EGG-WM--9052

DE91 006107

MODELING THE STEADY-STATE ISV PROCESS:
A 3-D FINITE ELEMENT ANALYSIS OF COUPLED THERMAL-ELECTIC FIELDS

M. A. Langerman

Published September, 1990

EG&G Idaho, Inc.
Idaho Falls, ID 83415

Prepared for the
U. S. Department of Energy
Idaho Operations Office
Under DOE Contract No. DE-AC07-76ID01570

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

MODELING THE STEADY-STATE ISV PROCESS: A 3-D
FINITE ELEMENT ANALYSIS OF COUPLED
THERMAL-ELECTRIC FIELDS
EGG-WM-9052

Approved By:

S. K. Merrill, Manager
In Situ Vitrification

Date:

9/18/90

Prepared By:

M. A. Langerman

Date:

9/14/90

Reviewed By:

M. O. Fryer
Technical Reviewer

Date:

9/19/90

ABSTRACT

Steady-state modeling considerations for simulating the in situ vitrification (ISV) process are documented based upon the finite element numerical approach. Recommendations regarding boundary condition specifications and mesh discretization are presented. The effects of several parameters on the ISV process response are calculated and the results discussed. The parameters investigated include: 1) electrode depth, 2) ambient temperature, 3) supplied current, 4) electrical conductivity, 5) electrode separation, and 6) soil/waste characterization.

TABLE OF CONTENTS

ABSTRACT	ii
ACKNOWLEDGMENTS	iii
TABLE OF CONTENTS.....	iv
LIST OF FIGURES.....	v
LIST OF TABLES	vi
NOMENCLATURE.....	vii
1.0 INTRODUCTION	1
2.0 ANALYTICAL MODEL DEVELOPMENT	3
2.1 Steady-State Field Equations	3
2.2 Numerical Solution	4
3.0 MODEL ASSESSMENT	8
3.1 2-D Analytic Solution	8
3.2 Domain Discretization	12
3.3 Joule Heat Calculation.....	14
3.4 Electrode Boundary Condition	15
3.5 3-D ISV Model	20
4.0 SENSITIVITY ANALYSIS	25
4.1 Electrode Depth.....	25
4.2 Ambient Temperature	27
4.3 Electrode Current	29
4.4 Electrical Conductivity	31
4.5 Electrode Separation	33
4.6 Soil/Waste Characterization	35
4.7 Summary	41
4.0 CONCLUSIONS	43
5.0 REFERENCES	44

LIST OF FIGURES

1.1 ISV process conditions.....	2
3.1 Electrostatic line charge configuration	9
3.2 Electrode surface configuration.....	10
3.3 Two electrode configurations used for determining the joule heating.....	11
3.4 Normalized voltage field (exact).....	13
3.5 Normalized joule heat (exact).....	13
3.6 Two-dimensional electrostatic field discretization	14
3.7 Comparison between the 2-D analytic and calculated joule heat solutions ($\xi=2.42\text{m}$).....	16
3.8 Comparison between the 2-D analytic and calculated joule heat solutions ($\xi=2.86, 1.54$, and $.22\text{m}$).....	17
3.9 Comparison between the 2-D analytic and calculated voltage solutions ($\xi=2.86, 1.54$, and $.22\text{m}$) ...	18
3.10 Three-dimensional thermal-electric discretization.....	19
3.11 Soil electrical conductivity.....	21
3.12 Soil thermal conductivity	21
3.13 Voltage at 0.6m and 5.4m depths.....	23
3.14 Joule heat as a function of ξ, η and z	24
3.16 Temperatures at $\xi=1.98\text{m}$	24
3.17 Temperatures at $\xi=0.22\text{m}$	24
4.1 Effect on temperatures at $\xi=2.42\text{m}$ due to electrode depth.....	26
4.2 Effect on temperatures at $\xi=2.42\text{m}$ due to ambient surface temperatures	28
4.3 Effect on temperatures at $\xi=2.42\text{m}$ due to supplied current	30
4.4 Typical effect on potential at $\xi=2.42\text{m}$ due to supplied current.....	31
4.5 Effect on temperatures at $\xi=2.42\text{m}$ due to electrical conductivity.....	32
4.6 Effect on temperatures at $\xi=2.42\text{m}$ due to electrode separation distance	34
4.7 Mesh diagram showing the soil/waste characterization at the 30% depth elevation.....	36
4.8 Mesh diagram showing the soil/waste characterization at the 50% depth elevation.....	36
4.9 Joule heat at $\xi=1.54\text{m}$	37
4.10 Joule heat at $\xi=0.66\text{m}$	38
4.11 Temperature at $\xi=1.54\text{m}$	39
4.12 Temperature at $\xi=0.66\text{m}$	40

LIST OF TABLES

4.1 Summary of results	42
------------------------------	----

NOMENCLATURE

<u>Symbol</u>	<u>Description of Symbol</u>
V	voltage
σ	electrical conductivity
σ'	Stefan-Boltzmann constant
n	outward normal unit vector
j	current density
ρ	material density
c_p	specific heat
U	velocity
k	thermal conductivity
T	temperature
q'''	joule heat
θ	temperature difference
W	weighting factor
R	residual
N	basis function
r	position vector
D	electrode diameter
b	electrode separation distance
h	heat transfer coefficient
A, B	constants

1.0 INTRODUCTION

The in situ vitrification (ISV) process is a waste remediation technology for treating hazardous materials by permanently reducing the toxicity, mobility and volume of the wastes on site or *in situ*¹. The ISV process involves electric melting of contaminated solids at high temperatures.

Figure 1.1 presents a cross-sectional view of the ISV process configuration.² Four electrodes are placed in a square pattern and at a preselected soil depth. The current, which is passed between diagonally opposed electrodes, is 90 degrees out of phase between electrode pairs. Joule heating melts the soil and surrounding wastes and the resulting off-gases flow upward to a collection hood. The process continues until the power is turned off or a steady-state condition is reached. Final disposition of contaminants depends upon several local parameters, particularly the soil/waste characterization. This local dependence makes it necessary to consider each remediation project individually. Site specific feasibility analyses become necessary to assure the system equipment and power rating are adequate to successfully remediate the wastes.

Feasibility studies of the type being considered here require analysis of coupled thermal-electric field phenomena and, due to the geometrical constraints and boundary conditions encountered, are amenable to finite element numerical solution techniques. Currently, however, there is no available analysis package describing the application of these analytical techniques to the problem of ISV modeling. The purpose of this study, therefore, is to investigate the general area of ISV modeling and establish guidelines for successful model applications. The modeling guidelines set forth are based upon calculations from several sensitivity analyses. These guidelines can be used to aid investigators in configuring models for estimating site specific power requirements for waste remediation.

