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THE PHYSICS OF TWO-PHASE FLOW:
CHOKED FLOW

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Choked Flow

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1. The Basic Governing Equations

1.1 Fundamental Assumptions

It will be shown that the basic equations governing the flow of a two-phase fluid, i.e., liquid and vapor, are natural extensions of those equations that describe single-phase flow. The exact form of the equations depends on the particular assumptions used to characterize the two-phase flow, i.e., the equations depend on the "model" chosen. In general the mathematical model is selected so as to retain as much of the physics of the phenomena as possible while at the same time permitting solutions to be obtained without undue complexity.

We shall now list our underlying assumptions.

- (a) Equilibrium exists between the two phases. That is, the liquid and vapor phases are in thermodynamic, mechanical and chemical equilibrium.
- (b) There is no "drift" between the phases. That is, the velocity of the liquid and vapor phases is the same, and the motion of the fluid can be described by a single velocity. Thus, the flow is visualized as being homogeneous.
- (c) The fluid is uniform. That is, the properties of the fluid are constant through any cross-section that is transverse to the direction of the flow.
- (d) The flow is one-dimensional even though it will be allowed to pass through a duct of varying cross-sectional area. That is, the change in the cross-sectional area of the flow is sufficiently gradual that the flow may be adequately described by a single, axial velocity at any point.

- (e) The flow is steady. That is, at any point in the flow the fluid properties are invariant with time.

From time to time we may specialize our basic equations to apply to more specific cases; we shall make clear our further assumptions at such times.

1.2 Continuity Equation

Subject to the foregoing assumptions, conservation of mass can be written in the form

$$\frac{d\dot{m}}{dz} = 0 = \frac{d}{dz} (\rho w A) = \frac{d}{dz} (\psi A) , \quad (1)$$

where $\psi \equiv \rho w$ is called the "flow density". The single spatial variable, z , is defined as positive in the direction of the flow. The fluid is characterized by a single velocity component, w , in the z -direction. Note that the mass flow rate is given by

$$\dot{m} = \rho w A = w A / v , \quad (2)$$

where v is the specific volume ($v=1/\rho$).

1.3 Energy Equation

This equation is found from applying the First Law of thermodynamics to the flow:

$$\frac{d}{dz} \left\{ \dot{m} \left(h + \frac{1}{2} w^2 + gz \right) \right\} = C \dot{Q} , \quad (3)$$

where C is the wetted circumference of the duct, here taken as a circular tube of diameter D , $C = \pi D$. The rate of heat transfer between the fluid and its surroundings per unit area is denoted by $\dot{Q}$ (i.e., $\dot{Q}$ is the heat flux).

The acceleration due to gravity, g , is taken as positive when it acts in the normal direction, i.e., downward. It ought to be noted that other forms of potential energy may be present such as in rotating systems, i.e., fluids confined within rotating vessels. In general whenever the gravitational term is opposed to the flow direction, it shall be taken as positive; when it is in the flow direction, it will be taken as negative. Finally, an element of surface area along the flow is given by

$$dA_z = Cdz \quad , \quad (4)$$

and the fluid is assumed to fill the pipe completely.

1.4 Entropy Equation

The entropy equation follows from the first part of the Second Law of thermodynamics [1], and includes two terms, one showing the entropy change caused by heat transfer and one showing the effect of dissipation (i.e., irreversibilities):

$$\frac{ds}{dz} = \frac{C\dot{Q}}{\dot{m}T} + \frac{C\dot{\Theta}}{\dot{m}T} \quad , \quad (5)$$

where the term, $\dot{\Theta}$, stands for the dissipated mechanical power (i.e., energy per unit time) per unit area. The absolute thermodynamic temperature, T , appears in both terms.

The single most challenging term is the dissipation function, $\dot{\Theta}$. It can be found exactly for some trivial cases. For example, in the case of pipe flow in which the velocity is everywhere uniform and a constant shear stress, τ_w , exists at the interface between the fluid and the pipe wall, then

$$\dot{\Theta} = w\tau_w \quad . \quad (6)$$

This corresponds to the case of solid friction in which a solid plug moves with friction through a tube. For this simple example, the dissipation turns out to be proportional to the fluid kinetic energy (or to the velocity squared):

$$\dot{\Theta} = \frac{1}{4} \lambda_w \frac{\dot{m}}{A} \left(\frac{1}{2} w^2 \right) , \quad (7a)$$

since

$$\tau_w = \frac{1}{4} \lambda_w \left(\frac{1}{2} \rho w^2 \right) , \quad (7b)$$

and where we have introduced the friction factor, λ_w .

The general entropy equation may be more conveniently expressed as

$$\frac{ds}{dz} = \frac{1}{T} q' + \theta' , \quad (8)$$

where the dissipation term, $\theta' \equiv C\dot{\Theta}/\dot{m}T$, and the specific heat flux, $q' \equiv C\dot{Q}/\dot{m}$, are now used. From the second part of the Second Law of thermodynamics,

$$\theta' \geq 0 . \quad (9)$$

For the special case just cited,

$$\theta' = \frac{1}{4} \frac{C}{A} \frac{1}{T} \lambda_w \left(\frac{1}{2} w^2 \right) \geq 0 , \quad (10)$$

or

$$\theta' = \frac{1}{D} \frac{1}{T} \lambda_w \left(\frac{1}{2} w^2 \right) \geq 0 , \quad (11)$$

since

$$C/A = \pi D / \left(\frac{1}{4} \pi D^2 \right) = 4/D \quad (12)$$

for a circular pipe.

1.5 Derivation of the Integrated Momentum Equation

The above equations, i.e., eqs. (1), (3), and (8), must be augmented by a thermodynamic equation of state for closure. When this is done, it can be shown that the momentum equation can be extracted from the set of

equations. Substitution of eq. (1) into (3) yields

$$\frac{dh}{dz} + \frac{d}{dz} \left(\frac{1}{2} w^2 \right) + g = q' \quad (13)$$

The fundamental thermodynamic equation in the form $h = h(s, P)$ may be written as one of the Gibbs equations

$$Tds = dh - \frac{1}{\rho} dP \quad (14)$$

Differentiating this with respect to z , and eliminating dh/dz with the aid of eq. (13), we obtain

$$- \frac{dP}{dz} = \rho g + \rho w \frac{dw}{dz} - \rho q' + \rho T \frac{ds}{dz} \quad (15)$$

Combining this with eqs. (8) and (10) results in the integrated momentum equation for one-dimensional fluid flow:

$$- \frac{dP}{dz} = \rho g + \rho w \frac{dw}{dz} + \frac{1}{4} \frac{C}{A} \rho \lambda_w \left(\frac{1}{2} w^2 \right) \quad (16)$$

This equation relates the pressure drop along the pipe to the gravitational acceleration, the fluid acceleration, and the frictional losses. It may be viewed as a mechanical energy equation having been obtained by subtracting the thermal energy equation, eq. (8), from the Gibbs (or caloric) energy equation, eq. (14).

1.6 Equation of State

An equation of state of the form

$$h = h(s, \rho) \quad (17)$$

is selected as being convenient for our purposes. It is not a fundamental equation [1], but does contain the appropriate variables to allow it to be used effectively with our earlier equations.

