \\ &= 2\pi R \cdot 2\pi r v_{gr} \omega \frac{\partial K}{\partial \omega} \frac{r}{\omega} \epsilon_k(r) \end{aligned} \quad (30)$$

$$\text{where } v_{gr} = \frac{\partial \omega}{\partial k_{i\perp}} = -\frac{c}{n_{||}} \frac{\omega_{LH}^2}{\omega^2} \left(\frac{m_e}{m_i} \right)^{1/2} \left(\frac{\omega^2}{\omega_{LH}^2} - 1 \right)^{3/2} \quad (31)$$

$$\text{and } \frac{\partial K}{\partial \omega} \frac{r}{\omega} = \frac{2}{\omega} \left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \right)$$

At each radius $F(w)$ is determined from Eq.(12) by integrating the exponent numerically. The function $F(w)$ is then normalized to obtain the

constant C . From $\epsilon_k(r)$, we find $D(n_{||}, r)$, and knowing C and the local plasma parameters we compute $k_{i\perp QL}(n_{||}, r)$, and the power absorbed from mode $n_{||}$ at r ,

$$\Delta P_r(n_{||}, r) = P_r(n_{||}, r) - P_r(n_{||}, r - \Delta r) \quad (32)$$

Equation (29) is then advanced another radial zone inward. The power absorbed goes to the electrons until the LMC point is encountered. At this point the rest of the power is absorbed by the ions. The total power density source terms in the transport equations (24) and (25) are found by summing over $n_{||}$

$$\begin{aligned} W_{LHe}(r) &= \sum_{n_{||}} \frac{\Delta P_r(n_{||}, r)}{2\pi R \cdot 2\pi r \cdot \Delta r} \Delta n_{||}, \quad r > r_{LMC}(n_{||}) \\ W_{LHi}(r) &= \sum_{n_{||}} \frac{P_r(n_{||}, r_{LMC})}{2\pi R \cdot 2\pi r_{LMC} \cdot \Delta r} \Delta n_{||} \end{aligned} \quad (33)$$

The quasilinear current is also calculated at each radius

$$J_{QL}(r) = -e n_e v_{the} \int_{-\infty}^{\infty} w F(w) dw \quad (34)$$

Alternatively, W_{LHe} and W_{LHi} could be calculated from Eq.(18) setting $W_{LHe} = \partial W_{KTOT} / \partial t$ for $r > r_{LMC}(n_{||c})$, and W_{LHi} equal to any unabsorbed power at $r = r_{LMC}(n_{||c})$, and J_{QL} could be found from Eq.(22).

This approach has been used in previous work ⁽¹⁴⁻¹⁶⁾ and is valid in the limit of high fields and a narrow spectrum. It does not take into account the change in ϵ_k and therefore in D and $k_{||}$ as the wave damps. As the modes with higher $n_{||}$ damp in the outer layer of the plasma, the spectrum narrows as the waves travel inward. Equations (18) and (22) neglect this change in $w_{||}$. Moreover, the approach we have used allows a more direct comparison between linear and quasilinear theory.

III. NUMERICAL RESULTS

A. PLT Simulations

We have simulated ion heating and electron heating experiments on

PLT.

1. PLT Ion Heating Simulations

The target plasma density and input spectrum were selected so that most of the energy was absorbed by the ions via linear mode conversion near the plasma center. The input spectrum is shown in Figure 3. The results are similar to our previous work in which a linear model of electron damping was employed⁽¹⁷⁾. Ion heating is effective over a range of plasma densities $n_e \approx 2.2-3.0 \times 10^{13} \text{ cm}^{-3}$. At higher densities edge heating occurs, at lower densities the waves are not absorbed. Quasilinear theory predicts slightly less ion heating than linear theory for this relatively broad spectrum because of enhanced electron losses at the edge. In Figures 4 and 5, we have plotted the central ion and electron temperatures $T_i(0)$ and $T_e(0)$ and the volume averaged temperatures $\langle T_i \rangle$ and $\langle T_e \rangle$ versus time for 0.5 and 1.0 MW of input power. We have chosen a target plasma with central density $n_e(0) = 2.5 \times 10^{13} \text{ cm}^{-3}$, which yields the most heating. The ion simulations are summarized in Table I, with linear results shown below for comparison. There is little difference between the results of the linear and quasilinear models. A small quasilinear current of $\approx 5\text{kA}$ is generated. An average ion temperature increase of approximately 2-3eV/kW of RF input power is achieved.

