FLUID SATURATIONS FROM ELECTRICAL
MEASUREMENTS BETWEEN WELLS

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by

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ABSTRACT

The feasibility of using electrical communication between wells to determine the average interwell fluid saturation was tested with a long core and three-dimensional model reservoirs. Two methods were proposed. The first employed an oxidizing solution in one well and a reducing solution in another to obtain an electric current through the reservoir from chemical reaction; the second, an impressed current between the wells and measurement of the potential difference. Experimental determination of the overall brine saturation using the electrical fields agreed with the volumetric amounts of brine and oil with which the model reservoirs were saturated. General mapping of the saturation distribution of the three-dimensional reservoir was also possible using a finite difference solution of Poisson's equation.

INTRODUCTION

The declining production of domestic oil has placed considerable emphasis on the development of new technology for production of residual oil remaining after the economic limit of oil production by primary and secondary methods has been reached. Research for the development of new technology for tertiary oil production has emphasized the importance of accurately determining the oil saturation and its distribution in an oilfield. Three principal methods currently are used for estimating fluid saturations in oilfields: (1) core analysis: extraction of cores followed by laboratory analysis of oil reservoir rock samples, (2) logging: interpretation of electrical, acoustical, and radioactive signals obtained from specialized tools lowered into the wells, and (3) material balance calculations: estimation of remaining fluid content by subtracting the produced fluid volumes from the initial estimates of the reservoir saturations. Each of these methods has varying degrees of accuracy depending upon laboratory and sampling techniques. Core analysis yields the fluid saturation of a small sample which generally is con-

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taminated with fluids used in drilling. Logging produces measurements of fluid saturation but only within a few feet from the wellbore and those measurements may yield grossly inaccurate estimates of the formation saturation if water coning has occurred at the well. Material balance calculations depend upon the accuracy of (1) initial oil saturation estimates, (2) production records that may be 25 to 40 years old, and (3) calculated estimates of pressure decline in the reservoir. None of these methods yield information about the interwell saturation distribution unless new wells are drilled throughout the field.

The ultra-long-spaced electric logs (ULSEL) employ spacing between the electrodes ranging from 30 to 300 meters (25)^{3/}. Information from the ULSEL and induction log readings are used to determine the profile of salt domes or cased wells in the vicinity by mapping the resistivity contrast between the salt or well casing and the other formations. Surface resistivity profiles of the earth, using long electrode spacings, are also used in prospecting for ores.

Mineral prospecting is conducted by measuring the natural earth currents produced by the galvanic action of the minerals and by measuring the isopotential lines developed from a current which is impressed directly into the ground. The mathematical interpretation of the pattern of current flow from the electrodes is based upon the Laplace equation (27).

This work was initiated to test the feasibility of using electric measurements between wells to obtain the average resistivity and from this the average fluid saturation between the wells. The two methods discussed in this report are similar to the two prospecting methods. In the first case, a potential is developed by introducing an oxidizing solution at one point and a reducing solution at another to develop an electric potential in a manner similar to the operation of a battery. In the second case, a current is impressed into the oil reservoir and the potential is measured to obtain the average resistivity between the electrodes.

LITERATURE REVIEW

Dry sandstones, unconsolidated sands, carbonate rocks, oil, and gas have such high resistances to an electric current that they are considered as insulators. Minerals, especially clay and pyrite, frequently associated with the sedimentary rocks that make up most petroleum reservoirs are conductive and, consequently, complicate the interpretation of resistivity logs. However, if

^{3/} Underlined numbers in parentheses refer to the list of references at the end of the paper.

we assume that the porous rock matrix is nonconductive, the electrical properties depend only upon the brine contained in the pores of the rock and the geometry of the pores. Then the conductivity of a fluid-saturated rock is due to the ions of the dissolved salts that make up the brine. Consequently, the magnitude of the electrical conductivity depends upon the salt concentration and temperature.

The resistivity of a material is the reciprocal of the conductivity and is defined as

$$\rho = \frac{r \cdot A}{L} \quad (1)$$

The value of the resistivity of a rock which is completely saturated with brine of a given concentration at a specific temperature is important for interpreting resistivity data. This value is the formation resistivity factor and was defined by Archie (3) as follows:

$$F = \frac{R_o}{R_w} \quad (2)$$

As an aid to interpretation of electric logs, Rall and Wright (24) published the mineral analysis and resistivity of 600 oilfield brines measured at 60°, 80°, 100°, and 120° F, and Arps (4) made a comprehensive study of the effect of temperature on the density and resistivity of sodium chloride solutions.

Guyod (11) noted that the formation factor increases as the sand becomes more cemented and established the dependence of the exponent, m , on the degree of cementation of the sand.

Archie gives an empirical relationship for water saturation. For clean, water-saturated sands the average value of n is 2; hence, the water saturation may be estimated with a fair degree of accuracy using the empirical relationship:

$$S_w = (R_o/R_t)^{1/n} = (FR_w/R_t)^{0.5} \quad (3)$$

The process of obtaining accurate data becomes increasingly difficult when the sedimentary formation being measured contains variable amounts of shale and silty materials. Early tests conducted by Patnode and Wyllie (22) developed conclusive data showing that wet rocks containing clay and other minerals have appreciable conductivity. The total measured conductivity was equal to the sum of the conductivity of the rock and brine. Perkins, Jr. and others (23) extended the work of Patnode and Wyllie by studying the abnormal resistivity associated with the

presence of shaly materials in sedimentary formations. They concluded that the abnormal electrical characteristics are due to the adsorption of ions from the brines by the shaly materials and derived a mathematical interpretation based on their laboratory results that verified their conclusions. Later, Wyllie and Southwick (36) reported on an experimental investigation of the effects of ion exchange materials on the electrical conductivity of porous media.

