J_{IC} Measured from Unstable Test Specimens with
Finite Differences in Crack Areas*

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by

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Introduction

Conventional linear elastic fracture mechanics predicts failure in elastic engineering structures using a single materials parameter. The critical strain energy release rate, $\mathcal{G}_{IC}$, is this parameter and it is most frequently measured at the onset of crack propagation in a linear elastic fracture test. Unfortunately, plane strain linear elastic test specimens are excessively large for materials with high toughness and low yield strengths for plane strain fracture [1]. Nonlinear elastic fracture mechanics is an attractive alternative to linear elastic fracture mechanics since small test specimens can be used to measure the pertinent material parameter(s).

Nonlinear fracture tests have concentrated on single test specimens [2,3] for which a nonlinear elastic solution is available even during crack growth [4]. Crack growth measurements in these specimens is, however, difficult to measure as it requires cumbersome heat tinting [5] or highly sensitive instrumentation that measures the crack length by partial unloading [6] during crack growth. Often in the nonlinear elastic specimen it is difficult to establish when crack initiation has occurred from the load versus loadpoint displacement record. If, however, an unstable specimen configuration was intentionally chosen then, when the crack first starts to propagate, it will propagate catastrophically and initiation conditions are easily observed. Unfortunately, nonlinear elastic solutions for unstable test geometries are not known.

The generalized force on a crack like feature in an elastic solid has been shown to be J [7]. J values have been determined from a series of test specimens that incrementally differ only in crack area [8-10]. Incremental differences in crack areas are used to approximate the derivative of the load on the specimen with respect to the crack area at constant loadpoint displacement. Such a testing scheme requires a large number of test specimens since an incremental limit is to be found. In the experimental aspects of this paper unstable, nonlinear elastic specimens with finite differences in

crack area are used to calculate the J value at the onset of crack propagation. The finite difference fracture test presented in this text may be accomplished with a minimum of two specimens. It will be shown in the analytic section of this paper that the area between the load-loadpoint displacement records of two specimens with a finite difference in crack area may be used to evaluate J at the onset of crack motion. It will be further shown that short cracks favor an unstable test geometry in slightly plastic specimens.

Nonlinear Fracture Mechanics

Figure 1 shows an elastic solid that contains a crack. The solid is loaded by the force, P . The loadpoint has a displacement, X , in the direction of P . The exposed crack area in the solid is $2A$. The solid is divided into two thermodynamic systems. System I contains only elastic material. This system surrounds the crack tip and has the generalized force, J , acting on System II. System I has been analyzed, in reference [11], for evaluation of J in nonlinear elastic problems. What follows below is an energy balance, on System I, that utilizes the concepts and notation of [11]. System II contains nonreversible material near the crack tip. The generalized force, J , is balanced by the material's fracture energy, R in System II. Kinetic processes at the crack tip have been used to investigate the rate of crack propagation in System II [12].

Referring to Figure 1, the incremental work done on System I is PdX . The incremental work done on System II by System I is $-JdA$. The incremental difference between these work terms is the incremental change in System I's strain energy, dW_1 . Thus,

$$dW_1 = PdX - JdA \quad (1)$$

The complementary strain energy, W_2 , is defined as $W_2 \equiv PX - W_1$. Thus,

$$dW_2 = XdP + JdA \quad (2)$$

W_1 and W_2 are state functions so from (1) and (2), the following definitions and Maxwell relationships immediately follow:

$$\frac{\partial W_1}{\partial X} \Big|_A \equiv P \quad ; \quad \frac{\partial W_1}{\partial A} \Big|_X \equiv -J \quad \text{and} \quad \frac{\partial P}{\partial A} \Big|_X = - \frac{\partial J}{\partial X} \Big|_A \quad (3)$$

$$\frac{\partial W_2}{\partial P} \Big|_A \equiv X \quad ; \quad \frac{\partial W_2}{\partial A} \Big|_P \equiv J \quad \text{and} \quad \frac{\partial X}{\partial A} \Big|_P = \frac{\partial J}{\partial P} \Big|_A \quad (4)$$

Integration of the extreme right side of equations (3) and (4), restricted to constant crack area during the integration, yield general expressions for J.

