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ION CYCLOTRON EMISSION CALCULATIONS USING A 2D FULL WAVE NUMERICAL CODE*

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ABSTRACT

Measurement of radiation in the HF band due to cyclotron emission by energetic ions produced by fusion reactions or neutral beam injection promises to be a useful diagnostic on large devices which are entering the reactor regime of operation.¹ A number of complications make the modelling and interpretation of such measurements difficult using conventional geometrical optics methods. In particular the long wavelength and lack of high directivity of antennas in this frequency regime make observation of a single path across the plasma into a viewing dump impractical. Pickup antennas effectively see the whole plasma and wall reflection effects are important. We have modified our 2D full wave ICRH code² to calculate wave fields due to a distribution of energetic ions in tokamak geometry. The radiation is modeled as due to an ensemble of localized source currents distributed in space. The spatial structure of the coherent wave field is then calculated including cyclotron harmonic damping as compared to the usual procedure of incoherently summing powers of individual radiators. This method has the advantage that phase information from localized radiating currents is globally retained so the directivity of the pickup antennas is correctly represented. Also standing waves and wall reflections are automatically included.

INTRODUCTION

A 2-D finite difference code has been developed to solve the full wave ICRF problem in tokamak and stellarators geometry.^{2,3} the basic equation is

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} - \frac{\omega^2}{c^2} \vec{E} - \frac{4\pi i \omega}{c^2} \sum_s \vec{J}_s = \frac{4\pi i \omega}{c^2} \vec{J}_{\text{ext}} \quad (1)$$

Where the plasma current due to species s , $\vec{J}_s$, is obtained from the warm plasma conductivity expanded to lowest significant order in $(k_{\perp} \rho_i)^2$, $\vec{J}_s = \sum_s \vec{\sigma}_s$

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$(\ell, k_{\perp}, k_{\parallel}) \cdot \vec{E}$ where ℓ = harmonic number $k_{\perp}$ is obtained from the local dispersion relation and $k_{\parallel}$ is approximately k_z . The source current $\vec{J}_{\text{ext}}$, normally taken to be the antenna current, is now the random, fluctuating current due to the radiating particles distributed throughout the plasma. We assume the equilibrium to be uniform in z and Fourier analyze in that direction. Since $\omega \ll \omega_{pe}$ we assume $E_{\parallel} = 0$ which reduces (1) to two-coupled second order equation for components of $\vec{E}$ in the $\vec{\nabla}\psi$ and $\vec{B} \times \vec{\nabla}\psi$ directions which are solved in a poloidal cross section, $\vec{x}_{\perp}$. The primary task now is to determine $\vec{J}_{\text{ext}}$ in terms of the assumed distribution of radiating α particles.

CALCULATIONS OF THE SOURCE CURRENT, $\vec{J}_{\text{ext}}$

At each grid point, p , we must determine the real and imaginary parts of $\vec{J}$ spatially averaged in a small area A_p about p . These are zero mean normal random variables whose statistics we must compute. For purposes of calculating these statistics we neglect the poloidal field and work in terms of $\vec{J}_{\perp}$ the x, y components of $\vec{J}$. We introduce the four-dimensional vector $\vec{I}(\vec{x}_{\perp}, k_{\parallel}, \omega) = (Re\{\vec{J}_{\perp}\}, Im\{\vec{J}_{\perp}\}) = (Re\{J_x\}, Re\{J_y\}, Im\{J_x\}, Im\{J_y\})$. The average current, I_p associated with grid point p is then

$$\vec{I}_p(k_{\parallel}, \omega) = \frac{1}{A_p} \int_{A_p} d^2 x_{\perp} \vec{I}(\vec{x}_{\perp}, k_{\parallel}, \omega) \quad (2)$$

The statistics of $\vec{I}_p$ are thus determined by the covariance matrix $\vec{Q}_{p,p'}(k_{\parallel}, \omega)$ where

$$\vec{Q}_{p,p'}(k_{\parallel}, \omega) = \left\langle \frac{1}{A_p A_{p'}} \int_{A_p} d^2 x_{\perp} \int_{A_{p'}} d^2 x'_{\perp} \vec{I}^{\top}(x_{\perp}, k_{\parallel}, \omega) \vec{I}(x'_{\perp}, k_{\parallel}, \omega) \right\rangle \quad (3)$$

$\langle \rangle$ denotes ensemble average and $\vec{I}^{\top}$ denotes matrix transpose. Expressing $\vec{I}(\vec{x}_{\perp}, k_{\parallel}, \omega)$ in terms of its k space transform $I(\vec{k}, \omega)$ we have

$$\vec{Q}_{p,p'}(k_{\parallel}, \omega) = \frac{1}{A_p A_{p'}} \int_{A_p} d^2 x_{\perp} \int_{A_{p'}} d^2 x'_{\perp} \int d^2 k_{\perp} d^2 k'_{\perp} e^{i(\vec{k}_{\perp} \cdot \vec{x}_{\perp} + \vec{k}'_{\perp} \cdot \vec{x}'_{\perp})} \vec{R}(\vec{k}, \vec{k}', \omega, \omega') \quad (4)$$

where $\vec{R}(\vec{k}, \vec{k}', \omega, \omega') = \langle \vec{I}^{\top}(\vec{k}, \omega) \vec{I}(\vec{k}', \omega') \rangle$ is the cross-spectral density matrix

In order to proceed we make a number of simplifying assumptions.

