Theoretical Light and Velocity Curves for Cepheid Variables

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ABSTRACT

New Research Initiatives Program W-318 was successfully completed with the calculation of a series of nonlinear Cepheid models with the new DYN code. The dynamical zoning feature of the DYN code makes it unique in that it allows reliable light curves, as well as velocity curves, to be calculated. The models are used to extend to light curves Christy's conclusion, based on velocity curves, that the observations of Cepheids cannot be matched unless the mass is significantly reduced (assuming a homogeneous composition). An examination of our results also leads us to the conclusion that observed light curves are considerably more complex than has previously been thought. The light and velocity curves we have calculated should be useful guides for interpreting the observations.

I. SUMMARY OF ACTIVITIES

We have used the new DYN code to calculate theoretical light and velocity curves for Cepheid variable stars during the period October 1977 - September 1978, under New Research Initiatives Program W-318 at the Los Alamos Scientific Laboratory (LASL). The calculations were undertaken to broaden our understanding of, and possibly resolve, the long-standing Cepheid mass anomaly.

A total of seven Cepheid models were calculated. Results of the study were presented at the Conference on Current Problems in Stellar Pulsation Instabilities, held at the NASA Goddard Space Flight Center in June 1978, and at a meeting of the American Astronomical Society, held at the University of Wisconsin (Madison) later in the same month. In addition, an article has been submitted

for publication to the Astrophysical Journal, and several internal memoranda were prepared during the program.

During this research program, we benefited greatly from consultations with J. I. Castor of the Joint Institute for Laboratory Astrophysics (JILA) and the Univ. of Colorado, and the originator of the DYN code; J. P. Cox (JILA- Univ. of Colorado); R. G. Deupree (Boston Univ.); and D. S. King (Univ. of New Mexico); as well as A. N. Cox (LASL, T-DOT) and many other colleagues at the Laboratory.

The remainder of this report is a discussion of the Cepheid mass anomaly and related problems, and of the progress made on these problems during this research program. This represents the final report on the program.

II. TECHNICAL INTRODUCTION

The theories of stellar evolution and stellar pulsation have both reached highly advanced states. It is therefore disturbing when these two theories conflict, as they seem to in regard to the "Cepheid mass anomaly." Cox¹ has summarized the various means by which the masses of Cepheid variables may be estimated. He has concluded that the mass anomalies involving the "beat Cepheids" (which show multiperiodic behavior) and the "bump Cepheids" (which show secondary peaks in their light and velocity curves) are the most serious ones. We would like to report here our efforts to study the bump Cepheids and their mass anomaly using the new stellar pulsation code, DYN, developed by Castor, Davis, and Davison.²

The bump Cepheids generally have periods in the range from 7 to 15 days. Christy³ computed nonlinear models for Cepheids with periods in this range. He found that he could not get bumps at the proper phase unless he reduced the masses of the models significantly below their "evolutionary masses." This is the mass anomaly for bump Cepheids. (The evolutionary mass is the mass that would be inferred from the luminosity using a standard mass-luminosity relation from the theory of stellar evolution.) The bump Cepheids were discussed again by Fricke, Stobie, and Strittmatter,⁴ who referred to models calculated by Stobie and found a similar anomaly.

In a certain respect, the models of Christy and Stobie were seriously incomplete. Their codes could produce theoretical velocity curves, but not light curves. This is a problem because velocity curves are much more difficult to obtain observationally than light curves. Thus, the theory gives information about velocity curves, while the bulk of the observations are of light curves. The DYN code can be used to produce reliable theoretical light curves. One of

our goals is therefore to study theoretically the relation between the light and velocity curves to see if it is reasonable to use them interchangeably.

The reason that theoretical light curves are difficult to compute with a typical Lagrangian code is that the emergent luminosity is contaminated by spurious "zoning bumps." Keller and Mutschlecner⁵ showed that the zoning bumps occur when the hydrogen ionization front moves from one zone to the next while the star is pulsating. Since the computational mesh is necessarily rather coarse, even though the ionization front is very thin, the light output of the star is significantly perturbed by the energy absorbed or released as an entire zone is ionized or recombines. In the DYN code, there are extremely fine (non-Lagrangian) zones that move with the hydrogen ionization front. Thus, with this code, the light curve is not affected by zoning bumps, and it is possible to study secondary features in the light curves without interference.