We shall need the total differential of eq. (17) with respect to the spatial variable:

$$\frac{dh}{dz} = \left(\frac{\partial h}{\partial s}\right)_\rho \frac{ds}{dz} + \left(\frac{\partial h}{\partial \rho}\right)_s \frac{d\rho}{dz} . \quad (18)$$

Note from eq. (14) that

$$\left(\frac{\partial h}{\partial P}\right)_s = \frac{1}{\rho} , \quad (19)$$

and

$$\rho \left(\frac{\partial h}{\partial \rho}\right)_s = \left(\frac{\partial P}{\partial \rho}\right)_s = a^2 , \quad (20)$$

where a is the sonic velocity for homogeneous flow.

1.7 Combined Working Equation

Let us now reduce the preceding set of equations, i.e., eqs. (1), (8), (13) and (18), to a single working equation. First we substitute eq. (18) into (13):

$$\left(\frac{\partial h}{\partial s}\right)_\rho \frac{ds}{dz} + \left(\frac{\partial h}{\partial \rho}\right)_s \frac{d\rho}{dz} + w \frac{dw}{dz} + g = q' , \quad (21)$$

in which we have used the identity

$$w \frac{dw}{dz} = \frac{d}{dz} \left(\frac{1}{2} w^2 \right) . \quad (22)$$

Next we use eq. (8) to eliminate ds/dz from eq. (21):

$$\left(\frac{\partial h}{\partial s}\right)_\rho \frac{q'}{T} + \left(\frac{\partial h}{\partial s}\right)_\rho \theta' + \left(\frac{\partial h}{\partial \rho}\right)_s \frac{d\rho}{dz} + w \frac{dw}{dz} + g = q' . \quad (23)$$

Rearranging terms with the aid of eq. (20) results in

$$-\frac{a^2}{\rho} \frac{d\rho}{dz} = g + \left(\frac{\partial h}{\partial s}\right)_\rho \theta' + \left[\left(\frac{\partial h}{\partial s}\right)_\rho \frac{1}{T} - 1 \right] q' + w \frac{dw}{dz} . \quad (24)$$

The continuity equation, eq. (1), may be expanded to yield

$$\frac{d}{dz}(\rho w A) = w A \frac{d\rho}{dz} + \rho A \frac{dw}{dz} + \rho w \frac{dA}{dz} = 0 \quad (25)$$

Solving this for dw/dz yields

$$\frac{dw}{dz} = -\frac{w}{\rho} \frac{d\rho}{dz} - \frac{w}{A} \frac{dA}{dz} \quad (26)$$

which in turn can be used in eq. (25), resulting in

$$-\frac{1}{\rho} \frac{d\rho}{dz} = \frac{1}{a^2 - w^2} \left\{ g + \left(\frac{\partial h}{\partial s} \right)_\rho \theta' + \left[\left(\frac{\partial h}{\partial s} \right)_\rho \frac{1}{T} - 1 \right] q' - \frac{w^2}{A} \frac{dA}{dz} \right\} \quad (27)$$

Let us focus on the term multiplying q' . It is a basic thermodynamic identity that

$$\left(\frac{\partial h}{\partial s} \right)_\rho = T + \frac{\gamma - 1}{\beta} \quad (28)$$

where γ is the specific heat ratio ($\gamma(s, P) \equiv c_p/c_v$), and β is the coefficient of volume expansion ($\beta \equiv \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p$). Thus the bracket term before q' is equal to $(\gamma - 1)/\beta T$. Another thermodynamic identity [2] of use here is:

$$\left(\frac{\partial T}{\partial v} \right)_s = -\frac{\gamma - 1}{v\beta} \quad (29)$$

From eq. (29) it follows that

$$\frac{\gamma - 1}{\beta} = \rho \left(\frac{\partial T}{\partial \rho} \right)_s \quad (30)$$

and with this we can write our final working equation in the form

$$-\frac{1}{\rho} \frac{d\rho}{dz} = \frac{1}{a^2 - w^2} \left\{ g + \left(\frac{\partial h}{\partial s} \right)_\rho \theta' + \frac{\rho}{T} \left(\frac{\partial T}{\partial \rho} \right)_s q' - \frac{w^2}{A} \frac{dA}{dz} \right\} \quad (31)$$

This is a first-order, non-linear ordinary differential equation which together with eq. (8) forms a system of equations that is amenable to solution by standard techniques. In the following section we shall examine the nature

of the solution for several special cases of interest.

We note that an alternate form of the working equation can be written by introducing the previously defined flow density, ψ :

$$-\frac{1}{\rho^3} \frac{d\rho}{dz} = \frac{1}{\psi_c^2 - \psi^2} \left\{ g + \left(\frac{\partial h}{\partial s} \right)_\rho \theta' + \frac{\rho}{T} \left(\frac{\partial T}{\partial \rho} \right)_s q' - \frac{\psi^2}{A\rho^2} \frac{dA}{dz} \right\}, \quad (32)$$

where the critical flow density, $\psi_c = \rho a$.

1.8 Nature of Solution for Special Cases

As a first step we observe the signs of the four terms comprising the bracket term on the right side of eq. (31). The first term, g , will be zero for flow in horizontal ducts, positive for upward-flowing fluids, and negative for downward-flowing fluids. In the second term, the factor θ' must always be non-negative (from eq. (9)), as must the factor $(\partial h / \partial s)_\rho$ for all normal substances and water except in its anomalous region which happens to be of no practical significance for geothermal systems (from eq. (28)). Thus the second term must always be non-negative. By similar reasoning we conclude that the factor $(\partial T / \partial \rho)_s$ in the third term is always non-negative, but the factor q' may be either positive (when heat is supplied to the fluid) or negative (when heat is removed from the fluid). Thus the third term may be positive (heat added), negative (heat removed), or zero (adiabatic). Finally the last term may be positive (expanding duct section), negative (contracting duct section), or zero (constant-area duct section).

Secondly we recognize that the choking velocity, a , (or the sonic velocity for a single-phase fluid) is a property of the fluid, being equal to $(\partial P / \partial \rho)_s$. This may be calculated for a two-phase fluid provided we know the properties of the mixture.

Thirdly it is clear that the solution of eq. (31) will depend on the nature of the dissipation function, θ' , except in the ideal case of frictionless flow, which can be represented as

$$\theta' = \frac{C}{A} \lambda_w(\rho, s, w) : \frac{w^2}{2} \geq 0, \quad (33)$$

where the functional form of $\lambda_w(\rho, s, w)$ remains to be determined.

Lastly the mathematical nature of the equation dictates that whenever $w = a$, the bracket term on the right side of the equation must vanish in order for the left side, i.e., dp/dz , to remain finite. Such a case corresponds to choked flow, and will be examined in more depth later.

Let us now investigate several special cases of interest.

Case A: $g = 0, \theta' = 0, q' = 0.$

These assumptions imply isentropic (i.e., reversible and adiabatic) flow in a horizontal duct. In such a case the flow is driven by the area variation along the duct. The governing equation is

$$\frac{1}{\rho} \frac{dp}{dz} = \frac{1}{a^2 - w^2} \frac{w^2}{A} \frac{dA}{dz}. \quad (34)$$

So long as $w < a$ ("subsonic"), the density and the cross-sectional area either increase or decrease together along the duct. When $w > a$, then the changes in ρ and A are of opposite sign. For the point where $w = a$, $dA/dz = 0$ and a throat exists, i.e., there is no change in area at this point. Clearly this is the familiar case of the De Laval nozzle.

