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MONTE CARLO STUDY OF THE MECHANISMS OF TRANSPORT
OF FAST NEUTRONS IN VARIOUS MEDIA

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1976

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Chapter 1

Introduction

The method of Monte Carlo has long been used for the study of neutron and gamma ray transport. The basic principle involved is the application of the random sampling technique to simulate collision histories of individual particles, since the interaction mechanics of particles with medium is statistical in nature. After following a large number of particle histories, statistically valid conclusions can be drawn about the average behavior of the particles, such as the transmitted dose, particle current or other weighted averages of flux spectrum in some region of interest. The method is particularly useful when the geometry of the problem is complex and requires detailed descriptions. To speed up computation and to reduce statistical errors, a technique of sampling particle histories with biased distributions of interaction probabilities is often used. In this technique the altered interaction probabilities may include the interaction per unit length of particle travel, the type of interaction, or the angular distribution after an interaction, etc., which are biased so as to favor those particles that contribute to the desired detector response. Such a procedure has often proved efficient and practical.

The method of Monte Carlo has, in addition, a unique advantage over the deterministic methods, such as $S_n^{(1)}$ or moments method⁽²⁾, in solving particle transport problems. It not only provides the statistical expectation on how many neutrons or photons would reach a detector defined by the energy sensitivity and geometrical location, but also can tell us what

happened to the neutrons or photons on their way to that specific detector. The life history of a particle may be conceived of in terms of a path or "trajectory" in a phase space of six dimensions, specifying the spatial position of the particle, its energy and the direction of travel in space. Those particles born at a specific point in phase space and which contribute to a given detector response correspond to a particular family of trajectories. The Monte Carlo tracking process which simulates particle motion in phase space can tell us the statistical properties that define this trajectory family. This information is difficult, if not impossible, to obtain by any deterministic methods.

Some use has been made in the past of the collision histories to discover significant events in the trajectories of interest. For example, L. Bourdet et. al.⁽³⁾ analyzed those neutrons contributing to the activation of secondary sodium in PHEONIX reactor. By making a "collision density" plot in the reactor containment "pot", they showed that most of the neutrons leading to activation of the secondary sodium loop had scattered around the lower part of the lateral neutron shield. Cramer et. al.⁽⁴⁾ used Monte Carlo to simulate the LLL experiment with a pulsed D-T neutron source located in the centre of an iron spherical shell, 22 cm in thickness. By examining the collision histories, they showed that while the exiting neutrons at high energy (> 1 MeV) suffered only a few collisions, one of the collisions was almost always an inelastic scattering to the continuum. This enabled them to conclude that the existing inaccuracy or inadequacy in the inelastic nuclear data of this type could result in serious errors in the integral

quantities calculated.

In this thesis, we have attempted to study the properties of trajectories followed by certain classes of "exceptional" neutrons, e.g. those which penetrate long distances in specific media. We have tried to find out how the macroscopic behavior of these neutrons is related to the interaction mechanics and to the microscopic properties of the medium. To do this we ask what special statistical properties of the individual collisions of the "exceptional" neutrons are distinguishable from those of the general sample of neutrons. An analogy to sociological studies may be drawn here. For instance, the sociologist may wish to know what factors give rise to "intellectual genius". He might do this by statistical examination of the life histories of such individuals in comparison to the general population as a whole. He might thus hope to disentangle which environmental or genetic factors can give rise to "genius". So too, here, we seek those factors in the nature of the microscopic collisions or in the cross section variations with energy which are likely to be correlated with "exceptional" behavior in a neutron trajectory through space and energy.

In the neutron transport applications considered in this investigation, the "general population" of neutron histories consisted of all neutrons that had slowed down to past a given energy somewhere in an infinite medium surrounding the source. The "exceptional neutrons" were those which were detected at or near the same given energy, but in a specific geometrical situation. Thus, the sample of exceptional neutrons may mean those that exited from a sphere of small radius in the specified energy range. Or it

may mean those neutrons that penetrated to large distances before being detected in the given energy bin. Such unusual neutron histories are often those of particular importance in practical applications. In analyzing their life histories we hope to find the factors that determine how many of the general population become exceptional neutrons.

For the purpose of analyzing what factors give rise to "exceptional" neutrons, the Monte Carlo method has both advantages and disadvantages. The great advantage, of course, is that only in Monte Carlo are the neutron histories directly accessible for such an analysis. This alone makes Monte Carlo the method of first choice. A second advantage is that the same set of histories can be used for a variety of geometrical situations. Thus, the histories generated from a point source in an infinite medium can be used for a bare sphere simply by dropping from the analysis that portion of the neutron history subsequent to the first outward passage through the given spherical surface. In this manner one set of histories has been used for a range of detector positions, both with and without reflectors.

The disadvantages of a stochastic method such as Monte Carlo are equally obvious. There are practical limitations on the number of histories that can be run or analyzed. If the "exceptional" neutrons of interest are indeed so exceptional that only a very small percentage of the source neutrons wind up in the category, then there is the danger the number of histories may be too small to analyze. In the situations investigated here, for example, it was found that neutrons from a fission source penetrating more than 20—30 cm of iron and remaining above 1 MeV were just too few

to be examined. We can say little therefore about the factors determining the high energy spectrum of neutrons penetrating deeply into iron. Biasing or importance sampling techniques could not be used because the final destinations of interest of the neutrons covered a wide volume of phase space. Biasing good for one final detector position would not be helpful for another.

The problems arising from the statistical nature of the Monte Carlo answers go even deeper. It appears that the life histories of "exceptional" neutrons often deviate from those of the normal population by slight and subtle differences. Hence even when it appears at first sight that there are enough histories to examine, one often is left with the uncertainty whether the small differences uncovered are really significant. The analysis of Monte Carlo histories is most meaningful when it turns up marked qualitative differences between histories of one class of neutrons and another. Perhaps one of the most important conclusions of this research is that in many practical situations such striking differences are rare.

The body of the thesis consists of the application of the Monte Carlo analysis of neutron histories to three examples, all of which involve an isotropic point source of neutrons in a homogeneous isotropic medium consisting of a single nuclide:

(1) Combined neutron slowing down and transport in a "featureless situation". The only interaction allowed is elastic scattering which is assumed to be isotropic in the center-of-mass system, and all microscopic

data are taken as independent of energy. In such a medium every lethargy interval looks alike to the neutron, regardless of the energy. Thus the neutron slows down and moves in a situation where the properties of one collision are the same as another—no particular energy or energy range is singled out. The statistical characteristics of the general population of neutrons are well known from the theory of the random walk. What is sought here are the features of the neutron history that distinguish the "exceptional" neutrons, those which either stay closer to the source, or penetrate farther from the source, than the average neutron.

(2) Neutron penetration in iron over a restricted energy region.

Because of its importance as a structural material—and as a neutron shield—iron is frequently used in reactor design, often in layers of substantial thickness. The factors that give rise to exceptional neutron behavior in iron, in both reflected and bare spherical geometries, have been studied first for a source monoenergetic at an energy of 150 keV, and the neutrons were followed down to 85 keV. The circumstances were chosen here so as to isolate and emphasize the role of deep cross section minima on neutron transport. Two deep minima are present in this energy range—at 138 keV and 129 keV, and the scattering is only elastic (neither absorption nor inelastic scattering need be taken into account).

(3) Penetration of fission neutrons in iron. In this more realistic situation, neutrons from an isotropic point fission source are followed down to about 10 keV in bare and reflected spheres. Here the dimensions were chosen to cover the range from the small bare sphere (~ 20 cm radius)

previously studied by Ching⁽⁵⁾ to the deep penetration through reflected layers of 1 m or more thickness, as investigated by Preeg⁽⁶⁾. Ching had found that the cross section minima did not significantly affect the exit spectrum from his small bare sphere. If anything, they tend to "drain off" the neutrons to the outer vacuum and thus decrease the number slowing down to energies below them. On the other hand, Preeg had found that transport in the minima was the dominant factor in determining the spectrum of neutrons penetrating to 1 meter or more. It is hoped to show the transition between these two behaviors and in addition to investigate the role played by inelastic scattering over the wide range of geometries.

While the general philosophy of this research has been recapitulated in the above description, the detailed analysis and discussion are presented in the four chapters that follow. Chapter 2 sketches how the cross sections are treated and how the histories are created. A brief description of the code SAM-CE used to create history file is also given.

Chapter 3 presents an analysis of the transport behavior of neutrons in a medium scattering only elastically and with cross sections constant in energy. The mass number of the medium used is 23 (Na). We examine the statistical properties of the parameters affecting the trajectories for neutrons which remain in the vicinity of the source or travel far from it. The life histories for such exceptional neutrons show only subtle differences from the histories experienced by the average neutrons.

The next two chapters are concerned with the studies of neutron penetration in iron. Chapter 4 considers the transport of neutrons as they

slow down from 150 keV to 85 keV. Not surprisingly, the trajectories of deeply penetrating neutrons in this situation exhibit most of their spatial transport at energies in the vicinity of cross section minima. Indeed, in these regions of low cross sections, the distance travelled between collisions is significantly longer than the mean-free-path length.

The more realistic problem of a fission source in an iron medium is studied in Chapter 5. Here the most striking conclusion is that detector responses are determined by different cross section phenomena as the geometry is changed from small bare spheres to very large spheres. This result has important implications for the validity of integral experiments as tests of evaluated data sets.

A general summary of the conclusions reached in the course of this investigation, along with some reflections on the usefulness and limitations of the technique, are presented in Chapter 6.

Chapter 2

The Methodology of History Generation

2.1. Introduction

In order to be able to investigate the neutron behavior in a given material and source specification following the proposed method in Chapter 1, one needs a set of sample histories. The samples are created by the Monte Carlo simulation in which all the events occurring in a neutron life span are recorded. Subsequent analysis is necessary to extract particular statistical information from these histories. In this chapter, we describe the procedure of obtaining the desired history file.

The main instrument used for the creation of such a file is the code SAM-CE⁽⁷⁾. The highlights of this code and its organization will be presented in Section 2.2. Prior to the creation of the Monte Carlo histories, several probability tables must be prepared. This is handled by the first part of the SAM-CE package, known as SAM-X. The way that these tables are prepared and the criteria we have used in preparing these tables are given in Section 2.3. SAM-F, the Monte Carlo tracking program is then introduced in Section 2.4. The method of specifying the geometry and the source spectrum is discussed. The re-organization of SAM-F history tape to achieve efficient handling is also described.

There are several approximations and errors introduced in the course of creating the neutron histories. For example, the scattering angular distributions are represented by histograms instead of continuous distributions. The word-packing process to save storage space also

causes loss of accuracy through loss of some significant figures. We have not attempted to study the effect of these approximations and errors. It will become clear later, however, that the physical conclusions drawn in the analysis are such that they will not be affected by these small and random inaccuracies.

2.2. The Monte Carlo Code SAM-CE, A General Description⁽⁸⁾

The SAM-CE package is a FORTRAN Monte Carlo code designed to solve the time dependent neutron and gamma ray transport equations in complex three-dimensional geometries. Some characteristics of this code are stated in the following:

(a) problem solved:

- (1) forward neutron calculations
- (2) forward and adjoint primary gamma ray calculations
- (3) forward and adjoint gamma ray calculations of the coupled neutron-secondary gamma ray problem

(b) scoring:

- (1) time-dependent fluxes and flux functionals are calculated as functions of energy, time and position
- (2) finite volume detectors and point detectors or both are allowed

(c) geometry:

- (1) the combinatorial geometry technique affords the user the capability of describing complex geometries accurately

(d) treatment of cross section:

- (1) all neutron and gamma ray cross section data and gamma ray production data are derived from ENDF libraries or libraries with ENDF format
- (2) cross sections are tabulated in point energy meshes
- (3) each distinct nuclide has its own unique and appropriate energy mesh
- (4) extremely precise and detailed descriptions of cross section behavior are permitted
- (e) weighting:
 - (1) importance sampling is available on energy, region and angle
- (f) thermal neutron option:
 - (1) thermal neutron cross section treatment and thermal neutron diffusion are available for low energy neutrons.

SAM-CE is composed of three separate programs:

- (a) SAM-X, a pre-processor for cross section data;
- (b) SAM-F, a program for forward Monte Carlo calculations;
- (c) SAM-A, a program for adjoint Monte Carlo calculations.

The organization of the SAM-CE system is outlined in Figure 2.1. Only a small portion of the SAM-CE capabilities was required in this research. It should be noted that no importance sampling was used. The branch which we have used in creating histories is separately marked by a boxed outline.

2.3. Preparation of Cross Section Data

Figure 2.1 SAM-CE System

Two sets of cross section data have been used in this dissertation. One is a fictitious constant cross section material with no absorption. The other is natural iron data in DNA library (details of the DNA iron data set are given in Chapter 5). Both sets are in the ENDF format so that they can be adapted directly by the SAM-X program. SAM-X organizes the cross section data and creates the organized data tape containing various scattering probability tables. This tape is subsequently used by the SAM-F to generate histories. The DNA natural iron set contains a huge number of data points, more than SAM-X was dimensioned to handle. To provide for a detailed description in the energy range of interest, SAM-X was therefore modified to include an increased number of energy points. During the Monte Carlo game (program SAM-F), the overall energy range of the problem is subdivided into small energy ranges, or "bands", each containing the reduced amount of cross section data appropriate to that range. These bands are shuttled into and out of computer core memory one at a time. Therefore in spite of the large amount of cross section data involved, the storage requirement for SAM-F can be kept within a reasonable size.

2.3.1. Treatment of Smoothed Cross Sections

Cross sections are organized into five tables: (1) total, (2) total scattering, (3) total inelastic scattering, (4) cumulative discrete level excitation and (5) multiplicity. The multiplicity $m(E)$ at energy E is defined as

$$m(E) = \frac{\sum_i N_i \sigma_i(E)}{\sum_i \sigma_i(E)},$$

where N_i is the number of neutrons emitted in reaction type i and the sum is taken over nonelastic, neutron-producing reactions. The elastic scattering cross section can be obtained by subtracting total nonelastic scattering cross section from total scattering cross section. Likewise, the absorption cross section can be obtained from the difference of total cross section and total scattering cross section.

The reactions (n, γ) , (n, p) , (n, d) , (n, t) , $(n, {}^3\text{He})$, (n, α) are treated as components of absorption cross section whereas (n, n') , $(n, 2n)$, $(n, n'\alpha)$ and $(n, n'p)$ etc. are treated as components of the nonelastic scattering cross section. The cross sections are tabulated at the energy points for which the elastic cross section is given in the DNA data set. Since the SAM-F code provided only for linear interpolation in energy, more points were added to the cross section tables so that the cross sections calculated at intermediate points by use of both linear interpolation and the interpolation scheme designated in the original data file agree to within some ϵ (typically 5%).

2.3.2. Treatment of Secondary Angular and Energy Distribution for Elastic and Discrete Level Inelastic Scatterings

The angular distributions, both for elastic scattering and discrete level inelastic scattering, are represented on the organized data tape by tables of percentiles (chi-tables) in the center-of-mass coordinates. A

new variable χ is defined to construct these percentile tables,

$$\chi = \frac{1-\mu}{2},$$

where μ is the cosine of scattering angle in the center-of-mass system.

Thus χ goes from 0 to 1 since μ goes from -1 to 1. For a given number of entries, N , the distribution of the scattering angle, $q(\chi)$ is calculated in $N+1$ equally probable bins. In each of the bins, the distribution is assumed to be uniform. Therefore for χ between χ_n and χ_{n+1}

$$q(\chi) = \frac{1}{N+1} \cdot \frac{1}{\chi_{n+1} - \chi_n}, \quad n=0, \dots, N \text{ and } \chi_0=0, \chi_{N+1}=1.$$

The χ_n values are determined from the solution of

$$\int_0^{\chi_n} q(\chi) d\chi = - \int_{-1}^{\mu_n} p(\mu) d\mu,$$

or equivalently,

$$\frac{n}{N+1} = \int_{1-2\chi_n}^1 p(\mu) d\mu,$$

where $p(\mu)$ is the angular distribution in the center-of-mass system given in the original data file. Note that for isotropic scattering $p(\mu) = 1/2$ and $q(\chi) = 1$.

The number of χ_n entries, N , is determined by SAM-X dynamically according to the following four criteria:

- (a) N shall be a non-decreasing function of energy,
- (b) N must not exceed the maximum number specified in the input (typically 30),

(c) the ℓ -th Legendre moment of the angular distribution M_ℓ ,

$$M_\ell = \sum_{n=0}^N \frac{1}{2(N+1)(\chi_{n+1} - \chi_n)} \cdot \int_{1-2\chi_{n+1}}^{1-2\chi_n} P_\ell(\mu) d\mu ,$$

where $P_\ell(\mu)$ is the Legendre polynomial of order ℓ , does not deviate from the ℓ -th Legendre moment of the given distribution f_ℓ

$$f_\ell = \int_{-1}^1 p(\mu) P_\ell(\mu) d\mu$$

by more than ϵ_ℓ for $\ell=1,2,\dots,L$. Our calculations have been performed with $L=3$, $\epsilon_\ell = 0.02$.

(d) the fractional deviation of the angular distribution from a given distribution does not exceed α , that is

$$1 - \frac{q(\chi)}{2p_m} \leq \alpha ,$$

where p_m is the maximum entry for the given distribution $p(\mu)$ in the interval $(1-2\chi_{n+1}, 1-2\chi_n)$ and $q(\chi)$ is the χ -distribution in (χ_n, χ_{n+1}) . $\alpha = 0.5$ has been used in our calculations.