$$J = \int_0^X - \frac{\partial P}{\partial A} \Big|_X dX \quad \text{and} \quad J = \int_0^P \frac{\partial X}{\partial A} \Big|_P dP \quad (5)$$

A complete energy balance description of System I requires three additional partial derivatives that are, in general, functions of the two independent state variables in the system. In [11] these functions have been chosen as

$$\alpha \equiv - \frac{\partial J}{\partial A} \Big|_X, \quad \beta \equiv - \frac{\partial P}{\partial A} \Big|_X \quad \text{and} \quad \gamma \equiv \frac{\partial P}{\partial X} \Big|_A.$$

α is the crack stability function, β is the strain energy coupling function and γ is the specimen stiffness. It has been shown in [11] that the equations in (5) are simply

$$J = \int_0^X \beta dX \quad (6)$$

and

$$J = \int_0^P (\beta/\gamma) dP \quad (7)$$

Of course, during integration of equations (6) and (7) the crack area must still be held constant. Equation (6) has been used in [10], to experimentally evaluate J at the onset of crack propagation. Note that this procedure requires integration of incremental differences in load and crack areas and therefore necessitates a large number of test specimens.

Figure 1 also shows System I in series with a linear elastic spring of stiffness k. k may be considered to represent a testing machine and fracture fixture spring constant. Frequently, in testing fracture specimens it is not convenient to directly measure the loadpoint displacement. The displacement measured is the total displacement of the loadpoint of the specimen plus the displacement due to machine compliance.

This total displacement is here termed, X_1 .

$$dX_1 = dX (1 + \gamma/k) \quad (8)$$

The new quantities α_1 , β_1 , and γ_1 are defined in reference to the displacement X_1 , (rather than X) and may be shown to be related to α , β , and γ of System I by

$$\alpha_1 = \alpha - \beta^2/(k+\gamma) \quad (9)$$

$$\beta_1 = \beta/(1+\gamma/k) \quad (10)$$

$$\gamma_1 = \gamma/(1+\gamma/k) \quad (11)$$

It is clear from equations (8) and (10) and from equations (7), (10) and (11) that J is the same when measured using X or X_1 displacements.

Figure 2 shows a schematic loading path in P - X space for two identical pre-cracked specimens that differ in crack area by an amount ΔA . The specimens are loaded into the plastic range before the onset of crack growth commences at the points ① and ②. Idealized, elastic, crack growth is assumed to occur for the specimen with the crack area A while the loadpoint displacement changes from ① to ② but maintaining a constant J . The equivalent nonlinear elastic specimen with a crack area, $A + \Delta A$, would then unload down the lower path from point ② to the origin in Fig. 2. It will now be shown that the energy contained in the loading cycle of Fig. 2 is simply $J\Delta A$. ΔA is the finite difference in the crack area between two specimens that are identical in every aspect except for their crack areas. Application of equation (1) along the loading curve from the origin to point ① shows that $W_1^{①}$ at point ① is the area under the loading curve, $W_1^{①} = \int_0^{X^{①}} P dX$. Application of equation (1) between point ① and point ② with J held constant gives,

$$W_1^{②} - W_1^{①} = \int_{X^{①}}^{X^{②}} P dX - J\Delta A \quad .$$

Unloading with $A + \Delta A$ held constant gives,

$$-W_1^{(2)} = \int_{X^{(2)}}^0 P \, dX.$$

The area enclosed between the three idealized curves in Fig. 2 is therefore found by adding all three contributions and is simply, $J\Delta A$. Or,

$$\oint P \, dX = J\Delta A. \quad (12)$$

It has therefore been established that if in P - X space, the curve describing constant J is known, then the loading curves for two specimens with finite differences in crack area may be used to calculate J . It however remains to be established on the P - X loading curve where the onset of crack propagation is occurring. It is further assumed that the crack starts to propagate at $J=R$ on each curve i.e. this J is a unique material parameter. It is probably a less serious approximation that in practice the J curve will be replaced by a straight line segment between the points ① and ② provided that ΔA is not excessively large. From the above considerations it is desirable to establish the conditions that will produce unstable crack propagation. An unstable crack system, at constant displacement, is desirable since large changes will occur in the load-loadpoint record at crack initiation for this system. This is discussed below and in the Experimental Section.