Case B: $g = 0, q' = 0.$

Here we have adiabatic flow through a horizontal duct with dissipation effects. The controlling equation is

$$-\frac{d(\ln \rho)}{dz} = \frac{1}{a^2 - w^2} \left\{ \left(\frac{\partial h}{\partial s} \right)_\rho \theta' - \frac{w^2}{A} \frac{dA}{dz} \right\}. \quad (35)$$

This equation allows for variable cross-sectional area along the channel. Since the first term in brackets is always positive, choked flow (i.e., $w=a$) can only occur in a section where $dA/dz > 0$, i.e., in the diverging section of the channel. For the special case of flow in a pipe (i.e., $dA/dz=0$), we see that choked flow can never occur inside the pipe ($w=a$ implies $d(\ln \rho)/dz$ tends to ∞). However, realizing that A refers to the cross-section of the flow, we see that if $w=a$ it can only occur infinitesimally beyond the end of the pipe, at which point $dA/dz > 0$ and the flow stream expands abruptly. Of course the flow at the end of the pipe need not be choked (e.g., when $w < a$), but if choked flow does exist (i.e., $w=a$), then it must choke at the end of the pipe.

Case C: $g > 0, q' = 0$.

This case represents upward flow in an adiabatic duct. The equation is

$$-\frac{d(\ln \rho)}{dz} = \frac{1}{a^2 - w^2} \left\{ g + \left(\frac{\partial h}{\partial s} \right)_\rho \theta' - \frac{w^2}{A} \frac{dA}{dz} \right\} \quad (36)$$

The interpretation of this case is similar to Case B since the term g simply adds on to the term $(\partial h / \partial s)_\rho \theta'$. Once again choking can only occur at the end (top) of a straight pipe. It might be pointed out that it is in principle possible to so contour the duct from the choking point upward that the flow remains choked from that point onward. The shape of the duct, $A(z)$, will be found from

$$\frac{a^2}{A(z)} \frac{dA(z)}{dz} = g + \left(\frac{\partial h}{\partial s} \right)_\rho \theta' \quad (37)$$

Case D: $g < 0, q' = 0.$

Here is the case of downward flow in an adiabatic channel. The equation is the same as eq. (36) except that the first term in brackets is now negative. The interesting point here is that it is in principle possible for the flow to choke somewhere inside a straight pipe ($dA/dz=0$) so long as the dissipation term cancels the gravitational term.

Case E: $g > 0, \theta' = 0, q' = 0.$

This corresponds to upward, isentropic flow. The governing equation is

$$-\frac{d(\mathcal{L}np)}{dz} = \frac{1}{a^2 - w^2} \left\{ g - \frac{w^2}{A} \frac{dA}{dz} \right\} \quad (38)$$

For a constant-area pipe, choking can only occur at the end (top) of the pipe. However, since the gravity term by itself is not large, one usually finds that extremely long pipes are needed to attain choking conditions.

2. Choking Velocity and Mass Flux in Liquid-Vapor Systems

2.1 Definitions and Assumptions

We shall first derive working equations for the isentropic compressibility, k_s , for a pure substance existing in equilibrium in the liquid and vapor phases. By definition, the compressibility is given as

$$k_s = - \frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_s, \quad (39)$$

where $v = 1/\rho$ is the specific volume. From eq. (20), we recall that the choking velocity, a , is given by

$$a^2 = \left(\frac{\partial P}{\partial \rho} \right)_s = - v^2 \left(\frac{\partial P}{\partial v} \right)_s. \quad (40)$$

Combining eqs. (39) and (40), we obtain

$$a^2 = \frac{v}{k_s}. \quad (41)$$

The critical flow density will be important:

$$\psi_c = \rho a. \quad (42)$$

Alternatively,

$$\psi_c^2 = \rho^2 a^2 = a^2 / v^2 = \frac{1}{v k_s} = - \left(\frac{\partial P}{\partial v} \right)_s. \quad (43)$$

It may be noted that $\psi_c = \rho a$ is also the "acoustic impedance".

So far all definitions are general. It remains for us to examine the case of a two-phase, i.e., liquid-vapor, system.

2.2 Special Case of Liquid-Vapor Systems

Here the system pressure is a function of the system temperature through the saturation vapor-pressure relation, $P = P(T)$. Also each extensive (or specific) property may be expressed as a linear function of the

saturation properties through the well-known "lever rule": e.g.,

$$v = (1 - x)v_f + xv_g \quad (44a)$$

$$= v_f + xv_{fg} \quad (44b)$$

$$= v(x, P) \quad ; \quad (44c)$$

$$s = (1 - x)s_f + xs_g \quad (45a)$$

$$= s_f + xs_{fg} \quad (45b)$$

$$= s(x, P) \quad . \quad (45c)$$

The saturation properties, v_f , s_f , v_g , s_g , $v_{fg} = v_g - v_f$, and $s_{fg} = s_g - s_f$, may be taken as functions of either pressure or temperature.

The familiar Clausius-Clapeyron equation [1] will be of use:

$$\left(\frac{dP}{dT} \right)_{\text{sat}} = \frac{s_{fg}}{v_{fg}} \quad (46)$$

This gives the slope of the vapor-pressure curve (P,T coordinates) as the ratio of the evaporation entropy to the evaporation specific volume.

We shall now determine the critical flow density for the two-phase case. We shall find it convenient to use Jacobians [3] to evaluate the partial derivatives involved. In brief summary, if two functions A and B each depend on only two variables, a and b, i.e., $A = A(a, b)$ and $B = B(a, b)$, then the Jacobian, J, is given by

$$J = \frac{\partial(A, B)}{\partial(a, b)} = \text{Det} \begin{vmatrix} \frac{\partial A}{\partial a} & \frac{\partial A}{\partial b} \\ \frac{\partial B}{\partial a} & \frac{\partial B}{\partial b} \end{vmatrix} \quad (47a)$$

$$= \frac{\partial A}{\partial a} \frac{\partial B}{\partial b} - \frac{\partial A}{\partial b} \frac{\partial B}{\partial a} \quad , \quad (47b)$$

where it is understood that whenever $\partial/\partial a$ appears b is held constant, and that whenever $\partial/\partial b$ appears a is held constant. Jacobians obey a chain rule, i.e.,

$$\frac{\partial(A,B)}{\partial(a,b)} = \frac{\partial(A,B)}{\partial(x,y)} : \frac{\partial(x,y)}{\partial(a,b)} , \quad (47c)$$

and subscripts on partial derivatives may be absorbed into the Jacobian, i.e.,

$$\left. \frac{\partial A}{\partial a} \right|_b = \frac{\partial(A,b)}{\partial(a,b)} . \quad (47d)$$

We start from eq. (43):

$$-\frac{1}{\psi_c^2} = \left(\frac{\partial v}{\partial P} \right)_s = \frac{\partial(v,s)}{\partial(P,s)} = \frac{\partial(v,s)}{\partial(P,x)} \cdot \frac{\partial(P,x)}{\partial(P,s)} \quad (48a)$$

$$= \frac{\partial(v,s)}{\partial(P,x)} \cdot \left[\frac{\partial(P,s)}{\partial(P,x)} \right]^{-1} = \left(\frac{\partial s}{\partial x} \right)_p^{-1} \frac{\partial(v,s)}{\partial(P,x)} . \quad (48b)$$

Now, using eqs. (45b) and (47b).

$$-\frac{1}{\psi_c^2} = s_{fg}^{-1} \left[\left(\frac{\partial v}{\partial P} \right)_x \left(\frac{\partial s}{\partial x} \right)_p - \left(\frac{\partial v}{\partial x} \right)_p \left(\frac{\partial s}{\partial P} \right)_x \right] . \quad (49)$$