1. We neglect correlations between different grid points ($\vec{Q}_{p,p'} = 0$ if $p \neq p'$).
2. We neglect particle-particle correlations (the probability density functions for the N particles of the ensemble is of the form $f^N(\vec{x}, \dots, \vec{x}_N, \vec{v}, \dots, \vec{v}_N) = \prod_{n=1}^N f^1(\vec{x}_n, \vec{v}_n)$)

4. Over the region A_p the plasma can be considered uniform ($f'(\vec{x}, \vec{V}) = 1/V f(\vec{V})$ where V is the plasma volume)
5. The domain of averaging A_p is a cylinder of radius r_A about the point p . After a somewhat tedious calculation, one can express Eq (4) as

$$\begin{aligned} \vec{Q}_{p,p}(k_z, \omega) = & \frac{\pi}{2} (2\pi)^3 \frac{N}{V} (2\pi q)^2 \frac{1}{\Delta\omega} \frac{Lz}{2\pi} \sum_{\ell} \int_0^{\infty} dk_{\perp} k_{\perp} \\ & \int d^3v f(\vec{v}) \frac{1}{\pi^2 k_{\perp}^2 r_A^2} J_1^2(k_{\perp} r_A) \\ & \times \vec{S}(\ell, k_{\parallel}, k_{\perp}, v_{\parallel}, v_{\perp}) \delta(\omega - \ell\Omega - k_{\parallel}v_{\parallel}) \end{aligned} \quad (5)$$

Where $\vec{S}$ is the symmetric matrix

$$\vec{S} = \begin{bmatrix} M_1 & 0 & 0 & M_2 \\ 0 & M_1 & -M_2 & 0 \\ 0 & -M_2 & M_1 & 0 \\ M_2 & 0 & 0 & M_1 \end{bmatrix}$$

$$M_1 = \pi v_{\perp}^2 \left[\frac{\ell^2}{k_{\perp}^2 \rho^2} J_{\ell}^2(k_{\perp} \rho) + J_{\ell}'^2(k_{\perp} \rho) \right] \text{ and } M_2 = 2\pi v_{\perp}^2 \frac{\ell}{k_{\perp} \rho} J_{\ell}(k_{\perp} \rho) J_{\ell}'(k_{\perp} \rho).$$

COMPUTATIONAL APPROACH

The discretization process converts Eq (1) into a linear algebraic system of the form

$$\vec{T} \vec{E} = \vec{J}_{\text{ext}} \quad (6)$$

where now $\vec{E}$ and $\vec{J}_{\text{ext}}$ are $4N$ dimensional vectors (2 field components $\times 2$ complex components $\times N$ grid points) and $\vec{T}$ is a $4N \times 4N$ matrix. After numerically inverting $\vec{T}$ the solution is obtained $\vec{E} = \vec{T}^{-1} \vec{J}_{\text{ext}}$. We can now take either of two computational approaches. (1) A Monte-Carlo approach in which random samples for $\vec{J}_{\text{ext}}$ are generated for each grid point from a multivariate normal distribution using the covariance matrix of Eq (5). The field $\vec{E}$ is computed and the power p coupled to the pickup antenna is computed from $\vec{E}$. Since $\vec{E}$ is a random variable many sample $\vec{E}$'s should be generated and average $\langle P \rangle$ determined. Of course, the matrix $\vec{T}^{-1}$ need only be computed once to generate the multiple $\vec{E}$ samples.

Alternatively, since the quantities of interest to us, such as the power P induced in the pickup antenna, can be written as a quadratic form in $\vec{E}$

$$(P = E_i P_{ij} E_j) \quad (7)$$

The ensemble average of P , can be written in terms of the covariance matrix of $\vec{E}$ ($\langle P \rangle = P_{ij} \langle E_i E_j \rangle$). We now take advantage of the fact that a linear trans-

formation of a multivariate normal distribution is again a normal distribution. The covariance matrix of $\vec{E}$, $\langle \vec{E} \vec{E} \rangle$, can be written

$$\langle \vec{E} \vec{E} \rangle = \vec{T}^{-1} \vec{Q} \left[\vec{T}^{-1} \right]^T, \quad (8)$$

where $\vec{Q} = \langle \vec{J}^T \vec{J} \rangle$ is the direct sum of matrices $\vec{Q}_{p,p}$ for each point.

CONCLUSIONS

We can model ion cyclotron emission in plasmas with our existing 2D full-wave code provided the statistics of the local fluctuating current spectrum $\vec{J}(\vec{x}_\perp, k_\parallel, \omega)$ can feasibly be computed. The advantage of this method over the use of geometrical optics is that global properties of standing waves and plasma reabsorption (i.e., cavity Q) are included and antenna directivity is naturally included. Many model source current distributions can be computed without need to recompute the inverse system matrix $\vec{T}^{-1}$. Such an approach is computationally feasible provided not too many values of ω and $k_\parallel$ need be considered. The principle limitation of the method is that the finite grid spacing places an upper limit on the frequencies (or lower limit on plasma wavelengths) which can be considered. The same method can of course be adapted to other radiation sources such as that generated by instabilities.

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