Neutral crack stability has been considered in [11] for the restricted conditions of constant load and constant displacement. It was shown that neutral crack stability at constant load occurs when $\frac{\partial J}{\partial A} \big|_P = -\alpha + \beta^2/\gamma = 0$; so J and P are both constant during crack propagation. This restriction on the general nonlinear theory is that $\alpha\gamma = \beta^2$. The specimen for which $\alpha\gamma = \beta^2$ is of particular importance in testing procedures; the J value is independent of the crack area. In linear elastic fracture mechanics, the specimen is a constant K specimen. It also follows for the general nonlinear specimen that the incremental work done on the specimen by P , during crack propagation is $P\Delta X = \Delta W_1 + J\Delta A$, where the Δ 's refer to finite changes occurring during crack propagation. In linear elastic fracture

mechanics $P\Delta X$ is equally partitioned between ΔW_I and $J\Delta A$ so $\frac{1}{2}P\Delta X = J\Delta A$. In nonlinear fracture mechanics constant displacement neutral stability will be discussed below.

Test specimens that have neutral stability in a perfectly stiff testing machine. i.e. those specimens for which $\partial J / \partial A|_X = -\alpha = 0$ are not widely understood but are of practical engineering significance; structures are often loaded with nearly constant displacement. For these specimen $\alpha = 0$ and both J and X are constant as the crack propagates. The loading force does no work and a decrease in the strain energy corresponds to an increase in $J\Delta A$. In linear elastic test specimens $J = -\frac{X^2}{2} \frac{dY}{dA}$ so $\alpha = \frac{X^2}{2} \frac{d^2 Y}{dA^2}$. It follows that $\alpha = 0$ will occur if the specimen stiffness versus crack area curve has an inflection point or is a linear function of A . Thus, a change in the sign of the curvature of the stiffness versus crack area curve gives $\alpha = 0$. In those regions where the curvature of the linear elastic stiffness function is positive, α is negative and an unstable crack system exists even at constant displacement. In the general nonlinear case α is a function of the two independent state variables and no simple conclusions may be reached.

Figure 3 shows three crack propagation curves at constant J ; the first has $\alpha > 0$ and the constant J curve deviates substantially from the loading portion when A is constant. The second curve has $\alpha = 0$ and for this case both X and J are constant. The last case has $\alpha < 0$ and has the widest difference in slope between the constant A loading curve and the constant J propagation curve. Experimentally it is desirable to choose a specimen geometry for which $\alpha < 0$ so the onset of crack propagation is more easily detected on the load-displacement record by a sudden change in slope. Experimentally this last example shows that for a deeper cracked specimen crack propagation starts at shorter displacements and lower loads. Equation (9) clearly indicates that with α positive α_I may be negative. Thus, consideration of where the displacement is held constant is an important aspect of crack stability in general.

In this text instability is defined (at constant displacement) as $\alpha < 0$. Although, the variation of J with crack area is an important part of crack instability, actual crack propagation instability is more complex; System II also contributes significantly

to crack stability though dR/dA terms [11]. Reference [13] has suggested that a sudden load drop at constant X be used to describe the actual crack instability point. It was further suggested that dR/dA terms could be described by a tearing modulus, T , with $dR/dA = T$ during crack propagation. The slope of the load-loadpoint displacement record may be found when crack propagation is proceeding with

$\frac{dJ}{dA} = \frac{dR}{dA} = T$. J and A may always be chosen to be functions of the independent variables P and X ; $J = J(P, X)$ and $A = A(P, X)$. Thus,

$$\left. \frac{\partial J}{\partial P} \right|_X dP + \left. \frac{\partial J}{\partial X} \right|_P dX = T \left\{ \left. \frac{\partial A}{\partial P} \right|_X dP + \left. \frac{\partial A}{\partial X} \right|_P dX \right\}$$

or

$$\frac{dP}{dX} = \frac{T\gamma - \beta^2 + \alpha\gamma}{\alpha + T} \quad (13)$$

The load-loadpoint slope is now determined during crack propagation. If $T = 0$ crack propagation proceeds with J constant and the corresponding $\frac{dP}{dX} = \gamma - \beta^2/\alpha$ (as has already been noted). When $\alpha = -T$, the load-loadpoint displacement is vertical; when $\beta^2/\gamma - \alpha = T$, then the load-loadpoint displacement is horizontal. The crack instability point for constant load occurs when $\beta^2/\gamma - \alpha = T$; constant displacement for $\alpha = -T$. Provided the restricted assumption that crack propagation occurs with $dR/dA = T$ is valid in general. Before the crack propagates $\partial P/\partial X|_A = \gamma$; a change in slope in the load-loadpoint displacement record to the slope given in equation (13) would indicate the onset of crack propagation provided $dR/dA = T$ during crack propagation.