The $\partial/\partial x$ terms in eq. (49) can be found from eqs. (44b) and (45b):

$$\left(\frac{\partial s}{\partial x} \right)_p = s_{fg} , \quad \left(\frac{\partial v}{\partial x} \right)_p = v_{fg} . \quad (50)$$

The $\partial/\partial P$ terms may be evaluated from eqs. (44a) and (45a):

$$\left(\frac{\partial v}{\partial P} \right)_x = (1-x) \frac{dv_f}{dP} + x \frac{dv_g}{dP} , \quad (51a)$$

$$\left(\frac{\partial s}{\partial P} \right)_x = (1-x) \frac{ds_f}{dP} + x \frac{ds_g}{dP} , \quad (51b)$$

where full derivatives may be used. Substitution of eqs. (50), (51a), and (51b) into (49) yields:

$$-\frac{1}{\psi_c^2} = s_{fg}^{-1} \left[\left\{ (1-x) \frac{dv_f}{dP} + x \frac{dv_g}{dP} \right\} s_{fg} - \left\{ (1-x) \frac{ds_f}{dP} + x \frac{ds_g}{dP} \right\} v_{fg} \right] \quad (52a)$$

$$= (1-x) \frac{dv_f}{dP} + x \frac{dv_g}{dP} - \frac{v_{fg}}{s_{fg}} \left\{ (1-x) \frac{ds_f}{dP} + x \frac{ds_g}{dP} \right\} . \quad (52b)$$

With the aid of the Clausius-Clapeyron equation, this may be written as:

$$-\frac{1}{\psi_c} = (1-x) \frac{dv_f}{dP} + x \frac{dv_g}{dP} - \left(\frac{dT}{dP}\right)_{\text{sat}} \left\{ (1-x) \frac{ds_f}{dP} + x \frac{ds_g}{dP} \right\} \quad (53a)$$

$$= (1-x) \left\{ \frac{dv_f}{dP} - \left(\frac{dT}{dP}\right)_{\text{sat}} \frac{ds_f}{dP} \right\} + x \left\{ \frac{dv_g}{dP} - \left(\frac{dT}{dP}\right)_{\text{sat}} \frac{ds_g}{dP} \right\} \quad (53b)$$

We now define the bracket terms as $-\phi_f$ and $-\phi_g$, respectively, and write

$$\frac{1}{\psi_c} = (1-x)\phi_f + x\phi_g, \quad (54a)$$

where

$$\phi_i = -\frac{dv_i}{dP} + \left(\frac{dT}{dP}\right)_{\text{sat}} \frac{ds_i}{dP}, \text{ with } i = f, g. \quad (54b)$$

Clearly ϕ is a function of pressure (or temperature) and may be computed for all two-phase states from a knowledge of the properties of the saturated liquid and saturated vapor.

We may introduce the entropy, s , via the thermodynamic identity for the first term in eq. (54b):

$$\frac{dv_i}{dP} = \left(\frac{\partial v_i}{\partial P}\right)_s + \left(\frac{\partial v_i}{\partial s}\right)_P \left(\frac{ds_i}{dP}\right)_{\text{sat}} \quad (54c)$$

Then, combining these we obtain

$$\phi_i = -\left(\frac{\partial v_i}{\partial P}\right)_s + \left(\frac{ds_i}{dP}\right)_{\text{sat}} \left\{ \left(\frac{dT}{dP}\right)_{\text{sat}} - \left(\frac{\partial v_i}{\partial s}\right)_P \right\}, \quad (54d)$$

and with the aid of the second Maxwell relation [1] (i.e., $\partial v / \partial s|_P = \partial T / \partial P|_s$), we get

$$\phi_i = -\left(\frac{\partial v_i}{\partial P}\right)_s + \left(\frac{ds_i}{dP}\right)_{\text{sat}} \left\{ \left(\frac{dT}{dP}\right)_{\text{sat}} - \left(\frac{\partial T}{\partial P}\right)_s \right\}. \quad (54e)$$

Now using the identity

$$\left(\frac{dT}{dP}\right)_{\text{sat}} = \left(\frac{\partial T}{\partial P}\right)_s + \left(\frac{\partial T}{\partial s}\right)_p \left(\frac{ds}{dP}\right)_{\text{sat}} \quad (54f)$$

and the second Maxwell relation for $\partial T/\partial P|_s$,

$$\left(\frac{dT}{dP}\right)_{\text{sat}} = \left(\frac{\partial v}{\partial s}\right)_p + \left(\frac{\partial T}{\partial s}\right)_p \left(\frac{ds}{dP}\right)_{\text{sat}} \quad (54g)$$

Substitution of eq. (54g) into (54d) yields

$$\phi_i = - \left(\frac{\partial v_i}{\partial P}\right)_s + \left(\frac{ds}{dP}\right)_{\text{sat}} \left\{ \left(\frac{\partial T}{\partial s}\right)_p \left(\frac{ds}{dP}\right)_{\text{sat}} \right\} \quad (54h)$$

$$\phi_i = - \left(\frac{\partial v_i}{\partial P}\right)_s + \left(\frac{\partial T}{\partial s}\right)_p \left(\frac{ds}{dP}\right)_{\text{sat}}^2 \quad (54i)$$

The first term can be replaced by $v_i k_s$ (from eq. (39)), and the term $\partial T/\partial s|_p$ can be shown from simple thermodynamic principles to be equal to T/c_p (as the slope of isobars in temperature-entropy coordinates). Thus,

$$\phi_i = v_i k_s + \frac{T}{c_p} \left(\frac{ds}{dP}\right)_{\text{sat}}^2 \quad (55)$$

In general, for both liquid and vapor, we may express $ds/dP|_{\text{sat}}$ as follows:

$$\left(\frac{ds}{dP}\right)_{\text{sat}} = \left(\frac{\partial s}{\partial P}\right)_T + \left(\frac{\partial s}{\partial T}\right)_p \left(\frac{dT}{dP}\right)_{\text{sat}} \quad (56a)$$

$$= - \left(\frac{\partial v}{\partial T}\right)_p + \frac{c_p}{T} \left(\frac{dT}{dP}\right)_{\text{sat}} \quad (56b)$$

$$= - v\beta + \frac{c_p}{T} \left(\frac{dP}{dT}\right)_{\text{sat}}^{-1} \quad (56c)$$

$$= - v\beta + \frac{c_p}{T} \frac{v_{fg}}{s_{fg}} \quad (56d)$$

where $\beta = (1/v)(\partial v/\partial T)_p$ is the coefficient of volume expansion.

For the case of liquids, $\beta \approx 0$ and $k_s \approx 0$, therefore

$$\left(\frac{ds}{dP}\right)_{\text{sat}} = \frac{c_p}{T} \left(\frac{dP}{dT}\right)_{\text{sat}}^{-1} \quad (57a)$$

and

$$\phi_f = \frac{c_p}{T} \left(\frac{dP}{dT} \right)_{\text{sat}}^{-2} = \frac{c_p}{T} \left(\frac{v_{fg}}{s_{fg}} \right)^2 \quad (57b)$$

This expression shows that ϕ_f may be taken as a function of either the temperature or the pressure, and can be computed and tabulated.