Van Nostrand and Cook (27) presented a comprehensive review of the interpretation of electrical resistivity data when applied to prospecting for minerals and survey of subsurface heterogeneities. They note that direct-current methods of prospecting are based on measurements of isopotential contours resulting from a direct current impressed into the ground. The theory is predicted by the Laplace equation for obtaining the electrical potential and pattern of flow of the current from the ground electrodes. The only differences between methods of direct-current prospecting are the spacing of electrodes, the electrical parameters determined, and the magnitude of the impressed current. Anomalies in resistivity are interpreted to define and quantify ore bodies; and interruptions of the isopotential lines are indicative of faults and other geologic heterogeneities. Since electrode spacing of 100 meters can be used and large areas can be examined, it seems feasible to apply this technique in determining the apparent resistivity between wells in an oilfield. If this can be done by focusing a current through an oil zone and then measuring the apparent resistivity between adjacent wells, some estimate of the saturation distribution of brine in the formation can be obtained. The saturation between wells and its distribution throughout an oilfield is vital to the initiation of an enhanced oil recovery program for any field.

THEORY

The theory of electric logging to determine fluid saturations is well documented (3,13,16,18,30,35). Equation (3) is used to determine the water saturation of porous media in the absence of clay.

The resistivity, R_o , of the rock when 100 percent saturated with brine is determined by one of two possible methods. Dunlap and others (8) describe a laboratory method using core samples from the oil reservoir. The hydrocarbons are removed from the formation rock samples by pumping acetone through the core. The salts are then removed by pumping water through the same core. The core sample is saturated with formation brine which is obtained from the production of a well in the formation. Resistivity measurements are made, and R_o is determined.

The second method for obtaining R_o is a logging method described by Fertl and Reynolds (9). Formation brine is pumped from the well and stored. An oil-miscible solution is injected into the bore hole in sufficient quantity to displace the oil in the formation surrounding the bore hole. The formation brine that was produced earlier is injected into the bore hole displacing the oil-miscible solution away from the bore hole. Logging measurements are made with the formation 100 percent saturated with brine, and R_o is determined.

To determine the apparent resistivity, R_t , between wells, a current must pass through the formation. By use of Archie's equation, the saturation distribution of brine throughout the field can then be mapped. An electric potential may be developed between wells using dc or ac emf impressed between electrodes in the wells or by forming an in situ battery by injecting a reducing solution (one capable of being oxidized), such as $FeCl_2$, into one well and an oxidizing solution (one capable of being reduced), such as $FeCl_3$, into another well within the same oil-producing formation (figure 1).

When an oxidizing solution is introduced in one well and a reducing solution into another well and the two wells are connected by means of an electric conductor at the surface, an electric current will flow between wells in the same manner as the operation of a storage battery (28). The electric current flows from the oxidizing solution to the reducing fluid through the brine and back through the conducting wire on the surface. The chemically induced electric current traveling through the interstitial fluids is used to measure the electrical resistivity of the formation, R_t , which is directly related to the fluid saturation using established correlations (3,6,8,14,16,18,21,30-33,35).

The process of causing dc or ac current to flow through a formation consists of impressing a voltage at one electrode and completing the electrical circuit at another electrode in the same formation.

For two-dimensional systems with one-dimensional paths (cores), R_t is calculated directly by the simple relationship:

$$R_t = \rho E_t A / (I_t \sqrt{2} L) \quad (4)$$

However, the two-dimensional path in the three-dimensional system is much more complicated. In this research a finite difference solution of the Poisson equation for calculating R_t was developed.

FIGURE 1. - Method used to measure the resistivity across an oilfield. The chemical reactions of oxidation and reduction are shown at each well.

Solution of the Poisson Equation

The apparent resistivity distribution between wells in a three-dimensional model of finite lateral extent was calculated by first assuming that the conductive medium between the measuring electrodes consists of a homogeneous, isotropic layer of constant thickness. In actual practice, this is undoubtedly an oversimplification of the system; however, provisions are made for computations with estimated variations of thickness. Obviously, this yields an estimation of resistivity variation in two horizontal dimensions which is used to determine the brine saturation distribution between wells in the field.

The behavior of an electric current in a nonuniform medium is governed by Poisson's equation:

$$-\nabla \cdot \left[\frac{1}{\rho(x,y,z)} \cdot \nabla V(x,y,z) \right] = \frac{\partial Q(x,y,z)}{\partial t} = q(x,y,z) \quad (5)$$

Using the assumptions introduced above, equation (5) is restricted to: (a) two independent variables (x and y) representing the surface, (b) an effective thickness, $\delta z_{i,j}$, which is specified at each node of the finite difference grid, and (c) an apparent homogeneous medium in which ρ is a constant value between a pair of measuring electrodes. This reduces equation (5) to a more manageable form:

$$-\left[\frac{\partial}{\partial x} \left(z_{i,j} \frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial y} \left(z_{i,j} \frac{\partial V}{\partial y} \right) \right] - \rho q(x,y)/\delta = 0 \quad (6)$$

The current supplied to the ground, by means of the electrodes, is expressed in amperes; whereas, the current density, q , represents the rate of charge generation (or adsorption) per unit area normal to the path of current flow. Therefore, it is desirable to express q in terms of current flowing into the ground. When the current is supplied to a two-dimensional system, the current density at the electrode is represented by I/A where A is the area of the outer surface of the electrode and, therefore, the term $(\rho q(x,y)/\delta)$ of equation (6) becomes:

$$\rho q(x,y)/\delta = \rho I/A\delta \quad (7)$$

Dirichlet boundary conditions ($V_{i,j} = 0$ at all boundaries) were used rather than von Neuman conditions because they are simpler and will yield an underestimation of potentials rather than an overestimation which is associated with the von Neuman boundary conditions. Furthermore, this model is designed to represent a small laboratory model having perfectly insulated boundaries far removed from the source and sink electrodes.