The chi-table is tabulated at each output energy point. When a collision occurs, a random number is generated to determine the angular bin after scattering. The angle after scattering is then selected uniformly in that bin. This procedure is carried out in SAM-F program.

Having determined the angle of scattering, the secondary energy after the scattering is calculated by the relation

$$E' = \frac{\bar{A}^2 + 2\bar{A}\omega_c + 1}{(A+1)^2} \cdot E ,$$

where ω_c is the cosine of the scattering angle in the center-of-mass

system, E is the energy before scattering and E' is the energy after scattering. The effective mass $\bar{A}$ (in unit of neutron mass) is equal to A , the mass number of the interaction medium, for elastic scattering and is equal to $A \sqrt{1 - \frac{A+1}{A} \frac{Q}{E}}$ for inelastic scattering with excitation energy Q . The excitation energy Q for discrete level inelastic scatterings is included in the organized data tape.

2.3.3. Treatment of Secondary Energy Distribution for Nonelastic Scattering to the Continuum

The secondary energy distributions of continuum components of nonelastic scatterings are combined into one energy distribution $q_c(E \rightarrow E')$ by SAM-X,

$$q_c(E \rightarrow E') = \frac{\sum_k \sigma_k^{in}(E) n_k \sum_{\ell} p_{\ell}^k(E) f_{\ell}^k(E \rightarrow E')}{\sum_k \sigma_k^{in}(E) n_k \sum_{\ell} p_{\ell}^k(E)}$$

where

$\sigma_k^{in}(E)$ is the nonelastic scattering cross section of reaction type k at energy E ,

n_k is the number of neutrons emitted in reaction type k ,

$p^k(E \rightarrow E') = \sum_{\ell} p_{\ell}^k(E) f_{\ell}^k(E \rightarrow E')$ is the probability of scattering from E to E' for reaction type k in the laboratory system (for details, see the reference 9).

For a given number N , the table of equally probable energies ENN(I) after scattering is established in $N-1$ bins. In each bin, the distribution is assumed to be uniform. At each output energy E , the entries ENN(I) are defined to be the solution of the equation

$$\int_0^{ENN(I)} q_c(E \rightarrow E') dE' = \frac{I-1}{N-1} \cdot \int_0^E q_c(E \rightarrow E') dE' .$$

The energy tables are used by SAM-F to determine the energy after a nonelastic collision event which excites the continuum. In the computation, an $\bar{E}$ is firstly selected from the secondary energy distribution. The effective mass $\bar{A}$ is then calculated according to the relation

$$\bar{A} = A \sqrt{\bar{E}/E} .$$

After sampling a cosine of scattering angle ω_c in the center of mass system (usually isotropic), the energy after scattering E' is computed from the equation

$$E' = \frac{\bar{A}^2 + 2\bar{A}\omega_c + 1}{(A+1)^2} E .$$

In our calculation, $N=11$ has been used.

2.4. Preparation of Neutron Histories

2.4.1. Creation of Neutron Histories

SAM-F, the second part of SAM-CE package, has been used to create the neutron histories. As the theory of Monte Carlo method in neutron transport calculations has been described in the literature in detail⁽¹⁰⁾, we shall state only briefly the program flow of SAM-F.

SAM-F begins with the processing of the input specifications such as geometry, source spectrum, material density, importance sampling weight, cross section banding and scoring energy bins, batch size and sample size, etc.. The energy meshes for cross section banding and

scoring bins are then combined defining a set of "superbins" (For purposes of generating neutron histories no "scoring" of the neutrons is needed.

Therefore only one output energy bin covering the entire energy range of interest was used. The structure of the superbins was the same as that of the cross section bands.). SAM-F then selects individual source neutrons, tracks the neutrons through a series of interactions within the geometry. The collision history for each neutron is followed within the current superbin. As the energy of the neutron is degraded below the lower energy boundary of the current superbin, the neutron is stored as "latent". All the histories for neutrons in a batch are followed in the current superbin before the latent neutrons are picked and followed in the next superbin. The tracking of neutron history then continues until it is terminated for any of five possible reasons,

- (a) the energy of the neutron after the collision degrades below the cutoff energy,
- (b) the elapsed time of the neutron exceeds the cutoff time,
- (c) the neutron escapes from the geometry,
- (d) the neutron is absorbed, or
- (e) the neutron is killed by importance sampling.

(in our calculations, only criteria (a) and (d) were ever invoked). When the particle traverses the designated scoring regions, the estimation of the flux is made. The whole process is repeated for all source particles one desires to follow. An edit of final result is then printed out.

During the tracking process, one can obviously record the information

concerning every interaction in the neutron life span with little extra work. Though it is not the major purpose of SAM-F to do this job, it does have an option which leads to the formation of an interaction tape. Only slight modification was needed to adapt this option so as to generate a history file.

To generate a set of histories which meets our requirement, we did not use any weighting. Further, a sufficiently large system was used so that the probability of a neutron history being terminated due to escape was negligibly small. For our purpose, a sphere with radius 50 meters has been used as the tracking region. Moreover, we also wished to have a point source. Since the program requires a volume source distribution, and does not have a point source option, a source region with radius 10^{-10} cm was adopted. As for the source spectrum specification, the Cranberg fission spectrum has been built into the SAM-CE program so that for a fission source no additional input source distribution was needed. For monoenergetic source, a step function with width 1 meV (10^{-3} eV) has been used.

The typical sample size created for each case study was 10,000 histories. The execution time required to create such a sample size depends on the complexity of collision mechanics and the collision numbers involved for each history. Including the time for creating the packed history tape (see the next sub-section), it requires about 0.12 second/history for the fictitious constant cross section problem, and about 0.28 second/history for the complete iron problem. Since the time cost for creating the histories

as well as the time required for subsequent analysis are approximately proportional to the sample size N , whereas the statistical error is proportional to $1/\sqrt{N}$, we would gain relatively small advantage by using too large of a sample. The number 10,000 chosen is in practice about the maximum feasible sample size that can be run within a reasonable amount of time.

2.4.2. History Handling, Storage and Retrieving

The history tape normally created by SAM-F is in the format of 14 words per collision, 490 words per record (CDC-6600 word). The 14 words at each collision give the three spatial coordinates, three incident direction cosines, one incident energy, one region number, one time variable, one detector number, one importance sampling weight, one history number, one carry-along weight and one interaction type. A history tape written in the way like this is neither economical nor efficient. In some cases, it requires perhaps more than ten 2400-foot magnetic tapes to meet the basic sample size (10,000 histories). To achieve efficient handling of the histories, organized files have been created in which unnecessary and redundant information such as region number, weight etc. on the original history tape are discarded. Only three spatial coordinates (in cm), incident energy (in eV) and interaction type are retained. The interaction type is indicated in the following fashion:

- (a) $M = 0$ history terminates, absorption
- (b) $M = 1$ elastic collision

(c) $M = 2$ inelastic collision

(d) $M = 3$ history terminates, degradation below cutoff energy.

These five variables are further packed into two words. The first word contains the X, Y coordinates. Each coordinate was truncated to 30-bit floating number with 18 bits of mantissa (originally there were 48 bits of mantissa for a 60-bit floating point number). This results in about 6 significant digits. The second word contains the Z coordinate, the energy and the interaction mode with 29, 29, 2 bits respectively. The interaction mode was stored as an integer whereas the other two were stored as floating numbers with 17 bits of mantissa. For each collision, the organized data file thus only requires one-seventh of the original data length. The organized data files are then stored on the magnetic tape at high density (800 bpi) and long record length (5120 words/record).

In the subsequent analysis of the organized data tape, the words are unpacked back into 60-bit words in which the 30 least significant bits for the X, Y coordinates and the 31 least significant bits for the Z coordinate and the energy are filled with zeros. The high order 58 bits (most significant bits) for the interaction mode are also filled with zeros. The proper order of the neutron collision sequence and the collision information can thus be restored.

$\left. \begin{array}{l} r \\ \cos\theta \\ E \\ M \end{array} \right\} \text{ pack in one word?}$

Chapter 3

Study of Neutron Penetration in A Constant Cross Section Medium

3.1. Introduction

In this chapter, we examine the behavior of neutron transport and slowing down in a medium having following properties;

- (a) constant cross section; $\Sigma_{\text{total}} = \Sigma_{\text{elastic}} = 1 \text{ cm}^{-1}$
- (b) zero absorption; $\Sigma_{\text{absorption}} = 0 \text{ cm}^{-1}$
- (c) scattering only elastically, and isotropically in the center of mass system,
- (d) single nucleus, atomic mass 22.7928 in unit of neutron mass (i.e. the element Na).

This situation can be described as featureless, as the properties of the medium are the same regardless of the location and energy at which an interaction of neutron with the medium occurs. Yet for given source energy some of the neutrons will penetrate deeply whereas others stay close to the source. What we seek in this study are the features in neutron life history that result in the unusual behavior of such exceptional neutrons.

Neutrons born at 10 MeV were followed down to 25 keV, for a total of 10,000 histories. Particular attention was paid to deeply penetrating neutrons, e.g. those which contributed to the "detector" 20 mfp from the source point. The "detector", if not otherwise specified, was taken to count all neutrons crossing a spherical surface in the energy interval from 45.5 keV to 25 keV (corresponding to the lethargy interval 5.4-6.0).

The width of this detector response is one tenth of the maximum lethargy

gain in slowing down from the source energy to 25 keV.

In the following discussion, the neutron current spectra and fluxes for some selected cases are compared in Section 3.2. The behavior of neutron trajectory conditioned by the final energy and location in a lethargy-radius phase space during the slowing down process are examined in Section 3.3. Since elastic scattering is the only collision type considered, three quantities must be determined at each collision—the polar and azimuthal angles of scattering, and the path length of the flight to the next collision. For a normal history, these three quantities are independent and unbiased and follow given probability distributions. In contrast, the probability distributions for the neutrons actually detected at a given energy interval and geometrical location are conditioned according to the source-detector distance and lethargy interval. We shall study in detail in Section 3.4, how the distributions of these three quantities are changed so as to select only those trajectories in phase space which satisfy the constraints.

3.2. Neutron Current Spectrum and Flux

The exit current J^+ can be estimated from the Monte Carlo histories.

The average exit current in lethargy bin j for $u \in (u_1, u_2)$ is defined as

$$J_j^+(r) = \frac{1}{u_2 - u_1} \int_{\Omega^+} \int_{u_1}^{u_2} N(r, \Omega, u) du d\Omega,$$

where $N(r, \Omega, u)$ is the angular flux of the neutron, and the integration on the solid angle Ω is carried over 2π of the outgoing direction. A com-

parison of exit current spectrum for 4 cm, 12 cm and 20 cm radius spheres in an infinite constant cross section medium is illustrated in Figure 3.1. Comparisons of the exit spectrum for a sphere in an infinite medium with the exit spectrum for a bare sphere are illustrated in Figure 3.2, 3.3, 3.4 for 4 cm, 12 cm and 20 cm radius sphere respectively. The spectra were computed in ten equal lethargy bins.

Neutrons contributing to the exit spectrum for any configuration were sampled from the same set of histories in one computer run. This advantage of correlated sampling permits us immediately to see the changes in spectrum due to either the change in boundary condition or the change in distance from the source. Other deterministic methods require separate calculations. For instance, it is clear from Figure 3.1 that the spectrum at 20 cm is soft; most of the histories which arrive at this distance are in low energy bins. The spectrum at 4 cm is still hard; there are more neutrons in high energy bins than in low energy bins. On the other hand, one can see from Figure 3.2, 3.3 and 3.4 that the difference in low energy current between a bare system and an infinite system is very substantial for small spheres. Obviously we are seeing the high leakage effect of small bare geometries. The exit current for the lowest lethargy bin for the 4 cm bare sphere is smaller by as much as a factor of 100 than that for the 4 cm sphere in an infinite medium. The difference between 20 cm bare and infinitely reflected spheres for the same lethargy bin however is only about a factor of 1.5.

The neutron flux can also be computed from the Monte Carlo

Figure 3.1 Comparison of Exit Current Spectra for 4 cm, 12 cm and 20 cm Radius Reflected Spheres in a Constant Cross Section Medium

Figure 3.2 Comparison of Exit Current Spectra for 4 cm Radius
Bare and Reflected Spheres

Figure 3.4 Comparison of Exit Current Spectra for 20 cm Radius
Bare and Reflected Spheres

histories. Several estimators for flux evaluation are available⁽¹¹⁾, but only the simplest one is used. The average flux density per unit source in lethargy interval (u_1, u_2) is obtained by the sum

$$\overline{\phi} = \frac{1}{N(u_2 - u_1)} \cdot \frac{1}{4\pi R^2} \sum_i \frac{1}{\cos \theta_i}$$

where N is the total number of source particles and the summation is taken over all the particles crossing the detector surface in the lethargy interval (u_1, u_2) . The angle θ_i is the angle between the direction of the i -th particle and the normal to the detector surface (see the following diagram)

The estimated average flux in the lowest lethargy bin (lethargy 5.4—6.0) is given in column 1 of Table 3.1. The results calculated on the basis of age theory is shown in column 2 of the same table. The average flux predicted by age theory in lethargy interval (u_1, u_2) is⁽¹²⁾

$$\overline{\phi}(r) = \frac{1}{u_2 - u_1} \int_{u_1}^{u_2} \phi(r, u) du$$

where

$$\phi(r, u) = \frac{e^{-r^2/4\tau}}{\xi (4\pi\tau)^{3/2}}$$

$$\tau = \frac{u}{3\xi(1-\bar{\mu})}$$

ξ = average lethargy gain per collision

$\bar{\mu}$ = average cosine of the scattering angle

Evaluating the integral gives

$$\bar{\Phi}(r) = \frac{1}{u_2 - u_1} \frac{3(1-\bar{\mu})}{2\pi r} \left\{ \bar{\Phi}(r/\sqrt{2u_1}) - \bar{\Phi}(r/\sqrt{2u_2}) \right\},$$

where

$$\bar{\Phi}(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-u^2/2} du$$

is the cumulative distribution function for the standard normal distribution.

Table 3.1. Comparison of Neutron Flux Density
Due to A Point, Isotropic Source in
An Infinite Medium. Detector Lethargy
Interval (5.4--6.0)

Detector Distance (cm)	$\bar{\Phi}^{\dagger\dagger}$ Monte Carlo	$\bar{\Phi}^{\dagger\dagger}$ Age Theory	$\bar{\Phi}^{\dagger\dagger}(\text{M.C.}) / \bar{\Phi}(\text{Age})$	$\bar{\Phi}^{**}(\text{P-3 Correction}) / \bar{\Phi}(\text{Age})$
	*	*		
4	2.436-03±0.15 [†]	2.012-03	1.17±0.18	1.044
8	1.229-03±0.03	1.195-03	1.03±0.03	1.015
10	8.250-04±0.04	8.013-04	1.02±0.04	0.975
12	4.611-04±0.05	5.031-04	0.92±0.05	0.972
16	1.600-04±0.11	1.498-04	1.07±0.12	0.974
20	3.727-05±0.22	3.189-05	1.17±0.26	1.083

† Fractional Standard Deviation

* Read 2.436×10^{-3}

†† Average flux in lethargy interval (5.4 — 6.0)

** Flux at $u = 5.7$

The flux ratios of the Monte Carlo calculation to the age-theory results are shown in column 3 of Table 3.1. Within the distance range of our calculation, the age-theory result in general does not differ very much from that of the Monte Carlo computation. This suggests that even the deepest penetration considered here does not select a group of particularly "exceptional" neutrons—they behave very much like the average "random-walk" histories. It may be noted, moreover, that age theory slightly underestimates the true flux at small distances, then slightly overestimates the true flux at intermediate distances, then again underestimates the true flux at large distances. That this behavior is not an artifact of the Monte Carlo sample is shown by computing the flux using a better approximation than the age theory. The last column of Table 3.1 gives the ratios of fluxes computed by means of Placzek's method⁽¹³⁾ to those computed by the age theory. The fluxes computed on the basis of Placzek's method are taken to the second correction term which is equivalent to taking into account the first four Legendre moment (0-th, 1-st, 2-nd, 3-rd) of the spherical harmonic expansion of the angular fluxes (Marshak's P-3 correction to age theory⁽¹⁴⁾). Because the flux varies slowly in the lethargy interval considered, the calculation is actually performed at the mid-point of the lethargy interval. If we let $\kappa = r/\sqrt{2\tau}$ with

$$\tau = \frac{u(u_1+u_2)/2}{3\xi(1-\bar{u})} ,$$

the ratio can be written as⁽¹⁵⁾

$$R(\kappa) = 1 + f(\kappa) - g(\kappa) ,$$

where

$$f(\kappa) = \frac{1}{4u} (s_{12}\kappa^4 + s_{11}\kappa^2 + s_{10}) + \frac{1}{16u^2} (s_{24}\kappa^8 + s_{23}\kappa^6 + s_{22}\kappa^4 + s_{21}\kappa^2 + s_{20})$$

$$g(\kappa) = \frac{1}{2u} (2s_{12}\kappa^2 + s_{11}) + \frac{1}{8u^2} (4s_{24}\kappa^6 + 3s_{23}\kappa^4 + 2s_{22}\kappa^2 + s_{21})$$

$$s_{12} = 0.1521, \quad s_{11} = -0.6293, \quad s_{10} = 0.1730,$$

$$s_{24} = 0.01156, \quad s_{23} = -0.22063, \quad s_{22} = 1.00357,$$

$$s_{21} = -0.94837, \quad s_{20} = 0.03328.$$

It is seen from Table 3.1 that the P-3 correction to age theory gives a flux with a spatial behavior very similar to that found by the Monte Carlo.