For the case of vapors, an interesting special case occurs when $ds/dP|_{\text{sat}} = 0$, i.e., when the saturated vapor curve passes through a vertical asymptote. Here,

$$\phi_g = v_g k_s \quad (58)$$

Another special case occurs whenever

$$\frac{\beta}{k-k_s} \cdot \frac{v_{fg}}{s_{fg}} \ll 1 \quad (59a)$$

or

$$s_{fg} \gg v_{fg} \frac{\beta}{k-k_s} \quad (59b)$$

which leads to

$$\left(\frac{ds}{dP} \right)_{\text{sat}} \gg \frac{c_p}{\beta T v_g} = \frac{\beta}{k-k_s} \quad (59c)$$

Then, from eq. (56d)

$$\frac{1}{\beta v_g} \left(\frac{ds}{dP} \right)_{\text{sat}} = -1 + \frac{c_p}{\beta T v_g} \frac{v_{fg}}{s_{fg}} \quad (59d)$$

in which the last term is very small compared to unity. Thus,

$$\left(\frac{ds}{dP} \right)_{\text{sat}}^2 = \beta^2 v_g^2 \quad (59e)$$

and

$$\phi_g = v_g k_s + \frac{T}{c_p} \beta^2 v_g^2 \quad (59f)$$

or

$$\phi_g = v_g \left[k_s + \frac{\beta^2 T v_g}{c_p} \right] \quad (59g)$$

$$\phi_g = v_g k \quad (59h)$$

In these equations, the term k represents the isothermal compressibility defined as

$$k = - \frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_T \quad (59i)$$

Physically, this special case corresponds to the condition in which small liquid droplets are finely dispersed through the vapor. These liquid droplets have a large latent heat and can absorb large amounts of energy at constant temperature. Thus the mixture stays at a constant temperature, and instead of dealing with the isentropic speed of sound, we have the isothermal speed of sound. The propagation of sound waves through ordinary atmospheric fog is a familiar example of this case.

The most general case for vapors gives

$$\phi_g = v_g k_s + \frac{T}{c_p} \left(\frac{ds}{dP} \right)_{\text{sat}}^2 \quad (60a)$$

Using eqs. (59c) and (59d), we can express

$$\left(\frac{ds}{dP} \right)_{\text{sat}}^2 = \frac{c_p^2 (k - k_s)^2}{\beta^2 T^2} \left[-1 + \frac{\beta}{k - k_s} \frac{v_{fg}}{s_{fg}} \right]^2, \quad (60b)$$

which in turn can be substituted into eq. (60a) to yield

$$\phi_g = v_g \left[k_s + \frac{c_p (k - k_s)^2}{v_g \beta^2 T^2} \left\{ -1 + \frac{\beta}{k - k_s} \frac{v_{fg}}{s_{fg}} \right\}^2 \right], \quad (60c)$$

or more simply,

$$\phi_g = v_g \left[k_s + (k - k_s) \left\{ -1 + \frac{\beta}{k - k_s} \frac{v_{fg}}{s_{fg}} \right\}^2 \right]. \quad (60d)$$

Following the usual convention for liquid-vapor mixtures, we adopt an evaporation function for ϕ , called ϕ_{fg} , and defined as

$$\phi_{fg} = \phi_g - \phi_f \quad (61a)$$

Recalling eq. (54a), we may write

$$\frac{1}{\psi_c^2} = \phi_f + x\phi_{fg} \quad , \quad (61b)$$

and observe that

$$\left. \frac{1}{\psi_c^2} \right|_{x=0} = \phi_f \quad , \quad (61c)$$

and

$$\left. \frac{1}{\psi_c^2} \right|_{x=1} = \phi_f + \phi_{fg} = \phi_g \quad . \quad (61d)$$

Alternatively, instead of eq. (61b) we may recall eq. (54a)

$$\frac{1}{\psi_c^2} = (1-x)\phi_f + x\phi_g \quad , \quad (54a)$$

and with the aid of eq. (53b), we may thus identify ϕ_f and ϕ_g as

$$\phi_f = - \frac{dv_f}{dP} + \left(\frac{dT}{dP} \right)_{sat} \frac{ds_f}{dP} \quad , \quad (62a)$$

and

$$\phi_g = - \frac{dv_g}{dP} + \left(\frac{dT}{dP} \right)_{sat} \frac{ds_g}{dP} \quad . \quad (62b)$$

Thus the evaporation term, ϕ_{fg} , may be expressed as

$$\phi_{fg} = \frac{dv_f}{dP} - \frac{dv_g}{dP} + \left(\frac{dT}{dP} \right)_{sat} \left[\frac{ds_g}{dP} - \frac{ds_f}{dP} \right] \quad (62c)$$

$$= - \frac{d(v_g - v_f)}{dP} + \left(\frac{dT}{dP} \right)_{sat} \frac{d(s_g - s_f)}{dP} \quad (62d)$$

$$= - \frac{dv_{fg}}{dP} + \frac{v_{fg}}{s_{fg}} \frac{ds_{fg}}{dP} \quad (62e)$$

$$= \frac{v_{fg}}{s_{fg}} \left[\frac{ds_{fg}}{dP} - \frac{s_{fg}}{v_{fg}} \frac{dv_{fg}}{dP} \right] \quad (62f)$$

$$= \frac{v_{fg}}{s_{fg}} \left[\left(\frac{dT}{dP} \right)_{sat} \frac{ds_{fg}}{dT} - \frac{s_{fg}}{v_{fg}} \left(\frac{dT}{dP} \right)_{sat} \frac{dv_{fg}}{dT} \right] \quad (62g)$$

$$= \frac{v_{fg}}{s_{fg}} \left[\frac{v_{fg}}{s_{fg}} \frac{ds_{fg}}{dT} - \frac{dv_{fg}}{dT} \right] \quad (62h)$$

$$= \frac{v_{fg}^2}{s_{fg}} \left[\frac{1}{s_{fg}} \frac{ds_{fg}}{dT} - \frac{1}{v_{fg}} \frac{dv_{fg}}{dT} \right] \quad (62i)$$

$$= \frac{v_{fg}^2}{s_{fg}} \left[\frac{d(\ln s_{fg})}{dT} - \frac{d(\ln v_{fg})}{dT} \right] \quad (62j)$$

$$= \frac{v_{fg}^2}{s_{fg}} \frac{d}{dT} \left[\ln \frac{s_{fg}}{v_{fg}} \right] \quad (62k)$$

$$= \frac{v_{fg}^2}{s_{fg}} \frac{d}{dT} [\ln P'] \quad (62l)$$

$$= \frac{v_{fg}^2}{s_{fg}} \frac{1}{P'} \frac{d}{dT} [P'] \quad (62m)$$

$$\phi_{fg} = \frac{v_{fg}^2}{s_{fg}} \frac{P''}{P'} \quad (62n)$$

The Clausius-Clapeyron equation, eq. (46), was used to get to eq. (62l).

2.3 Summary of Working Equations

In summary we have the following working formula for the case of liquid-vapor, two-phase equilibrium systems:

$$\frac{1}{\psi_c^2} = \frac{1}{(\rho a)^2} = \phi_f + x \phi_{fg} \quad (63a)$$

$$= \frac{v_{fg}^2}{s_{fg}} \left[\frac{c_{p,f}}{s_{fg} T} + x \frac{P''(T)}{P'(T)} \right] \quad (63b)$$

When $x = 0$, we retrieve the saturated liquid case, and when $x = 1$, we retrieve the saturated vapor case. In between (at constant temperature and pressure), ψ_c^{-2} varies linearly with x .