III. COMPUTATIONAL RESULTS

We have used the DYN code to calculate three 7.5-day and four 10-day Cepheid models. The properties of the models are summarized in Table I. All seven models have homogeneous population I compositions ($X = 0.70$, $Y = 0.28$, $Z = 0.02$). The model parameters have been chosen so that the models lie close to the center of the Instability Strip in the HR Diagram. The models at each period differ principally in the ratio of their actual mass to their evolutionary mass. The values of this ratio for the various models are also given in Table I. Note that the evolutionary masses vary from model to model since the models have different luminosities (in order to keep them near the center of the Instability Strip). The mass-luminosity relation and the slope of the Instability Strip were taken from King, Hansen, Ross, and Cox.⁶

The procedure for computing the models was generally the same as described by Castor, Davis, and Davison² in their report on DYN. The calculation is begun by constructing a static model, which is then given a velocity distribution proportional to $r^{6.3}$, normalized to 10 km s^{-1} at the surface. As the calculation progresses, the amplitude is allowed to grow, augmented with occasional intervals of artificial pumping, until it reaches a limiting value. The pumping is done by increasing the velocities everywhere by 5 per cent at each kinetic energy maximum (i.e., twice per period). Quadratic artificial viscosity (with a Stellingwerf⁷ cutoff at 10 per cent of the isothermal sound speed) was used. In addition, linear artificial viscosity was used in the first (deepest) zone in

TABLE I
MODEL PARAMETERS

Model Number	Period (days)	Mass ($M_{\odot}$)	Luminosity ($L_{\odot}$)	Effective Temp. (K)	M_{evol}^* ($M_{\odot}$)	M/M_{evol}^*
6A	7.52	6.70	3587	5700	6.70	1.00
6B	7.55	4.00	2404	5717	5.97	0.67
6C	7.56	3.40	2105	5717	5.75	0.59
5A	9.78	7.40	5210	5682	7.46	0.99
4A	9.79	6.00	4421	5687	7.11	0.84
3A	9.78	5.00	3849	5690	6.84	0.73
1B [†]	9.79	4.00	3187	5700	6.48	0.62

* $\log (L/L_{\odot}) = 3.48 \log (M_{\text{evol}}/M_{\odot}) + 0.68.$

[†] The "Goddard model".

order to avoid the numerical instability involving spurious oscillations deep in the core that was discussed by Christy.⁸ The code was run with an intermediate opacity table built from the Stellingwerf⁷ fit to the "King4a" opacity table (published by Cox and Tabor⁹).

The limiting amplitudes of the models appear to be determined, in our computations, by the development of shocks. The problem of the limiting amplitude will be discussed in more detail below. Here, we simply present Table II, in which the limiting amplitudes are given in terms of the peak kinetic energy, the full amplitude in radial velocity (in the rest frame of the star, without projection factors), and the full amplitude in bolometric magnitude. The full amplitudes in effective temperature are also given.

The light and velocity curves for the 7.5-day models are shown in Figs. 1 and 2, respectively. The light and velocity curves for the 10-day models, presented by Adams, Castor, and Davis¹⁰ at the Goddard Conference on Current Problems in Stellar Pulsation Instabilities, are reproduced here in Figs. 3 and 4, respectively. In Fig. 5, the light and velocity curves for the lowest mass 7.5-day model (number 6C) are replotted together for direct comparison.

TABLE II
CALCULATED LIMITING AMPLITUDES

Model Number	Period (days)	Mass ($M_{\odot}$)	C_Q	Peak K.E. * (10^{42} erg)	$(\Delta M_{\text{bol}})_{\text{max}}$ (mag)	$(\Delta v_r)_{\text{max}}$ (km s^{-1})	T_e Range (K)
6A [†]	7.52	6.7	1.0	9.8	1.61	77.1	4780 - 7110
6B	7.55	4.0	1.0	2.10	1.27	63.8	4850 - 6510
6C	7.56	3.4	1.0	1.58	1.26	62.8	4700 - 6390
5A	9.78	7.4	2.0	9.70	1.26	65.3	4930 - 6690
4A	9.79	6.0	2.0	5.30	1.14	61.8	4990 - 6480
3A	9.78	5.0	2.0	3.30	1.18	54.9	4850 - 6450
1B	9.79	4.0	1.0	1.78	1.09	47.0	4880 - 6350

* Peak kinetic energy at expansion.

[†] The amplitude for model 6A was still increasing slowly at the end of the calculation.

Fig. 1.