The influence of image electrodes, as described in detail by van Nostrand (27), was used. The finite difference grid was expanded and image electrodes identical to the original source and sink in magnitude and sign are included outside the actual boundary at a distance equal to the distance from the boundary to the actual electrode. This solution, including the computer program, is shown in detail in the appendix.

EXPERIMENTAL APPARATUS AND PROCEDURES

General Apparatus

Two multimeters were used for the current and voltage measurements: a Fluke Model 8600a digital multimeter with both ac and dc capability and a Keithley Model 160B digital multimeter.

Figure 2 shows the schematic for both ac and dc measurements for either the core or the three-dimensional reservoir. This is essentially Rust's (26) configuration for two- and four-electrode resistivity measurements. For ac measurements, a Hewlett Packard Model 200B ac generator with a frequency range of 20 Hz to 20KHz and 0 to 20 v output was used as the signal generator. A Hewlett Packard Model 211A generator was used to generate square wave dc. The frequency range of the 211A is 1 Hz to 1 megaHz with 0 to 20 v output.

The current in the circuit was determined by measuring the voltage drop across a precision resistor (figure 2) with the voltmeter switch in position one. Input electrodes 1 and 2 can also be used as measuring electrodes when switch 2 is in position 2 (two-electrode method). The four-electrode measurement occurs when input is to electrodes 1 and 2 and measurements are taken at electrodes 3 and 4 (switch 2 in position 1). In the three-dimensional model, the 4-electrode measurements were unsatisfactory, and a 2- and 3-electrode method was used by having one side of switch 2 always in position 2. In this way, at least one input electrode is always used as a measuring electrode.

The fluids used to saturate the core and three-dimensional reservoir were an artificial brine containing 30.7 grams NaCl per liter of water with a resistivity of 0.215 ohm meters and a 31.9° API crude oil obtained from the Bartlesville Sandstone at a depth of 664 to 677 feet in Nowata County, Oklahoma. The only treatment of the oil was centrifugation to remove bottom sludge and water.

Core Tests (Two-Dimensional System With One-Dimensional Path)

A 76-cm-long by 3.8-cm-diameter core was obtained from an outcrop of the Cottage Grove Sandstone about 2 miles north of Dewey, Okla. This formation is a very uniform sandstone having a porosity of about 26 percent.

The core was equipped with electrodes by painting a band of silver print conductive paint around the core 15.2 cm from each end (figure 3). A bare end of an insulated lead was inserted in the silver print with the other end on the core end piece. The end pieces have platinum screens and leads for the end electrodes which are isolated from the silver band electrodes by plexiglass end pieces and a rubber sleeve.

FIGURE 2. - Signal input and measurement for ac or dc measurements of the core or the model reservoir. Numbered circles indicate electrodes.

FIGURE 3. - Method used to determine the resistivity of a core.

The core was dried and weighed (1,604.0 grams). The core was saturated with 227 ml of brine and assembled with end pieces, wrapped with Saran wrap, and inserted in a Hassler-type holder (12). Water was pumped into the annulus between the rubber tube and the plexiglass outer tube to maintain a confining pressure of 50 psi.

Five pore volumes of brine were pumped through the core to ensure 100 percent brine saturation. The resistivity was measured using the four-electrode method (26) with $R_o = 0.425$ ohm meter.

Oil was pumped into the core, keeping a careful material balance, until the core was at residual water saturation. Brine was pumped through the core stopping periodically to obtain 2- and 4-electrode resistivity measurements until residual oil saturation was reached.

The volume of all displaced fluids was measured and recorded as follows:

Volume of brine initially in core, ml	227.00
Volume of brine displaced by oil, ml	<u>128.5</u>
Volume of brine remaining in core, ml	98.5

Initial volume of oil in the core is equal to the volume of brine that was displaced by the oil (128.5 ml).

Initial volume of oil in core, ml	128.5
Volume of oil displaced by brine, ml	<u>57.1</u>
Volume of oil remaining in core, ml	71.4

A reducing solution was injected into one end of the core, and an oxidizing solution was placed in the other end. This was accomplished in one case by pumping 50 ml of 0.5 molar ferrous chloride into one end and 25 ml of 0.5 molar ferric chloride into the other end of the core sample. Thus, 25 ml of each solution remained in the core. No additional oil was removed from the core during injection of these solutions.

Platinum screen electrodes, in contact with the two ends of the core, were connected to a commutator, voltmeter, ammeter, and 51,000 ohm resistor, as shown in figure 3. The resistivity of the core containing the residual oil saturation was determined using the electric current that was generated by the reducing and oxidizing solutions.

When the measurements were complete, 10 pore volumes of brine were pumped through the core to remove the oxidizing and reducing solutions, and the core was charged once more with 25 ml of 0.5 molar solutions of cuprous and cupric chloride. Resistivity measurements were taken.

The copper solutions were removed from the core, and a third set of solutions was introduced by injecting 25 ml samples each of a 0.05 molar solution of mercurous chloride in hydrochloric acid and a 0.3 molar solution of mercuric chloride from which resistivity measurements were obtained.

Single Layer Three-Dimensional Model Reservoir (Two-Dimensional Path)

A simple model was prepared by removing the large particles from building sand by topping it through a No. 14 sieve. A 6-cm layer of the sand was added to a wooden box which was constructed 127 cm square and 40 cm high with the joints sealed with epoxy resin and the interior painted with a nonconducting marine paint.

Cylindrical copper electrodes, 3-cm long with a diameter of 0.635 cm, with insulated leads soldered at one end were inserted at the corners and center of a 40-cm square grid in the center of the 127-cm square box. Two electrodes were imbedded in the grid halfway between the center and each of two adjacent corner electrodes. The seven electrodes were inserted vertically with the top 1.5 cm below the surface of the sand.