This equation may be written in dimensionless form as follows:

$$\frac{s_{fg} T}{(\rho a)^2 v_{fg}^2} = \frac{c_{p,f}}{s_{fg}} + x T \frac{P''(T)}{P'(T)} \quad (64a)$$

$$\left(\frac{v}{v_{fg}} \right)^2 \frac{h_{fg}}{a^2} = \frac{c_{p,f}}{s_{fg}} + x T \frac{P''(T)}{P'(T)} \quad (64b)$$

Notice that we obtain the two limiting cases:

$$\frac{a}{h_{fg}^{1/2}} = \frac{v_f}{v_{fg}} \cdot \left(\frac{s_{fg}}{c_{p,f}} \right)^{1/2}, \quad \text{for } x = 0, \quad (65a)$$

and

$$\frac{a}{h_{fg}^{1/2}} = \frac{v_g}{v_{fg}} \cdot \left[\frac{c_{p,f}}{s_{fg}} + T \frac{P''(T)}{P'(T)} \right]^{-1/2}, \quad \text{for } x = 1. \quad (65b)$$

The appropriate scaling factor for the choking velocity is thus seen to be the square root of the latent heat, leading to a reduced choking velocity

$$a^+ = a/h_{fg}^{1/2}. \quad (66)$$

In general we can express a^+ as

$$a^+ = \frac{v}{v_{fg}} \cdot \left[\frac{c_{p,f}}{s_{fg}} + xT \frac{P''(T)}{P'(T)} \right]^{-1/2}, \quad (67a)$$

$$= \left(\frac{v_f}{v_{fg}} + x \right) \left(\phi'_f + x\phi'_{fg} \right)^{-1/2}, \quad (67b)$$

where we have introduced nondimensional functions ϕ'_f and ϕ'_{fg} defined as

$$\phi'_f = c_{p,f}/s_{fg}, \quad (67c)$$

and

$$\phi'_{fg} = T \frac{P''(T)}{P'(T)} = \frac{d(\ln P')}{d(\ln T)}. \quad (67d)$$

In the next section we will discuss the numerical evaluation of these various functions for two specific working fluids.

3. Calculations and Presentation of Results

3.1 Scope of the Calculations

The following quantities have been computed:

- (a) Choking velocity, a_f and a_g , for saturated water substance over the temperature range 100-250°C, and for saturated Refrigerant-114 over the range (-20) - (+70)°C;

- (b) Critical or choking flow density, $\psi_{c,f}$ and $\psi_{c,g}$, for the same fluids and temperature ranges given in (a);
- (c) Saturation properties including the functions ϕ'_f , ϕ'_{fg} and ϕ'_g for both fluids.

Tables of the results are given along with graphs of selected functions.

3.2 Thermodynamic Property Correlations

All calculations were performed on a Hewlett-Packard Model-85 desk-top computer system. The required thermodynamic properties for water substance and refrigerant-114 were calculated from simple correlations obtained from primary sources.

- (a) Saturation pressure, $P(T)$

$$\log_{10} P(t) = a_0 + \frac{a'}{t+273} + a_1 t + a_2 t^2 + a_3 t^3 \quad (68)$$

- (b) Saturation pressure times evaporation volume change, $Pv_{fg}(t)$

$$Pv_{fg}(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 \quad (69)$$

- (c) Saturated liquid volume, $v_f(t)$

$$v_f(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 \quad (70)$$

- (d) Saturated liquid enthalpy, $h_f(t)$

$$h_f(t) = d_0 + d_1 t + d_2 t^2 + d_3 t^3 \quad (71)$$

- (e) Latent heat, $h_{fg}(t)$

$$h_{fg}(t) = e_0 + e_1 t + e_2 t^2 + e_3 t^3 \quad (72)$$

- (f) Saturated liquid entropy, $s_f(t)$

$$s_f(t) = f_0 + f' \ln(t + 273) + f_1 t + f_2 t^2 + f_3 t^3 \quad (73)$$

- (g) Saturated liquid isothermal compressibility, $k_f(P)$

$$k_f(P) = g_0 + g_1 P \quad (74)$$

The coefficients used in the correlating equations are given in Table 1. The units of the various coefficients have been chosen such that the units of the various quantities are as follows:

$$\{t\} = ^\circ\text{C} \quad (75a)$$

$$\{P\} = \{k_f^{-1}\} = \text{kP} \quad (75b)$$

$$\{v_f\} = \{v_{fg}\} = \{1/\rho\} = \text{m}^3/\text{kg} \quad (75c)$$

$$\{h_f\} = \{h_{fg}\} = \text{kJ/kg} \quad (75d)$$

$$\{s_f\} = \{s_{fg}\} = \{c_{p,f}\} = \text{kJ/kg}\cdot\text{K} \quad (75e)$$

$$\{a\} = \text{m/s} \quad (75f)$$

$$\{\psi\} = \text{kg/s}\cdot\text{m}^2 \quad (75g)$$

All quantities needed for the calculation of the choking conditions in two-phase flow may be found from the above correlations. The first and second derivatives of pressure with respect to temperature, $P'(T)$ and $P''(T)$, may be obtained analytically, at first using eq. (68) and then evaluated numerically.

The property values calculated from the simple correlations agree very well with those given for water substance by Keenan, Keyes, Hill and Moore [4], and for Refrigerant-114 by Oguchi [5].

3.3 Summary of Results

Tables 2 and 3 contain saturation properties, including the nondimensional functions ϕ'_f , ϕ'_{fg} , and ϕ'_g , for water and Refrigerant-114, respectively. Using the values in these tables together with eqs. (66) and (67b), one may readily compute the choking velocity as a function of the dryness fraction x . For computational purposes it should be noted that the units of enthalpy must be joules/kilogram in order to obtain units of meters/second for choking velocity. The results of choking velocity and flow density for saturation conditions for water substance and Refrigerant-114 are listed in Tables 4-7.

Figures 1-4 are graphical representations of the same results.

Tables 8 and 9 contain a and ψ_c as a function of dryness fraction x for several isotherms for the two fluids. Figures 5 and 6 show lines of constant choking velocity and flow density in temperature-entropy coordinates for the fluids.

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Table 1
Coefficients in correlating equations for
thermodynamic properties

<u>Coefficient</u>	<u>Water substance</u>	<u>Refrigerant-114</u>
a_0	7.5445	7.979809665
a'	-2.1178E+03	-1.649188E+03
a_1	2.73E-03	-4.974952E-03
a_2	-1.5945E-05	8.487E-06
a_3	2.6739E-08	0
b_0	126	12.75321704
b_1	0.433	3.04701443E-02
b_2	4.41E-04	-2.28733374E-04
b_3	-3.9E-06	-1.1629186E-06
b_4	0	-3.2E-09
c_0	0.001	6.539242079E-04
c_1	0	1.192772E-06
c_2	3.97E-09	3.6841033E-09
c_3	0	3.4858654E-11
d_0	0	4.186793049E+02
d_1	4.218	0.954609102
d_2	-7.98E-04	1.281856E-03
d_3	5.17E-06	-3.21E-07
e_0	2.5015E+03	1.374105791E+02
e_1	-2.4335	-0.331240625
e_2	1.8339E-03	-1.158802E-03
e_3	-1.8616E-05	-3.32E-06

(cont.)