Bolometric light curves for three 7.5-day Cepheids. Zero phase is at maximum light, except for the $3.4 M_{\odot}$ model, where zero phase was shifted 0.2 units earlier for easier comparison. The model parameters are given in Table I.

Fig. 2.

Velocity curves for three 7.5-day Cepheids. The velocities are interpolated values at optical depth two-thirds. They are given in the rest frame of the star (positive for radial expansion) without any correction for projection effects. The phase convention is as in Fig. 1. The model parameters are given in Table I.

Fig. 3.

Bolometric light curves for four 9.8-day Cepheids. Zero phase is at maximum light, except for the 4.0 $M_{\odot}$ (Goddard) model, where zero phase was shifted 0.2 units earlier for easier comparison. The model parameters are given in Table I.

Fig. 4.

Velocity curves for four 9.8-day Cepheids. The velocities (positive for radial expansion) are interpolated values that show the motion of the matter at optical depth two-thirds. The phase convention is as in Fig. 3. The model parameters are given in Table I.

Fig. 5

Light and velocity curves for the 3.4 $M_{\odot}$ 7.5-day model. The dip in the light curve and the bump in the velocity curve that are referred to in the text are marked with arrows.

An independent test of the accuracy of the DYN code can be obtained by comparing the light and velocity curves for our $4.0 M_{\odot}$ 10-day model (number 1B), the "Goddard model," with the curves calculated for the same model by Fischel, Sparks, and Karp.¹¹ Those investigators attempted to calculate a reliable light curve with their Lagrangian code by using a large number of zones (100 versus 65 in our calculation). In fact, our light and velocity curves agree well with theirs. This confirms the accuracy of our calculation and, in retrospect, confirms the accuracy of theirs, as well.

IV. DISCUSSION OF THE LIGHT AND VELOCITY CURVES

The light and velocity curves for the 7.5- and 10-day models are qualitatively very similar. For the evolutionary mass models, the velocity curves are nearly featureless, while the light curves show weak bumps low on the descending branch. As the mass is reduced below the evolutionary mass, the velocity curve develops a bump, and the bump moves earlier in phase. At the same time, the bumps in the light curve become more prominent, and they move earlier in phase, as well.

The relation between the light and velocity curves is illustrated for the lowest mass 7.5-day model (number 6C) in Fig. 5. Overall, the curves are similar. (If the velocity curve were plotted according to the usual astronomical convention, with positive velocity for inward motion, the curves would be nearly mirror images of each other.) Both curves show a rapid rise from minimum, followed by a descending branch with prominent bumps.

The sharp dip on the rising branch of the light curve is the "artificial viscosity dip." It is caused by the passage of a compression wave through the photosphere. The wave, which is broadened by the artificial viscosity, precedes the shock that stops the infall of the stellar envelope at this phase. There is a nearly discontinuous rise in the velocity curve just before this dip occurs in the light curve. The artificial viscosity dip and the corresponding structure in the light curve are almost surely artifacts due to the approximate treatment of shocks (with artificial viscosity) in the calculations.

The broader bumps on the descending branch are apparently due to the interaction of compression waves with the photosphere, and are unrelated to the artificial viscosity. In the velocity curve, there is only one prominent bump on the descending branch (labelled "bump" in Fig. 5). In the light curve, there is a double peak near maximum, and then a dip (labelled "dip" in Fig. 5) midway down

the descending branch. The origin of the double peak in the light curve, which has no counterpart in the velocity curve, is not understood.

The bump in the velocity curve can be clearly identified as a "Christy bump." This is possible because, in a "Christy plot," where the velocity curves for many zones are plotted together, the occurrence of this bump coincides with the arrival at the surface of a compression wave that has been reflected off the core. Cox¹² has published a Christy plot for one of his models, in which he shows how the compression wave is generated near minimum radius and can be followed as it propagates into the core, reflects, and propagates to the surface. This sequence of events was described earlier by Christy.^{3,13}

The Christy bump in the velocity curve does have a counterpart in the light curve, but in the light curve, it should more properly be called a "Christy dip" (Fig. 5). The dip is caused by the rapid increase in the effective gravity in the photosphere as the reflected compression wave passes through. The mechanism by which this happens has been discussed by Castor¹⁴ in the context of his RR Lyrae star models. Since it is the dip in the light curve that coincides with the Christy bump in the velocity curve, we would expect the bump in the light curve (the recovery from the dip) to lag systematically behind the bump in the velocity curve by as much as 0.2 in phase.