Fourteen liters of oil was mixed with the sand to make the sand oil-wet and to ensure a relatively uniform saturation. Brine was added making the volumetrically measured brine saturation for the entire reservoir 56 percent of pore volume.

Measurements were obtained with every possible combination of input and measuring electrodes. However, measured voltages were very small and showed much scatter for the four-electrode method (figure 2). As a result, only two- and three-electrode measurements were obtained as mentioned earlier. Both 20 Hz ac and dc inputs were used to measure the voltage in the reservoir. Calculations were made with the Poisson equation (Computer Program).

Multiple Layer Three-Dimensional Model Reservoir
(Two-Dimensional Path)

Wells which also served as electrodes were constructed by capping five 75-cm lengths of 0.64-cm-diameter stainless steel (SS) tubing at one end. These capped tubes were placed in 1.91-cm-diameter tygon tubing and held in a vertical position. The annular space between the SS tubing and the tygon tubing was filled with concrete.

After the concrete hardened (3 days), a diamond saw was used to cut slots through the cement and into the SS tubing to act as perforations. The cuts were made over a 6-cm portion of the tubing to coincide with the center of the sand zone. Four cuts were made in a radial fashion. The tygon tubing was removed, and an 8-cm-long piece of 50-50 mesh SS screen extending 1 cm past the slots at both ends was used to prevent sand intrusion. The screen was silver soldered at spots along the seam.

Shale tailings were obtained from drilling sites in Nowata County and immersed in a tank of water to form a mud shale. A 17-cm layer of the mud shale was placed in the empty 127-cm box, and the wells were set in position in the shale forming a five-spot with the outer wells forming a 1-meter square.

Building sand, which had been topped through a No. 14 sieve, was added to form a 10-cm layer as the middle formation in the box. Oil (26.6 liters) and brine (26.6 liters) were poured simultaneously onto the sand giving a water saturation of 0.50. The measured porosity of the packed sand was 33 percent. A 9-cm layer of shale was added as the top formation in the miniature model reservoir.

Resistivity of the sand when 100 percent saturated with brine (R_o) was determined using a 1.87-cm-diameter, sand-packed tube fitted with end electrodes. The two-electrode ac method (26) was used to determine an R_o of 0.275 ohm meter.

Measurements of voltage and current for each set of wells were made with ac input. Values for S_w were calculated using Poisson's equation.

The potential was checked between the wells with no impressed emf to determine the possibility of spontaneous potentials resulting from the difference in ion concentration at the sand-shale interfaces. Voltages on the order of 0.1 v were measured. The electrodes were shorted every night to discharge these as much as possible.

One hundred milliliters of 0.5 molar ferric chloride was pumped into one well, and 100 ml of 0.5 molar ferrous chloride was pumped into another well. Resistivity was measured using the emf from the oxidizing and reducing solutions.

Brine was pumped into the three remaining wells while producing the wells where oxidizing or reducing solutions were located. Background voltages were again obtained. The ferrous and ferric chloride solutions were pumped into two other wells. Voltages and currents were again obtained for resistivity calculations.

These solutions were removed, and the other oxidizing and reducing solutions described in the core tests were used. Values for S_w were calculated for all the tests using Poisson's equation.

RESULTS AND DISCUSSION

Core Tests (Two-Dimensional System With One-Dimensional Path)

Archie proposed a value of 2 for the constant, n , in equation (1) after correlations of experimental data on resistivity measurements published by several authors (15,17,19,34). The value of n for the results of the two- and four-electrode resistivity measurements is 1.95 (figure 4). These results were obtained at various water saturations while displacing oil from the core with a brine solution and using impressed ac current to obtain the resistivity measurements.

Measurements and calculations for the resistivity data from the oxidizing and reducing solutions are given in table 1.

TABLE 1. - Results of resistivity measurements using oxidizing and reducing solutions.

Solutions	R_t , ohm meter	S_{or} , % Figure 4	S_{or} , ml	S_{or} , ml volumetric	Relative error	n
Fe^{++}/Fe^{+++}	0.89	32.0	72.6	71.4	0.0168	2.0
Cu^{+}/Cu^{++}	1.064	37.0	84.0	71.4	0.01765	2.4
Hg^{+}/Hg^{++}	1.018	35.0	79.4	71.4	0.1120	2.3

$$\text{Relative error} = \frac{S_{or\text{-volumetric}} S_{or}}{\text{volumetric } S_{or}}$$

FIGURE 4. - Resistivity as a function of brine saturation determined using a 76-cm-long by 3.8-cm-diameter core.

The average of the values of n obtained from the oxidizing and reducing solutions is 2.2. Thus the experimental data from both ac input and oxidizing and reducing solutions are in close agreement with Archie's empirical correlations which are used extensively for electric logging calculations.

Single Layer Three-Dimensional Model Reservoir (Two-Dimensional Path)

A simple reservoir with only one layer and impressed emf instead of the oxidizing and reducing solutions was used to develop experimental procedures, apparatus, and mathematics, and for experience.

Part of the experience gained involved determining electrode effective area for the finite difference modeling. Adams (1) suggests that this area may be considerably different from the electrode geometric area. However, our computer model gave very good results using the actual geometric area of one electrode, or since the electrodes were identical and geometrically symmetrical and parallel, one-half the geometric area of two electrodes multiplied by porosity and water saturation.

Calculations of the water saturations by the solution of Poisson's equation indicate an average saturation of 49.4 percent for 20 Hz square wave dc input and 44.0 percent for the 20 Hz ac input compared to a volumetric balance of 56.0 percent. Table 2 gives the saturation distribution between wells, and figure 5 is a saturation profile for the calculated water saturations for the solutions of Poisson's equation. The saturation profile indicates higher water saturations toward electrodes W-1, W-2, and W-4 and lower values near the center and toward electrode W-3.