Table 1, continued

<u>Coefficient</u>	<u>Water Substance</u>	<u>Refrigerant-114</u>
f_0	-23.8403	-1.168052836
f'	4.25	0.954609102
f_1	- 2.14E-04	0
f_2	- 5.73E-07	4.693E-06
f_3	7.61E-09	-1.7E-08

Table 2

Saturation properties for water substanceincluding ϕ'_f , ϕ'_g and ϕ'_{fg}

t °C	v_{fg} m ³ /kg	h_{fg} kJ/kg	s_f kJ/kg·K	s_g kJ/kg·K	ϕ'_f	ϕ'_{fg}	ϕ'_g
0	2.06E+02	2501.5	.000	9.158	.458	17.385	17.843
10	1.07E+02	2477.3	.151	8.900	.479	16.586	17.065
20	5.82E+01	2453.4	.296	8.665	.500	15.831	16.331
30	3.32E+01	2429.6	.436	8.451	.522	15.117	15.639
40	1.97E+01	2405.9	.572	8.255	.544	14.440	14.984
50	1.21E+01	2382.1	.704	8.075	.567	13.798	14.366
60	7.71E+00	2358.1	.831	7.909	.591	13.189	13.781
70	5.06E+00	2333.8	.955	7.756	.616	12.612	13.227
80	3.41E+00	2309.0	1.075	7.614	.642	12.063	12.705
90	2.36E+00	2283.8	1.193	7.481	.668	11.544	12.212
100	1.67E+00	2257.9	1.307	7.358	.696	11.052	11.748
110	1.21E+00	2231.2	1.419	7.242	.726	10.586	11.312
120	8.90E-01	2203.7	1.528	7.133	.757	10.147	10.904
130	6.68E-01	2175.2	1.634	7.030	.790	9.733	10.523
140	5.09E-01	2145.7	1.739	6.932	.824	9.344	10.169
150	3.93E-01	2114.9	1.842	6.840	.861	8.981	9.842
160	3.08E-01	2082.8	1.943	6.751	.901	8.642	9.543
170	2.44E-01	2049.3	2.042	6.666	.943	8.329	9.272
180	1.95E-01	2014.3	2.140	6.585	.989	8.040	9.029
190	1.58E-01	1977.7	2.236	6.506	1.038	7.777	8.815
200	1.29E-01	1939.2	2.331	6.430	1.092	7.538	8.630
210	1.06E-01	1898.9	2.425	6.355	1.150	7.325	8.475
220	8.73E-02	1856.7	2.518	6.283	1.213	7.137	8.350
230	7.25E-02	1812.3	2.610	6.212	1.282	6.975	8.257
240	6.05E-02	1765.7	2.702	6.143	1.359	6.839	8.197
250	5.06E-02	1716.9	2.793	6.074	1.443	6.728	8.171

Table 3

Saturation properties for Refrigerant-114

including ϕ'_f , ϕ'_g and ϕ'_{fg}

t °C	v_{fg} m ³ /kg	h_{fg} kJ/kg	s_f kJ/kg·K	s_g kJ/kg·K	ϕ'_f	ϕ'_{fg}	ϕ'_g
-20	3.26E-01	143.6	4.1162	4.6834	1.591	9.589	11.180
-15	2.63E-01	142.1	4.1340	4.6845	1.663	9.288	10.951
-10	2.14E-01	140.6	4.1517	4.6860	1.739	9.000	10.739
- 5	1.76E-01	139.0	4.1693	4.6878	1.817	8.725	10.542
0	1.46E-01	137.4	4.1868	4.6899	1.899	8.462	10.361
5	1.21E-01	135.7	4.2042	4.6922	1.983	8.212	10.195
10	1.02E-01	134.0	4.2216	4.6948	2.072	7.973	10.044
15	8.62E-02	132.2	4.2389	4.6975	2.164	7.745	9.908
20	7.32E-02	130.3	4.2560	4.7005	2.259	7.528	9.787
25	6.26E-02	128.4	4.2731	4.7036	2.359	7.322	9.681
30	5.37E-02	126.3	4.2901	4.7069	2.463	7.127	9.590
35	4.63E-02	124.3	4.3070	4.7102	2.572	6.942	9.514
40	4.01E-02	122.1	4.3237	4.7136	2.686	6.767	9.453
45	3.49E-02	119.9	4.3404	4.7171	2.805	6.602	9.407
50	3.04E-02	117.5	4.3570	4.7207	2.929	6.448	9.378
55	2.66E-02	115.1	4.3734	4.7242	3.061	6.304	9.365
60	2.33E-02	112.6	4.3897	4.7278	3.198	6.171	9.369
65	2.05E-02	110.1	4.4058	4.7314	3.344	6.048	9.392
70	1.80E-02	107.4	4.4219	4.7349	3.498	5.937	9.434

Table 4

Choking velocity of saturated liquid
and saturated vapor for water substance

t °C	a_f m/s	a_g m/s
100	1.1204E+00	4.3867E+02
110	1.5212E+00	4.4451E+02
120	2.0264E+00	4.5010E+02
130	2.6529E+00	4.5539E+02
140	3.4183E+00	4.6033E+02
150	4.3407E+00	4.6483E+02
160	5.4391E+00	4.6884E+02
170	6.7331E+00	4.7227E+02
180	8.2429E+00	4.7505E+02
190	9.9901E+00	4.7709E+02
200	1.1998E+01	4.7831E+02
210	1.4290E+01	4.7863E+02
220	1.6895E+01	4.7798E+02
230	1.9843E+01	4.7630E+02
240	2.3169E+01	4.7355E+02
250	2.6913E+01	4.6970E+02

Table 5

Choking velocity for saturated liquid and
saturated vapor for Refrigerant-114

t °C	a_f m/s	a_g m/s
-20	5.8157E-01	1.1355E+02
-15	7.0725E-01	1.1420E+02
-10	8.5229E-01	1.1477E+02
- 5	1.0184E+00	1.1527E+02
0	1.2074E+00	1.1568E+02
5	1.4210E+00	1.1601E+02
10	1.6611E+00	1.1625E+02
15	1.9297E+00	1.1640E+02
20	2.2288E+00	1.1645E+02
25	2.5604E+00	1.1641E+02
30	2.9268E+00	1.1626E+02
35	3.3305E+00	1.1602E+02
40	3.7740E+00	1.1566E+02
45	4.2601E+00	1.1520E+02
50	4.7919E+00	1.1463E+02
55	5.3729E+00	1.1395E+02
60	6.0070E+00	1.1316E+02
65	6.6984E+00	1.1225E+02
70	7.4522E+00	1.1124E+02

Table 6

Choking flow density for saturated liquid
and saturated vapor for water substance

t $^{\circ}\text{C}$	$\psi_{c,f}$ $\text{kg/s}\cdot\text{m}^2$	$\psi_{c,g}$ $\text{kg/s}\cdot\text{m}^2$
100	1.0776E+03	2.6239E+02
110	1.4514E+03	3.6768E+02
120	1.9168E+03	5.0503E+02
130	2.4861E+03	6.8106E+02
140	3.1715E+03	9.0305E+02
150	3.9848E+03	1.1789E+03
160	4.9373E+03	1.5171E+03
170	6.0401E+03	1.9266E+03
180	7.3035E+03	2.4171E+03
190	8.7379E+03	2.9987E+03
200	1.0354E+04	3.6822E+03
210	1.2161E+04	4.4789E+03
220	1.4172E+04	5.4012E+03
230	1.6399E+04	6.4624E+03
240	1.8857E+04	7.6771E+03
250	2.1563E+04	9.0618E+03

Table 7

Choking flow density for saturated liquid
and saturated vapor for Refrigerant-114

t °C	$\psi_{c,f}$ kg/s.m ²	$\psi_{c,g}$ kg/s.m ²
-20	9.2127E+02	3.4754E+02
-15	1.1107E+03	4.3290E+02
-10	1.3269E+03	5.3392E+02
- 5	1.5715E+03	6.5245E+02
0	1.8463E+03	7.9038E+02
5	2.1531E+03	9.4967E+02
10	2.4932E+03	1.1323E+03
15	2.8684E+03	1.3403E+03
20	3.2799E+03	1.5758E+03
25	3.7292E+03	1.8408E+03
30	4.2176E+03	2.1375E+03
35	4.7465E+03	2.4679E+03
40	5.3172E+03	2.8342E+03
45	5.9313E+03	3.2386E+03
50	6.5901E+03	3.6833E+03
55	7.2955E+03	4.1706E+03
60	8.0492E+03	4.7029E+03
65	8.8534E+03	5.2827E+03
70	9.7106E+03	5.9126E+03