At very large distances from a point source in an infinite medium, the flux deviates from the Gaussian distribution and approaches an exponential distribution. The flux computed on the basis of pure age theory should therefore grossly underestimate the true flux at very large distances. Some signs of this behavior are apparent in Table 3.1, but at the deepest penetration age theory underestimates the flux by only the order of 10–20%. Here is more clear evidence that 20 cm at this energy is not a deep enough penetration to require very unusual properties for the neutron to get out that far. However, of all the source neutrons, only about 3% of them contribute to the response of the detector at 20 cm. Any substantially deeper penetrations can not be examined since the Monte Carlo sample as constituted would run out of neutrons.

3.3. Neutron Motion in Phase Space

The motion of general neutron population in the medium studied is considered to satisfy the following conditions:

- (a) each collision is independent,

- (b) there is no correlation among polar and azimuthal angle distributions and path length distribution,
- (c) the azimuthal angle of scattering is isotropically distributed,
- (d) path lengths are independent of lethargy and are distributed exponentially.
- The mean square displacement from the source point under these conditions can be derived explicitly for the general neutron population by well known arguments (see Appendix A). For sufficiently large number of collisions, it can be expressed as

$$\overline{r^2} = \frac{2\bar{n}\lambda^2}{1-\bar{\mu}} = \frac{2u\lambda^2}{\xi(1-\bar{\mu})} \quad , \quad (3.1)$$

where $\bar{n}$ = mean number of collisions in gaining lethargy u

λ = mean path length

$\bar{\mu}$ = mean scattering angle in the LAB system

ξ = average lethargy gain per collision

If we let τ be $\frac{u\lambda^2}{3\xi(1-\bar{\mu})}$, then $\overline{r^2}$ is simply 6τ , the age-theory prediction⁽¹⁶⁾.

The root-mean-square radial displacement for the general neutron population as a function of lethargy was estimated from the Monte Carlo histories. The result is shown on Figure 3.5. The calculation based on the expression 3.1 is also plotted on the same graph. It is clear from the plot that after about 7 cm (corresponding to about 30 collisions), the age-theory prediction is in excellent agreement with the "measured" result for the complete sample. Only for the first few collisions is there a noticeable deviation.

Figure 3.5 Root-Mean-Square Distance of Slowing Down for

For neutrons contributing to the detector at a given distance, the mean-square radial displacement from the source in their life history can also be estimated from the Monte Carlo histories. Two such conditional cases have been examined. One is for those contributing to the detector at 4 cm, the other is for those contributing to the detector at 20 cm, both in an infinite medium. The results are shown in Figure 3.5. Comparison of the slowing down curve for the conditional neutrons to that for the general neutron population reveals changes in characteristics of the neutron trajectories in phase space. For example, neutrons contributing to the detector at 4 cm, on the average, move across the 4 cm-radius detector surface in the early stage of their life. They eventually come back to the detector surface by moving inward. On the other hand, neutrons contributing to the detector at 20 cm have consistently larger displacement at each lethargy than do the normal histories. The curve, moreover, can not be approximated by any age-like theory with an "age" directly proportional to the Fermi age τ ($\tau = \frac{u}{3\xi(1-\bar{\mu})}$), as the curve is not well represented by any formula of the form $r^2 = au$.

Clearly for the 4 cm and 20 cm cases, the neutrons contributing to the detector do not satisfy all of the conditions (a) — (d). Therefore, in the following sections, these conditions are tested and the bias introduced into the distributions of collision parameters is examined for these two cases.

3.4. Study of Transport Parameters -- Polar Angle of Scattering, Path Length and Azimuthal Angle of Scattering

3.4.1. Angle of Scattering and Number of Collisions

For a given lethargy interval of slowing down, the distribution of scattering angle is directly associated with the distribution of collision numbers. In general, the average number of collisions $\bar{n}$ required for a neutron to gain lethargy u is

$$\bar{n} = \int_0^u \frac{du'}{\xi(u')},$$

where $\xi(u')$ is the mean lethargy increment of a neutron upon scattering at u' . In general, if the mass number A of the interaction medium is large and if the scattering is elastic, $\xi(u')$ can be written approximately as⁽¹⁷⁾

$$\xi(u') \approx \xi_0 \{1 - \bar{\mu}_0(u')\},$$

where

$\xi_0 = 0.0845$, is the mean lethargy increment if the

scattering is elastic and isotropic in the CM system,

$\bar{\mu}_0$ = the average cosine of scattering in the CM system.

For the general neutron population in this study, $\xi(u')$ is independent of u' and $\bar{\mu}_0$ is 0, therefore the average number of collisions is simply u/ξ_0 .

For conditional cases, that is when only those neutrons contributing to the detector at a specific position are of concern, it is not clear a priori whether $\xi(u')$ is independent of u' , nor is it clear a priori if the scatterings are isotropic. To study these phenomena from the Monte Carlo histories, it is convenient to break $\bar{n}$ into two quantities:

$$\bar{n} = \bar{n}_1(0) + \bar{n}_2, \quad (3.2)$$

where $\bar{n}_1(0)$ is the average number of collisions required for a neutron slowing down from the source energy to the upper limit of detector energy response (lethargy u_d), and $\bar{n}_2$ is the average number of collisions suffered

by the neutron between the time it reaches the upper energy limit of the detector response interval and the time of detection.

Instead of computing $\overline{n}_1(0)$, it is more instructive to determine the function $\overline{n}_1(u)$, defined as

$$\overline{n}_1(u) = \int_u^{u_d} \frac{du'}{\xi(u')} \quad (3.2.1)$$

which gives the average number of collisions generated in the interval $u_d - u$ for $0 \leq u \leq u_d$. Clearly for $u=0$, $\overline{n}_1(u)$ becomes $\overline{n}_1(0)$. In Figure 3.6, the behavior of the mean collision number, $\overline{n}_1(u)$, is plotted as a function of lethargy difference $u_d - u$ for the general population and for the neutrons contributing to the detector at 4 cm and 20 cm in an infinite medium. The reciprocal of the slope indicates the average lethargy gain per collision, it is therefore a measure of the average angle of scattering. For general neutron population, the slope is closely given by the predicted $1/\xi_0$.

For 4 cm and 20 cm cases, it is seen the relation between the average number of collisions and the lethargy is linear, that is the slope is a constant. This shows that $\xi(u)$, the average lethargy gain per collision for conditional neutrons, is independent of the lethargy in the medium studied. This result allows us to write

$$\xi(u) = \xi \approx \xi_0 \cdot (1 - \overline{\mu}_d)$$

for the conditional cases. The mean angle of scattering then can be computed from the slope using the relation

$$\overline{n}_1(u) = \frac{u_d - u}{\xi_0(1 - \overline{\mu}_d)} \quad 0 \leq u \leq u_d$$

The slope shows that the distribution of scattering angle for the conditional neutrons is biased in such a way that neutrons contributing to a larger

Figure 3.6 Lethargy Versus Average Number of Collision

detector sphere have smaller average angle of scattering, and vice versa, but in any case, the bias on the distribution is small. The average cosine of scattering in the center-of-mass system, $\bar{\mu}_0$, for the 20 cm case, for example, is only 0.025. Correspondingly, the mean number of collisions is 2.6% greater than that of the general population.

We still have to find values for $\bar{n}_2$. It will be remembered that for neutrons contributing to the detector at a specific position, $\bar{n}_2$ is simply the average number of collisions occurred after the neutrons have been slowed down below the upper energy limit of the detector response but before the neutrons are detected. However, for the general neutron population, no detector geometric boundary is imposed. Thus in order to be able to compare the average number of collisions for the conditional cases to that for the general neutron population, we have to choose a reference lethargy at which to compute a suitable $\bar{n}_2$ for the general population. A logical choice of this reference would appear to be the mid-point of the detector lethargy interval. With $u = u_d + \frac{u_{d1} - u_d}{2}$, where u_{d1} is the upper lethargy limit of the detector response ($u_{d1} = 6.0$ and $u_d = 5.4$ for the detector being used), the second term of (3.2) can be written as

$$\bar{n}_2 = \int_{u_d}^{u_d + (u_{d1} - u_d)/2} \frac{du'}{\xi(u')} \quad (3.2.2)$$

for the general neutron population, and the average number of collisions, $\bar{n}$, calculated by (3.2) along with (3.2.1) and (3.2.2) is 67.5.

Table 3.2 shows the mean number of collisions per history actually observed for neutrons contributing to the 4 cm, 12 cm and 20 cm-radius detectors with vacuum boundary and in an infinite medium. For neutrons

contributing to the detector at 20 cm in an infinite medium, the overall increase in the mean number of collisions over that of the general population is about 2.5%. This is consistent with the increase in the average number of collisions computed by using the first term of (3.2) only. The result thus indicates that the choice of the mid-point of the detector lethargy interval as the base for comparison is appropriate.

Neutrons contributing to the detector for the case of 4 cm-radius bare sphere constitute a special case. These neutrons must slow down over a large lethargy interval, yet they must confine themselves within the small sphere, consequently they suffered considerably less collisions than did the normal neutrons. Also the mean path length between successive collisions shown in the next sub-section was considerably shorter than the normal neutrons.

As noted, the mean number of collisions for neutrons contributing

Table 3.2 Average Number of Collisions for Neutrons Contributing to The Detector in Lethargy Interval (5.4 - 6.0)

Detector Distance (cm)	$\bar{n}$	$\bar{n}$
	Vacuum Boundary	Infinite Medium
4	57.67 $\pm$ 1.02	65.96 $\pm$ 0.24
12	66.59 $\pm$ 0.18	67.15 $\pm$ 0.13
20	68.18 $\pm$ 0.37	69.01 $\pm$ 0.35
Reference Neutrons*	—	67.45

* The average number of collisions for reference neutrons is computed from equation (3.2) with $u=5.7$.

to the response with a 20cm-radius sphere in an infinite medium is only about 2.5% (or 1-2 collisions) larger than the result for normal neutrons. Nonetheless, this small increase in the mean number of collisions is quite significant statistically. Assuming normal distributions, there is less than 1% chance that the observed number of collisions is consistent with the hypothesis that the mean number of collisions is the same as for the general population.

The average cosine of scattering angle in the LAB system, $\bar{\mu}$, relates to the average cosine of scattering angle in the CM system, $\bar{\mu}_0$, in the following way⁽¹⁸⁾

$$\bar{\mu} \approx \frac{2}{3A} + (1 - \frac{3}{5A^2}) \bar{\mu}_0 + \dots$$

For the case of 20 cm reflected sphere, $\bar{\mu}_0 = 0.025$ and $A = 22.7928$, thus $\bar{\mu}$ is 0.055. Other things being equal, more collision means larger spatial displacement from the source. With 2.5% increase in the mean number of collisions and $\bar{\mu} = 0.055$, equation (3.1) shows that the increase in the root-mean-square displacement is 2.4%. Since on the average, neutrons contributing to the detector at 20 cm have traveled about twice as much in radial distance than the normal neutrons, the 2.4% increase is clearly not enough.

The result that there is no substantial increase in the number of collisions for neutrons penetrating deeply is significant for discovering the mechanism of penetration. One way that a neutron could travel far is to make very many small angle collisions (which is the basis of "straight-ahead" approximation). The investigation presented here shows that this

can not be the major mechanism leading to moderately deep penetration of neutrons in a constant cross section medium.

3.4.2. Path Length Distribution

For the general neutron population, the probability that a neutron emerging from a collision travels a distance x before making the next collision in dx about x is

$$p(x)dx = e^{-\Sigma_t(E)x} \Sigma_t(E)dx,$$

where $\Sigma_t(E)$ is the macroscopic total cross section. The mean path length is

$$\bar{\ell}(E) = \int_0^{\infty} x \cdot p(x)dx = \frac{1}{\Sigma_t(E)}.$$

If we define $y = \Sigma_t(E) \cdot x$, then y is the number of mean free paths that a neutron could travel before next collision takes place. The distribution of y is e^{-y} , independent of the neutron energy and cross section $\Sigma_t(E)$. It is the distribution of y which has been studied for the Monte Carlo histories generated here. In this particular case, since Σ_t is 1, the distribution of y is the same as the distribution of x .

The distribution of path length for the Monte Carlo histories can be studied by dividing the distribution into N bins. The bin boundaries y_n are chosen in such a way that each bin would have equal probability if the distribution of y were exponential with mean 1. These y_n 's thus satisfy the equation

$$\frac{n}{N} = \int_0^{y_n} e^{-y} dy \quad \text{for } n=0, \dots, N$$

or equivalently

$$y_n = -\ln\left(1 - \frac{n}{N}\right)$$

In this investigation, $N=4$ has been used. The four bins are therefore

Bin #1 : (0.000, 0.288), Bin #2 : (0.288, 0.693),

Bin #3 : (0.693, 1.386), Bin #4 : (1.386, ∞).

The observed distribution of path length for the general neutron population is shown in Table 3.3. It agrees quite well with the base distribution.

For neutrons contributing to the detector at a given position, the distribution of path length may not be exponential with mean 1. If the same bin structure is used to study the conditional neutrons, the distribution then may no longer be uniform in each of the bins. The observed distributions for neutrons contributing to the detector at 4 cm, 12 cm and 20 cm in an infinite medium are given in Table 3.3.

The table shows that the distribution skews towards shorter path length for the 4 cm case, and skews towards longer path length for the

Table 3.3 Path Length Distribution for Neutrons Contributing
to The Detector in Lethargy Interval (5.4 - 6.0)
in An Infinite Medium

<u>Detector Distance</u>	<u>Bin #1</u>	<u>Bin #2</u>	<u>Bin #3</u>	<u>Bin #4</u>
4 cm	0.2569	0.2532	0.2504	0.2395
12 cm	0.2500	0.2504	0.2501	0.2495
20 cm	0.2408	0.2408	0.2496	0.2688
Reference Neutrons	0.2502	0.2499	0.2501	0.2498

Bin boundary: (0.0, 0.288), (0.288, 0.693), (0.693, 1.386)
(1.386, ∞)

20 cm case. Whether the deviations from the base distribution are significant can be tested by the chi-square statistics⁽¹⁹⁾. For instance, the computed χ^2 value for detector at 20 cm is 60, whereas the value of χ^2 for the 1% significant level is 11. In other words, the deviation from the base distribution for the 20 cm case is clearly statistically significant. The computed χ^2 value for the 4 cm case is 53, and thus the deviation here is also not by chance.

The mean path length, $\bar{\ell}$, can also be computed from the Monte Carlo histories. For the general population, $\bar{\ell}$ should be equal to $1/\Sigma_t = 1$ cm. The $\bar{\ell}$ actually computed from the Monte Carlo histories is 1.000 ± 0.001 cm. The mean path lengths for neutrons contributing to the 4 cm, 12 cm and 20 cm-radius detector with vacuum boundary and in an infinite medium are listed in Table 3.4.

It has been mentioned in Section 3.4.1 that neutrons contributing to the 4 cm-radius bare sphere constitute a special case. The mean path

Table 3.4 Mean Path Length (cm) for Neutrons Contributing
to The Detector in Lethargy Interval (5.4 - 6.0)

<u>Detector Distance</u> <u>(cm)</u>	$\bar{\ell}$ <u>Vacuum Boundary</u>	$\bar{\ell}$ <u>Infinite Medium</u>
4	0.862 ± 0.030	0.962 ± 0.003
12	0.976 ± 0.004	1.001 ± 0.003
20	1.057 ± 0.007	1.074 ± 0.008
Reference Neutrons	—————	1.000 ± 0.001

length is 14% shorter than that of the reference neutrons. In general, the mean path length consistently increases with detector distance. For neutrons contributing to the case of 20 cm-radius sphere in an infinite medium, the mean path length is about 7% longer than that of the reference neutrons.

Though the path length distribution for neutrons contributing to the detector at 20 cm in an infinite medium deviates significantly from the base distribution, the deviations are not large. This is more clearly shown in Figure 3.7 in which more bins has been used to plot the distribution. The small but significant deviations in the path length distribution for this 20 cm case may arise from two possibilities (among others):

- (a) the distribution is an exponential but with mean λ not equal to but close to 1.
- (b) the distribution is composite consisting of an exponential distribution with mean $\lambda_1 = 1$ cm, but with an additional component at long path length.

Either one of these could be made to fit the observed distribution reasonably well. In Figure 3.7, an exponential distribution with mean 1.074 cm and a composite distribution $0.9e^{-x} + 0.1 \frac{e^{-x/1.74}}{1.74}$ are shown as examples.