Figure 6 is an isopotential map of the miniature reservoir calculated by the solution of Poisson's equation. The impressed dc input is at electrodes W-1 and W-2. These isopotential lines agree in shape with the isopotential lines presented in Van Nostrand (27).

Multiple-Layer, Three-Dimensional Model Reservoir (Two-Dimensional Path)

After experience in the modeling procedure had been gained using the simple reservoir, a three-layer model was constructed. Data were taken using impressed emf; then oxidizing and reducing solutions were introduced into the wells to generate an emf, but the results were inconclusive. The measured voltages, with no oxidizing or reducing solutions and no impressed emf, were on the order of 0.1 volt. The measured voltages with the oxidizing and reducing solutions did not differ significantly from the values recorded without the solutions present. Apparently, the solutions migrated along the bottom shale layer and came in contact with each other causing anomalous measurements. Only the impressed emf's were used in the calculations, and, because of time limitations, no additional experiments with oxidizing and reducing solutions were run.

TABLE 2. - Water saturation between electrodes in the single layer three-dimensional model reservoir calculated by the finite difference solution of Poisson's equation.

Electrodes	Water saturations	
	Poisson Equation	
	ac input	dc input
W-1--W-2	0.490	0.571
W-1--W-3	.414	.492
W-1--W-4	.406	.516
W-1--W-5	.453	.475
W-1--W-6	.370	.482
W-1--W-7	.406	.469
W-2--W-3	.369	.557
W-2--W-4	.657	.596
W-2--W-5	.556	.542
W-2--W-6	.412	.506
W-2--W-7	.478	.531
W-3--W-4	.519	.457
W-3--W-5	.427	.473
W-3--W-6	.347	.476
W-3--W-7	.368	.431
W-4--W-5	.513	.449
W-4--W-6	.441	.503
W-4--W-7	.482	.484
W-5--W-6	.370	.445
W-5--W-7	.394	.429
W-6--W-7	.377	.480
Average water Saturation	.440	.494

FIGURE 5. - Brine saturation distribution calculated from the Poisson equation for a single-layer, three-dimensional model reservoir.

FIGURE 6. - Isopotential lines calculated from the solution of Poisson's equation with input at wells W-1 and W-2 in a single layer-model.

FIGURE 7. - Brine distribution calculated between wells in a multilayered, three-dimensional model reservoir using Poisson's equation.

With the impressed emf, the average water saturation calculated from the Poisson equation was 49.1 percent compared to 50.0 percent by material balance. Figure 7 shows the saturation distribution between wells. The water saturation is higher toward the center and lower near the boundaries of the box.

CONCLUSIONS

This work was an attempt to develop a novel approach for accurate determination of fluid saturations throughout the formation. It has proved successful in a laboratory model. More work should be done in a field situation by persons familiar with electrode design and configurations. Electrode design and methods of introducing an electric current between wells in a formation remain to be worked out, but dc currents have been used over large distances in mineral prospecting and in ultra-long-spaced logging techniques.

The two-dimensional system (core) presents no problems in determining oil and brine saturations from resistivity data. The low emf from the oxidizing and reducing solutions provides adequate current for determining resistivities in these systems, and Archie's relationship and value for n allow accurate calculation of water saturation from the resistivity data.

Poisson's equation gave excellent results. This method of computer modeling is directly applicable to field situations since the computer program can be modified to include known heterogeneities.

NOMENCLATURE

A, A_a	area, m^2
E	electric potential, volts
E_o	electric potential measured with rock 100 percent saturated with brine, volts
E_t	electric potential measured when more than one fluid is in porous matrix, volts
F	formation resistivity factor
I	current, ampere
I_o	current when rock fully saturated with brine, ampere
I_t	current when more than one fluid is in the porous matrix, ampere
L, L_a	length of current path, m
m	empirical constant
n	empirical constant
Q	charge density, coulombs/ m^3
q_{xyz}	current density, amps/ m^3
r	resistance, ohms
ρ	resistivity, ohm-m
R_o	resistivity of rock 100 percent saturated with brine, ohm-m
R_t	resistivity of rock with brine and another fluid, ohm-m
R_w	resistivity of brine, ohm-m
S_{or}	residual oil saturation
S_w	brine or water saturation
S_{wi}	residual brine saturation

NOMENCLATURE—Continued

V electric potential, volts
Z formation thickness, m
Ø porosity

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APPENDIX

Combining equations (5)^{1/} and (6) yields:

$$- \left[\frac{\partial}{\partial x} \left(z \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left(z \frac{\partial v}{\partial y} \right) \right] - \rho I / A \emptyset = 0 \quad (A-1)$$

which is then discretized as follows:

$$\begin{aligned} \frac{1}{Dx} \left[\frac{z_{i,j}}{Dx} (v_{i+1,j} - v_{i,j}) - \frac{z_{i,j}}{Dx} (v_{i,j} - v_{i-1,j}) \right] + \frac{1}{Dy} \\ \left[\frac{z_{i,j}}{Dy} (v_{i,j+1} - v_{i,j}) - \frac{z_{i,j}}{Dy} (v_{i,j} - v_{i,j-1}) \right] + \\ \rho I / A \emptyset = 0 \end{aligned} \quad (A-2)$$

If a grid with equal spacing in the X and Y directions is used and the following terms are defined, equation (A-2) is reduced to:

$$Dx = Dy, \quad PI = PJ = MI = MJ = z_{i,j} / (Dx)^2, \text{ and } \alpha = (PI + MI + PJ + MJ)$$

$$\alpha \cdot v_{i,j} + PI \cdot v_{i+1,j} + MI \cdot v_{i-1,j} + PJ \cdot v_{i,j+1} + MJ \cdot v_{i,j-1} + \rho I / A \emptyset = 0 \quad (A-3)$$