The scope of this study is not intended to cover a complete envelope of possible ISV operating conditions but rather to assess and present results from several selected parametric evaluations of variables considered important in modeling the process response and determining the system power requirements. This investigation considers only the steady-state disposition of the ISV melt. Transient power and control techniques are not covered.

Section 2 of this report presents a detailed discussion of the solution method employed for this investigation. Section 3 presents an assessment of the numerical model and describes specific modeling techniques. Section 4 presents the results from several sensitivity analyses including the effects on process response from varying the: 1) electrode depth, 2) ambient temperature, 3) electrode current, 4) electrical conductivity, 5) electrode separation and 6) soil/waste characterization. Finally, Sections 5 and 6 present conclusions reached from this investigation and the cited references, respectively.

Figure 1.1 ISV process conditions.

2.0 ANALYTICAL MODEL DEVELOPMENT

This section presents a discussion of the analytical modeling and solution techniques developed for this investigation. Both the thermal and electric fields and the coupling between the two are discussed.

The solution of the potential field is coupled to the solution of the temperature field through the nonlinear dependence of electrical conductivity on temperature and through the heat source term arising from passing a current source through a medium of finite electrical resistance (i.e., joule heat). The solution method employed is to solve the electrostatic field with an assumed temperature field and determine the joule heat for input to the energy equation. The energy equation is then solved for the temperature field and the electrostatic field is resolved with the new temperature field and so on to convergence. The following discussion presents the electrostatic and energy equations used in this analysis.

2.1 Steady-State Field Equations

The potential field is governed by the charge conservation equation,³ which in vector form is

$$\nabla \cdot \sigma \nabla V = 0, \quad (2.1)$$

and boundary conditions

$$V = \hat{V}(\text{boundary}), \quad (2.2)$$

$$-\sigma \nabla V \cdot \mathbf{n} = j_n, \quad (2.3)$$

where V is the voltage and σ is the electrical conductivity which is specified as a scalar function of the temperature, T . In (2.2) V is specified at points along the boundary. In (2.3), the normal outward component of the current density, j , is specified at the boundary where j is defined as

$$\mathbf{j} = -\sigma \nabla V. \quad (2.4)$$

The temperature field is governed by the energy equation given below as

$$\rho c_p (\mathbf{U} \cdot \nabla T) = \nabla \cdot \mathbf{k} \nabla T + q'''. \quad (2.5)$$

Neglecting convection, equation (2.5) reduces to

$$-\nabla \cdot k \nabla T = q''', \quad (2.6)$$

with the steady-state boundary conditions

$$T = T(\text{boundary}), \quad (2.7)$$

$$-k \nabla T \cdot \mathbf{n} = h_n \theta, \quad (2.8)$$

and at the melt front

$$k_l \nabla T \cdot \mathbf{n} = k_s \nabla T \cdot \mathbf{n}, \quad (2.9)$$

where the subscripts *l*, *s* mean, respectively, liquid and solid and where $\mathbf{U}=[u,v,w]$ is fluid velocity (equal to zero in the solid region), ρ is material density, k is thermal conductivity, c_p is specific heat, T is temperature, q''' is joule heat $=j^2/\sigma = \sigma |\nabla V|^2$, and θ is the temperature difference $(T - T_{\text{ambient}})$.

Again, the two field equations are coupled through the joule heat term and the dependence of σ on T . The following section discusses the numerical approach used to solve the above equations subject to the stated boundary conditions.

2.2 Numerical Solution

This section presents a brief account of the numerical approach used in solving the thermal-electric field equations discussed in the previous section. A numerical algorithm similar to that used in the TRIPLE⁴ code is employed. TRIPLE is a three dimensional finite element code developed to solve the heat conduction equation and includes phase change solution capabilities. The overall procedure uses the finite element method⁵(FEM) to solve a method of weighted residuals⁶(MWR) statement of the governing equations and boundary conditions.

Briefly, in the FEM, the temperature or potential field throughout the continuum of interest is approximated by a discretization procedure which divides the solution domain into finite regions called elements. The field variable is then expressed in terms of an assumed approximating function within each element. The approximating functions, also known as interpolation functions or basis functions, are defined in terms of the values of the field variable at specified points called nodes. These nodes

usually lie on the element boundaries where adjacent elements are considered to be connected. Hence, in the FEM, the dependent field variables at the nodes are the unknowns. Once the unknowns are determined, the interpolation functions define the variable throughout the assemblage of elements.

One of the important features of the FEM is the ability to formulate solutions for individual elements and then to assemble the individual solutions to represent the entire problem solution. A method used frequently to formulate the properties of individual elements is called the method of weighted residuals (MWR). In the MWR, the general functional behavior of the field variable is assumed so as to approximately satisfy the general field equation and boundary conditions. Substitution of this approximation into the original differential equation and boundary conditions then results in some error called a residual. This residual is required to vanish in some average sense over the entire solution domain. This can be represented mathematically as

$$\int_{\text{vol}} W_j R \, d\text{vol} = 0 \quad j = 1, 2, 3, \dots, nn \quad (2.10)$$

where R is the residual, W_j is the j^{th} weighting factor, and nn is the number of undetermined parameters in the system. For example, in the case of the charge conservation equation,

$$R = \nabla \cdot \sigma \nabla \hat{V} \quad (2.11)$$

where $\hat{V}$ is a semi-discrete approximation of V (for convenience the $\hat{\cdot}$ is dropped in the following discussion).

In the FEM one of the essential steps taken is to integrate equation (2.10) by parts and in doing so part of the differentiability is shifted from V to W_j (V goes from C^2 to C^1 continuity). The theorem relating the surface integral for the flux of a vector field to the volume integral of its divergence is known as the Gauss or divergence theorem and is equivalent to integration by parts in multidimensions. Applying this theorem to equation (2.10) with equation (2.11) substituted gives (after some rearranging):

$$-\int_{\text{vol}} (\sigma \nabla V) \cdot (\nabla W_j) \, d\text{vol} + \int_{\text{surf}} W_j \sigma \nabla V \cdot \mathbf{n} \, d\text{surf} = 0. \quad (2.12)$$

Note that the process of shifting part of the differentiability requirement from V to W has opened the solution up to an entirely new set of functions that have to be only once differentiable. Mathematically speaking, the above form of the original problem is called a "weak" form. Note also that the differentiability shift has resulted in the inclusion of the second type of boundary condition within the equation set. This is an obvious advantage of the FEM over other numerical procedures. For example, on regions where $-\sigma \partial V / \partial n \neq 0$, the second term in equation (2.12) is used to input the required values. An insulated boundary condition (ie, $-\sigma \partial V / \partial n = 0$) is satisfied by doing nothing on those boundaries.