To see to what extent the small deviations in path length distribution such as those given above could affect the mean distance of penetration, we use the expression for $\overline{r^2}$ obtained in Appendix A which reads

$$\overline{r^2} = \frac{2\bar{n}\lambda^2}{1-\bar{\mu}} + \bar{n}(\sigma^2 - \lambda^2) \quad (3.3)$$

In deriving this equation, the only condition different from those given for equation (3.1) is that the path length distribution is arbitrary with mean λ and standard deviation σ . Equation 3.3 is for an unconstrained set of histories, but given suitable values for the averages, it might represent the histories of the neutrons detected at a given distance, e.g. 20 cm. If the path length distribution is further assumed to have the form

$$p(x) = (1-\alpha) p_1(x) + \alpha \cdot p_2(x) ,$$

where $p_1(x)$ and $p_2(x)$ are two normalized distributions with means λ_1, λ_2 and standard deviations σ_1, σ_2 respectively, the (3.3) can be written as

$$\overline{r^2} = \frac{2\bar{n} \lambda_o^2}{1 - \overline{\mu}} + \bar{n} \epsilon , \quad (3.4)$$

where

$$\lambda_o = (1-\alpha) \lambda_1 + \alpha \lambda_2$$

$$\epsilon = (1-\alpha)(\lambda_1^2 + \sigma_1^2) + \alpha(\lambda_2^2 + \sigma_2^2) - 2\lambda_o^2 .$$

The first term in (3.4) gives a value of $\overline{r^2}$ as though the path length distribution were exponential with mean λ_o . The $\bar{n} \epsilon$ is then a correction term arising from the presence of the second distribution. It is important to note that this correction term is linear in $\bar{n}$.

For the 20 cm case, possibility (a) given above implies $\alpha = 0$ and $\lambda_o = 1.074$ cm (from Table 3.4) and $\epsilon = 0$. With $\bar{n} = 69$, $\overline{\mu} = 0.055$ and $\lambda_o = 1.074$ cm, the root-mean-square displacement computed by (3.4) is 13 cm, which is about 10% greater than the root-mean-square radial distance that an average neutron travels. For possibility (b), suppose $p_1(x)$ is exponential with $\lambda_1 = 1$ cm so that $\sigma_1 = \lambda_1$. If $p_2(x)$ is also assumed

to be exponential, then σ_2 is equal to λ_2 , and ε can be written as

$$\varepsilon = 2 \cdot \left\{ (1-\alpha) \cdot \lambda_1^2 + \alpha \lambda_2^2 - \lambda_0^2 \right\}$$

α and λ_2 must satisfy the constraints that

$$(1-\alpha) + \alpha \lambda_2 = \lambda_0 = 1.074 \text{ cm}$$

and that the distribution in each bin should be consistent with what has been observed above in Table 3.3 or in Figure 3.7. The values of α and λ_2 which best satisfy these constraints are $\alpha = 0.1$ and $\lambda_2 = 1.74 \text{ cm}$ (cf. Figure 3.7). With $\bar{n} = 69$, $\bar{\mu} = 0.055$ and $\lambda_1 = 1 \text{ cm}$, $\lambda_2 = 1.74 \text{ cm}$, $\alpha = 0.1$, the root-mean-square distance of slowing down is found to be 13.25 cm, or is about 12% greater than the root-mean-square distance that an average neutron travels. If the bias in path length distribution were the only mechanism which accounted for those histories contributing to the detector at 20 cm, then the root-mean-square displacement should be close to 20 cm. The discussion thus demonstrates that for this featureless medium, the deviation in path length distribution is small, and the biased distribution of path length can not be the only characteristic of the neutrons that penetrate deeply. Note also that the observed biased distribution of path lengths can be represented by a variety of models.

3.4.3. Distribution of Azimuthal Angle of Scattering

The azimuthal angle of scattering is generally assumed to be isotropically distributed in the LAB system for neutrons slowing down in a medium. The Monte Carlo histories created for the general neutron population specifically make use of this assumption. For the selected class

of neutrons under the constraint of detection at a given position, however, the distribution of azimuthal direction of "successful" collisions is found to be no longer uniform but is biased such that the neutron moves in a favorable direction.

To study the bias in the azimuthal angle of scattering, we define the reference direction for the azimuth as that which gives the maximum chance of detection. In the case of spherical geometry, the reference direction is that of the projection of the radius vector on the plane normal to the incident vector for the scattering (see the sketch below). The radius vector which goes through the origin and the point of collision is taken to be outward if the scattering takes place inside the detector surface, and is taken to be inward if the scattering takes place outside the detector surface. The mathematical computation of the azimuthal angle of scattering relative to the preferential direction is described in Appendix B.

The azimuthal angle of scattering, Φ , ranges from $-\pi$ to π , with the angle 0 in the direction of the reference specified above. Since the distribution of the scattering azimuth can be assumed to be an even function of Φ , the absolute value of Φ , $|\Phi|$, was studied for the Monte Carlo histories. The distribution of $|\Phi|$ was examined by dividing the angle between 0 and π into N bins. The bin boundaries are chosen such that each bin would have equal probability if the distribution of azimuthal angle of scattering were uniform. The bin boundaries thus satisfy the following equation

$$\frac{n}{N} = \int_0^{|\Phi_n|} P(|\Phi|) d|\Phi| \quad n=0, \dots, N$$

with

$$P(|\Phi|) = 1/\pi.$$

In this study, $N=4$ has been used. The four bins for $|\Phi|$ are therefore

Bin #1 : (0 , $\pi/4$), Bin #2 : ($\pi/4$, $\pi/2$),

Bin #3 : ($\pi/2$, $3\pi/4$), Bin #4 : ($3\pi/4$, π).

The distributions computed from the Monte Carlo histories for the general population and for the detector at 4 cm, 12 cm and 20 cm in an infinite medium are given in Table 3.5*. To test whether the distribution of azimuthal angle of scattering deviates significantly from the base

* Note that the distribution for constrained neutrons is always high in bin #1 and always low in bin #4. This is because the reference direction always shifts for the constrained neutrons. For the general population, however, the reference direction is always outward.

Table 3.5 Distribution of Azimuthal Angle of Scattering for
Neutrons Contributing to The Detector in Lethargy
Interval (5.4 - 6.0) in An Infinite Medium

<u>Detector Distance</u>	<u>Bin #1</u>	<u>Bin #2</u>	<u>Bin #3</u>	<u>Bin #4</u>
4 cm	0.276	0.254	0.239	0.231
12 cm	0.267	0.256	0.244	0.234
20 cm	0.285	0.262	0.238	0.215
Reference Neutrons	0.251	0.250	0.249	0.250

The distributions are for the absolute value of the scattering azimuth, with angle 0 in the direction of reference. Bin boundary: $(0, \pi/4)$, $(\pi/4, \pi/2)$, $(\pi/2, 3\pi/4)$, $(3\pi/4, \pi)$

distribution, the chi-square method can be applied. The computed χ^2 value for the general population is about 4. Thus the slight deviations in the distribution for the general population (0.4% in bin #1 and bin #3) are a result of statistical fluctuation, and the distribution is uniform in the four bins as expected. For the neutrons contributing to the detector at a given position, the selection of azimuthal direction of scattering is biased towards the detector location. In particular, for the 20 cm case, the computed χ^2 value is 370. We must therefore certainly reject the hypothesis of uniformity in the distribution of azimuthal angle of scattering. The demonstration of non-uniformity in the azimuthal angle distribution for such unusual neutrons is welcome since we have seen that while there is no substantial increase either in the collision number or in the path length for these neutrons, yet they are still able to penetrate relatively deeply

compared to the general population. The number of collision azimuths in the most favorable angular bin (bin #1) is increased by about 14% relative to the uniform distribution.

In Appendix A, an estimation of the effect of biased distribution of azimuthal angle of scattering on the mean-square radial distance of slowing down for a set of unconstrained histories is derived. It is more convenient in this treatment to measure the scattering azimuth not relative to the previously defined reference direction but with respect to the planes consisting of the vectors of previous scatterings (see Appendix A for detail). If the average cosine of the azimuthal angle of scattering for the j -th collision relative to the plane consisting of the $(j-1)$ th vector of scattering and of any of the $(j-2)$ th, , $(j-(j-1))$ th vector of scattering is assumed to be a small quantity ϵ , independent of j , and if all the other properties regarding the neutron slowing down remain unchanged, it can be shown that the mean square displacement of a neutron from the source after $\bar{n}$ collisions is

$$\overline{r^2} = \frac{2\bar{n}\lambda^2}{1-\bar{\mu}} \left\{ 1 + \frac{\bar{n}c\epsilon}{2} \right\}, \quad (3.5)$$

where

$$c = (\pi/4)^2.$$

Note that where ϵ is zero, as for the general neutron population, the expression then reduces to equation (3.1). For a non-zero ϵ , the correction due to non-uniformity in the distribution of the azimuthal angle of scattering is proportional to $\bar{n}^2$. Thus even for a small ϵ , the

correction could be substantial if $\bar{n}$ is very large.

The quantity $\bar{\epsilon}$ for neutrons contributing to the detector at 20 cm in an infinite medium has been estimated from the Monte Carlo histories to be $1.26 \pm 0.10 \%$. Notice that $\bar{\epsilon}$ is the average cosine computed from the Monte Carlo histories for the j -th scattering azimuth relative to the plane consisting of the $(j-1)$ th and $(j-i)$ th directions of scattering for $i=2, \dots, (j-1)$ and for all possible values of j . However, if only the average cosine of the scattering azimuth for j -th scattering relative to the plane consisting of the $(j-1)$ th and $(j-2)$ th directions of scattering is computed for all possible j , the result is also about the same as $\bar{\epsilon}$. What this indicates is that there might exist some long-persistence correlation among the collisions in the life history of the exceptional neutrons. Without considering this correlation in the first place, and with $\bar{n}=69$ (the mean number of collisions suffered by the neutrons contributing to the detector at 20 cm in an infinite medium), the average increase in the root-mean-square displacement due to the biased azimuthal angle of scattering is about 13%. Clearly, the bias is too small, and the increase in radial transport by this amount is not sufficient by itself to account for the increased distance that the neutrons contributing to the detector at 20 cm traveled.

To consider the overall effect of the independent biased distributions of the three parameters we've discussed so far on the radial distance of slowing down, we write, for a set of unconstrained histories,

$$\overline{r^2} = \frac{2\bar{n}\lambda^2}{1-\bar{\mu}} \left\{ 1 + \frac{\bar{n}\bar{\epsilon}}{2} \right\} + \bar{n}\bar{\epsilon}, \quad (3.6)$$

where

$\bar{n}$ = average number of collisions

λ = mean path length

$\bar{\mu}$ = average cosine of the polar angle of scattering in
the LAB system

$$c = (\pi/4)^2$$

$\bar{\epsilon}$ = average cosine of the azimuthal angle of scattering

$\epsilon' = \sigma^2 - \lambda^2$, where σ is the standard deviation of
the path length distribution.

The conditions of independence between collisions and among the three factors discussed are still assumed to hold. The increase in the root-mean-square radial distance of slowing down for a set of histories with "altered" distributions of polar and azimuthal angles of scattering and of path length relative to a set of histories with original distributions, δ , can be written as

$$\delta = \frac{\sqrt{\bar{r}^2} - \sqrt{\bar{r}_0^2}}{\sqrt{\bar{r}_0^2}} = \left\{ \frac{1 - \bar{\mu}_0}{1 - \bar{\mu}} \cdot \frac{\bar{n}}{\bar{n}_0} \cdot \frac{\lambda^2}{\lambda_0^2} \left(1 + \frac{\bar{n} c \bar{\epsilon}}{2} \right) + \frac{\bar{n}}{\bar{n}_0} \cdot \frac{\epsilon' (1 - \bar{\mu}_0)}{2 \lambda_0^2} \right\}^{1/2} - 1,$$

where the subscript 0 indicates the statistical parameters for histories with original distributions. The general neutron population represents this set of histories with unaltered original distributions. For histories with statistical properties the same as those for neutrons contributing to the detector at 20 cm in an infinite medium, the root-mean-square distance of slowing down is about 15 cm, or is about 25% greater than that for the general neutron population (for the general neutron population, the root-mean-square displacement is 11.8 cm).

3. 4. 4. Dependency Between Azimuthal Angle of Scattering and Path Length

It has been shown, in the above discussions, that the three independent biased distributions of transport parameters are not sufficient to account for the penetration of exceptional neutrons. However, there is the further possibility that neutron histories selected by given detector energy conditions may exhibit correlations among the three factors discussed and/or between successive collisions. An example might be in a correlation between the azimuthal angle of scattering and the distance to the next collision, i. e. selection of a favorable scattering direction coupled by a favorable selection of path length. To investigate this possibility, the joint distribution of the azimuthal angle of scattering and the path length for the Monte Carlo histories was studied.

The computation of the joint distribution of the azimuthal angle of scattering and the path length was performed by using 4 angular bins and 4 path length bins, with bin boundaries the same as those used in Section 3. 4. 2. and 3. 4. 3. If the base distributions were true (that is the distribution of path length is exponential with mean 1 and the distribution of scattering azimuth is isotropic), and if there were no correlation between the selection of azimuthal angle of scattering and of path length, then each cell would have probability $1/16$ (0.0625). Table 3. 6 gives the distributions for the general population and for the histories contributing to the detector at 20 cm in an infinite medium; the distribution in each cell has been normalized to that for the base distribution. No correlation can be observed from Table 3. 6 for the general population. This is of course expected.

Table 3.6 Normalized Scattering Azimuth-Path Length Distribution
for Neutrons Contributing to The Detector in Lethargy
Interval (5.4 - 6.0)

(A) Reference Neutrons :

<u>Angular</u> <u>Path Length</u>	<u>Bin #1</u>	<u>Bin #2</u>	<u>Bin #3</u>	<u>Bin #4</u>
Bin #1	1.002	1.000	0.997	1.005
Bin #2	1.005	0.997	0.998	1.000
Bin #3	1.006	0.998	1.000	0.997
Bin #4	1.003	0.998	0.995	0.998

(B) Neutrons contributing to the detector at 20 cm in an infinite medium:

<u>Angular</u> <u>Path Length</u>	<u>Bin #1</u>	<u>Bin #2</u>	<u>Bin #3</u>	<u>Bin #4</u>
Bin #1	0.997	0.949	0.974	0.923
Bin #2	1.048	0.950	0.962	0.896
Bin #3	1.134	1.059	0.936	0.859
Bin #4	1.382	1.227	0.938	0.763

1. The distribution in each cell is normalized to 0.0625.
2. For bin boundaries, see Table 3.3 and Table 3.5.

For neutrons contributing to the detector at 20 cm in an infinite medium, the distribution suggests that scatterings having better direction are more likely to be followed by longer path lengths. Chi-square test of statistical independence⁽²⁰⁾ shows that the 1% level of significance is 26, whereas the computed χ^2 value for the 20 cm case is 87. The effect of this correlation towards increasing in the mean-square radial distance of slowing down, however, can not be calculated analytically.

3.5. Conclusion

The life histories of two sets of "exceptional" neutrons in a featureless medium have been studied in detail in comparison to the general population of neutrons that have slowed down to the same energy. In the root-mean-square sense the two classes of neutrons travelled 35% and 170%, respectively, as far as the average neutron before detection. These are large deviations, and the two kinds of neutrons may seem to be aptly described as "exceptional". On the other hand the flux and current densities of the two classes agree very well with the predictions of age theory. Since age theory is based on the behavior of large numbers of collisions with the same statistical properties as the average neutron, it is perhaps not surprising that the modifications in the collision characteristics of our "exceptional" neutrons are slight and subtle. The three characteristics of each collision—path length to collision, the polar and azimuthal angles of scattering—differ by only a few percent in mean values from the general average. There are somewhat more evident biases in the statistical

distributions. But these statistical changes fall far short of explaining by themselves the gross differences in root-mean-square distance travelled by the two sets of neutrons.

It seems likely that the different behavior of the "exceptional" neutrons arises rather out of correlated biases in the individual collision properties and more especially in correlations between successive collisions. Some evidence for these correlations has been presented here. Bias towards favorable azimuths was found to be correlated with exceptional path lengths between collisions. There appear to be significant correlations in the azimuth scattering angles among successive collisions.

Where the cross section properties of the medium show a wide range of behavior with energy, it is possible for a neutron history to be "exceptional" by exploiting a particular cross section phenomenon in a narrow energy range. An example would be deep penetration by means of long-range transport in a cross section minimum, as discussed in the next chapter. But here, where the cross sections are the same at all collisions, moderately deep penetration for example occurs rather as the cumulative effects of small modifications in all collision that are difficult to detect in statistical tests. It is likely the changes in collision behavior are larger for really deep penetration distances, e.g. to 5 or more times the general root-mean-square distance. But it is not possible to obtain a sufficient sample of such histories with the "straight" Monte Carlo simulation used here. To study such unusual neutron histories it would be necessary to construct a "biased" Monte Carlo procedure which still permits statistically

significant conclusions to be drawn about the properties of the histories.

Chapter 4

Study of Neutron Penetration in Iron from 150 keV to 85 keV

4.1. Introduction

A "small" problem designed to isolate the effect of cross section minima on neutron transport is studied in this chapter. The problem consists of a monoenergetic 150 keV source in an infinite bulk of iron with a detector sensitive in the energy range from 100 keV to 85 keV. A total of 10,000 histories was used. A neutron on the average needs only about 16 collisions to degrade below the cutoff energy (namely the lower limit of detector energy). This relatively small number of collisions reduces the computer running time considerably (hence "small"). On the other hand, however, there are two deep cross section minima (at about 138 keV and 129 keV respectively) in the energy range of neutron slowing down. The ratio of the average cross section to the cross section with the lowest value is about 10. Preeg's analysis⁽²¹⁾ showed that under such cross section situation, the deep penetration of neutrons both within and below the cross section minima would be determined predominantly by the transport in the minima. For neutrons that penetrate very deeply, the transport taking place within the deepest cross section minimum is so long and so important in comparison to the transport taking place outside the region of that minimum that the relaxation length of the neutron flux below the minimum is dominated by that of the flux within the minimum. Apparently, the behavior of the life history for such exceptional neutrons is quite different from that for the general population. The study presented

here therefore demonstrates that the analysis of the Monte Carlo histories can indeed tell us the special features in the life history of the exceptional neutrons. Meanwhile, the study also provides some more direct information concerning the collision behavior of the neutrons penetrating in a medium containing deep cross section minima.