In fact, a systematic difference between the bump phases in observed light and velocity curves has been reported by Stobie.¹⁵ However, Stobie's difference has the opposite sign, with the bump in the light curve occurring earlier. Stobie suggests that the phase delay has to do with the sound travel time through the photosphere, since the lines are formed higher than the continuum. Our velocity curves give the velocities at optical depth $2/3$. However, Karp¹⁶ gives velocity curves referred to optical depths 1.0, 0.1, and 0.001 for his 12.05-day model, and the phase of the bump hardly changes at all.

The source of the discrepancy appears rather to have to do with the traditional way in which observed light and velocity curves are interpreted. They are usually seen as having a single bump superposed on a smooth underlying curve, and the bump in the light curve and the bump in the velocity curve are implicitly assumed to be due to the same physical phenomenon. In fact, our computed light curves show structures that are not reflected in the velocity curves (the double peaks). Thus, one could identify one of the double peaks as the bump in the light curve and erroneously assume it was physically related to the Christy bump in the velocity curve. Consider, for example, how the curves in Fig. 5 would be

interpreted if they were sampled at 20 points per period, and they had some observational scatter. We would probably identify the second peak of the double peak in the light curve as "the bump." We would then associate it with the Christy bump in the velocity curve and reach the same conclusion that Stobie did! The conclusion to be drawn is that, guided by our computed light and velocity curves, we should expect the observed curves to be more complex than we have previously allowed. As one consequence of this conclusion, observers should cease the practice of drawing smooth lines through their data ("to guide the eye"), since in doing so, they are almost surely obscuring what information is present in the data.

We began by comparing the light and velocity curves for model 6C. Actually, the discussion applies equally well for the other models. Our conclusion is that the theoretical light and velocity curves are very similar overall, but that they differ significantly in detail. Great caution should therefore be exercised in using theoretical velocity curves to interpret observational results based on light curves.

We now return to the systematic behavior of the light and velocity curves in Figs. 1-4 as the mass is reduced. We see that the Christy bump (dip) can be followed from model to model along the sequence at each period. The identification of these features as "Christy bumps" can be confirmed by looking at the Christy plots for the models. We therefore conclude that the Christy bump (dip) systematically increases in strength and moves earlier in phase as the mass is reduced. This confirms Christy's³ original conclusion concerning the velocity curves and extends it to the light curves. It means that the mass anomaly for the bump Cepheids cannot be resolved simply by improving the treatment of the hydrogen ionization front in the model calculations.

The fact that the light curves for the 7.5- and 10-day models show the same qualitative behavior is a bit perplexing in view of the observed Hertzsprung progression (Cox¹⁷), which shows a systematic change in the light curves with period. If we include Karp's 12.05-day model ($M = 5 M_{\odot}$, $L = 5000 L_{\odot}$, $T_e = 5730$ K, $M/M_{\text{evol}} = 0.68$), the situation is even more perplexing. According to the Hertzsprung progression, a 12-day Cepheid should have a bump on the ascending branch. Nevertheless, what is almost certainly the analog of our Christy bumps is still seen to be high on the descending branch. The first peak of the double peak in the light curve has moved to a position high on the ascending branch, though. This would presumably be the "bump" now. The theoretical light curve is

thus fairly complex, but in fact, examination of Pel's¹⁸ data for Cepheids in this period range shows that the observed light curves are rather complex, too. This reinforces our earlier conclusion that the observations show more than simply a peak and a bump.

V. THE PROBLEM OF THE LIMITING AMPLITUDE

The limiting amplitude in real Cepheids is probably reached as a result of the saturation of the driving mechanism. There is little observational evidence for the presence of strong shocks in Cepheids.^{19,20} In our models, however, rather strong shocks develop as the amplitudes approach their limiting values. It thus appears that the dissipation associated with shocks limits the amplitudes in our calculations.