In cases where the reservoir thickness, $z_{i,j}$, varies within the zone under investigation, $z_{i,j}$ can be entered as input data at each node of the finite grid network. The reservoir used in the experiments described in this report had a constant thickness. In the program, equation (A-3) was multiplied through by $(DX)^2/z$ and the coefficients PI, MI, PJ, and MJ were equated to 1.0 and retained in the equation to facilitate modification if variable thickness or unequal spacing was used later, consequently:

$$PI=MI=PJ=MJ=1.0$$

$$\alpha = -4.0$$

^{1/} Equation numbers without the A-prefix refer to equations occurring in the main text.

$$\alpha \cdot V_{i,j} + PI \cdot V_{i+1,j} + MI \cdot V_{i-1,j} + PJ \cdot V_{i,j+1} + MJ \cdot V_{i,j-1} +$$

$$\rho I (Dx)^2 / \phi ZA = 0 \quad (A-4)$$

Referring to figure A-1, $V_{i,j}=0$ for all boundary nodes, and the objective of the model is to evaluate the potential function, $V_{i,j}$, for all elements within the null boundaries. With these conditions, the problem is solved by successive line overrelaxation (10) which is an implicit method by which a line of components of V^n are defined simultaneously in such an interrelated manner that it is necessary to solve a linear sub-system of the whole set of components at once before a single one can be determined. That is, the equation is solved implicitly for each row, sweeping down the columns (figure A-1) in a row-by-row fashion. The equation (A-5) incorporates an iteration parameter, W , in the tridiagonal matrix call. The reader is referred to the literature for a detailed description of the successive line overrelaxation technique for solving matrices (5,7,10,28). Figure A-2 illustrates the relationship between the known and unknown values that is used to transform equation (A-4) to (A-5). This equation is written for each point within the boundaries of figure A-1 to create the set of simultaneous algebraic equations that must be solved.

FIGURE A-1. - Finite difference grid.

FIGURE A-2. - Relationship between the known (filled-in circles) and unknown (open circles) values in the grid.

$$\left[MI \cdot V_{i-1,j} + PI \cdot V_{i+1,j} + \alpha V_{i,j} \right]^{n+1} = \left[(1-W) (MI \cdot V_{i-1,j} + PI \cdot V_{i+1,j} + \alpha V_{i,j}) \right]^n - W \left[MJ \cdot V_{i,j-1}^{n+1} + PJ \cdot V_{i,j+1} + \rho I (Dx)^2 / \emptyset ZA \right] = 0 \quad (A-5)$$

Defining the last term: $ZAP_{ij} = \rho I (Dx)^2 / \emptyset ZA$

$$PI \cdot V_{i+1,j} + \alpha V_{i,j} = (1-W) \left[MI \cdot V_{i-1,j} + PI \cdot V_{i+1,j} + \alpha V_{i,j} \right] - W \left[MJ \cdot V_{i,j-1}^{n+1} + PJ \cdot V_{i,j+1} + ZAP_{i,j} \right] - MI \cdot V_{i-1,j} =$$

$D_2 \text{ ----- } \begin{matrix} I=2, m-1 \\ j=2 \end{matrix}$ (A-6)

Moving the known to the right-hand side and writing equation (A-6) for the nodes of row $J=2$, the generalized equation for any row and node emerges as equation (A-7):

$$MI \cdot V_{i-1,j} + PI \cdot V_{i+1,j} + \alpha V_{i,j} = (1-W) \left[MI \cdot V_{i-1,j} + PI \cdot V_{i+1,j} + \right.$$

$$\alpha V_{i,j} \Big] - W \left[MJ \cdot V_{i,j-1}^{n+1} + PJ \cdot V_{i,j+1} + ZAP_{i,j} \right] =$$

D_n ----- $I=3, m-2$
 $j=2$

(A-7)

There is an equation of the form of equation (A-7) for each column of the nodes shown in figure A-1, making a tridiagonal set which is solved by Gaussian elimination (subroutine TRIDAG). Figure A-3 is a listing of the main program and subroutine TRIDAG; statement number 159 is the expression of equation (A-7). Figure A-4 is a flow diagram of the program showing the main characteristics.

FIGURE A-3. - Computer Program for calculation of
voltages and water saturation in
a laboratory model.

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BOX.FTN      /TR:BLOCKS/WR

      C      BOX11.FT
      C
      C      THREE-D RESISTIVITY EXPERIMENT--POTENTIAL FIELD
      C      ....SUBROUTINE TRIDAG USED
      C

0001      DIMENSION A(45),B(45),C(45),D(45),V(45,45),VNEW(45,45),
1      TEMP(45),E(45,45),Q(45,45),VOBS(7,7),VCAL(7,7)
0002      COMMON M
0003      REAL IP,IM,JP,JM
0004      CALL ASSIGN(1,'BOX10.DAT')
0005      CALL ASSIGN(6,'TT10:')
0006      CALL ASSIGN(2,'DC100B.DAT')
0007      CALL ASSIGN(3,'DC10.DAT')
0008      TTI=SECNDS(0.0)
0009      W=1.1
0010      NX=45
0011      PHI=.33
0012      Z=.06
0013      RHO0=.2751
0014      EMAX=1.E-3
0015      DO 1 KWELL=1,6
0016      DO 1 LWELL=KWELL+1,7
0017      DENOM=1.
0018      RT=.887
0019      READ(2,102)VOBS(KWELL,LWELL)
0020      READ(1,102)AMP
0021      IF((KWELL .NE. 1).OR.(LWELL .NE. 5)) GO TO 1
0022      M=45
0023      KMAX=500
0024      DX=.05
0025      WRITE (6,201) AMP,RT,EMAX,Z,RHO0,KWELL,LWELL