Equation (2.12) can be put in matrix form as

$$-\int_{\text{vol}} \sigma(W_j \underline{\nabla}) (\underline{\nabla}^T V) \, d\text{vol} + \int_{\text{surf}} W_j (-j_n) \, d\text{surf} = 0, \quad (2.13)$$

noting that

$$(\sigma \nabla V) \cdot (\nabla W_j) = \sigma(W_j \underline{\nabla}) (\underline{\nabla}^T V) \quad (2.14)$$

where an underbar denotes a row vector while an underbar with a transpose symbol (T) denotes a column vector.

The next step in the FEM is the discretizations of the region of interest into subregions called elements. Note that in general only a subset of the total elements will have a portion of their boundary that coincide with the external boundary of the domain of interest. The MWR-FEM requirement now becomes

$$\sum_{i=1}^{nb} \int_{\text{surf}} W_j (-j_n) \, d\text{surf}_i - \sum_{i=1}^{ne} \int_{\text{vol}} \sigma(W_j \underline{\nabla}) (\underline{\nabla}^T V) \, d\text{vol}_i = 0, \quad (2.15)$$

where ne is the number of elements and nb is the number of elements in the subset of all elements which have a portion of their boundary that coincides with the domain of the external boundary.

Next it is necessary to determine a general interpolation function that allows a description of V throughout the element given V at nodes within the element. These discrete points are called nodes. The required function can be represented as

$$V(x,y,z)_{\text{in element}} = \sum_{i=1}^{nne} N_i V_i = \underline{N} \underline{V}^T \quad (2.16)$$

where nne is the number of nodes per element. Substituting (2.16) into (2.15) and, as a final step, setting the weighting functions (W 's) equal to the interpolation functions (N 's) results in

$$\sum_{i=1}^{ne} \int_{vol} \sigma(N_i \nabla) (\nabla^T N) \underline{V}^T dvol_i = - \sum_{i=1}^{nb} \int_{surf} \underline{N}^T (j_n) dsurf_i. \quad (2.17)$$

Remark 1: Setting W equal to N is the Galerkin⁷ method. In going from equation (2.16) to equation (2.17), the problem statement changes from 1 equation with nn unknowns to nn equations with nn unknowns.

Equation (2.17) can be written in a general manner as

$$[K] \underline{V}^T = \underline{F}^T, \quad (2.18)$$

where the $[]$ denotes a square matrix. Hence, once $[K]$ and $\underline{F}^T$ are constructed, $\underline{V}^T$ may be obtained in a standard elimination manner. Note that in equation (2.18) above

$$[K] = \sum_{i=1}^{ne} \int_{vol} \sigma(N_i \nabla) (\nabla^T N) \underline{V}^T dvol_i \quad (2.19)$$

and

$$\underline{F}^T = - \sum_{i=1}^{nb} \int_{surf} \underline{N}^T (j_n) dsurf_i. \quad (2.20)$$

Once the electrical potential distribution is obtained from the solution of equation (2.18), calculation of the joule heat distribution is straight forward. As noted previously

$$q''' = \frac{j^2}{\sigma} = \sigma |\nabla V|^2.$$

Hence,

$$q_i''' = \sigma_i \left[\left(\frac{\partial V}{\partial x} \right)_i^2 + \left(\frac{\partial V}{\partial y} \right)_i^2 + \left(\frac{\partial V}{\partial z} \right)_i^2 \right]. \quad (2.22)$$

The above discussion considered only the charge conservation equation. In general, however, the method also applies to the energy (heat conduction) equation, the solution to which has been extensively applied and tested⁴. The following section presents an application of the electrostatic model and compares results from a numerical solution with results from an exact solution for a 2-dimensional field problem.

3.0 MODEL ASSESSMENT

To assess the accuracy and completeness of the numerical solution to the equations discussed in the preceding section, a calculation was conducted of a 2-D potential field problem in a material having a constant electrical conductivity. This problem has an analytic solution to compare with the numerical results and thus provides a means of judging the accuracy of the numerical solution and a means of assessing modeling techniques. The problem consists of determining the potential field produced by four electrodes held at a constant voltage. The solution includes the joule heating produced by the flow of current.

3.1 2-D Analytic Solution

Consider first the electrostatic potential produced from an infinitely long line charge. The potential so produced satisfies Laplace's equation given as

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) = 0 \quad (3.1)$$

where r is the magnitude of the position vector from the line charge to some point in the field. The solution to equation (3.1) is

$$V = A \ln(r) + B \quad (3.2)$$

where A and B are constants determined by boundary conditions. Similarly, four line charges produce an electrostatic potential of the form

$$V = A_1 \ln(r_1) + A_2 \ln(r_2) + A_3 \ln(r_3) + A_4 \ln(r_4) + B \quad (3.3)$$

where r_1 is the distance from line charge 1 to some point in the field, etc. If line charge 1 and line charge 2 have positive charges and 3 and 4 have negative charges of the same magnitude then

$$V = A \ln \left(\frac{r_1 r_2}{r_3 r_4} \right) + B. \quad (3.4)$$

A top view of the four line charge configuration is shown in Figure 3.1. The following relationships are derived from the geometry shown in Figure 3.1.

$$r_1 = \sqrt{\left(x + \frac{b}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2}, \quad (3.5)$$

$$r_2 = \sqrt{\left(x - \frac{b}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2}, \quad (3.6)$$

$$r_3 = \sqrt{\left(x - \frac{b}{2}\right)^2 + \left(y + \frac{b}{2}\right)^2}, \quad (3.7)$$

and

$$r_4 = \sqrt{\left(x + \frac{b}{2}\right)^2 + \left(y + \frac{b}{2}\right)^2}. \quad (3.8)$$

Note that surfaces of constant potential satisfy

$$M = \frac{r_1 r_2}{r_3 r_4} \quad (3.9)$$

where M is a constant.

Figure 3.1 Electrostatic line charge configuration.

Now to relate the four line charge problem described above to the four electrode problem of interest, consider the situation where r_1 is much smaller than b . Then

$$M \approx \frac{r_1 b}{b \sqrt{2} b} \quad (3.10)$$

or

$$\left(x + \frac{b}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = 2 M^2 b^2. \quad (3.11)$$

Equation (3.11) is the equation of a circle centered at line 1 and represents an isopotential line coincident with the electrode surface as shown in Figure 3.2. The potential distribution for the four electrode configuration shown in Figure 3.2 is

$$V = V_0 \frac{1}{\ln\left(\frac{D}{2\sqrt{2}b}\right)} \ln\left(\frac{r_1 r_2}{r_3 r_4}\right), \quad (3.12)$$

where

V_0 = applied voltage,

D = electrode diameter,

and

b = distance between electrode centers.

Figure 3.2 Electrode surface configuration.