The strongest shock that develops is the one that stops the infall of the envelope near the phase of maximum compression. This shock gives rise to the "artificial viscosity dip" discussed above. In most of the models, another shock develops a short time later at the phase of rapid expansion. This shock occurs near the hydrogen ionization front. It is associated with a brief transition of the front from "D type" to "E type" as it moves rapidly inward in mass. This inward-moving shock stays below optical depth unity and therefore has no effect on the light and velocity curves. The ionization front transition is discussed in more detail by Adams and Castor.²¹

It is possible that the development of shocks limits the amplitude in real Cepheids, but that the amplitudes are limited in such a way that the shocks are weaker than we calculate. Alternatively, some details of the physics may have been omitted, for example, some subtle feature in the opacity, that limits the amplitude. Whatever the cause, our calculated limiting amplitudes, especially in bolometric magnitude, are too large. Pel²² infers bolometric light amplitudes of up to about 0.5 mag from his five-color photometry, while our calculated amplitudes (Table II) are more than one magnitude, even for the reduced mass models. The calculated velocity amplitudes, corrected for projection effects, are not quite so large compared with the observations. For example, Evans²³ found a full velocity amplitude of 43 km s^{-1} (61 km s^{-1} after multiplication by 24/17) for the 7.2-day Cepheid η Aql, while Parsons²⁴ found a full amplitude of 35 km s^{-1} (49 km s^{-1} after multiplication by 24/17) for the 9.8-day Cepheid, β Dor.

The calculated effective temperature amplitudes also seem large. The amplitude of about 1500 K for the reduced mass 10-day models is considerably larger than the amplitude of 950 K observed by Parsons²⁴ for β Dor, although it is consistent with the earlier measurement of 1550 K for the same star by Estes and Wood.²⁵ However, we have not yet calculated any colors, line profiles, or other empirical temperature diagnostics for our models with a suitable atmospheres code. Until these calculations are completed, it is impossible to say how serious the differences between the theoretical and observed temperature amplitudes are.

VI. SUMMARY AND CONCLUSIONS

We have computed reliable theoretical light curves for a comprehensive set of Cepheid models. With these calculations, we are able to extend to the light curves Christy's³ conclusion that computed velocity curves for Cepheids with periods near 10 days do not show bumps like those that are observed unless the masses of the models are significantly reduced. Thus, the mass anomaly for the bump Cepheids remains a problem. It cannot be resolved simply by treating the hydrogen ionization front more accurately.

We have also reached some important conclusions by comparing the calculated light and velocity curves for the various models. We find that, while the light and velocity curves are similar overall, they differ significantly in detail. In particular, the "Christy bump" in the velocity curve coincides with a dip that precedes the bump in the light curve. We also find that the light curve shows some features (the double-peaked structure) that have no counterpart in the velocity curve. One conclusion that can be drawn from this is that it is dangerous to use theoretical velocity curves to interpret observational results based on light curves. The other, more basic conclusion is that the observed light curves are probably more complex than we have previously thought. Our calculated light curves suggest the possibility of multiple bumps and shoulders, and we should be alert for such possibilities in the observations.

These conclusions should hold even if the occurrence of bump Cepheids is not due to reduced stellar masses, since our computed light and velocity curves are self-consistent. Thus, they contain information about the interaction of compression waves in the outer envelope of a pulsating star and should not depend greatly on the mechanism for generating the waves.

Aside from the mass anomaly, one fundamental problem remaining is that of the limiting amplitude. The calculated amplitudes, especially the light amplitude, seem to be too large. Some important mechanism for limiting the driving or for damping may have been omitted. It is possible that the problem involves the assumption of a homogeneous composition. In particular, Cox et al.²⁶ have suggested that the presence of a helium-rich layer on the surface would resolve the mass anomaly; perhaps it would reduce the limiting amplitude, as well.

The calculation of seven models with the DYN code should certainly just be considered a first step. The next step should be to compute models with shorter and longer periods, and to compute models away from the center of the Instability Strip in the HR Diagram. It would be very interesting to see if significant qualitative changes occur as a model is moved across the Instability Strip at constant period. It would also be interesting to compute an inhomogeneous model with a helium-rich surface layer. Besides computing new models, it would be worthwhile to apply a sophisticated atmospheres code to the DYN output to predict diagnostics, like colors and line profiles, for a more detailed comparison with the observations.