      C
0026      GO TO (31,32,33,34,35,36,37),KWELL
0027      31      I=20
0028              J=20
0029              I2=2
0030              J2=20
0031              I3=I
0032              J3=I2
0033              GO TO 38
0034      32      I=28
0035              J=20
0036              I2=I
0037              J2=2
0038              I3=44
0039              J3=J
0040              GO TO 38
0041      33      I=28
0042              J=28
0043              I2=44
0044              J2=J
0045              I3=I
0046              J3=44
0047              GO TO 38
0048      34      I=20
0049              J=28

```

FIGURE A-3 (Cont.)

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0050		I2=2
0051		J2=J
0052		I3=I
0053		J3=44
0054		GO TO 38
0055	35	I=24
0056		J=24
0057		I2=I
0058		J2=J
0059		I3=I
0060		J3=J
0061		GO TO 38
0062	36	I=26
0063		J=22
0064		I2=I
0065		I3=I
0066		J2=J
0067		J3=J
0068		GO TO 38
0069	37	I=26
0070		J=26
0071		I2=I
0072		I3=I
0073		J2=J
0074		J3=J
0075	38	CONTINUE
0076		GO TO (41,42,43,44,45,46,47),LWELL
0077	41	I4=20
0078		J4=20
0079		I5=2
0080		J5=J4
0081		I6=I4
0082		J6=I5
0083		GO TO 48
0084	42	I4=28
0085		J4=20
0086		I5=I4
0087		J5=2
0088		I6=44
0089		J6=J4
0090		GO TO 48
0091	43	I4=28
0092		J4=28
0093		I5=44
0094		J5=J4
0095		I6=I4
0096		J6=I5
0097		GO TO 48
0098	44	I4=20
0099		J4=28
0100		I5=2
0101		J5=J4
0102		I6=I4
0103		J6=44
0104		GO TO 48
0105	45	I4=24

FIGURE A-3 (Cont.)

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0106      J4=24
0107      I5=I4
0108      I6=I4
0109      J5=J4
0110      J6=J4
0111      GO TO 48
0112  46    I4=26
0113      J4=22
0114      I5=I4
0115      I6=I4
0116      J5=J4
0117      J6=J4
0118      GO TO 48
0119  47    I4=26
0120      J4=26
0121      I5=I4
0122      I6=I4
0123      J5=J4
0124      J6=J4
0125  48    CONTINUE
0126  100    FORMAT(2I2)
0127          WRITE(6,210) I,J,I4,J4
0128          WRITE(6,260) I2,J2,I3,J3,I5,J5,I6,J6
0129  260    FORMAT(' IMAGE ELECTRODES ARE AT (' ,I2,' ' ,I2,' '),( '
1          ,I2,' ' ,I2,' '),( ' ,I2,' ' ,I2,' ' ) & (' ,I2,' ' ,I2,' ' ),('
0130  55      SW=SQRT(RH00/RT)
0131          DO 2 IK=1,M
0132          DO 2 JK=1,M
0133              Q(IK,JK)=0.
0134              V(IK,JK)=0.
0135  2        VNEW(IK,JK)=0.
0136          AREA=0.0006*SW
0137          ZAP=RT*AMP*DX*DX/(Z*PHI*SW)
0138          Q(I,J)=ZAP/AREA
0139          Q(I2,J2)=Q(I,J)
0140          Q(I3,J3)=Q(I,J)
0141          Q(I4,J4)=-Q(I,J)
0142          Q(I5,J5)=Q(I4,J4)
0143          Q(I6,J6)=Q(I4,J4)
0144  210    FORMAT(' INPUT CURRENT IS BETWEEN COORDINATES',
1          I2,' ' ,I2,' AND ' ,I2,' ' ,I2/)
          C
0145  3        K=0
0146  4        K=K+1
0147          IOK=0
0148          ING=0
          C
0149          DO 8 JK=2,M-1
0150          DO 6 IK=2,M-1
0151              IP=1.
0152              IM=1.
0153              JP=1.
0154              JM=1.
0155              AL=-IP-IM-JM-JP
0156              A(IK)=IM
0157              B(IK)=AL

```

FIGURE A-3 (Cont.)

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BOX.FTN      /TR:BLOCKS/WR

0158      C(1K)=IP
0159      D(1K)=(1.-W)*(IM*V(1K-1,JK)+IP*V(1K+1,JK)+AL*V(1K,JK))
          1      -W*(JM*VNEW(1K,JK-1)+JP*V(1K,JK+1)-Q(1K,JK))
0160      6      CONTINUE
0161      CALL TRIDAG(A,B,C,D,TEMP,NX)
0162      DO 8 1K=2,M-1
0163      VNEW(1K,JK)=TEMP(1K)
0164      E(1K,JK)=ABS(VNEW(1K,JK)-V(1K,JK))
0165      8      CONTINUE
          C
0166      DO 14 1K=2,M-1
0167      DO 14 JK=2,M-1
0168      IF (E(1K,JK) .LT. EMAX) GO TO 10
0169      ING=2
0170      GO TO 12
0171      10      IOK=1
0172      12      V(1K,JK)=VNEW(1K,JK)
0173      14      CONTINUE
          C
0174      IF (ING .LT. IOK) GO TO 16
0175      IF (K .LT. KMAX) GO TO 4
          C
0176      WRITE (6,200)
0177      GO TO 1
0178      16      VCAL(1,2)=ABS(V(20,20)-V(28,20))
0179      VCAL(1,3)=ABS(V(20,20)-V(28,28))
0180      VCAL(1,4)=ABS(V(20,20)-V(20,28))
0181      VCAL(1,5)=ABS(V(20,20)-V(24,24))
0182      VCAL(1,6)=ABS(V(20,20)-V(26,22))
0183      VCAL(1,7)=ABS(V(20,20)-V(26,26))
0184      VCAL(2,3)=ABS(V(28,20)-V(28,28))
0185      VCAL(2,4)=ABS(V(28,20)-V(20,28))
0186      VCAL(2,5)=ABS(V(28,20)-V(24,24))
0187      VCAL(2,6)=ABS(V(28,20)-V(26,22))
0188      VCAL(2,7)=ABS(V(28,20)-V(26,26))
0189      VCAL(3,4)=ABS(V(28,28)-V(20,28))
0190      VCAL(3,5)=ABS(V(28,28)-V(24,24))
0191      VCAL(3,6)=ABS(V(28,28)-V(26,22))
0192      VCAL(3,7)=ABS(V(28,28)-V(26,26))
0193      VCAL(4,5)=ABS(V(20,28)-V(24,24))
0194      VCAL(4,6)=ABS(V(20,28)-V(26,22))
0195      VCAL(4,7)=ABS(V(20,28)-V(26,26))
0196      VCAL(5,6)=ABS(V(24,24)-V(26,22))
0197      VCAL(5,7)=ABS(V(24,24)-V(26,26))
0198      VCAL(6,7)=ABS(V(26,22)-V(26,26))
0199      DIF=VOBS(KWELL,LWELL)-VCAL(KWELL,LWELL)
0200      IF((DIF .GT. .1).OR.(DIF .LT. -.1))GO TO 57
0201      GO TO 59
0202      57      DENOM=DENOM+1.
0203      IF(DIF)61,61,62
0204      61      RT=RT-RT/DENOM
0205      GO TO 55
0206      62      RT=RT+RT/DENOM
0207      GO TO 55
0208      59      DELTA=SECNDS(TTI)
0209      WRITE(6,250)DENOM,SW,RT,DIF,DELTA,AREA,ZAP,Q(I,J)