From Ohms law, the current density vector is

$$\mathbf{j} = \sigma \nabla V. \quad (3.13)$$

Finally, the volumetric joule heat produced by the current of magnitude j is given in 2-D by

$$q''' = \frac{j^2}{\sigma} = \sigma \left[\left(\frac{\partial V}{\partial x} \right)^2 + \left(\frac{\partial V}{\partial y} \right)^2 \right]. \quad (3.14)$$

For the ISV process the electrodes have time varying voltages with 90 degrees of phase difference between electrodes, so

$$V_1 = V_0 \cos(\omega t), \quad (3.15)$$

$$V_2 = V_0 \cos(\omega t + \frac{\pi}{2}), \quad (3.16)$$

$$V_3 = V_0 \cos(\omega t + \pi), \quad (3.17)$$

and

$$V_4 = V_0 \cos(\omega t + \frac{3\pi}{2}). \quad (3.18)$$

The volumetric joule heat needed for the heat conduction calculation is obtained from the concept of superposition and is equal to the sum of the volumetric joule heat terms obtained from the two electrostatic configurations shown in Figure 3.3.

Figure 3.3 Two electrode configurations used for determining the joule heating.

From equation (3.12), the following result is obtained

$$\frac{\partial V}{\partial x} = \frac{V_0}{\sqrt{2} \ln(\frac{D}{2\sqrt{2}b})} \left[\frac{1}{r_1 r_2} \frac{\partial(r_1 r_2)}{\partial x} - \frac{1}{r_3 r_4} \frac{\partial(r_3 r_4)}{\partial x} \right] \quad (3.19)$$

with a similar relationship for $\partial V / \partial y$ and where

$$\frac{1}{r_1 r_2} \frac{\partial(r_1 r_2)}{\partial x} = \frac{g(x,y)}{g'(x,y)} \quad (3.20)$$

with

$$g(x,y) = (x+\frac{b}{2})(x-\frac{b}{2})^2 + (x-\frac{b}{2})(x+\frac{b}{2})^2 + (x-\frac{b}{2})(y-\frac{b}{2})^2 + (x+\frac{b}{2})(y-\frac{b}{2})^2 \quad (3.21)$$

and

$$g'(x,y) = (x+\frac{b}{2})^2(x-\frac{b}{2})^2 + (y-\frac{b}{2})^2(x-\frac{b}{2})^2 + (y-\frac{b}{2})^2(x+\frac{b}{2})^2 + (y-\frac{b}{2})^4. \quad (3.22)$$

Similarly,

$$\frac{1}{r_3 r_4} \frac{\partial(r_3 r_4)}{\partial x} = \frac{h(x,y)}{h'(x,y)} \quad (3.23)$$

with

$$h(x,y) = (x+\frac{b}{2})(x-\frac{b}{2})^2 + (x-\frac{b}{2})(x+\frac{b}{2})^2 + (x+\frac{b}{2})(y+\frac{b}{2})^2 + (x-\frac{b}{2})(y+\frac{b}{2})^2 \quad (3.24)$$

and

$$h'(x,y) = (x+\frac{b}{2})^2(x-\frac{b}{2})^2 + (y+\frac{b}{2})^2(x+\frac{b}{2})^2 + (y+\frac{b}{2})^2(x-\frac{b}{2})^2 + (y+\frac{b}{2})^4. \quad (3.25)$$

Similar relations hold for the $\partial V/\partial y$.

The above formulation allows calculation of an exact solution in terms of both potential, V , and joule heat, q''' , for the 2-D electrostatic problem. The exact solution can then be used to assess the accuracy of the numerical solution and to assess the sensitivity of the results to electrode modeling and boundary condition specification. Results from this assessment are discussed in the following paragraphs.

3.2 Domain Discretization

Several calculations were conducted to assess the effects of varying the location of the electrodes within the numerical mesh. This sensitivity study was performed to help establish the minimum domain of discretization necessary to maintain a reasonable level of accuracy in the calculated results. In one case the electrodes were placed on the four extreme corners of the mesh. As might be expected, the numerical solution became less accurate in the vicinity of the boundary, particularly in the vicinity of the electrodes. The loss in accuracy near the boundary is the obvious compensation for specifying a Dirichlet boundary condition along the outer boundary and adjacent to the electrodes. Although this boundary condition is necessary to obtain a unique solution, it must be supplied at a sufficient distance so as not to impact the accuracy of the results in the domain of interest.

Figures 2.4 and 2.5 show respectively, the normalized magnitudes of the voltage field and the joule heat obtained from the analytic solution. The results shown are based upon a relative radius from the electrodes. The relative radius is the magnitude of the ratio of the distance from an electrode to a point in the field divided by the distance between the electrode and the center (origin) of the ISV melt. The results shown are for a line passing through the origin and one of the four electrode positions. As indicated, the voltage diminishes to less than 25% of the maximum value beyond a relative radius of about 2 in a direction away from the origin. The divergence of the voltage field (ie, the electric field, which is indicated here by the joule heat shown in Figure 3.5) is negligible at and beyond a relative radius of 1. Therefore, to assure that the heating around the electrodes was adequately simulated, the external boundary condition (ie, $V=0$) was specified at a relative radius of about 2.5 or, for this problem, at a distance of approximately 6.5m from the origin. This distance also provided for an adequate far field temperature boundary condition for the heat conduction calculations necessary in the 3-D, variable property, ISV problem.

Figure 3.4 Normalized voltage field (exact).

Figure 3.5 Normalized joule heat (exact).

The resulting problem discretization is shown in Figure 3.6 with the four electrode locations indicated by shading. The element size in the region of primary interest was limited by the electrode size. The element size on the boundary was based primarily upon convenience and maintaining compatibility with the square symmetric constraint discussed in the following paragraph.

Figure 3.6 Two-dimensional electrostatic field discretization.

3.3 Joule Heat Calculation

As mentioned in the previous section, the voltage is 90 degrees out of phase between adjacent electrode pairs. This means that the volumetric joule heat source is time varying but, because the time scale of the joule heat oscillations (16ms) is small relative to the time scale of the ISV process (hours or days), the process is adequately modeled using the time average joule heat. This time average value is determined from the principle of superposition. Normally this requires the solution of two independent linear problems with the final solution obtained as the sum average of the two independent solutions. If the discretization, however, is square symmetric around the origin, and if the material medium is

uniformly distributed, then the average joule heat can be obtained from one solution. This is accomplished recognizing that, under the constraints stated, the joule heat source terms will be symmetric about the geometric center of the mesh. Starting at a corner of the mesh, a matrix of joule heat values, q'''_{ij} , can be assembled where ij represents the determinant location corresponding to the element of the i^{th} row and the j^{th} column of the numerical mesh. The average joule heat distribution is then obtained by adding the symmetric members of the matrix and dividing by 2. Mathematically this is represented simply as

$$\bar{q}'''_{ij} = \frac{1}{2} (q'''_{ij} + q'''_{ji}), \quad (3.26)$$

where $\bar{q}'''$ is the average volumetric joule heat source of the element.