In the following discussion, the data set which has been used is described briefly in Section 4.2. The transport phenomenon in the cross section minima is then explored in Section 4.3, where the dominance of the cross section dip in determining deep penetration is shown. As has been described in Chapter 3, three quantities describe the nature of each collision—the polar angle of scattering, the path length of next flight and the azimuthal angle of scattering. In order to understand the influence of cross section on the transport process, it is essential to understand the influence of cross section on these quantities. These interrelations are detailed in Section 4.4.

4.2. Description of Data Set

Iron cross sections in the energy range from 150 keV to 85 keV were taken from DNA 4180 MOD.2 and they were subsequently processed by the code SAM-X. A plot of the total cross section from 90 keV to 150 keV is presented in Figure 4.1.

There are two interference minima in this region at about 138 keV and 129 keV respectively. The 138 keV dip, which is only 1.2ξ (where ξ is the maximum lethargy change upon scattering) below the source is

Figure 4.1 Total Cross Section for Iron

of particular importance in the deep penetration, for it is both deeper and wider than the 129 keV dip. The lowest cross section in the 138 keV dip is only 0.3 barns equivalent to a mean-free-path length 40 cm, whereas the lowest cross section in the 129 keV dip is 0.6 barns. At the source energy, the total cross section is about 5 barns, rising up to 20 barns just above the 138 keV dip. Below the minima, the total cross section is roughly constant at about 3 to 4 barns, with occasional spikes of narrow p-wave resonances of negligible width. The absorption cross section averages out to about 0.3% of the total cross section. Elastic scattering is the only scattering process involved, and is isotropic in the center-of-mass system.

4.3. Study of Transport Phenomena Dominated by Effects of Cross Section

Minima

We have noticed that, for a detector sensitive from 100 keV to 85 keV, the 138 keV cross section minimum represents a wide and deep open "channel"* in phase space. The transport of neutrons contributing to the response of a detector far from the source is expected to be determined by travel within that dip, i.e. the cross section minimum dominates the deep transport. More specifically, a cross section dip is said to dominate when most of the penetration is done at an energy in the vicinity of that dip and the average distance between collisions is much

* A "channel" means a region of (r, E, Ω) phase space which provides a particular easy route for the neutron point to traverse from the source to the detector region.

longer than the mean free path length. Furthermore, the minimum is narrow enough so that almost all of the collisions in the minimum remove the neutron from the region of the minimum. Preeg⁽²²⁾ showed that, for deeply penetrating neutrons in such a medium, the relaxation length of the neutron flux at energies below the minimum would be determined by that of the flux at the energy of the minimum. Figure 4.2 shows the total outgoing current computed from the Monte Carlo samples in the 138 keV dip and below both dips as a function of distance. It can be seen that below the dips, the exit current builds up and then decays as the detector distance increases, and eventually decays at about the same rate as does the current in 138 keV minimum.

When a cross section minimum dominates the deep penetration, the neutrons contributing to a detector with response energy below the minimum travel most of their spatial distance within the region of the minimum. The transport in that minimum is much longer than that of the general population. One way to show this phenomenon is to examine, from the Monte Carlo histories, the root-mean-square distance of slowing down in the life history of those neutrons detected far from the source. In Figure 4.3, the root-mean-square displacement from the source during the course of slowing down is plotted against neutron lethargy (with the source at 150 keV used as the zero of the lethargy) for the general population and for the detector at 80 cm in an infinite medium. It is observed that when the detector is located at 80 cm in an infinite medium, the average radial distance traveled by the detected neutrons in

Figure 4.2 Exit Current As a Function of the Radius
of Detector Sphere

Figure 4.3 Root-Mean-Square Distance of Slowing Down
from A 150 keV Point, Isotropic Source in An Infinite Iron Medium

the 138 keV minimum is about 60 cm. Thus about 75% of the total transport is accomplished in that minimum. For the reference case (i. e. the histories of all neutrons detected irrespective of their geometric location of detection), the average radial transport in the 138 keV dip is only about 25 cm.

If age theory held, the mean-square radial distance of transport can be computed by the relation⁽²³⁾

$$\overline{r^2(u)} = \int_0^u \frac{2 \, du'}{\xi (1 - \bar{\mu}) \Sigma_s(u')},$$

where

u = lethargy

ξ = mean lethargy increment per collision

$\bar{\mu}$ = average cosine of the scattering angles in the LAB system

Σ_s = macroscopic scattering cross section.

The results are also plotted on Figure 4.3, and are seen to fit quite well with the mean-square radial distances of slowing down observed for the reference histories, at least below 138 keV. At energies above 138 keV, on the average, only about 2—3 collisions take place. Age theory is not expected to be satisfactory for such small number of collisions.

Another way in which the Monte Carlo analysis can show the effect of the minima is in terms of the probability that a given history has collisions in the minima and subsequently contributes to the detector response at certain specific distance from the source. This probability was computed by arbitrarily defining the minima as regions having total cross section less than 1 barn. Since there are two places (129 keV and

138 keV) having cross section less than 1 barn, there are four possible sets of "dip combinations", each of which was examined individually.

Suppose we use binary representation (c_1, c_2) , where the index 1 represents the 138 keV dip and the index 2 represents the 129 keV dip, and where the c 's are assigned the value 1 if a given history has undergone collisions in that dip and 0 if it hasn't. Then each neutron history is characterized by one of the four possible combinations: $(1, 0)$, $(0, 1)$, $(1, 1)$ and $(0, 0)$. A plot of the probability for each of these combinations versus detector distance reveals the relative importance of the separate minima at each distance. This is illustrated in Figure 4.4 in which the $(1, 0)$ and $(1, 1)$ cases have been collapsed together. Clearly, the chance that a given detected neutron has had a collision in the 138 keV dip markedly increases as the detector has moved away from the source whereas the chance of having no collision in either of the dips quickly diminishes. The probability of going through the 129 keV dip alone, shows up a broad maximum around 25 cm, and then decreases as the detector distance increases. Therefore only the deepest channel eventually dominates the deepest penetration. In any case, the probability that the neutron has gone through at least one minimum increases from 40% at 10 cm to 98% at 80 cm.

If again we consider the cross section minima as regions having cross section less than 1 barn, we can calculate the mean path lengths both inside and outside the minima for neutrons contributing to the detector response at various distances. The results obtained for a detector in an infinite medium are shown in Table 4.1. Clearly, the mean path lengths

Figure 4.4 Probability of Having Collision in the 138 keV and 129 keV
Cross Section Minima for Neutron Detected at Various
Distances from the Source in An Infinite Medium

Table 4.1 Comparison of Mean Path Length (cm) for Neutrons

Detected at Various Radial Distances in an Infinite Iron Medium

Detector Distance	Case (1,0)		Case (0,1)		Case (1,1)		Case (0,0)
	Inside	Outside	Inside	Outside	Inside	Outside	
10	7.83±0.47	3.10±0.05	6.91±0.51	2.92±0.07	8.18±0.53	2.98±0.10	3.10±0.03
20	12.37±0.88	3.54±0.06	10.45±0.55	3.55±0.13	11.96±0.52	3.48±0.06	3.72±0.04
30	19.18±0.89	3.67±0.08	14.42±0.94	3.83±0.11	15.80±0.99	3.46±0.10	4.57±0.08
40	23.36±1.14	3.92±0.11	22.75±2.10	4.13±0.14	19.70±0.48	3.39±0.07	4.91±0.11
50	32.43±1.68	3.93±0.16	33.38±2.08	4.13±0.17	25.13±1.87	3.64±0.14	6.29±0.27
60	43.17±2.07	3.70±0.16	37.50±5.30	4.36±0.43	27.23±1.80	3.86±0.26	5.98±0.30
70	53.64±2.90	4.08±0.44	40.53±9.42	5.76±0.89	36.98±4.67	3.26±0.17	7.40±0.67
80	59.52±6.47	3.72±0.28	51.49±4.90	2.53±0.10	33.00±4.32	3.98±0.34	10.24±1.24
90	66.91±5.93	3.39±0.24	69.17±9.08	3.68±0.52	35.79±3.82	2.79±0.41	- -
100	72.95±9.28	3.16±0.31	- -	- -	45.61±6.88	4.35±0.54	- -
Reference Case	24.57±0.67	3.69±0.03	16.06±0.46	3.79±0.03	21.26±0.28	3.57±0.03	3.90±0.02

of travel within the dips increase significantly as the source-detector separation increases. In the (1,0) case, for example, the mean path length in the 138 keV dip at detector distance 40 cm from the source is about the same as that for the reference case. When the detector is at 10 cm, the mean path length in that dip is only 8.0 cm. When the detector is at 80 cm, the mean path length however has risen to about 60 cm. It might be noticed that, for the histories involving the minima (i.e. the (0,1), (1,0) and (1,1) histories), the mean path lengths outside the minima do not change significantly with detector distance. On the other hand, for the (0,0) histories, which do not traverse a minimum, the mean path length shows significant increase with detector distance.

4.4. Relation Between Cross Sections and Transport Parameters

4.4.1. Relation Between Cross Sections and Collision Numbers

For the energy range being analyzed, the only slowing down process is through elastic scattering, which is moreover isotropic in the center-of-mass system. Therefore, considering the general population of neutrons reaching the detector energy range, the average number of collision in a unit lethargy interval should be a constant ($du/d\bar{n} = \xi = 0.0357$) independent of the lethargy. It is not clear, however, if this is still true when only a selected class of neutrons conditioned both by detector energy and geometric location is considered. To investigate any effect of fluctuating cross sections on the collision number for such constrained neutrons, the following analysis method has been used. Cross sections in

the region of interest were divided into "levels", e.g. 1 barn, 2 barns etc..

For a particular "level", a total lethargy width is defined corresponding

to the sum of all lethargy segments for which the cross section is below

that level (cf. Figure 4.5). The lethargy width so obtained for a "level"

then includes those contained in lower levels. At the lowest level, the

corresponding lethargy width is zero, while at the other extreme, for a

level corresponding to the maximum cross section the lethargy width

corresponds to the entire lethargy range of the region of consideration.

For neutrons contributing to the detector at a given distance from the source,

the average number of collisions in the history made below a given level

and the fraction of the detected neutrons which had had collisions below

that level were estimated. Only those collisions with energy above the

Figure 4.5

upper limit of the detector sensitivity (namely $E \geq 100$ keV) were considered. The average number of collisions and the fractions obtained were then plotted versus lethargy interval. A secondary scale is included on the lethargy interval axis corresponding to the cross section levels. If there is any biasing in the number of collision in the vicinity of a particular cross section "level", we would expect to see an increase or a decrease in the slope of the collision number in the vicinity of that "level" relative to that for the reference case. The results of the analysis for the collision numbers are illustrated in Figure 4.6 for detector at 80 cm and 10 cm in an infinite medium. A magnified version of the low cross section level section of Figure 4.6 is given in Figure 4.7a, and the results for the fraction of the detected neutrons which have collisions in the low cross section regions are presented in Figure 4.7b. Again the results are for detector at 80 cm and 10 cm in an infinite medium. In addition, the results for all neutrons detected irrespective of their geometric location of detection, i.e. the reference case, are also shown on both Figure 4.6 and Figure 4.7.

From the curves shown in the Figure 4.6 and 4.7, one can see that

- (a) The average number of collision versus lethargy interval is a straight line with slope 28 collisions/lethargy in the reference case. This is of course expected, and indeed it agrees well with the prediction ($1/\xi = 28.01$).
- (b) The number of collision per unit lethargy interval for histories detected at 80 cm is larger than that for the reference case only in low cross section regions, e.g. below 1 barn. There is practically no difference

Figure 4.6 Average Number of Collision .VS. Lethargy
Interval Corresponding to Various Cross
Section Levels
Cross Section (Barn)

Figure 4.7 (a) Average Number of Collision .VS . Lethargy Interval Corresponding to Various Cross Section Levels

(b) Fraction of The Detected Neutrons Having Collisions in Lethargy Interval Corresponding to Various Cross Section Levels

Figure 4.7 (a)

Figure 4.7 (b)

in the slopes above about 1.5 barns between 80 cm case and the reference case. Thus all the increase in the average number of collisions for the 80 cm case is due to increased collisions below about 1.5 barns. Conversely, neutrons detected at 10 cm have fewer collisions per unit lethargy interval only in the region below about 1.5 barns.

Again there is no significant difference from the result for the reference case above 1.5 barns.

- (c) Neutrons contributing to the detector at 80 cm have a higher chance of undergoing collisions in the low cross section regions than do reference neutrons, whereas histories contributing to the detector at 10 cm have a smaller chance. For instance, 70% of the histories have collisions in lethargy regions with cross section less than 1.2 barns when there is no constraint corresponding to the detector geometric location. Yet nearly all neutrons detected at 80 cm have collisions in those regions. The corresponding fraction for detector at 10 cm is only about 50%.

Thus, the above observations suggest that as the neutrons penetrate deeply, more and more of the collisions occur in the low cross section regions. The fact that the only biasing in the collisions is in the low cross section regions emphasizes the essential role of cross section minima in deep penetration.

4.4.2. Relation Between Cross Sections and Path Length Distributions

The distribution of path lengths between collisions has also been

studied. By normalizing each path length to the corresponding total macroscopic cross section, all path lengths for the general reference population should follow an independent exponential distribution with mean 1. To analyze possible deviations in the histories of the detected neutrons, four "cells" in the normalized path length were used with boundaries (0.0, 0.288), (0.288, 0.693), (0.693, 1.386), (1.386, ∞). For the exponential distribution with unit mean, these four cells have an equal probability of 25%. This distribution serves as base distribution for comparison. The energy region was then arbitrarily divided into subregions that gave appropriate emphasis on the energies near the minima. The normalized path lengths in each subregion were grouped together and their fractional division among the four cells was computed. Table 4.3 illustrates the path length distribution for four such subregions for detector locations at 80 cm and 10 cm in an infinite medium. Also included in this table is an entry of the χ^2 values computed from the Monte Carlo samples. Note that the χ^2 value for the 1% significance level is 11.3.

As the table shows, in the energy regions containing cross section minima (region 2 and 3), the path length distributions are very much biased for both very large and very small penetration distances. For the 80 cm case, the distributions are strongly biased to favor the longest path length bin (cell #4). Conversely, for the detector at 10 cm, the distributions are strongly biased toward the shortest path length bin (cell #1). It may also be noticed that, for energy regions other than those containing cross section minima, the deviations from the base distribution are

medium case. But, if we now use a 5% significance level, corresponding to a χ^2 value of 7.8, we would have to reject the hypothesis of uniformity in the distribution of scattering azimuth for collisions taking place at regions with cross section less than 1 barn and greater than 2 barns. The distribution for collisions at cross section regions between 1 barn and 2 barns is not certain because of poor statistics.

It is seen from Table 5.4 that the distribution for azimuthal scattering angle is preferentially biased away from the reference direction. The biasing is strongest for collisions at low cross sections, but appears definitely present even for high cross section collisions. There seems little doubt as to the reason for this behavior. Almost all the collisions occur at energies above the upper detector limit—with a bare assembly once a neutron is "counted" by the detector it is lost from the system. After emerging from a collision at any cross section level a neutron has a good chance of leaking from the small bare sphere. This is especially true for

Table 5.4 Distribution of Scattering Azimuth for Neutrons Contributing to Detector 17 for A 20 cm-Radius Bare Sphere

Cross Section Interval $\sigma_1 - \sigma_2$ (Barns)	Bin #1	Bin #2	Bin #3	Bin #4	χ^2
0.0 - 1.0	0.189	0.238	0.250	0.323	9.1
1.0 - 2.0	0.245	0.229	0.242	0.284	2.5
2.0 - 4.0	0.216	0.253	0.253	0.278	11.3
4.0 - ∞	0.233	0.249	0.250	0.268	12.4

distribution biased toward longer flights between collisions in the cross section minima. The collisions outside the minima play no extraordinary role in the deep penetration.

4.3.3. Relation Between Cross Sections and Azimuthal Angle Distributions

The procedure for estimating the distribution of azimuthal scattering angle relative to the preferential direction has been described above in Chapter 3, Section 3.4.3. Because the cross sections vary with energy, we divide the magnitude of cross section ranging from zero to infinity into several intervals. Collision histories are studied separately in the energy regions for which the magnitude of cross section is in a given interval. Four such intervals have been used here: $(0, 1)$, $(1, 2)$, $(2, 5)$ and $(5, \infty)$, where the unit is in barns. Table 4.4 shows the distributions of the scattering azimuth in four angular bins with the detector at 80 cm in an infinite medium. The bin boundaries are the same as those used in Chapter 3, Section 3.4.3.