2. PLT Electron Heating Simulations

For the following set of PLT simulations, a lower density target plasma and input spectrum with higher n_e were chosen, so that the dominant energy absorption mechanism was electron Landau damping rather than linear mode conversion. The input spectrum is drawn in Figure 6. In Figures 7 and 8, the central and average electron and ion temperatures are shown as a function of time, for RF input powers of 0.5 and 1.0 MW. The results

of a case with $n_e(0) = 1.8 \times 10^{13} \text{ cm}^{-3}$ are listed in Table II. Comparing linear and quasilinear models, we see that the quasilinear theory predicts about the same increase in $\langle T_e \rangle$ but lower central electron temperatures, and greater edge losses. The increase in the volume averaged electron temperature is about 1.2eV/kW of input power. The quasilinear current is about 155kA for 0.5MW and 400kA for 1.0MW of input power. It should be possible to observe quasilinear current drive on PLT for high input power.

B. INTOR Reactor Simulations

1. Constraints on the RF Input Spectrum

The RF input spectrum must satisfy several constraints. First, the accessibility condition $n_{||}^2 > 1 + \omega_{pe}^2 / \omega_{ce}^2$ must be met in order for the waves to penetrate.

Secondly, adequate energy must be supplied to the central plasma ions over a broad range of temperatures as the reactor is brought to ignition. Ideally, this would be achieved by choosing the spectrum so as to maximize direct heating of the central ions, and minimize edge losses to the electrons. A less efficient method of igniting the plasma is to heat the central electrons, which transfer the energy collisionally to the ions. Let us examine these constraints for a standard case with parameters given in Table III.

a. Accessibility

The parallel index of refraction $n_{||}$ must satisfy the accessibility condition; e.g., $n_{||} > (1 + \omega_{pe}^2 / \omega_{ce}^2)^{1/2} = 1.16$ for the case in Table III.

b. Energy Supply to the Central Ions.

The ideal spectrum is one which undergoes linear mode conversion heating the ions at the plasma center, but loses little energy to the

electrons. Let us define $n_{i0}(T)$ to be the mode, which is converted at the plasma center when $T_i(0) = T_e(0) = T$. From Equation (3), we find,

$$n_{i0}(T) = \frac{\left[\frac{\omega^2}{\omega_{LH}^2(0)} - 1 \right] \frac{c}{v_{the}(0)}}{\left[12 + 3 \left(\frac{\omega^2}{\omega_{ce}\omega_{ci}} \right)^2 \right]^{\frac{1}{2}}} \propto \frac{1}{T^{\frac{1}{2}}} \quad (35)$$

In Figure 9, we have sketched $n_{i0}(T)$ versus T for several frequencies.

Waves with $n_{ii} > n_{i0}(T)$ have their LMC point nearer the plasma edge; those with $n_{ii} < n_{i0}(T)$ do not undergo mode conversion. If we require that the waves remain accessible as the plasma heats; e.g., $n_{i0}(T) \geq 1.16$, for $T \leq T_f = 20\text{keV}$, then the frequency must satisfy

$$\frac{\omega}{\omega_{LH}(0)} \geq \left[\left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \right)^{\frac{1}{2}} \left(\frac{T_f}{m_e} \right)^{\frac{1}{2}} \frac{1}{c} + 1 \right]^{\frac{1}{2}} \left[12 + 3 \left(\frac{\omega^2}{\omega_{ce}\omega_{ci}} \right)^2 \right]^{\frac{1}{4}} \quad (36)$$

For our standard case, $\omega \geq 1.34 \omega_{LH}(0)$. At a given central temperature T , modes with $n_{ii} \approx n_{i0}(T)$ are absorbed by the central ions, via linear mode conversion, provided $\omega \geq 1.34 \omega_{LH}(0)$.

We estimate electron Landau damping losses for these modes by checking to see where the spatial attenuation factor $2k_{i1}(n_{ii}, r)\Delta r$ in Eq.(29) becomes large. Define $n_{ie}(T)$ to be the lowest wavenumber for which electron Landau damping is significant in the center

$$2k_{i1}(n_{ie}(T), 0) \Delta r \approx 1 \quad (37)$$

Since the linear and quasilinear theories predict different attenuation factors, from Eqs. (16) and (19), we plot $n_{iL}(T)$ and $n_{iQL}(T)$ versus T .