3.4 Electrode Boundary Condition

With the exception of the electrodes, the upper surface of the domain was represented as an insulated boundary. The electrode boundary conditions were specified in two manners. In one case, a Dirichlet boundary condition (voltage) was specified at the four mesh points adjacent to the electrodes via equation (3.12) above. In the other case a Neumann boundary condition (current density) was specified at the top surface of the electrode mesh. Figure 3.7a compares the joule heat obtained from the analytic solution described above with the joule heat obtained from the numerical solution over the mesh shown in Figure 3.6. The results shown were obtained applying the Dirichlet boundary condition at the electrodes and are for those points lying on a line parallel to the η axis and passing through $\xi=2.42m$. Figure 3.7b presents a similar comparison but for the current density boundary condition at the electrodes. Note that the magnitude of the heating is highest in the vicinity of the electrodes. The higher heating rate near the electrodes is due to the higher current density near the electrodes.

In both boundary condition cases, the numerical solution is in reasonable agreement with the exact solution. Figure 3.8 shows the joule heat for the case of the voltage specification on the electrode boundary for points lying on lines parallel to that for Figure 3.7. Again the agreement between the calculated and exact solutions is good. Figure 3.9 compares the voltage distribution between the exact and the numerical solutions and shows that the numerical calculation provides an adequate representation of the exact voltage. Similar agreement exists in the case of the Neumann boundary condition.

It has been shown that either method of specifying the boundary conditions (ie, electrode

voltage or current density) provides an adequate simulation for the 2-D problem with constant material properties. This is not the case when the full 3-D solution is sought in a material(s) where the properties are strongly nonlinear functions of temperature. For this latter case, the current density method of specifying the electrode boundary conditions was determined to be superior. When using this approach, the electrode was modeled in the electrostatic static solution by specifying a material with an electrical conductivity several orders of magnitude higher than the medium between the electrodes.

a) Dirichlet boundary condition

b) Neumann boundary condition

Figure 3.7 Comparisons between the 2-D analytic and calculated joule heat solutions ($\xi=2.42\text{m}$).

Figure 3.8 Comparisons between the 2-D analytic and calculated joule heat solutions ($\xi=2.86, 1.54$, and 0.22m).

a) $z_i = 2.86$ m

b) $z_i = 1.54$ m

c) $z_i = 0.22$ m

+ calculated + exact
 (Note: The legend in the image uses square markers for 'calculated' and circle markers for 'exact', despite the text showing plus signs.)

Figure 3.9 Comparisons between the 2-D analytic and calculated voltage solutions ($\xi = 2.86, 1.54$, and 0.22 m).

Using the information obtained from the 2-D analytic analysis discussed above, a 3-D ISV model was constructed. The model discretization is shown in Figure 3.10. This discretization was used for both the electrostatic and heat conduction calculations.

For the ISV calculation, some of the material properties become functions of temperature making the problem nonlinear and necessitating iteration in solution of both field equations. Nonlinearity is also introduced in the heat conduction equation through the surface radiation boundary condition. The complete model is discussed in the following subsection. The following section presents results from several parametric analysis.

Figure 3.10 Three-dimensional thermal-electric discretization.

3.5 3-D ISV Model

The ISV problem differs from the 2-D analytic problem in that the solution is now manifested in 3 dimensions and the material properties are no longer constant but strong functions of temperature. As mentioned previously, this latter difference renders the problem nonlinear and necessitates a coupled solution involving the energy equation.

The model used to perform the 3-D ISV calculations is discussed in the following paragraphs. Also included is the algorithm used to solve the coupled thermal-electric field equations and a sample calculation based upon a supplied current of 4000 amps and an electrode separation and depth of 3.66m. The overall solution domain is 12.32m x 12.32m x 6m with the upper surface radiating as a black body to an ambient temperature of 300K. No waste material was simulated and the soil material properties were obtained from reference 8. It should be noted that all calculations performed in this analysis were done on an IBM system 2 386 PC.

The soil electrical conductivity and thermal conductivity are shown as functions of temperature in Figures 3.11 and 3.12, respectively. As mentioned, the material properties were obtained from reference 8 and are based upon the following relationships:

$$\sigma = \left\{ 10 \exp \left[a \left(\frac{10^4}{T} \right)^2 + b \left(\frac{10^4}{T} \right) + c \right] \right\} \quad T < 1428K \quad (3.27)$$

$$\sigma = \left\{ 10 \exp \left[m \left(\frac{10^4}{T} - X_0 \right) \right] \right\} \quad T > 1428K, \quad (3.28)$$

and

$$k = m^*(T - T^*) + k^*, \quad (3.29)$$

where σ and k are the electrical and thermal conductivity, respectively and where the constants are:

$m = -\frac{1}{1.9}$, $X_0 = 6.9$, $a = -8.28 \times 10^{-2}$, $b = 0.61$, $c = -0.288$, $m^* = 2.13 \times 10^{-3}$ ($T^* = 293K$), $k^* = 0.13$ ($T < 1173K$), $m^* = 25.5 \times 10^{-3}$ ($T^* = 1173K$), and $k^* = 2.00$ ($T > 1173K$).

It has been shown⁹ that radiation heat transfer dominates the upper surface heat loss. For the analysis discussed herein, this boundary condition has been linearized in the form

$$q'' = h_{\text{rad}}(T_{\text{surface}} - T_{\text{ambient}}) \quad (3.30)$$

where q'' is surface heat flux, T is temperature, h_{rad} is the radiation heat transfer coefficient $= \sigma'(T_{\text{ambient}} + T_{\text{surface}})(T_{\text{ambient}}^2 + T_{\text{surface}}^2)$, and σ' is the Stefan-Boltzmann constant.

Figure 3.11 Soil electrical conductivity.

Figure 3.12 Soil thermal conductivity.

The overall solution scheme is as follows:

1. Assume a temperature distribution.
2. Solve the electrostatic field problem.
3. From the potential field distribution calculate the current density and joule heat distribution.
4. With the joule heat source distribution, calculate the temperature distribution from the heat conduction equation.
5. Compare the calculated temperature distribution with that used in the solution of the electrostatic field problem and return to step 2 if necessary.
6. Continue until temperature convergence is reached.

In step 1 above, it was found that the number of iterations required for convergence decreases if a relatively high temperature is assumed initially (e.g., $T = \text{constant} = 2000\text{K}$). This decrease is due to the relatively low values of joule heat calculated at this temperature which, in turn, are a result of the relatively low resistivity of the soil at this temperature. Conversely, if a lower temperature is assumed (eg, $T = 1000\text{K}$), then very large values of joule heating are calculated which result in high temperatures from solution of the conduction problem. Since under relaxation of the temperatures is performed, the larger the difference between the input temperature (electrostatic problem) and output temperature (conduction problem) the more iterations required to converge.