EXPERIMENTAL

Materials

Two materials were chosen for the experimental study, a titanium alloy Ti-6Al-4V and ASTM-A471 Ni-Cr-Mo-V rotor steel. These alloys were selected for the initial testing as their mechanical, chemical and fracture properties were established in previous works. Further, they represent two classes of alloys that are of current interest in the fracture field.

Table I gives the elemental chemical analysis of the Ti-6Al-4V [14] and the ASTM A471 [9]. Table II shows the mechanical and fracture properties of both the Ti-6Al-4V [15] and the ASTM A471 [9] alloys.

Procedure

All testing was conducted at room temperature with three-point bend specimens. The experimental apparatus is shown in Fig. 4. Both fatigue precracking and testing were done in a closed loop hydraulic testing instrument. Crack lengths were measured with a cathetometer during fatigue cycling crack growth so that load ranges could be chosen that kept the minimum and maximum stress intensity constant for each specimen and below fifty percent of the estimated K_{IC} . Several specimens of each material were precracked to different crack lengths. The ratios of crack length to specimen height were approximately 0.48 and 0.61 for the Ti-6Al-4V samples and 0.34, 0.42 and 0.56 for the A471 steel.

The J test consists of loading each specimen to failure while recording the load versus displacement. This was accomplished under constant displacement rate conditions.

The Onset of Crack Propagation

A serious problem in any nonlinear fracture specimen is to determine

on the load-displacement record where crack growth initiates. Clearly, this calls for a value judgement of crack initiation. However, since the fracture process is typically characterized by heterogenous nucleation on exact onset point of crack extension may not really exist. In this study we have examined two different techniques of establishing what we believe represents the extreme values along the load-displacement curve at which a crack may be said to extend. Two specimens of equal dimensions are loaded in three-point bend. One contains a blunted notch (0.25 mm in diameter) and the other a fatigued precrack. Initially their load-displacement records are identical but where they deviate represents the point of crack extension in the fatigued precrack specimen. This is the first detectable crack growth and is the minimum value characterizing crack initiation. The second point under consideration is to be the first drop in load detected or a sudden change in the slope of the load-displacement curve. This point represents the onset of crack propagation with J or K held constant and occurs later in the load-displacement record and thus is a maximum value. There is little argument that the crack is propagating at, and past, this point as crack motion is unambiguously detected macroscopically. The value (minimum) of load and displacement at the first onset of crack propagation though will still be determined by experimental technique. For the moment, let it suffice to take the above mentioned minimum and maximum values and discuss their engineering and crack propagation implications.

In obtaining the toughness, J_{IC} , for a specific material using the sharp slope change or the maximum load, displacement point (load drop) results in a maximum value. While the minimum load, displacement point provides a lower, or in terms of engineering design, a safer toughness

value. For this reason we may be inclined to select the test that gives the lower value. But, for a testing standpoint, the load-drop, sharp slope change criterion is simpler to perform and analyze. Therefore, the experimental goal is to design the test for instability to decrease the difference between the minimum and maximum measured toughness and to allow an easier test without yielding an optimistically high value of toughness. It is possible to alter the testing stability of a sample of a particular material by using short cracks in the sample.

Data Analysis

For each set of specimens in question the individual specimen's load-displacement record was replotted on a single master graph. The points on two specimens load-displacement curve where crack propagation were said to initiate were connected by a straight line. This straight line, drawn between the selected points, is to represent a constant J line. Clearly however, this is an approximation as the line need not be straight. This could lead to error in specimens whose a/w differential is very large. But, it was seen that even in the worst cases of short and long crack lengths the difference in calculated J 's was relatively small, always less than five percent.