The quasilinear curves were calculated for a narrow spectrum, $\Delta\omega/\bar{\omega} = 1/16$.

and 10, 25 and 100MW of input power. All modes with $n_{||} > n_{||e}(T)$ are electron damped in the plasma. Modes with $n_{||} < n_{||e}(T)$ do not suffer much attenuation. The values of $n_{||L}$, $n_{||QL}$ are insensitive to ω . We have taken $\omega = 1.34 \omega_{LH}(0)$.

Figure II combines the information in Figures 9 and 10 to give a range of $n_{||}$, which is not absorbed by the electrons, but heats the central ions. The linear model implies that wavenumbers which are mode converted in the center are also heavily electron damped. It appears that electron heating is the only possible ignition method in the context of the linear model. A broad range of wavenumbers $n_{||} = 3.5 \rightarrow 1.2$ is necessary to supply energy to the central plasma as it heats. The quasilinear results indicate that for a narrow spectrum, it is possible to heat only the ions. In order to stay in tune with the rapidly heating plasma, it would be necessary to sweep the central value of the narrow spectrum, following the linear mode conversion point $n_{||c}(t) = n_{||o}(T)$. For a broad spectrum quasilinear theory predicts higher electron losses than linear theory, and greater edge heating.

In summary, linear theory requires electron heating as the primary absorption method. Quasilinear theory implies that direct ion heating can be achieved with a narrow time-dependent spectrum.

2. Simulation Results

We have simulated ignition in INTOR using both the linear and quasilinear models. In Figure 12, we show the results of a linear simulation. The input spectrum, Figure 12(a), was fixed in time with a large spread of wavenumbers, so that adequate power was available as the temperature increased. Virtually all the wave energy was absorbed by the electrons, which then transferred the energy to the ions. Ignition was reached with 75MW of power. Figures 12(b) and (c) show $T_i(0)$, $T_e(0)$, $\langle T_i \rangle$ and $\langle T_e \rangle$ as a function of time. Figure 12 (d) is a 3-D plot of the ion temperature

profile $T_i(r,t)$ as a function of time. When this same broad spectrum case was run with the quasilinear model, electron edge losses were so severe that almost no heating of the main plasma occurred.

The results of a quasilinear simulation with a narrow time-dependent spectrum are illustrated in Figure 13. The input spectrum was calculated from Eq.(28), setting $n_{e,c}(t)$ to have its LMC point at the plasma center, and taking $N = 16$. The input spectrum is sketched at several times in Figure 13 (a). Ignition was easily reached for 25MW of input power. About 95% of the incident power was absorbed by the ions. Figures 13 (b), (c), and (d) are plots of $T_i(0)$, $T_e(0)$, $\langle T_i \rangle$, $\langle T_e \rangle$, and $T_i(r,t)$ as functions of time. As expected, the power required to ignite the plasma is substantially reduced, from ≈ 75 MW to 25 MW, when it is possible to heat the ions directly, rather than heating the electrons.

The question arises as to whether it would be more efficient to use a narrower time-dependent spectrum tuned to heat central electrons for the linear model. Simulations show that only a small decrease in the power required for ignition is realized, because most of the power needed to bring the plasma from $T = 5$ keV to 20 keV is already in a small range of $n_{e,c} \approx 1.5, 2.2$.

IV. SUMMARY AND CONCLUSIONS

We have added a simple quasilinear model of lower hybrid heating to the BALDUR 1-D tokamak transport code. The program was used to simulate PLT and the INTOR reactor design. For PLT a temperature increase of ≈ 2 keV/kW of RF input power was predicted. Quasilinear theory gave slightly less heating than linear theory, because of greater electron edge losses. For PLT ion heating cases, it will probably be hard to discern between linear and quasilinear theory experimentally. In experiments with mostly

electron heating, the predicted quasilinear current should be observed (e.g., $\geq 30\%$ for the Ohmic current) for input powers above 0.5MW. The electron temperature increase according to quasilinear theory is almost the same as the linear result.

For INTOR the linear and quasilinear theories gave very different results. Linear theory implied that electron Landau damping and subsequent collisional ion heating was the only effective method for heating the central plasma to ignition. A broad input spectrum was needed to cover changing absorption characteristics. About 75MW of RF power was required to reach ignition. According to the quasilinear model, direct ion heating was possible for a narrow time-dependent spectrum, which was tuned so that mode conversion occurred near the plasma center. The power necessary to ignite the plasma was reduced to about 25MW. For a broad spectrum, quasilinear theory predicted more edge electron losses and less main plasma heating than linear theory.

ACKNOWLEDGEMENTS

It is a pleasure to acknowledge useful conversations with Drs. E. Valeo, S. Bernabei, N. Fisch, C. Karney, C. Singer, and D. Post.