As mentioned, for the calculation performed, the medium is soil and it is assumed to be uniformly distributed. In the actual case, the soil will contain various wastes distributed nonuniformly. Consequently, for the actual case, various materials will have to be simulated. In addition, for the real case, soil/waste subsidence occurs as the waste decomposes and off-gasing results. No attempt was made to simulate the decrease in volume associated with this subsidence and the effect it may have on the resulting process response.

Figure 3.13a shows the calculated voltage as a function of η at the 0.6m depth for several ξ coordinates. The results shown are the average element voltage which is simply the average of the eight element nodal-values. Figure 3.13b shows the voltage at the 5.4m depth. Based upon the diminishing value of the voltage at this lower depth, it might be expected that the joule heat here would be lower. This expectation is, in fact, the case as indicated by the results shown in Figure 3.14 which shows the heat source as a function of ξ and η at several elevations. Note that the change in joule heat with elevation is most pronounced near the electrodes (where the maximum heating occurs).

Figures 3.15 and 3.16 show the calculated temperature stratification in the ISV melt. The temperature stratification is a result of the varying joule heat, the material property dependence upon temperature, and the upper surface heat loss. Based upon the calculated temperatures and a soil fusion temperature of 1428 K, a cold cap would form to about the 1m elevation and the final melt would be contained between the 1 and 5m depths.

It is interesting to note here that the total calculated power is about 700kw (4000 amps per electrode pair at a potential of 85 volts). The power system that this particular calculation was based upon has a total rating of 3700kw (4000 amps per electrode pair at a potential of 400 volts). The steady-state power available therefore, may be substantially higher than that which the system can deliver. The power the system can deliver is a function of the current available and the resistivity (temperature) of the melt. The following section presents results from studies showing the effects that varying several important parameters have on the final state of the ISV melt.

a) 0.6m depth

b) 5.4m depth

Figure 3.13 Voltage at 0.6m and 5.4m depths.

a) $z_i=2.42\text{m}$

b) $z_i=1.54\text{m}$

• .6m depth • 1.8m • 3.0m
• 4.2m • 5.6m

c) $z_i=0.66\text{m}$

Figure 3.14 Joule heat as a function of ξ, η and z

Figure 3.15 Temperatures at $\xi=1.98\text{m}$.

Figure 3.16 Temperatures at $\xi=0.22\text{m}$.

4.0 SENSITIVITY ANALYSIS

In this section the steady-state ISV process response sensitivity to selected parameters is analyzed. The parameters analyzed include: 1) electrode depth, 2) ambient surface temperature, 3) supplied current, 4) material electrical conductivity, 5) electrode separation and 6) soil/waste characterization. The model used in the analysis is the same as that discussed in the previous two sections.

4.1 Electrode Depth

The first response parameter discussed is the electrode depth. For this analysis, three electrode depths were investigated and included 3.6, 7.2 and 14.4m. The total domain depths in these calculations were set at 6, 12, and 24m, respectively. The supplied current used in each calculation was 4000 amps and the ambient hood temperature was 300K.

Figure 4.1 shows the calculated temperature at three relative depths, representing percents of the total domain depth, and indicates the effect of varying the electrode depth on the resulting temperature. As expected, the lower the electrode depth the lower the calculated temperature reached. The steady-state power for these calculations, however, was not proportionately affected by the different depths and ranged from approximately 500kw for the 14.4m depth to 700kw for the 3.6m depth. This power insensitivity is the result of off-setting effects. Although the lower depths resulted in lower temperatures and therefore, higher melt resistivities, the lower depths exposed more electrode surface area thereby reducing the local current densities. These two off-setting effects resulted in a total power that remained relatively constant at values of 15-20% of the total system power rating of 3700kw.

The next section presents a brief discussion of the effect of the hood temperature on the process response.

a) 10% depth

b) 50% depth

c) 90% depth

Figure 4.1 Effect on temperatures at $\xi=2.42\text{m}$ due to electrode depth.

4.2 Ambient Temperature

The configuration of the hood which collects the off-gases as well as the temperature of the gases within the hood affects the energy loss at the surface of the melt. As mentioned previously, the energy loss at the surface manifests itself primarily in the form of radiant heat loss. For the calculations performed, only radiation heat transfer was considered. The following paragraphs present a brief discussion of the effect of varying this boundary condition on the ISV process response.

The results from two calculations are compared. For both calculations, a current of 4000 amps was specified at the electrodes. The electrode depth for each calculation was specified at 3.6m. In one calculation, the ambient temperature was set at 300K. For the other calculation, a hood covering a portion of the surface was simulated by specifying an ambient temperature of 800K over a portion of the surface (6.16m x 6.16m) and an ambient temperature of 300K over the remaining portion of the surface.

Figure 4.2 shows the calculated temperatures at various depths. As indicated the effect on the calculated melt temperature of changing the ambient temperature boundary condition by 500K is relatively insignificant. The maximum difference is less than 100K at any elevation. For the lower ambient temperature with no hood simulation, the depth of the surface cold cap is increased over that of the higher ambient temperature calculation. In fact, for the latter case, little or no cold cap is calculated to occur in the center portion of the melt. This observation is based upon a soil fusion temperature of about 1400K and is indicated in Figure 4.2 by the higher calculated temperatures at the 10% depth in the case of the hood simulation. The surface boundary condition had little effect on the steady-state system power. In both calculations, the calculated power was about 700kw.

The next discussion presents the effect of varying the supplied current on the resulting process response.

a) 10% depth

b) 50% depth

• T=300K • T=800K
 c) 90% depth

Figure 4.2 Effect on temperatures at $\xi=2.42\text{m}$ due to ambient surface temperature.

4.3 Electrode Current

During the ISV transient, the current to the electrodes is varied so as to maintain the total power approximately constant and as close as possible to the power rating of the system, thereby achieving a high power efficiency. To maintain a relatively constant power to the melt, the current supply to the electrodes is increased as the process proceeds and the soil temperatures rise and the average melt resistance falls. At some point during the later periods of the transient, however, when the maximum current is being supplied, the melt will reach an equilibrium state and the power supplied will be offset by that lost in the form of heat to the surroundings. The resistance of the melt at this time will depend upon the temperature of the melt. Given a supplied current, the potential drop across the electrodes will be proportional to the resistance of the melt and therefore, the power supplied will also be proportional to the resistance or the temperature of the melt. The problem becomes one of calculating the state of equilibrium and the final power achievable with the system and the soil/waste characterization. The following discussion shows the effect of varying the supplied current on the final process state and the power achievable.

For the calculations, an off-gas hood was simulated, as discussed in the previous section, and the electrode depth was set at 3.6m. For one calculation, the supplied current was set at 4000 amps and for the other calculation the supplied current was set at 2950 amps.