Table 8

Choking flow density and velocity in two-phase water
systems at 100, 150, 200 and 250°C

<u>x</u> <u>%</u>	<u>ψ_c</u> <u>kg/s.m²</u>	<u>a</u> <u>m/s</u>
(t = 100°C)		
0	1.0776E+03	1.1204E+00
10	6.7003E+02	1.1264E+02
20	5.2750E+02	1.7682E+02
30	4.4900E+02	2.2552E+02
40	3.9757E+02	2.6612E+02
50	3.6054E+02	3.0157E+02
60	3.3224E+02	3.3341E+02
70	3.0970E+02	3.6254E+02
80	2.9121E+02	3.8954E+02
90	2.7568E+02	4.1482E+02
100	2.6239E+02	4.3867E+02
(t = 150°C)		
0	3.9848E+03	4.3407E+00
10	2.7882E+03	1.1267E+02
20	2.2687E+03	1.8088E+02
30	1.9614E+03	2.3350E+02
40	1.7525E+03	2.7755E+02
50	1.5987E+03	3.1605E+02
60	1.4794E+03	3.5064E+02
70	1.3833E+03	3.8226E+02
80	1.3038E+03	4.1157E+02
90	1.2367E+03	4.3899E+02
100	1.1789E+03	4.6483E+02

(cont.)

Table 8, continued

<u>x</u> <u>%</u>	ψ_c <u>kg/s.m²</u>	<u>a</u> <u>m/s</u>
(t = 200°C)		
0	1.0354E+04	1.1998E+01
10	7.9628E+03	1.1174E+02
20	6.7095E+03	1.8053E+02
30	5.9073E+03	2.3499E+02
40	5.3377E+03	2.8105E+02
50	4.9064E+03	3.2150E+02
60	4.5651E+03	3.5791E+02
70	4.2864E+03	3.9125E+02
80	4.0532E+03	4.2214E+02
90	3.8544E+03	4.5105E+02
100	3.6822E+03	4.7831E+02
(t = 250°C)		
0	2.1563E+04	2.6913E+01
10	1.7808E+04	1.1231E+02
20	1.5511E+04	1.7629E+02
30	1.3923E+04	2.2866E+02
40	1.2740E+04	2.7367E+02
50	1.1814E+04	3.1356E+02
60	1.1065E+04	3.4966E+02
70	1.0443E+04	3.8281E+02
80	9.9149E+03	4.1361E+02
90	9.4597E+03	4.4247E+02
100	9.0618E+03	4.6970E+02

Table 9

Choking flow density and velocity in
two-phase Refrigerant-114 at -20, -10, 0, 10,
20 and 30°C

<u>x</u> <u>%</u>	<u>ψ_c</u> <u>kg/s·m²</u>	<u>a</u> <u>m/s</u>
(t = -20°C)		
0	9.2127E+02	5.8157E-01
10	7.2771E+02	2.4190E+01
20	6.2036E+02	4.0851E+01
30	5.4977E+02	5.4131E+01
40	4.9884E+02	6.5383E+01
50	4.5986E+02	7.5270E+01
60	4.2879E+02	8.4167E+01
70	4.0327E+02	9.2309E+01
80	3.8182E+02	9.9851E+01
90	3.6347E+02	1.0691E+02
100	3.4754E+02	1.1355E+02
(t = -10°C)		
0	1.3269E+03	8.5229E-01
10	1.0771E+03	2.3776E+01
20	9.3010E+02	4.0464E+01
30	8.3047E+02	5.3928E+01
40	7.5724E+02	6.5402E+01
50	7.0050E+02	7.5514E+01
60	6.5485E+02	8.4628E+01
70	6.1711E+02	9.2975E+01
80	5.8522E+02	1.0071E+02
90	5.5781E+02	1.0795E+02
100	5.3392E+02	1.1477E+02

(cont.)

Table 9, continued

x %	ψ_c kg/s.m ²	a m/s
(t = 0°C)		
0	1.8463E+03	1.2074E+00
10	1.5356E+03	2.3378E+01
20	1.3425E+03	4.0000E+01
30	1.2077E+03	5.3582E+01
40	1.1068E+03	6.5231E+01
50	1.0276E+03	7.5533E+01
60	9.6324E+02	8.4839E+01
70	9.0964E+02	9.3373E+01
80	8.6410E+02	1.0129E+02
90	8.2478E+02	1.0870E+02
100	7.9038E+02	1.1568E+02

(t = 10°C)		
0	2.4932E+03	1.6611E+00
10	2.1187E+03	2.3022E+01
20	1.8742E+03	3.9482E+01
30	1.6986E+03	5.3108E+01
40	1.5646E+03	6.4877E+01
50	1.4580E+03	7.5329E+01
60	1.3706E+03	8.4793E+01
70	1.2972E+03	9.3487E+01
80	1.2345E+03	1.0156E+02
90	1.1801E+03	1.0912E+02
100	1.1323E+03	1.1625E+02

(cont.)

Table 9, continued

x %	ψ_c kg/s·m ²	a m/s
(t = 20°C)		
0	3.2799E+03	2.2288E+00
10	2.8406E+03	2.2729E+01
20	2.5408E+03	3.8933E+01
30	2.3194E+03	5.2524E+01
40	2.1474E+03	6.4352E+01
50	2.0087E+03	7.4904E+01
60	1.8939E+03	8.4488E+01
70	1.7967E+03	9.3308E+01
80	1.7131E+03	1.0151E+02
90	1.6401E+03	1.0920E+02
100	1.5758E+03	1.1645E+02
(t = 30°C)		
0	4.2176E+03	2.9268E+00
10	3.7143E+03	2.2523E+01
20	3.3567E+03	3.8380E+01
30	3.0858E+03	5.1853E+01
40	2.8715E+03	6.3670E+01
50	2.6963E+03	7.4266E+01
60	2.5498E+03	8.3922E+01
70	2.4248E+03	9.2829E+01
80	2.3165E+03	1.0112E+02
90	2.2216E+03	1.0891E+02
100	2.1375E+03	1.1626E+02

Figure Captions

- Fig. 1. Choking velocity for saturated liquid water.
- Fig. 2. Choking velocity for saturated water vapor.
- Fig. 3. Choking velocity for saturated liquid Refrigerant-114.
- Fig. 4. Choking velocity for saturated Refrigerant-114 vapor.
- Fig. 5. Lines of constant choking velocity and critical flow density
in temperature-entropy coordinates for water substance.
- Fig. 6. Lines of constant choking velocity and critical flow density
in temperature-entropy coordinates for Refrigerant-114.

Fig. 1. Choking velocity for saturated liquid water.

Fig. 2. Choking velocity for saturated water vapor.

Fig. 3. Choking velocity for saturated liquid Ref.-114.

Fig. 4. Choking velocity for saturated
Ref.-114 vapor.

Fig. 5. Lines of constant choking velocity and critical flow density in temperature-entropy coordinates for water substance.

Fig. 6. Lines of constant choking velocity and critical flow density in temperature-entropy coordinates for Ref.-114.