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FIGURE A-3 (Cont.)

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FORTRAN IV-PLUS V02-51      13:17:56      28-SEP-77      PAGE 5
BOX.FTN          /TR:BLOCKS/WR

0210      WRITE(3,1000)((V(I,J),I=1,M),J=1,M)
0211      1000      FORMAT(1X,10(1PE12.4))
0212      WRITE(6,202)V(20,20),V(28,20),V(28,28),V(20,28),V(24,
1          24),V(26,22),V(26,26)
0213      250      1      FORMAT(1X,'ITERATIONS--',F5.0,' SW = ',F7.3,' RT= ',F10.2,'
1          DIF=',F7.2/' ELAPSED TIME IS ',F15.0,' SECONDS'//
2          AREA=',E11.3,'ZAP=',E11.3,'Q=',E11.3//)
0214      WRITE(6,230)VCAL(1,2),VCAL(1,3),VCAL(1,4),VCAL(1,5),VCAL
1          (1,6),VCAL(1,7),VCAL(2,3),VCAL(2,4),VCAL(2,5),VCAL(2,6)
2          ,VCAL(2,7),VCAL(3,4),VCAL(3,5),VCAL(3,6),VCAL(3,7),VCAL
3          (4,5),VCAL(4,6),VCAL(4,7),VCAL(5,6),VCAL(5,7),VCAL(6,7)
0215      230      FORMAT(' V12=',1PE12.4,5X,'V13=',1PE12.4,5X,'V14=',1PE12.4/'
1          V15=',1PE12.4,5X,'V16=',1PE12.4,5X,'V17=',1PE12.4/' V23=',1PE12.4
2          ,5X,'V24=',1PE12.4,5X,'V25=',1PE12.4/' V26=',1PE12.4,5X,'V27=
3          ',1PE12.4,5X,'V34=',1PE12.4/' V35=',1PE12.4,5X,'V36=',1PE12.4,
4          5X,'V37=',1PE12.4/' V45=',1PE12.4,5X,'V46=',1PE12.4,5X,'V47=',
5          1PE12.4/' V56=',1PE12.4,5X,'V57=',1PE12.4,5X,'V67=',1PE12.4)
0216      WRITE(6,205)K
0217      SW=100.*SQRT(RH00/RT)
0218      WRITE(6,206)SW
0219      206      FORMAT(' WATER SATURATION IS ',F8.4,' PERCENT'/////////)
0220      1      CONTINUE
C
0221      102      FORMAT (F10.7)
C
0222      200      FORMAT(' NO CONVERGENCE, THE VOLTAGES AND RESIDUALS ARE:'/)
0223      201      FORMAT(1X,'AMP=',F8.5,'RT=',F5.3,'EMAX=',F7.5,
1          ' Z=',F6.3,5X,'RH00=',F7.4/' INPUT IS BETWEEN WELLS',I3,'& ',I3)
0224      202      FORMAT(1X,6(1PE12.4))
0225      204      FORMAT(1X//)
0226      205      FORMAT (1X,'ITERATIONS',I4//)
0227      20      CONTINUE
0228      END

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FORTRAN IV-PLUS V02-51      13:18:41      28-SEP-77      PAGE 1
TRIDAG.FTN       /TR:BLOCKS/WR

0001      SUBROUTINE TRIDAG(A,B,C,D,V,NX)
0002      DIMENSION A(NX),B(NX),C(NX),D(NX),V(NX),BETA(101),GAMMA(101)
0003      COMMON M
0004      L=M-1
0005      IF=2
0006      BETA(IF)=B(IF)
0007      GAMMA(IF)=D(IF)/BETA(IF)
0008      IFP1=IF+1
0009      DO 1 I=IFP1,L
0010      BETA(I)=B(I)-A(I)*C(I-1)/BETA(I-1)
0011      1      GAMMA(I)=(D(I)-A(I)*GAMMA(I-1))/BETA(I)
0012      V(L)=GAMMA(L)
0013      LAST=L-IF
0014      DO 2 K=1,LAST
0015      I=L-K
0016      2      V(I)=GAMMA(I)-C(I)*V(I+1)/BETA(I)
0017      RETURN
0018      END

```

FIGURE A-4. - Flow diagram of the computer program showing the main characteristics.