In this 80 cm case, the distributions of azimuthal angle of scattering in both low cross section ($\sigma \leq 2$ barns) and high cross section ($\sigma > 5$ barns) regions are not significantly biased. A possible reason for this behavior is suggested by examination of the geometry of the situation. As we have seen, the high cross section region ($\sigma > 5$ barns) is just below the source energy. Thus for collisions in this region, no particular preference in the scattering azimuth would be expected. Also the neutrons have not moved far from the source prior to scattering into the energies

Table 4.4 Distribution of Scattering Azimuth for Neutrons
Contributing to the Detector at 80 cm in An
Infinite Medium

Cross Section Interval $\sigma_1 < \sigma \leq \sigma_2$ (Barns)	Bin #1	Bin #2	Bin #3	Bin #4	χ^2
0.0 - 1.0	0.295	0.254	0.221	0.230	1.6
1.0 - 2.0	0.316	0.219	0.219	0.246	2.9
2.0 - 5.0	0.330	0.310	0.187	0.173	47.6
5.0 - ∞	0.315	0.254	0.193	0.238	6.0

* The distributions are for the absolute value of the scattering azimuth ranging from 0 to π with angle 0 in the direction of reference. Bin boundaries: (0, $\pi/4$), ($\pi/4$, $\pi/2$), ($\pi/2$, $3\pi/4$), ($3\pi/4$, π).

of the cross section minima. Thus for the scatterings which take the neutrons into the minima, all azimuthal angles of scattering correspond to directions pretty much along the radius vector from the source. Hence no particular preference in azimuth angle of scattering would be expected too. These expectations are indeed born out by the data in Table 4.4. On the other hand, most of the collisions in the energy regions for which the cross section is less than 5 barns but greater than 2 barns occur towards the end of the trajectories. Here collisions which scatter towards the direction of the detector have a higher chance of being detected, and therefore one would expect to see some bias in the distributions of azimuthal scattering angles. Table 4.4 again bears out this expectation. For collisions in the energy regions with cross section between 2 barns and 5 barns, the observed distribution of scattering azimuth deviates significantly from the uniform distribution. The population in Bin #1 (towards the detector) for the observed distribution is 32% higher than that for the reference

distribution, whereas the population in Bin #4 (away from the detector) is 31% lower.

4.5. Conclusion

The problem studied here was designed to single out the effect of deep cross section minima on the transport of neutrons to great distances. In this, the problem is certainly a success. Without the presence of the minima, the root-mean-square travel of neutrons reaching the mid-detector energy would have been about 15 cm. The actual detector position studied was 80 cm, or five times as far. For any significant chance of getting out to such distances the neutrons must travel some of the time in one or both of the minima. But more than that, the transport behavior of the "successful" neutrons in the region of the minima must be out of the normal. They make more than the average number of collisions in the minima, and the distance travelled between collisions is considerably longer than normal. Such biased trajectories reach the detector by doing most of the penetration in the region of the minima—the minima can truly be said to dominate the transport. On the other side, if the detector is very close to the source, it tends to pick up only those neutrons whose trajectories avoid the minima.

The results of this chapter show that the analysis of Monte Carlo histories does clearly distinguish situations of dominance by cross section minima. More generally, if neutrons can reach a detector in significant numbers only by exploiting one particular set of cross section features, then the Monte Carlo technique will be able to pick out the resulting narrow

band of favored trajectories. But, as the conclusions of the previous chapter indicate, where there is no great advantage to be gained from the cross section behavior in any narrow energy region, it will be much more difficult to distinguish the unusual characteristics of the desired neutron trajectories.

Chapter 5

Study of Fission Neutron Penetration in Iron from 15 MeV to 10 keV

5.1. Introduction

Fast neutron penetration in iron has been investigated at Columbia University previously by Preeg⁽²⁴⁾ and Ching⁽²⁵⁾. Preeg analysed the effect of cross section minima on the deep penetration of neutrons at high energies, specifically in the range 1.57-2 MeV. He showed that at detector distance of 1 meter or more, the penetrating flux in this energy region is dominated by the transport in the cross section minima. Ching on the other hand, analyzed the neutron leakage from a 14 MeV source through a small bare sphere (radius 23 cm). He concluded that below about 300 keV the leakage spectrum was greatly influenced by inelastic scattering phenomena. Cross section minima were shown not to be a dominant factor in neutron transport. If anything, the minima correspond to energy regions of high leakage with resultant depletion of the flux slowing down to lower energies. That Ching and Preeg observed quite different phenomena is a consequence of the markedly different sources and geometries examined in each case. The conclusions that each drew may be valid only for the particular source-detector configuration involved. In this chapter we report on studies of neutron penetration in iron due to a fission source, for a broad range of source-detector configurations. Questions such as the importance of transport in cross section minima and the role played by inelastic scattering in neutron slowing down are explored as a function of

the source-detector geometry.

Histories generated from a fission source were followed in the energy range 15 MeV-10 keV. The iron microscopic cross section data were taken from DNA library. The nuclear density used was 0.08487 nuclei/cm-barn. In the energy range of consideration, the cross sections exhibit many peaks and valleys. Scatterings other than isotropic elastic collisions are also involved. Consequently, the mechanisms of neutron penetration become very complicated. A general description of the cross section data set used is presented in Section 5.2. In this section, some special features in the iron cross section such as the deep interference minima, and inelastic scattering from the lowest excited level are examined. In Section 5.3, the current spectra of exiting neutrons are compared for various detector geometries, and physical reasons for the observed rapid slowing down of high energy neutrons are given.

The next two sections are devoted to a study of the influence of varying cross sections on neutron transport. In Section 5.4, the average radial distance of slowing down as a function of neutron lethargy is examined for neutrons contributing to detectors at different distances and for various energy responses. Such a study essentially identifies the "average" trajectory in lethargy-radius phase space traveled by the neutron from the point of birth to the point of detection. By comparison with the average trajectories of unconstrained neutrons, the importance of cross section minima on neutron transport as well as the effect due to other cross section properties can be observed. In Section 5.5, we then investigate the

statistical properties of three neutron collision parameters, i.e. the collision number, the path length and the scattering azimuth. The study is carried out by dividing the cross section into different levels of magnitude, as in the previous chapter. The parameters are examined for all collisions with cross sections less than each such cross section level or between two cross section levels to isolate the effect of different cross section magnitudes.

Finally, in Section 5.6, we shift to the study of the life history of low energy leakage neutrons from a small bare sphere. This investigation is important since most of the integral measurements relating to cross section sensitivity study inherently involve small bare assemblies. In Sub-section 5.6.1, we present some results showing the energy regions in the source fission spectrum which are most likely to give rise to the low energy leakage neutron spectrum. In Sub-section 5.6.2, we then elucidate the detailed effect of inelastic scattering on the trajectory of those low energy leakage neutrons that are born above the inelastic excitation threshold.

Most of the following analyses are based on the information extracted from 10,000 histories created in an infinite iron medium following neutrons born in a fission source down to 10 keV. However, because of the high probability of inelastic scattering, neutrons born in the MeV region are rapidly slowed down into the keV region. In addition, therefore, 50,000 histories were simulated down to 850 keV in an effort to improve the analysis of the trajectories contributing to those detectors sensitive at energies

above this limit.

5.2. Description of Data Set

The basic data set used in the analysis is DNA 4180 Mod. 2⁽²⁶⁾ for natural iron*. The total cross section of iron shows marked fluctuations, particularly in the region from 100 keV to 1 MeV. To provide a detailed representation of the cross sections, including all resonance peaks (both S and P-wave) and interference minima, the data set therefore requires an energy mesh of about 5000 points to span the range from 15 MeV to 10 keV.

A plot of the total cross section for iron is shown in Figure 5.1. The most prominent feature in the total cross section is the famous interference minimum at 24 keV. In this data set the minimum goes down to 0.4 barns at its lowest point and is very broad, relative to the average energy loss of a 24 keV neutron upon scattering. However, other cross section minima at higher energies, such as the one at 138 keV (which goes down to 0.3 barns) as well as those around 300 keV and 600 keV, are also important in affecting the trajectories in neutron penetration.

Elastic scatterings are isotropic in the center-of-mass system below 300 keV. Above 300 keV, the anisotropic angular distribution of scattering

* The errors involved in calculations using this data set instead of ENDF/B version 4 are small. In the DNA data set, pointwise cross sections are given for total, elastic and (n, γ) reactions. In the ENDF/B data set, these cross sections are given in two parts: a pointwise background plus resonance parameters.

Figure 5.1 Total Cross Section for Iron
(DNA 4180, Mod. 2)

is represented by Legendre polynomial expansion coefficients in the CM coordinates. These coefficients are used by SAM-X to reconstruct equal-probability scattering tables for each energy mesh point. The methods of constructing these bins of equal probability have been described above in Chapter 2.

Inelastic scattering cross sections to 26 discrete levels are given explicitly. The lowest threshold corresponds to the ^{56}Fe level at 846 keV. The next level observed is the lowest level in ^{54}Fe at 1.4 MeV. Discrete level inelastic scattering cross sections account for about one third of the total cross section in the range from 1 MeV to 4 MeV. Inelastic scattering from the ^{56}Fe first level is particularly significant in many neutron slowing down processes. The cross section for this type of scattering increases to an average level of about 0.8 barns in the range 1-2 MeV, but exhibits a strong resonance structure with peaks as high as 1.6 barns. The angular distribution of inelastic scatterings is handled by SAM-X in a way similar to that for elastic scattering.

Continuum inelastic scatterings may occur above a threshold energy of 4.613 MeV and the $(n, 2n)$ reaction may occur above a threshold energy of 11.4 MeV. In the DNA data file, the secondary energy distributions for each non-elastic reaction are given separately. SAM-X combines the distributions for all non-elastic reactions into one equal-probability energy distribution table at each energy mesh point (cf. Chapter 2).

Absorption cross sections, including (n, γ) , (n, α) , (n, p) , (n, d) , (n, t) , and $(n, {}^3\text{He})$ reactions, are in general small. Above about

3 or 4 MeV, the major contribution to the absorption process is due to (n, p) reaction; and at energies higher than 10 MeV, an additional significant contribution comes from (n, α) reaction. The absorption cross section in this high energy region ranges from about 20 millibarns at about 4 MeV to about 174 millibarns at 15 MeV. This is about 0.5% to 7% of the total cross section. Below 3 MeV, the absorption is primarily radiative capture. According to the data set used, the absorption cross section between 3 MeV and 60 keV is smooth, averaged to about 4 millibarns or about one-tenth of a percent of the total cross section. Below 60 keV, many (n, γ) resonances exist. These resonances were included in the data file and were actually used in the calculation*. The overall effect of absorption is that about 30% of the histories are terminated by this process before neutrons degrade below the cutoff energy at 10 keV. Of these histories, about 96% are absorbed below 600 keV.

Source neutrons are generated from tabulated Cranberg fission spectrum. A list of this spectrum is given in Appendix C. One can see from the distribution that about one quarter of the source neutrons are emitted with energy below the lowest inelastic scattering threshold.

5.3. Neutron Current Spectrum

The average outgoing current, $J^+(u)$, in a lethargy interval (u_i, u_{i+1}) has been computed for various distances from the source. $J^+(u)$ is

* It has been reported that the absorption cross section below 60 keV may be overestimated in the present data set⁽²⁷⁾.

defined as the number of neutrons per unit lethargy per unit source crossing the detector boundary in lethargy interval (u_i, u_{i+1}) in an outgoing direction (i.e. $\hat{n} \cdot \Omega > 0$, where $\hat{n}$ is a unit vector in the direction outward normal to the surface and Ω is the direction of a neutron moving across the boundary. See also Chapter 3.). The energy spectrum of J^+ was sampled in 18 energy bins from 15 MeV to 10 keV. These bins act as energy selective detectors. The energy boundaries for these detectors are listed in Table 5.1. The choice of the bin boundaries was such that neutrons contributing to a particular detector bin would be able to reflect the influence of cross section peculiarities, for example the cross section minima, in that bin, yet each bin was still wide enough to have sufficient number of neutrons to provide reasonable statistics.

Table 5.1 Detector Energy Limits

<u>Detector Number</u>	<u>Energy Range (MeV)</u>
1	15.000 - 9.000
2	9.000 - 5.000
3	5.000 - 3.000
4	3.000 - 2.500
5	2.500 - 2.000
6	2.000 - 1.300
7	1.300 - 1.000
8	1.000 - 0.860
9	0.860 - 0.680
10	0.680 - 0.500
11	0.500 - 0.400
12	0.400 - 0.280
13	0.280 - 0.200
14	0.200 - 0.120
15	0.120 - 0.060
16	0.060 - 0.027
17	0.027 - 0.020
18	0.020 - 0.010

The exit current spectra of iron spheres of radii 20, 60 and 100 cm are shown in Figure 5.4 for an infinite medium, while the exit current spectra of bare and reflected spheres of the same radii are compared in Figure 5.3—5.5. The plots indicate that neutrons above 1 MeV disappear rapidly from the spectra as the radius of the detector sphere increases. Even at a distance of 20 cm from the source the peak in the energy spectrum of J^+ has already shifted to about 500—600 keV (the peak in the source fission spectrum is at about 1 MeV). The rapid disappearance of high energy neutrons is due to following two reasons:

- (a) The probability of rapid slowing down by means of inelastic scattering is very high. Indeed, about 90% of the neutrons born above the lowest inelastic excitation threshold have at least one inelastic scattering. Of these neutrons, about 90% have an inelastic scattering that carries them to an energy below the lowest threshold for inelastic scattering. Such interactions tend to occur early in the neutron life. On the average, neutrons suffer only about 4—5 elastic collisions before the inelastic interaction taking place.
- (b) The mean-free-path length in the MeV region is only about 4 cm, which is relatively short. There are no particularly deep cross section minima in this region. Hence the inelastic collisions which take a fast neutron to below 1 MeV occur most of the time close to the source.

On the other hand, it is also clear from Figure 5.3 that the exit spectrum of a 20 cm-radius bare sphere falls off strongly below about 500 keV, the kind of phenomenon that also has been found in the case of 14 MeV

Figure 5.2 Comparison of Exit Current Spectra for 20, 60, 100 cm- Radius Reflected Iron Spheres, Fission Source

Figure 5.3 Comparison of Exit Current Spectra for 20 cm-Radius Bare and Reflected Iron Spheres, Fission Source

Figure 5.4 Comparison of Exit Current Spectra for 60 cm-Radius Bare and Reflected Iron Spheres, Fission Source

Figure 5.5 Comparison of Exit Current Spectra for 100 cm-Radius Bare and Reflected Iron Spheres, Fission Source

neutrons⁽²⁸⁾. Indeed, comparison of Figure 5.3 and 5.4, 5.5 shows that there are proportionately many fewer neutrons below 100 keV for the 20 cm-radius bare sphere than for either the reflected 20 cm sphere or the larger spheres. Clearly, we are seeing the high leakage effect for a small bare geometry. The ratio of J^+ of a reflected system to that of a bare system, which we denote as Γ , is about 5 for the 20 cm-radius sphere in the energy bins covering 20—100 keV. Γ rises to about 10 for the lowest bin covering 10—20 keV region. The larger Γ for the lowest bin arises from the fact that most of the neutrons slowing down to this bin have collisions in the region of 24 keV minimum, collisions which are likely to cause them to leak out of the system.

For larger spheres, it is seen from Figure 5.4 and 5.5 that Γ is in general smaller in comparison to that for the smaller spheres in a given detector bin at energies below 500 keV. However, for a given sphere radius, Γ is relatively larger in a detector energy region containing high cross sections, particularly the region of 27—60 keV, than in a detector energy region containing cross section minima. For a 60 cm-radius sphere (Figure 5.4), Γ is 4.5 in Bin 16 (covering 27—60 keV), but is 2 in Bin 17 (covering the 24 keV minimum). When the detector radius increases to 100 cm (Figure 5.5), Γ is still about 4.5 in Bin 16, but decreases to about 1.5 in Bin 17. This behavior may be understood by an examination of the way that neutrons contribute to the exit current. As we know, collisions in the region of high cross section do not result in much spatial transport. Hence most of the neutrons which contribute to J^+ in the detector

interval containing high cross sections are likely to be those arriving at the vicinity of the detector location by means of transport in the cross section minima at higher energies. These neutrons then subsequently scatter around the detector geometrical location and slow down into the detector response interval. In the calculation of neutron exit current, J^+ for a bare sphere is constituted only by those neutrons which make their first passage through the detector surface in the response interval. However, J^+ for reflected sphere takes into account every outward passage of the neutrons through the detector surface in the response interval. A neutron which scatters into and out of the detector surface several times in the response interval contributes to the J^+ as many times as it passes outward through the surface. Thus, in the detector regions containing high cross sections, the neutrons suffer many collisions around the detector surface and tend to increase the detected J^+ value of a reflected system. Correspondingly Γ is relatively larger. On the other hand, for the detector regions containing cross section minima, most of the neutrons contributing to J^+ for a large sphere are likely to be those travelling long distances outwards in the regions of the minima. Since the minima regions are inherently narrow, if these neutrons happen to scatter out of the minima in the vicinity of the detector location, they may make some more contributions to the J^+ of the reflected system by the multiple scatterings at high cross section regions which occur below the minima but are still within the detector response interval. But if they don't, it is highly unlikely that these neutrons will make further contributions to the J^+ of the same detector for a reflected

system. Consequently Γ' is relatively smaller.

5.4. Role of Cross Section Minima in Neutron Penetration

It has been pointed out that there are many cross section minima below 500 keV, some of them very deep. To reveal the influence of these interference minima on neutron trajectories at different penetration distance, an analysis similar to that applied above to the "small" iron problem has been used here. However, to simplify the computation, the average radial distance of slowing down was calculated instead of the root-mean-square value. Since the root-mean-square radial transport, $\sqrt{\overline{r^2}}$, relates to the mean radial transport, $\overline{r}$, in the following way

$$\sqrt{\overline{r^2}} = \overline{r} \sqrt{1 + \sigma^2/\overline{r}^2},$$

where σ is the standard deviation for the radial transport, for $\overline{r} \gg \sigma$

(which is generally true), it makes little difference using either $\overline{r}$ or $\sqrt{\overline{r^2}}$.