REFERENCES

1. D. Duchs, D.E. Post, P.H. Rutherford, Nucl.Fus., 17, 565, (1977).
2. D.E. Post, C.E. Singer, A.M. McKenney and the PPPL Transport Group, Working Draft of the BALDUR Documentation, TFTR Physics Group, Report #33, PPPL, Jan. 1981.
3. S. Bernabei and J. Fidone, Third Symposium on Plasma Heating in Toroidal Devices, Varenna, Sept.6-17, (1976), 92...
4. N.J. Fisch, Phys. Rev. Lett., 41, 873, (1978).
5. C.F.F. Karney, Phys.Fluids, 22, (1979).
6. A.H.Glasser, "Toroidal Mode Structure of Lower Hybrid Waves", Auburn U. Report, PR-3, to appear in Phys.Rev.Lett., (1981).
7. P.T. Bonoli and E. Ott, U. of MD, Plasma Preprint PL #81-017, (1981).
8. D.W. Ignat, "Toroidal Effects on Propagation, Damping and Linear Mode Conversion of Lower Hybrid Waves", PPPL- #1843, (1980).
9. S. Bernabei, W. Hooke, private communication.
10. INTOR Design Workshop Proceedings (1979).
11. T.H. Stix, Phys.Rev.Lett., 15, 878, (1965).
12. T.H.Stix, The Theory of Plasma Waves, McGraw-Hill:New York, (1962).
13. M. Brambilla, Nucl.Fus., 16, 47, (1976).
14. D.A. Ehst, Nucl.Fus., 19, 1369, (1979).
15. J. Brooks, et.al., "A Lower Hybrid Heating System for an Ignition Tokamak", Argonne Nat'l Labl, ANL/FPP/TM - 103, (1978).
16. R. Englade, P.T. Bonoli, T. Antonsen, M. Porkolab, Bull.Am.Phys.Soc., 25, 1002, (1980).
17. J.M. Ogden and S. Bernabei, Nucl.Fus., 20, 909, (1980).

TABLE I.
PLT ION HEATING SIMULATIONS

Plasma Parameters

$$D^+, I = 450\text{kA}, B_T = 32\text{kG}$$

$$n_e(0) = 2.2 \times 10^{13} \text{ cm}^{-3}$$

RF Power Parameters

$$f = 0.8 \text{ GHz}$$

$$P = 0.5, 1.0\text{MW}$$

$$\Delta t = 0.2\text{s}$$

		Before RF $t = 0.1\text{s}$				After RF $t = 0.3\text{s}$					
Power (MW)		$T_i(0)$ (eV)	$\langle T_i \rangle$ (eV)	$\langle T_e \rangle$ (eV)	OH (kW)	$T_i(0)$ (eV)	$\langle T_i \rangle$ (eV)	$\langle T_e \rangle$ (eV)	OH (kW)	RF (kW)	J_{LH} (kA)
0.5	QL	754	431	865	470	3458	1729	961	278	454	3.73
	L	754	431	865	470	3788	1767	951	277	450	-
1.0	QL	760	427	867	459	4091	2514	1058	239	858	5.50
	L	750	431	857	486	4453	2600	1058	245	877	-

TABLE II

PLT ELECTRON HEATING SIMULATIONS

Plasma Parameters

$$D^+, I = 450\text{kA}, B_T = 32\text{kG}$$

$$n_e(0) = 1.8 \times 10^{13} \text{ cm}^{-3}$$

RF Power Parameters

$$f = 0.8\text{GHz}$$

$$P = 0.5, 1.0\text{MW}$$

$$\Delta t = 0.2\text{s}$$

Before RF t = 0.1s					After RF t = 0.3s						
Power (MW)		T _e (0) (eV)	<T _e > (eV)	<T _i > (eV)	OH (kW)	T _e (0) (eV)	<T _e > (eV)	<T _i > (eV)	OH (kW)	RF (kW)	J _{LH} (kA)
0.5	QL	1615	898	312	449	2678	1424	280	178	396	155
	L	1615	898	312	449	3178	1379	287	174	352	-
1.0	QL	1615	898	312	449	3134	2110	265	118	891	400
	L	1615	898	312	449	3942	2155	252	115	849	-