Recall from the earlier discussion that two separate arguments were presented that gave different points for the onset of crack motion. These were termed the minimum and maximum J initiation points. For the Ti-6Al-4V tests the points at sharp slope change (maximum) were selected. Figure 5 shows the load-displacement curves for two Ti-6Al-4V specimens and the constant J line is drawn in. Figure 6 is a low magnification optical fractograph of these two specimens whose dimensions were 0.787 by 1.448 cm. For the A471 rotor steel the maximum in load was used for

the maximum point but the case for the minimum value was also examined. Figure 7 shows the load-displacement curves for the three A471 specimens and the constant J lines drawn in for both the maximum and minimum crack initiation points. Figure 8 is the optical fractograph for these specimens whose dimensions were 0.609 by 1.201 cm. Two blunt notch specimens corresponding in notch length to two precracked specimens were made and their load-displacement records were taken. Figure 9 shows the load-displacement curves for the A471 steel specimen with an a/w of 0.34 and the blunted notch specimen with a similar crack length. It is clear from this figure that the crack is propagating long before maximum load is reached i.e. propagation occurs at the deviation point of the two curves. This then provides a minimum point for a minimum J calculation.

Calculating J_{IC}

The enclosed area between two load-displacement curves were measured by a planimeter. This area is the $\int PdX$. The crack areas are measured by planimeter from the optical fractographs.[†] The difference in crack area for any two specimens is now known, and this value is ΔA .

J_{IC} is calculated from the integrated area $\int PdX$, and the measured ΔA between any two specimens. J_{IC} is calculated by substitution into equation (12).

The experimental results for the maximum and minimum value of J_I are listed in Table III. For each J_I calculated there is an equivalent K_I , the stress intensity factor, given by the relation,

[†]A planimeter is used because the crack fronts typically are not straight, this however, is a small correction, thus the crack areas could be easily calculated from an average crack length and specimen width.

$$J_I = \frac{k^2}{E} (1-\nu^2) .$$

Where E is the modulus of elasticity and ν is Poisson's ratio. The equivalent K_I 's are also listed in Table III.

Discussion

The calculated K_{Ieq} from the J test may now be compared to the predetermined experimental K_{IC} 's. In the case of the Ti-6Al-4V alloy the K_{Ieq} (from $J_{I_{max}}$) was $60 \text{ MNm}^{-3/2}$, the K_{IC} was $54 \text{ MNm}^{-3/2}$, the K_{Ieq} is 11 percent higher. For the A471 rotor steel the $K_{Ieq_{max}}$ was $154 \text{ MNm}^{-3/2}$ and the $K_{Ieq_{min}}$ $107 \text{ MNm}^{-3/2}$. Respectively, these values are 17 percent higher and 19 percent lower than the K_{IC} . If we accept the argument that J_{IC} (and K_{IC}) should represent the state at which crack propagation first initiates, a critical point, then the conservative minimum value of J_I should be taken as J_{IC} and its corresponding K_{Ieq} as K_{IC} . Note, that the onset point here is more clearly defined than the the K_{IC} test requirement that utilizes the 95% secant line technique [1][†]. We are therefore inclined to accept the minimum experimental J_I as J_{IC} .

One advantage of the J_{IC} test is the greatly reduced plane-strain specimen thickness requirement. For this J_{IC} test the A471 rotor steel specimen was nearly ten times smaller than required for the K_{IC} test [1]. These thickness requirements are listed in Table IV. The Ti-6Al-4V alloy specimens could have been much smaller, however, this was unnecessary in our tests.

Conclusions

The J_{IC} test described here is simple to perform. It requires only four specimens. These are two fatigue cracked specimens with different finite crack lengths and two blunted notch specimens each with a notch

[†] The P_a point for the K_{IC} test probably lies somewhere between the J_I minimum and the load maximum. It's not surprising then to find the K_{Ieq} from $J_{I_{min}}$ and $J_{I_{max}}$ straddle the K_{IC} as determined by ref. [1], the E-399 method.

depth matching one of the fatigue cracked specimens. The experimentally determined value of J_{IC} and its equivalent K_{IC} are here argued to be correct (in terms of engineering significance) and conservative with respect to the current K_{IC} test.