Furthermore, the lethargy range from 15 MeV down to the cutoff energy (10 keV) was divided into 86 subgroups, with particularly narrow subgroups in the regions of deep interference minima. The zero lethargy was chosen to be at 15 MeV. From the histories of those neutrons which contribute to a given detector response at a given distance from the source, the average radial distance, $\overline{r}$, at which neutrons left each subgroup and the associated standard error $\sigma_{\overline{r}}$ were calculated. A plot of the location of $\overline{r}$ versus the lethargy of the subgroup center represents roughly the trajectory of the center of neutron cloud in phase space as it travelled from source to detector. It has been established in Chapter 4 that if any particular cross section

minima dominated and provided a favored trajectory for neutron transport, most of the spatial motion of the center of neutron cloud would be at the lethargy of the minimum with little transport at any other lethargy. Some of the results for detectors D17, D14, D12 and D7 in an infinite medium are illustrated in Figure 5.7—5.8 and Figure 5.10—5.13. The radial distance of slowing down for fission neutrons in an infinite medium is shown in Figure 5.6. On these plots, the horizontal bars represent one $\pm \sigma_T$ limit.

From Figure 5.6, it is seen that the average neutron travels a "mean" distance of about 87 cm from the source by the time it slows down past the 24 keV minimum. Very little of this distance, barely about 17 cm, is covered at energies above the inelastic threshold of 860 keV. In the region between 79 and 27 keV, where the cross section is always 5 barns or higher, no appreciable transport takes place. Thus the majority of the transport occurs for the energies between 800 and 100 keV, where the average neutron travels about 45 cm. A few of the breaks in the curve can be identified with specific cross section minima, the most prominent being the 25-20 keV minimum. In this region the average transport according to Figure 5.6 is about 17 cm, which, considered as a mean-free-path length, corresponds to a microscopic cross section of 0.7 barns. This figure fits in reasonably well with the average cross section in the 20-25 keV range.

When we select only those histories of neutrons constrained to reach a given detector at a given distance, the differences in the lethargy versus mean-radial distance curves are mostly quantitative for detectors at lower energies and at distances of practical interest (about 1 meter or so).

The curves retain the same general shape and emphasize the same energy regions. Thus, consider the curve for neutrons reaching a 20—27 keV detector located at a radius of 120 cm (Figure 5.7), which is not much farther than "normal" neutrons would have traveled by this energy. These doubly-constrained neutrons travel almost the same distance as the average neutron by the time they have reached the inelastic threshold—20 .vs. 17 cm. Again no transport occurs between 79 keV and 25 keV. At other energies the curve in Figure 5.7 is somewhat stretched out compared to Figure 5.6—about 60 cm is traveled between 800 and 100 keV, and about 30 cm in the 24 keV minimum. About one-quarter of the total transport thus occurs in the lowest minimum. While this is a healthy chunk of the total distance travelled, the minimum certainly cannot be said to dominate the transport. Since the lowest cross section in this minimum corresponds to a mean-free-path length of about 30 cm and the region of the minimum is wider than the maximum lethargy change upon scattering, it is clear that no unusual or improbable neutron trajectories within the region of the minimum need be involved.

When the same detector is stationed at 200 cm from the source, cf. Figure 5.8, the transport at energies above the inelastic threshold remains practically unaltered. The ratio of the distance travelled between 800 and 100 keV to that in the 20—25 keV minimum also remains roughly the same (from 2:1 at 120 cm to 2.2:1 at 200 cm). But the individual distances involved have increased so much that it is clear we are dealing with rather out-of-the ordinary neutron histories. Transport in the 20—25 keV

minimum alone amounts on the average to about 50 cm, which is quite long compared to even the mean-free-path length at the very minimum of the dip. Probably, another 30—40 cm of transport takes place in the vicinity of the 138 and 129 keV minima. It cannot be said that a minimum dominates the transport for this detector-distance combination, but it does seem as if a number of minima between them account for most of the distance travelled.

This picture is confirmed by the plot in Figure 5.9 which shows the location of the average radial coordinate of neutrons just before entering the region of the 24 keV minimum versus the radius of Detector 17 in an infinite medium. Neutron detection occurs along the 45-degree line. Thus, the vertical distance from the points to the 45-degree line indicates how transport takes place in the 24 keV minimum. To within the statistical accuracy, Figure 5.9 shows that for detector distance of 80 cm or more, about 25% of the total travel occurs in this dip. For a detector distance of 3 meter this corresponds to about 75 cm, which is far in excess of the mean-free-path length in the dip. Thus it is only for detector distances much greater than 1 meter that the favored trajectories involve improbably long paths in the minima (i. e. $\gg 30$ cm), magnifying the sensitivity of the penetrating flux to the values of the lowest cross section. For distances of 1 meter or less, this prominent minimum plays no special role in determining the penetrating flux at this and lower energies.

For Detector 14 (responding to neutrons between 120 and 200 keV) a location at 120 cm means proportionately greater penetration than for the detector at lower energy. Figure 5.6 shows that the average "unconstrained"

neutron penetrates only to a distance of about 60 cm by the time it reaches this energy. At all comparable energies the neutron observed by this detector (cf. Figure 5.10) travels farther than the neutron detected at 20—27 keV (cf. Figure 5.7). Even above the inelastic threshold there is a slight increase in the transport. But the greatest "stretch-out" occurs in the region of the minima below 400 keV, with about 50—60 cm of travel taking place in the regions of the minima around 300 keV and around 138 keV. The dependence on transport in the region of minima is accentuated in the response of Detector 12 (280—400 keV) to when located at 120 cm, cf. Figure 5.11. This distance is 3—4 times longer than the distance an "average" neutron travels by the time the detector energy range is reached. Figure 5.11 shows that about 75% of the transport takes place below 400 keV, and more than half of this penetration (~ 40 —50 cm) occurs in the region of the 300 keV minima.

On the basis of these results one would expect that for detectors of still higher energy, in the MeV range, the minima would play an important, even dominating role at closer distances, well within those encountered in practice. Unfortunately, the strong inelastic scattering process rapidly slows the neutrons to below the MeV region, and few survive to penetrate even moderate distances at these energies. Statistical errors in locating the center of the neutron "cloud" are large even at distances of 40 cm, and become overwhelming from 60 cm on. The measures of neutron location versus lethargy are therefore hard to depict and the interpretations become doubtful. For the analysis of the high energy detector responses

an altered subgroup structure was used, with the lethargy range from 15 MeV to 860 keV being divided into 37 subregions. Figure 5.12 shows the analysed mean distance at various lethargies of those neutrons which eventually make up the response of Detector 7 (1.0—1.3 MeV) at 20 cm from the source in an infinite medium, and Figure 5.13 shows the same analysis for the detector at 40 cm. Figure 5.13 shows the same analysis for the detector at 40 cm. Figure 5.12 for the 20 cm detector is not far different from the appropriate portion of Figure 5.6 for the "average" neutron. But in Figure 5.13 at twice the distance the plot shows a pronounced kink in slope first below 2 MeV. It hardly seems coincidence that pronounced minima first appear in the total cross section data set at and below this energy. Again no one minimum can be said to dominate; the added transport appears to come from the sum of the contributions provided by many minima.

When a low energy detector (in keV region) is placed very close to the source, the mean-radial distance versus lethargy curve will be qualitatively different from those discussed above. Consider, for example, the curve for neutrons reaching a 20—27 keV detector located at a radius of 20 cm (Figure 5.14), which is $\sim 23\%$ of the distance that "normal" neutrons would have travelled by this energy. These constrained neutrons travel about the same distance as the average neutron at energies above the inelastic threshold. Thus by the time they reach the threshold, the mean radius coordinate has already been about 17 cm. However, in the 800—100 keV region, the region with many deep cross section minima, these

Figure 5.6 Average Radial Distance of Slowing Down in An Infinite Medium
for Neutrons Emitted from A Point Fission Source

Figure 5.7 Average Radial Distance of Slowing Down for Neutrons Contributing to Detector 17 at 120 cm in An Infinite Medium

Figure 5.8 Average Radial Distance of Slowing Down for Neutrons Contributing to Detector 17 at 200 cm in An Infinite Medium

Figure 5.9 Center of Neutron Cloud at 27 keV Resonance
Versus Distance of Detector 17

Figure 5.10 Average Radial Distance of Slowing Down for Neutrons Contributing to Detector 14 at 120 cm in An Infinite Medium

Figure 5.11 Average Radial Distance of Slowing Down for Neutrons Contributing to Detector 12 at 120 cm in An Infinite Medium

Figure 5.12 Average Radial Distance of Slowing Down
for Neutrons Contributing to Detector 7
at 20 cm in An Infinite Medium

Figure 5.13 Average Radial Distance of Slowing Down
for Neutrons Contributing to Detector 7
at 40 cm in An Infinite Medium

Figure 5.14 Average Radial Distance of Slowing Down for Neutrons Contributing to Detector 17 at 20 cm in An Infinite Medium

neutrons only travel a mean radial distance of about 12 cm, which is considerably less than the 45 cm that an average neutron would travel. Note that the average radial distance of the neutrons goes beyond the detector radius. Below ~ 100 keV, the center of the neutrons gradually moves inwards, with about 5 cm travelling in the 24 keV minimum.

5.5. Relation of Cross Section to the Transport Parameters

The three collision parameters concerning neutron transport, i.e. the number of collisions, the path length and the scattering azimuth, have been investigated independently for the constant-cross-section problem in Chapter 3 and for the "small" iron problem in Chapter 4. It has been demonstrated that in a constant cross section medium with a monoenergetic source, no very unusual behavior can be observed in these parameters even at penetrating distance as far as twice of the distance that a "normal" neutron would have traveled. However, all the distributions of the parameters were shown to be somewhat biased so as to favor penetration toward the detector. In the small iron problem, we've shown that in deep penetration more collisions occurred in low cross section regions. The most important conclusion, however, is that the path length distributions in regions containing cross section minima are severely skewed toward long flights. Here the three parameters are again examined independently. The analysis results for Detector 17, which covers 20--27 keV region, are taken as typical examples.

5.5.1. Relation of Cross Section to the Number of Collisions

To investigate the effect of varying cross section on the behavior of collision numbers, we follow the method used in Chapter 4. The range of cross section variation was divided up into intervals by specifying 20 levels of magnitude. For those neutrons contributing to a given detector response the average number of collisions per history made below each cross section "level" was computed, along with the fraction of histories having collisions below that "level". As in Chapter 4, in order to give a common base for comparison with the reference case, the collisions were counted up to the upper energy limit of a given detector response. Figure 5.15 illustrates the analysis result for collision number for neutrons contributing to Detector 17. The abscissa "lethargy interval" again corresponds to the sum of the lethargy ranges in which the cross section is less than a given level. The lethargy ranges are those between 15 MeV and upper energy limit of a detector response. The corresponding cross section levels are also indicated on the abscissa as a secondary scale. The large increment in the lethargy range between 3 and 4 barns is due to the lethargy interval from 3 MeV to 15 MeV in which most cross sections are greater than 3 barns but less than 4 barns. Since neutrons have relatively large energy loss upon scattering (inelastic scattering) in this interval, the collision curves are flatter in the 3 and 4 barn region than for either the very high or the very low cross section regions. Because of the resultant S shape of the collision curves, the plot is not very useful in showing the effect of a particular cross section level on the collision number. To overcome

Figure 5.15 Average Number of Collision .VS. Lethargy Interval

— Detector 17 in An Infinite Medium

this, a similar analysis was carried out for those neutrons contributing to Detector 17 in which only those collisions made below 860 keV were taken into consideration. Thus the inelastic collisions which cause relatively large lethargy change upon scattering are removed and the lethargy width between 3 and 4 barns is considerably reduced. The resulting curves are more nearly straight lines and therefore easier to examine. Note that as a corresponding change, the lethargy interval for a given level now refers only to the energy range between 860 keV and the upper energy limit of a detector response. Figure 5.16 illustrates the result for Detector 17 over this restricted energy range.

It is seen from Figure 5.16 that there is no particular emphasis on the collisions at low cross section regions for both large and small penetrating distances relative to the "reference distance" (cf. Chapter 4). That this is not a result of excluding the collisions in the 20—27 keV region is evidenced in Figure 5.17 in which a plot similar to Figure 5.16 is given for Detector 18. Although the results for Detector 18 include the collisions in the 24 keV minimum, the shapes of the curves are still similar to those corresponding curves for Detector 17 in Figure 5.16. It is clear therefore that the results for Detector 17 represent the general features valid for any of the keV detector responses. To demonstrate even more clearly that there is no particular emphasis on the collisions at low cross section regions, Figure 5.18 presents the plot of the difference in the average number of collisions between the reference case and the case for a given distance of Detector 17 as a function of lethargy interval defined by the

Figure 5.16 Average Number of Collision .VS. Lethargy Interval — Detector I7 in An Infinite Medium, Collisions Taking Place above 860 keV Excluded

Figure 5.17 Average Number of Collision .VS. Lethargy Interval
 —Detector 18 in An Infinite Medium. Collisions
 Taking Place above 860 keV Excluded.

cross section levels. Figure 5.18 shows that there are about 16 additional collisions taking place at a detector distance of 200 cm compared to the reference case of the same detector response (D17). However, while about 5 of these additional collisions take place below 2 barns, there are also 5—6 additional collisions occurring above 4 barns. Similarly, at a detector distance of 20 cm there are about 20 fewer collisions taking place compared to the reference. Of these, about 5 fewer collisions are omitted below 2 barns whereas about 7 are lost above 4 barns. Therefore, for this fission problem, while there are significant changes in collision numbers for neutrons contributing to a given detector at distances both far away and close to the source, these changes occur as frequently in high cross section regions as they do in low cross section regions.

It may be noted that the additional collisions in low cross section regions do not by themselves account for all difference in spatial transport. Consider those neutrons contributing to Detector 17 at 200 cm for example. Even if we assume that all the additional scatterings below 2 barns were along the radius vector, the average additional radial transport due to the increase in collisions below 2 barns would still be less than 60 cm^{*}. From

* This may be realized by the following argument:

Since below 1 barn, there are about 2 additional collisions and below 2 barns there are about 5 additional collisions, we may approximate the additional collisions as a linear function of cross section, i. e. $\sigma/\Delta\bar{n} \sim 2/5$ for $0.3b \leq \sigma \leq 2b$, where $0.3b$ is the lowest cross section for iron. The average additional radial transport Δr is

$$\Delta r = \int \lambda(\sigma) d\Delta\bar{n} = \int \frac{1}{N\sigma} d\Delta\bar{n},$$

where $\lambda(\sigma)$ is the mean-free-path length at cross section σ and N is the nuclear density. Since $1/N = 12$ cm-barn for iron, so

$$\Delta r = \int \frac{12(\text{cm-barn})}{\sigma(b)} d\Delta\bar{n} = 30 \int_{0.3}^2 \frac{d\tau}{\tau} (\text{cm}) = 57 (\text{cm}).$$

Figure 5.18 Difference of Mean Collision Number
Between Constrained Neutrons and
Reference Neutrons — Detector 17

Figure 5.6 it is seen that an average neutron contributing to the same detector response travels about 87 cm in radial distance. Therefore even under the above assumption, the further transport still leaves the neutrons about 50 cm short of the detection.

As a final note, it is interesting to compare Figure 5.16 to Figure 3.6. Figure 5.16 can be viewed as giving the average number of collisions per unit lethargy at various cross section levels for elastic collision event in the life histories of those neutrons contributing to Detector 17. Figure 3.6 showed the average number of collisions per unit lethargy for neutrons slowing down in a featureless medium and contributing to a given detector. The curves in both figures have similar character, namely the average number of collisions per unit lethargy is constant or nearly so irrespective of the cross section condition for a given detector distance.

5.5.2. Relation of Cross Section to the Path Length Distribution

The distribution of path length for histories contributing to various energy detectors in an infinite medium have also been studied. The range of cross section variations was divided into several intervals. From the histories of those neutrons contributing to a given detector response at a given distance from the source, the distribution of path length in the unit of mean free path was computed separately in each of the cross section intervals. Four such cross section intervals were used: $(0, 1)$, $(1, 2)$, $(2, 4)$ and $(4, \infty)$, where the unit is in barns. The distribution was grouped into 4 bins, with bin boundaries the same as those used in Chapter

3 and 4. Again if the distribution were exponential with mean 1, each bin would have equal probability of 25%. This exponential distribution serves as the base for comparison. Some of the results for Detector 17 are tabulated in Table 5.2. Table 5.2 includes an entry for the χ^2 values computed from the Monte Carlo samples. The 1% significant level for 3 degree of freedom gives χ^2 value 11.3.

It is seen from Table 5.2 that the distributions for the reference case fit quite well with the base distribution as one would expect. However, at the detector distance of 20 cm, which is very close to the source, it is quite clear that when the cross section is below 1 barn, the distribution is skewed toward shorter path lengths. When the cross section is greater than 1 barn on the other hand, the distributions deviate only slightly from the base distribution. At the detector distance of 1 meter, which, as we recall, is not far from the "reference distance", the distributions resemble the base distribution. A possible exception is the distribution for cross sections less than 1 barn, in which there maybe statistically significant evidence for a small bias to account for the slightly larger transport. When the detector is moved to 200 cm, which is more than twice of the distance that an average neutron would travel, the Monte Carlo result clearly shows that the selection of path length by the detected neutrons is biased toward long flight for collisions in the regions of cross section minima. For instance, 32% of the collisions below 1 barn are in the longest bin (Bin #4), which is about 30% greater than the base distribution. This phenomenon is more accentuated at even deeper penetrating distances.