REFERENCES

1. A. N. Cox, "A Discussion of Cepheid Masses," to appear in the proceedings of the Conference on Current Problems in Stellar Pulsation Instabilities, held in June 1978 at the NASA Goddard Space Flight Center.
2. J. I. Castor, C. G. Davis, and D. K. Davison, "Dynamical Zoning Within a Lagrangian Mesh by Use of DYN, a Stellar Pulsation Code," Los Alamos Scientific Laboratory report LA-6664 (February 1977).
3. R. F. Christy, "The Theory of Cepheid Variability," *Quarterly Journal of the Royal Astronomical Society* 9, 13-39 (1968).
4. K. Fricke, R. S. Stobie, and P. A. Strittmatter, "The Masses of Cepheid Variables," *Astrophys. J.* 171, 593-604 (1972).
5. C. F. Keller and J. P. Mutschlecner, "Hydrodynamic Envelopes of Classical Cepheid Variables," *Astrophys. J.* 167, 127-136 (1971).
6. D. S. King, C. J. Hansen, R. R. Ross, and J. P. Cox, "Applications of Linear Pulsation Theory to the Cepheid Mass Problem and the Double-Mode Cepheids," *Astrophys. J.* 195, 467-474 (1975).
7. R. F. Stellingwerf, "Modal Stability of RR Lyrae Stars," *Astrophys. J.* 195 441-466 (1975).
8. R. F. Christy, "The Hertzsprung Progression in Cepheid Calculations," in Cepheid Modeling (NASA Publication SP-383), D. Fischel and W. H. Sparks, eds. (NASA, Washington, DC, 1975) pp. 85-98.

9. A. N. Cox and J. E. Tabor, "Radiative Opacity Tables for 40 Stellar Mixtures," *Astrophys. J. Suppl.* 31, 217-312 (1976).
10. T. F. Adams, J. I. Castor, and C. G. Davis, "Light Curves for 'Bump Cepheids' Computed with a Dynamically Zoned Pulsation Code," to appear in the proceedings of the Conference on Current Problems in Stellar Pulsation Instabilities, held in June 1978 at the NASA Goddard Space Flight Center.
11. D. Fischel, W. H. Sparks, and A. H. Karp, in "A Reexamination of the Goddard Model," by R. G. Deupree, pp. 243-270, in Proceedings of the Solar and Stellar Pulsation Conference (held in August 1976 at Los Alamos), Los Alamos Scientific Laboratory report LA-6544-C (1976).
12. A. N. Cox, "The Anomalous Cepheid Masses," *Sky and Telescope* 55, 115-118 (1978).
13. R. F. Christy, comment in Cepheid Modeling (NASA Publication SP-383) (NASA, Washington, DC, 1975) p.237.
14. J. I. Castor, "Atmospheric Dynamics of a Model RR Lyrae Star," Ph.D. Thesis, California Institute of Technology, Pasadena (1966).
15. R. S. Stobie, "Cepheids," in Multiple Periodic Variable Stars, (Proceedings of IAU Colloquium No. 29, Budapest, Hungary, September 1975), W. S. Fitch, ed. (D. Reidel Publ. Co., Dordrecht, Holland, 1976) pp. 87-113.
16. A. H. Karp, "Hydrodynamic Models of a Cepheid Atmosphere," in Cepheid Modeling (NASA Publication SP-383), D. Fischel and W. H. Sparks, eds. (NASA, Washington, DC, 1975) pp. 99-114).
17. J. P. Cox, "Cepheids," *Reports on Progress in Physics* 37, 563-698 (1974).
18. J. W. Pel, "Five-Channel Photometry of Cepheids and Supergiants in the Southern Milky Way," *Astronomy and Astrophysics Suppl.* 24, 413-471 (1976).
19. R. P. Kraft, "The Spectra of Supergiants and Cepheids of Population I," ch. 9 in Stellar Atmospheres, G. P. Kuiper and B. M. Middlehurst, eds. (Univ. of Chicago Press, Chicago, 1960), pp. 370-410.
20. J. Lub, J. van Paradijs, J. Pel, and P. R. Wesselius, "Ultraviolet Photometry of the Cepheid β Doradus from the ANS Satellite," *Astronomy and Astrophysics*, in press.
21. T. F. Adams and J. I. Castor, "A Transitory R-Type Ionization Front in a Cepheid Model," *Astrophys. J.*, in press.
22. J. W. Pel, "Intrinsic Colours and Physical Properties of Cepheids," *Astronomy and Astrophysics* 62, 75-94 (1978).
23. N. R. Evans, "Light and Radial Velocity Observations of Classical Cepheids," *Astrophys. J. Suppl.* 32, 399-407 (1976).
24. S. B. Parsons, "Temperature and Gravity Variations of the Classical Cepheid Beta Doradus," *Astronomical Journal* 76, 562-566 (1971).
25. J. K. Estes and H. J. Wood, "The G-Band Variation of Beta Doradus," *Astronomical Journal* 75, 999-1000 (1970).
26. A. N. Cox, G. Michaud, and S. W. Hodson, "Cepheid Masses and cepheid Winds," *Astrophys. J.* 222, 621-626 (1978).