Figure 4.3 shows the effect that varying the current had on the final temperatures of the ISV melt. As expected, the higher current resulted in higher temperatures but the effect is not very significant with the maximum calculated temperature difference being less than 130K. Figure 4.4 shows the typical effect that the current variation had on the potential drop across the electrodes. As indicated, the effect is negligible. Since the power is equal to the current times the potential drop, the steady-state power differs between the two calculations almost in direct proportion to the applied current and is equal to 700kw and 530kw for the 4000 amp calculation and the 2950 amp calculation, respectively.

The effect of varying the medium electrical conductivity was analyzed and is discussed briefly below.

a) 10% depth

b) 50% depth

$\bullet$ $I=4000a$ $\circ$ $I=2950a$
 c) 90% depth

Figure 4.3 Effect on temperatures at $\xi=2.42$ m due to supplied current.

Figure 4.4 Typical effect on potential at $\xi=2.42\text{m}$ due to supplied current.

4.4 Electrical Conductivity

Probably the least quantified and yet most important material property needed for investigations of this type is the electrical conductivity. For most materials, particularly fluids, this property is strongly affected by temperature and normally increases dramatically with increasing temperature (see Figure 3.11). Consequently, it was considered appropriate to assess the effect that varying this property has on the resulting ISV process response. The results obtained are discussed in the following paragraphs.

Three calculations were performed. For each calculation the electrode current was set at 4000 amps and a surface hood was simulated in the manner discussed previously. The electrode depth was set at 3.6m. For the reference calculation, the electrical conductivity was determined from equations (3.27) and (3.28). For the second calculation the electrical conductivity was that of the reference calculation divided by 2 and for the third calculation the electrical conductivity was that of the reference calculation multiplied by 2.

Figure 4.5 shows the effect that varying the electrical conductivity had on the resulting calculated temperatures. In the calculation with the relatively low electrical conductivity, the temperatures are significantly higher than in the calculation with the relatively high conductivity with

a) 10% depth

b) 50% depth

$\sigma = \text{ref}$ $2 \times \text{ref}$ $.5 \times \text{ref}$
 c) 90% depth

Figure 4.5 Effect on temperatures at $\xi = 2.42$ m due to electrical conductivity.

the magnitude of the temperature for the reference calculation falling in between. The maximum calculated temperature reached in the case of the former was approximately 2530K whereas in the case of the latter was 2270K. The power for the high electrical conductivity case was calculated to be 510kw or about 50% that of the low electrical conductivity case which was calculated to be 986kw.

The following section summarizes the effects on calculated temperature from varying the electrode separation distance.

4.5 Electrode Separation

The distance separating the electrodes affects the total melt electrical resistance and the local melt current density. Consequently, the electrode separation can significantly affect the process response and therefore, was considered an important parameter to assess from a sensitivity standpoint.

Results from two calculations are compared to provide insight into the effect that the electrode separation has on the calculated steady-state temperature response. For each calculation, a current of 4000 amps was specified and the electrodes were buried to a depth of 3.6m. The medium simulated was soil and an ambient hood temperature of 800K was used. For one calculation, the distance separating the electrodes was set at 3.6m and for the other calculation this distance was set at 7.2m.

Figure 4.6 compares the calculated temperatures between the two cases just described. As might be expected the greater the electrode separation, the lower the temperatures. Although the greater separation produced a higher steady-state power (1300kw versus 703kw), this power was insufficient to heat the soil to temperatures similar to that of the 3.6m electrode separation calculation.

Next, the effect of the soil/waste characterization is presented.

a) 10% depth

b) 50% depth

← 3.6m separation ← 7.2m separation
c) 90% depth

Figure 4.6 Effect on temperatures at $\xi=2.42$ m due to electrode separation distance.

4.6 Soil/Waste Characterization

The previous analyses provide insight into the calculated sensitivity of the ISV process response to variation in several important variables. These results, however, provide little insight into the integral behavior of the process. As a final analysis therefore, a calculation was conducted which more realistically simulates the soil/waste characterization encountered in low-level waste sites. The calculation was based on data from pit 9 of the disposal area discussed in references 10 and 11.

The buried wastes described in references 10 and 11 range from various metals and chemicals to wood, water, paper, oil and asbestos. The waste is interspersed with voids and the overall waste volume has a void fraction approaching 25%. Only a limited amount of electrical conductivity data was available for the various waste constituents.^{12,13} In general, however, the electrical conductivity can be categorized as high, intermediate, or low. For example, 13% of the waste volume is made up of wood, paper, oil and asbestos with the electrical conductivity of each constituent being low, on the order of $10^{-3} \text{ (ohm-m)}^{-1}$ or less. Six percent of the waste volume is comprised of water which has an electrical conductivity on the order of 1 (ohm-m)^{-1} . Nine percent of the waste volume is comprised of metals and chemical compounds which are characterized by relatively high electrical conductivities on the order of $10^4 \text{ (ohm-m)}^{-1}$ or greater. The remaining waste volume ($\approx 47\%$) is soil with the material properties described by equations (3.27)-(3.28). For the site described in reference 10, an overburden of 1m of soil and an underburden of about 1.4m of soil surround approximately 3.6m of waste volume.

The basic model used for this calculation is the same as that described in the previous sections with a current specification of 4000 amps and an ambient hood temperature of 800K. For the present calculation, it was assumed that the voids within the waste volume would be displaced by the soil overburden such that the electrode depth was set at 3.6m. The top layer of elements (10% depth) simulated the soil overburden. The next two layers of elements (30 and 50% depths) simulated the waste volume. The bottom two layers of elements (70 and 90% depths) simulated the soil underburden. The distribution of the three categories of wastes was arbitrarily chosen such that the metals and compounds occupied the lower elevations or the 50% depth elevation and the wood/paper/oil/and asbestos occupied the higher elevations or the 30% depth elevation. The water was distributed at both depths. Figures 4.7 and 4.8 show the distribution pattern at the two depth elevations. Note that distribution symmetry is maintained such that the joule heat calculation constraint is not violated (see

Figure 4.7 Mesh diagram showing the soil/waste characterization at the 30% depth elevation.

Figure 4.8 Mesh diagram showing the soil/waste characterization at the 50% depth elevation.

Section 3.3). Results obtained from this calculation are compared with results obtained with a soil only medium. The comparisons are discussed briefly in the following paragraphs.

Figures 4.8 and 4.9 compare joule heat values from the calculations described above between the 10 and 50% depth elevations for a ξ coordinate of 1.54 and 0.66m, respectively. The results generally show a significant difference in the value of the calculated source terms which can be correlated to the location of the wastes shown in Figures 4.6 and 4.7. This correlation, however, is not obvious in Figure 4.10c which compares the joule heat at the 50% depth elevation and at $\xi=0.66$ m. With reference to Figure 4.7, it is noted that no waste products are simulated at this value of ξ . The electric field and therefore the joule heat, however, are affected by surrounding elements most of which do contain simulated wastes. The results from this calculation indicate that the relatively low heating values at the 30% depth elevation are offset by the relatively high values at the 50% depth elevation. This offsetting effect is further apparent in the calculated temperature field.