TABLE III
INTOR REACTOR PARAMETERS

PLASMA

Major radius (m), R	6.0
Plasma radius (m), a	1.5
Aspect ratio, A	3.9
Elongation, κ	1.5
Av. Ion Temperature (KeV), $\langle T_i \rangle$	10.0
Av. DT Ion Density (10^{-14}cm^{-3}), $\langle n_i \rangle$	0.7
Effective Charge, Z_{eff}	1.0
B_T on Axis (T)	5.0
Safety Factor, q	3.0
Burn Av. $\langle \beta_T \rangle$ (%)	6.0
Plasma Current (MA), I_p	5.6
Edge Toroidal Field Ripple (%) (peak to peak average)	0.75

FIGURES

- Fig. 1 Wave path in the tokamak.
- Fig. 2 Quasilinear electron distribution function $F(w)$.
- Fig. 3 Input spectrum for PLT ion heating simulations.
- Fig. 4 PLT ion heating simulation. $T_i(0)$, $T_e(0)$, $\langle T_i \rangle$, $\langle T_e \rangle$ vs. time for 0.5MW of input power.
- Fig. 5 PLT ion heating simulation $T_i(0)$, $T_e(0)$, $\langle T_i \rangle$, $\langle T_e \rangle$ vs. time for 1.0MW of input power.
- Fig. 6 Input spectrum for PLT electron heating simulations.
- Fig. 7 PLT electron heating simulation. $T_e(0)$, $T_i(0)$, $\langle T_e \rangle$, $\langle T_i \rangle$ vs. time for 0.5MW of input power.
- Fig. 8 PLT electron heating simulation. $T_e(0)$, $T_i(0)$, $\langle T_e \rangle$, $\langle T_i \rangle$ vs. time for 1.0MW of input power.
- Fig. 9 Linear mode conversion curves for several frequencies, $n_{r0}(T)$ vs. T for $\omega = 1.1 \omega_{LH}(0)$, $1.3 \omega_{LH}(0)$, $1.7 \omega_{LH}(0)$.
- Fig.10 Electron Linear and Quasilinear Landau damping curves. $n_{rL}(T)$, $n_{rQL}(T)$ vs. T .
- Fig.11 Range of Effective Wavenumbers for Linear and Quasilinear Models.
- Fig.12 INTOR Simulation. Linear Model. Broad Spectrum.
- Fig.13 INTOR Simulation. Quasilinear Model. Time-Dependent Narrow Spectrum.

FIG. 1. Wave path in the tokamak

FIG. 2 Quasilinear electron distribution function $F(w)$

FIG. 3. Input spectrum for PLT ion heating simulations.

FIG. 4 PLT ion heating simulation. $T_i(0)$, $T_e(0)$, $\langle T_i \rangle$, $\langle T_e \rangle$ vs. time for 0.5MW of input power

FIG. 5 PLT ion heating simulation. $T_i(0)$, $T_e(0)$, $\langle T_i \rangle$, $\langle T_e \rangle$ vs. time for 1.0MW of input power.

FIG. 6. Input spectrum for PLT electron heating simulations.

FIG. 7 PLT electron heating simulation. $T_e(0)$, $T_i(0)$, $\langle T_e \rangle$, $\langle T_i \rangle$ vs. time for 0.5MW of input power.

FIG. 8 PLT electron heating simulation. $T_e(0)$, $T_i(0)$, $\langle T_e \rangle$, $\langle T_i \rangle$ vs. time for 1.0MW of input power.

FIG. 9 Linear Mode Conversion curves, $n_{||0}(T)$ vs. T for $\omega = 1.1 \omega_{LH}(0)$, $1.34 \omega_{LH}(0)$, $1.7 \omega_{LH}(0)$.

FIG. 10 Electron linear and quasilinear damping curves. $n_{||L}(T)$, $n_{||QL}(T)$; vs. T for $\omega = 1.34 \omega_{LH}(0)$.

FIG. 11 Range of effective wavenumbers for linear and quasilinear models.

FIG. 12a Input Spectrum. INTOR Linear Simulation

FIGS. 12 b,c. INTOR simulation. $T_i(t)$, $T_e(t)$, $\langle T_i \rangle$, $\langle T_e \rangle$ vs. time, linear model, 75MW input power.

FIG.12d. INTOR linear simulation. $T_i(r, t)$ vs. radius and time.

FIG. 13a. INTOR quasilinear simulation. Time dependent input spectrum at various times.

FIG. 13 b,c. INTOR quasilinear simulation. $T_i(0)$, $T_e(0)$, $\langle T_i \rangle$, $\langle T_e \rangle$ vs. time for 25MW input power.

FIG. 13d. INTOR quasilinear simulation. $T_i(r, t)$ vs. radius and time.