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TABLE I. Elemental chemical analysis
of the Ti-6Al-4V alloy [14], and the
ASTM-A471 Ni-Cr-Mo-V rotor steel [9].

Ti-6Al-4V
Element (percent)

Al	v	Fe	Sn	Mo	Cu	Mn	Cr	Si	C	N	O	H
6.6	4.0	0.10	<0.05	0.01	0.01	0.01	<0.01	0.02	0.01	0.015	0.143	0.007

A471
Element (percent)

C	Mn	P	S	Si	Ni	Cr	Mo	V	Sn	Sb
0.28	0.29	0.010	0.008	0.02	3.80	1.76	0.40	0.14	0.019	0.001

TABLE II. Room temperature mechanical properties and K_{IC} of the experimental alloys [15, 9].

Alloy	Yield Strength (0.2% offset) MN/m^2	Tensile Strength MN/m^2	K_{IC} $MNm^{-3/2}$
Ti-6Al-4V	930	970	54
ASTM-A471	916	1020	132

TABLE III. Experimental Results for J_I and calculated equivalent K_I .

Alloy	$J_{I\max}$ MN/m	$K_{Ieq(\max)}$ MNm ^{-3/2}	$J_{I\min}$ MN/m	$K_{Ieq(\min)}$ MNm ^{-3/2}
Ti-6Al-4V	.0294	60	-	-
A471	0.108	154	0.052	107

TABLE IV. Thickness requirements for
plane-strain J_{IC} and K_{IC} tests.

Test	Thickness requirement	Ti-6Al-4V	ASTM-A471
K_{IC}	$B \geq 2.5 \left(\frac{K_{IC}}{\sigma_y} \right)^2$ (ref.[1])	8 mm	52 mm
J_{IC}	$B \geq 25 \frac{J_{IC}}{\sigma_y}$ (ref.[16])	1 mm*	3 mm*/1 mm**
	Actual B	8 mm	6 mm

* maximum J_I

** minimum J_I

Figure Captions

- Figure 1: A specimen, not necessarily linear elastic, in series with a linear elastic spring of spring constant, k . The elastic specimen is System I. All non-reversible processes in the crack tip region are included in System II.
- Figure 2: A schematic load-loadpoint deflection curve. The crack areas are held constant on the loading curves that go through the origin and the points ① or ②. Between the points ① and ② J is assumed to be constant. It is shown in the text that $\int P dX = J \Delta A$, the lightly dotted lines shows a constant X condition and a loading curve for crack area $= A + \frac{\Delta A}{2}$. The limiting forms in the incremental limit of the areas enclosed are obvious (see equations (5))
- Figure 3: A schematic load deflection curve for initially constant crack area then for constant J . $\partial P / \partial X|_J = \gamma - \beta^2 / \alpha$. $\alpha > 0$ will follow a curve similar to ①, $\alpha = 0$ follows curve ② and $\alpha < 0$ follows curve ③.
- Figure 4: The experimental apparatus for fracture toughness determination. Load versus displacement (from either the machines or externally mounted LVDT) is recorded for the three-point bend specimens. The specimen in tw's figure has a height, w , of 12 mm.
- Figure 5: Load vs. displacement for two Ti-6Al-4V test specimens of different crack areas. A line is drawn connecting the points at sharp slope change which is here assumed to represent crack motion. The enclosed area ($\int P dx$) is equal to $J \Delta A$.
- Figure 6: Optical fractograph of the specimen of Fig. 5. Specimen dimensions are 0.787 x 1.448 cm. The light areas are the fatigue cracks.
- Figure 7: Load vs. displacement for three A471 steel specimens of different crack length. Lines are drawn connecting points of maximum load for all three specimens and also the minimum points for specimens 1 and 2. The enclosed areas represent $J \Delta A(\max)$ and $J \Delta A(\min)$ respectively.
- Figure 8: Optical fractograph of the specimens of Fig. 7. Specimen dimensions are 0.609 x 1.201 cm. The light areas are the fatigue cracks.
- Figure 9: Load vs. displacement for two A471 steel specimens. One specimen has a blunted notch and the other a fatigue crack of equivalent length. Where the two curve deviate represents the onset of crack motion in the fatigue cracked specimen.

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5

Fig. 6.

Fig. 7.

Fig. 8.

Fig. 9.