Figures 4.10 and 4.11 show the calculated temperatures corresponding to the location of the calculated joule heat given in Figures 4.8 and 4.9. As indicated, the difference in the results when a distributed waste is or is not simulated is negligible. The power in the case of the soil/waste simulation was calculated to be

Figure 4.9 Joule heat at $\xi=1.54$ m.

a) 10% depth

b) 30% depth

c) 50% depth

Figure 4.10 Joule heat at $\xi=0.66\text{m}$.

a) 10% depth

b) 50% depth

c) 90% depth

→ soil → soil/waste

Figure 4.11 Temperature at $\xi=1.54$ m.

a) 10% depth

b) 50% depth

c) 90% depth

Figure 4.12 Temperature at $\xi=0.66$ m.

approximately 677kw which is within 5% of the power in the case of the soil only simulation which was 703kw.

It should be emphasized that the calculations above did not attempt to simulate any of the transient process response which would produce soil subsidence, waste redistribution, and off-gassing. Each of these processes may prove to be important to the final or steady-state response of the system but the simulation of which is beyond the scope of this investigation.

The following section summarizes the results from the calculations presented above.

4.7 Summary

Presented in the following paragraphs is a short summary of the calculated results presented in the previous sections.

Several assumptions were employed in the numerical analysis discussed herein. First, it was assumed that the electrical field can be considered electrostatic. Second, convection was neglected in the energy equation. Third, no effect from soil subsidence was considered and fourth, the soil/waste consistency was assumed to be uniform and distributed symmetrically about the geometric center of the melt.

Table 4.1 lists a summary of pertinent parameters regarding the calculated ISV process response. The results presented indicate that the maximum temperature reached is affected most by the effective melt electrical resistance and the distance between electrodes. The calculated power is affected most by the melt electrical resistance, which is a function of the material electrical conductivity and electrode separation. The electrode depth had relatively minor effect on the calculated power and the calculated temperatures, with the lower powers and temperatures occurring at the lower electrode depths. The power and the maximum temperature were affected by the magnitude of the supplied current; the power being in direct proportion to the current. The ambient temperature had only a minor affect on the calculated temperature and power. For the calculation performed, the steady-state temperatures were unaffected by soil/waste characterization. Finally, none of the powers calculated were near the total system power rating of 3700kw. During the transient this power may be

reached but for the conditions of these calculations, the steady-state power is only on the order of 20% of the maximum power.

Table 4.1 Summary of results

T_{max} , K	Power, kw	depth,m	separation,m	T_{amb} ,K	Current,amps	σ ,1/ohm-m	V, volts	soil/waste
2360	680	3.6	3.6	300	4000	ref ¹	88	soil
2270	472	7.2	3.6	300	4000	ref	62	soil
2190	464	14.4	3.6	300	4000	ref	60	soil
2370	703	3.6	3.6	800	4000	ref	92	soil
2270	530	3.6	3.6	800	2950	ref	92	soil
2270	510	3.6	3.6	800	4000	2*ref	66	soil
2530	986	3.6	3.6	800	4000	0.5*ref	128	soil
2007	1300	3.6	7.2	800	4000	ref	102	soil
2327	677	3.6	3.6	800	4000	ref	91	soil/waste

¹The reference σ was determined from equations (3.27) and (3.28).

4.0 CONCLUSIONS

It is generally concluded from the analysis performed that the finite element method provides a robust solution technique for simulating the ISV process. From the results and discussions presented, specific conclusions reached include:

- 1) The current density is the appropriate electrode boundary condition.
- 2) The ISV discretization domain should extend at least to a relative radius of 2 in the direction away from the melt.
- 3) The steady-state power reached is affected most by the resistivity of the melt.
- 4) The depth of the electrodes affects the magnitude of the temperatures reached within the melt with higher temperatures associated with smaller electrode depths.
- 5) The magnitude of the ambient temperature does not appreciably affect the ISV process response.
- 6) The steady-state power is directly proportional to the supplied current.
- 7) The electrode separation distance affects both the power and the steady-state temperatures with lower temperatures associated with greater separation distances.
- 8) The power and temperatures attained in soil are not appreciably affected if the soil is interspersed with materials having different electrical conductivities.
- 9) The resistivity of the melt is the most important material property for determining the ISV process response.

The last conclusion is important because to simulate the ISV process response requires reliable soil characterization data. If this data is not available then an accurate analysis becomes difficult and reliance on engineering judgement becomes necessary.

5. REFERENCES

1. Arrenholtz, D.A., et.al., "In Situ Vitrification Program Treatability Investigation Workplan," EGG-WM-9072, June 1990.
2. MacKinnon, R.J., et.al., "In Situ Vitrification Model Development and Implementation Plan," Revision 1.0, EGG-WM-9036, August 1990.
3. Corson, D.R. and P. Lorrain, Introduction to Electromagnetic Fields and Waves, W.H. Freeman and Company, San Francisco and London, 1962.
4. E. C. Lemmon, "TRIPLE: A Three-dimensional Finite Element Heat Conduction Program."
5. Huebner, K.H. and E.A. Thornton, The Finite Element Method for Engineers, Second Edition, John Wiley & Sons, 1982.
6. Finlayson, B.A. and L.E. Scriven, "The Method of Weighted Residuals-A Review," Appl. Mech. Rev., Vol. 19, no. 9, Sept, pp. 735-748, 1966.
7. Aral, M.M., et.al., "Finite Element Galerkin Method Solutions to Selected Elliptic and Parabolic Differential Equations," Proceedings of the 3rd Conference on Matrix Methods in Structural Mechanics, Wright-Patterson Air Force Base, Dayton, Ohio, Oct. 1971.
8. Buelt, et.al., "In-situ Vitrification of Transuranic Wastes: An Updated Systems Evaluation and Applications Assessment," PNL-4800 Suppl.1 UC-70, Pacific Northwest Laboratory, March 1987.
9. P.E. Murray, et.al., "A Preliminary Numerical Study of Heat Transport During In Situ Vitrification of Soil," EGG-WM-9038, June, 1990.
10. C.E. Slater, et.al., "Off-Gas Release Rate Calculations for the Large Scale Test in Pit 9", EG&G Engineering Design File ERP-ISV-038, July 1990.
11. T.E. Wierman, "ISV Field Test Siting and Source Term", EG&G Idaho Engineering Design File BWP-ISV-024, March 1990.
12. R.C. Wiest and M.J. Astle, Eds., CRC Handbook of Chemistry and Physics, CRC Press, Inc., Boca Raton, 1981.
13. J.H. Perry, et.al., Eds., Perry's Chemical Engineers' Handbook, 4th ed., McGraw-Hill Book Company, Inc., New York, 1963.

END

DATE FILMED

02 / 26 / 91