Table 5.2 Distribution of Normalized Path Length for
Neutrons Contributing to Detector 17 in
An Infinite Medium

Reference Case

Cross Section Interval $\sigma_1 - \sigma_2$ (Barns)	Bin #1	Bin #2	Bin #3	Bin #4	χ^2
0.0 - 1.0	0.252	0.248	0.250	0.249	2.0
1.0 - 2.0	0.251	0.250	0.250	0.249	0.7
2.0 - 4.0	0.250	0.249	0.250	0.251	1.8
4.0 - ∞	0.250	0.249	0.251	0.250	2.7

20 cm-Radius Sphere

Cross Section Interval $\sigma_1 - \sigma_2$ (Barns)	Bin #1	Bin #2	Bin #3	Bin #4	χ^2
0.0 - 1.0	0.291	0.292	0.271	0.146	374.1
1.0 - 2.0	0.250	0.265	0.258	0.227	34.3
2.0 - 4.0	0.256	0.251	0.251	0.242	14.8
4.0 - ∞	0.252	0.252	0.252	0.244	12.0

100 cm-Radius Sphere

Cross Section Interval $\sigma_1 - \sigma_2$ (Barns)	Bin #1	Bin #2	Bin #3	Bin #4	χ^2
0.0 - 1.0	0.247	0.242	0.251	0.260	16.8
1.0 - 2.0	0.249	0.252	0.250	0.249	0.7
2.0 - 4.0	0.249	0.248	0.250	0.253	5.5
4.0 - ∞	0.249	0.249	0.253	0.249	6.7

200 cm-Radius Sphere

Cross Section Interval $\sigma_1 - \sigma_2$ (Barns)	Bin #1	Bin #2	Bin #3	Bin #4	χ^2
0.0 - 1.0	0.223	0.212	0.244	0.321	77.6
1.0 - 2.0	0.249	0.226	0.260	0.265	12.7
2.0 - 4.0	0.246	0.245	0.252	0.257	4.1
4.0 - ∞	0.246	0.247	0.249	0.258	5.2

260 cm-Radius Sphere

Cross Section Interval $\sigma_1 - \sigma_2$ (Barns)	Bin #1	Bin #2	Bin #3	Bin #4	χ^2
0.0 - 1.0	0.194	0.177	0.248	0.381	47.1
1.0 - 2.0	0.292	0.228	0.225	0.255	7.5
2.0 - 4.0	0.253	0.237	0.270	0.240	5.0
4.0 - ∞	0.254	0.262	0.218	0.266	14.6

* Bin boundaries : Bin #1: (0.000, 0.288), Bin #2: (0.288, 0.693)
Bin #3: (0.693, 1.386), Bin #4: (1.386, ∞)

For example, at penetrating distance of 260 cm, the corresponding fraction is about 50% greater than the base distribution, although the statistics are less significant. In both 200 cm and 260 cm cases, the distributions at high cross section regions are not significantly different from the base distribution.

We conclude therefore that, in neutron deep penetration, the distribution of path length for scatterings in the cross section minima is skewed toward long flight. This skewed behavior becomes more and more pronounced as the penetrating distance increases relative to the reference population. Conversely, for neutrons staying close to the source location, the distribution of the travelling distance between two collisions at energies of cross section minima is skewed toward shorter flights.

5.5.3. Relation of Cross Section to the Distribution of Scattering Azimuth

In Chapter 3 and 4, we studied the distribution of scattering azimuth relative to a pre-defined reference direction. Here we use the same definition of reference direction to study the azimuthal angle of scattering for fission neutron penetrating in iron. The reference direction is that which gives the maximum probability of detection for a scattering. In spherical geometry, the reference direction is along the projection of the radius vector on the plane normal to the incident vector for the scattering (cf. Chapter 3). If the scattering takes place inside the detector sphere the outward radius vector is used, otherwise the reference direction is defined relative to the inward radius vector. In this manner the reference

direction is always oriented towards the detector spherical surface. Again as in Chapter 3 and 4, four bins are used to study the distribution. The choice of the bin boundaries was such that if the distribution were isotropic, each bin would have equal probability of 25%. Bin #1 is designated as the one which gives the preferential direction toward the reference vector. A detailed description of the bins was given in Chapter 3. To unravel any effect of varying cross section, the distribution was separated according to the cross section magnitudes at which the scattering takes place. As in the preceding section for the path length distribution, the differential data are shown in the following discussion.

Table 5.3 lists the distributions of scattering azimuth for four cross section intervals for Detector 17 under various circumstances in an infinite medium. The computed χ^2 values are also shown. The 1% significance level is 11.3. Clearly, the reference case agrees quite well with the uniform distribution. It may be mentioned here again that since no geometric boundary is imposed for the reference case, the reference direction for the scattering azimuth is always outward.

As evidenced by the χ^2 values in Table 5.3, the deviations for fixed detector locations from the base distribution are most significant when collisions take place in regions having cross section magnitude less than 1 barn. Furthermore, the distributions of scattering azimuth in these low cross section regions show the largest differences from uniformity, being always skewed toward the reference direction, i.e. the distribution is always high in Bin #1 and always low in Bin #4. Such a bias is more pronounced

Table 5.3 Distribution of Scattering Azimuth for Neutrons
Contributing to Detector 17 in An Infinite Medium

Reference Case

Cross Section Interval $\sigma_1 - \sigma_2$ (Barns)	Bin #1	Bin #2	Bin #3	Bin #4	χ^2
0.0 - 1.0	0.250	0.248	0.249	0.253	3.6
1.0 - 2.0	0.253	0.249	0.248	0.250	4.7
2.0 - 4.0	0.250	0.251	0.249	0.250	1.8
4.0 - ∞	0.250	0.250	0.249	0.251	2.2

20 cm-Radius Sphere

Cross Section Interval $\sigma_1 - \sigma_2$ (Barns)	Bin #1	Bin #2	Bin #3	Bin #4	χ^2
0.0 - 1.0	0.322	0.262	0.215	0.201	226.8
1.0 - 2.0	0.278	0.259	0.240	0.223	69.4
2.0 - 4.0	0.263	0.257	0.245	0.235	66.2
4.0 - ∞	0.263	0.254	0.244	0.239	85.0

100 cm-Radius Sphere

Cross Section Interval $\sigma_1 - \sigma_2$ (Barns)	Bin #1	Bin #2	Bin #3	Bin #4	χ^2
0.0 - 1.0	0.278	0.266	0.237	0.219	184.5
1.0 - 2.0	0.262	0.261	0.242	0.235	67.5
2.0 - 4.0	0.256	0.255	0.247	0.242	51.7
4.0 - ∞	0.253	0.254	0.247	0.246	27.7

200 cm-Radius Sphere

Cross Section Interval $\sigma_1 - \sigma_2$ (Barns)	Bin #1	Bin #2	Bin #3	Bin #4	χ^2
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for penetrating distances that are either quite small or very large in comparison to the penetrating distance of an average neutron. For Detector 17 at a distance of 100 cm from the source, a distance only slightly larger than the "reference distance" of penetration, the fractional distribution in Bin #1 is only about 11% larger than uniformity would predict for collisions taking place at very low cross section values (less than 1 barn). The percent deviation for the same bin and cross section condition is about 29% when the detector is at a distance of 20 cm and about 33% when the detector is at a distance of 200 cm. When the detector is moved to 260 cm, this percentage deviation increases to about 48% but with reduced statistical significance.

When the collisions occur at cross sections higher than 1 barn, our Monte Carlo samples still show significant deviations from the base distribution, at least for detector distances of 200 cm or less. For the most part these distributions exhibit some bias towards the reference direction, but as can be seen from Table 5.3, the deviations are considerably less than when the collisions occur in the regions of the minima.

Therefore, along with the observations for the path length distribution in Section 5.5.2, the analysis of the scattering azimuths in the Monte Carlo histories shows that collisions in the cross section minima play a far more important role than those in high cross section regions in determining the trajectories of exceptional neutrons despite the fact that there are still significant changes in collision numbers at high cross section regions. In particular, in deep penetration, neutrons not only have a skewed distribution

of path length favoring long flight paths at the energy of cross section minima, but under the same conditions also have higher probability of picking up the "right" direction to get them to the detector.

It might seem strange that even when the detector is close to the source, there is a strong bias towards the reference direction for collisions in the regions of the minima. However it may be remembered from Figure 5.14 that for a detector at 20 cm in an infinite medium neutrons at the energies of the deep minima are near or beyond the radius of the detector. If they make collisions at low cross sections the only chance they have of still reaching the detector is if they scatter backward toward the source. Because most of them are already beyond the detector this requires an azimuth close to the reference direction.

That this reasoning is correct can be seen by looking at the distribution of azimuth directions for the same detector location, but in a bare system. Neutrons contributing to a low energy detector on the surface of a small bare assembly must be confined within the geometrical boundary in the process of slowing down. Consequently, the direction of scattering azimuths is unlikely to have higher probability towards a direction surrounding the reference vector defined above. Table 5.4 gives the distributions for scattering azimuth for neutrons contributing to Detector 17 for a 20 cm-radius bare sphere. The distributions are again computed according to the cross section magnitude. Because very few histories manage to reach the detector for this system, the statistics are rather poor. The χ^2 values shown in Table 5.4 are therefore not as impressive as those for the infinite

medium case. But, if we now use a 5% significance level, corresponding to a χ^2 value of 7.8, we would have to reject the hypothesis of uniformity in the distribution of scattering azimuth for collisions taking place at regions with cross section less than 1 barn and greater than 2 barns. The distribution for collisions at cross section regions between 1 barn and 2 barns is not certain because of poor statistics.

It is seen from Table 5.4 that the distribution for azimuthal scattering angle is preferentially biased away from the reference direction. The biasing is strongest for collisions at low cross sections, but appears definitely present even for high cross section collisions. There seems little doubt as to the reason for this behavior. Almost all the collisions occur at energies above the upper detector limit—with a bare assembly once a neutron is "counted" by the detector it is lost from the system. After emerging from a collision at any cross section level a neutron has a good chance of leaking from the small bare sphere. This is especially true for

Table 5.4 Distribution of Scattering Azimuth for Neutrons Contributing to Detector 17 for A 20 cm-Radius Bare Sphere

Cross Section Interval $\sigma_1 - \sigma_2$ (Barns)	Bin #1	Bin #2	Bin #3	Bin #4	χ^2
0.0 - 1.0	0.189	0.238	0.250	0.323	9.1
1.0 - 2.0	0.245	0.229	0.242	0.284	2.5
2.0 - 4.0	0.216	0.253	0.253	0.278	11.3
4.0 - ∞	0.233	0.249	0.250	0.268	12.4

scatterings in the region of the minima. Hence those histories in which the neutron managed to slow down to the detector energy without leaking are likely to involve collisions which sent the neutron preferentially back to the source, away from the reference direction which here points outward. The mechanisms whereby a neutron survives to low energies for small detector-source distances are thus quite different for a reflected sphere compared to a bare sphere. The next sections will analyse these mechanisms for the bare assembly in greater detail.

5.6. Study of Low Energy Leakage Neutrons from a Small Bare Assembly

In this section we use the Monte Carlo histories to analyse in detail the characteristics of neutrons leaking at low energies from a small bare assembly—an "exceptional" situation almost the opposite extreme to deep penetration in an infinite medium. Most integral experiments used to "test" evaluated cross section sets have been made with relatively small systems, and the analysis of such a geometry therefore has considerable practical importance. The unusual features of the leakage spectrum from a 20 cm-radius bare sphere have previously been touched on in the preceding section when discussing the distribution of scattering azimuths. It was already noted there that high leakage in such a system makes it very difficult for a neutron to remain within the sphere down to low energies. Here we examine how neutrons reach low energy detectors on the bare sphere in such a manner as to minimize prior leakage.

With a fission source, neutrons can be born either below or above

the lowest inelastic excitation threshold of iron. Each of these two classes of neutrons contributes in some proportion to the response of a detector responding at energies below this threshold. Since quite different phenomena may be involved in the histories of these two categories of neutrons, we shall discuss them separately in what follows.

5.6.1. Effect of Source Energy on the Neutrons Detected at Energies below 500 keV

Of all source neutrons, 26% are born below the lowest inelastic excitation threshold of iron (^{56}Fe 1-st excited level at threshold energy 860 keV). The only mechanism that slows these neutrons down to still lower energies is elastic collision at a rate of about 28 collisions per unit lethargy. In the flight between each collision the neutron has a good chance of leakage, especially when its energy corresponds to one of the many cross section minima in the keV region. The probability that such a neutron survives to be picked up by a low energy detector will therefore be negligible unless it is born at an energy not too far above the detector energy range. Table 5.5 demonstrates how well this prediction is fulfilled. The mean source energies are shown for those neutrons born at energies below 860 keV which subsequently contribute to various detectors with response energies below 500 keV. Four geometrical situations are considered: 20 cm bare sphere, 20 cm reflected sphere, 100 cm reflected sphere, and the reference population, where detection is not constrained to occur at any particular distance in an infinite medium. For the reference population,

Table 5.5 Mean Source Energy (MeV) for Neutrons Having Initial Energy $\leq$ 860 keV
and Contributing to Detectors with Response Energy $\leq$ 500 keV

Detector Number	Detector Energy (MeV)	Reference Case	20 cm Sphere Bare	20 cm Sphere Reflected	100 cm Sphere Reflected
11	0.500 - 0.400	0.636 $\pm$ 0.003	0.587 $\pm$ 0.006	0.639 $\pm$ 0.007	0.726 $\pm$ 0.021
12	0.400 - 0.280	0.577 $\pm$ 0.004	0.463 $\pm$ 0.003	0.537 $\pm$ 0.005	0.666 $\pm$ 0.013
13	0.280 - 0.200	0.540 $\pm$ 0.003	0.358 $\pm$ 0.011	0.469 $\pm$ 0.012	0.648 $\pm$ 0.019
14	0.200 - 0.120	0.505 $\pm$ 0.003	0.236 $\pm$ 0.008	0.391 $\pm$ 0.014	0.587 $\pm$ 0.009
15	0.120 - 0.060	0.483 $\pm$ 0.003	0.171 $\pm$ 0.007	0.346 $\pm$ 0.016	0.582 $\pm$ 0.019
16	0.060 - 0.027	0.474 $\pm$ 0.003	0.109 $\pm$ 0.004	0.305 $\pm$ 0.030	0.570 $\pm$ 0.024
17	0.027 - 0.020	0.476 $\pm$ 0.003	0.080 $\pm$ 0.005	0.290 $\pm$ 0.012	0.529 $\pm$ 0.009
18	0.020 - 0.010	0.475 $\pm$ 0.004	0.044 $\pm$ 0.013	0.357 $\pm$ 0.019	0.517 $\pm$ 0.016

the mean source energy of the detected neutrons increases as the detector energy limits decreases, asymptotically approaching a value of about 475 keV, which is about the same as the average energy of fission neutrons born below 860 keV. This is of course as it should be.* However, with the 20 cm bare sphere the leakage probability during slowing down is so high that the mean source energy of the detected neutrons can not be more than tens of keV above the detector upper limit. Thus, with detector 17 sensitive up to 27 keV, the mean source energy is only about 80 keV, even though fewer than 1% of the fission neutrons being considered are born below 80 keV. The behavior of the 20 cm bare detector sphere is in marked contrast to the other geometries analysed. Thus with the 20 cm sphere in an infinite medium the mean source energy for Detector 17, while still below that for the reference population, is almost 4 times higher than for a bare sphere of the same radius. With the 100 cm reflected sphere the mean source energy is significantly above that for the reference population. What is taking place in these infinite medium cases is easily understandable. There is no loss mechanism here, but in the 20 cm case neutrons at the higher source energy tend to go so far out that there is a lessened chance of their diffusing back to the detector by the time they have slowed down

* For the reference population, absorption is the only loss in the slowing down process. The absorption generally occurs at very low energies. Thus the reference population contributing to detectors responding to energies below 100 keV practically do not have bias in selecting the source energies.

to the detector energy. Contrariwise for the 100 cm detector, the higher energy source neutrons have a better chance of going out that far before they have been scattered to below the detector energy. The behavior at these two distances is in complete accord with the conclusions drawn above (Section 5.4) from the mean-distance curves relative to the reference population.

Similar calculations can be performed for neutrons born above the lowest excitation threshold. However, the results are found not very useful, for the mean source energies computed for the 20 cm-radius bare sphere case are not very much different from those for the reference case even for very low energy detectors such as Detector 17 or 18. This is actually a coincident, for the source distributions are very different. Instead therefore, the distribution of the source energy of all detected neutrons was computed for two low energy detectors (Detector 17 and 18). (This distribution also provides valuable information for those neutrons born below the inelastic threshold). A comparison of the distribution of neutrons contributing to Detector 17 for 20 cm-radius bare sphere case to that for the reference case is shown in Figure 5.19; and a similar comparison for the 100 cm-radius reflected sphere case is shown in Figure 5.20.

Since the life histories of those neutrons contributing to Detector 17 located at 100 cm are not very different from the life history of the reference neutron as was seen above, p. 99, the distribution of source energy for these constrained neutrons also resembles that for the reference

Figure 5.19 Distribution of Source Energy for
Neutrons Contributing to Detector 17
20 cm-Radius Bare Sphere Case .VS.
Reference Case

Figure 5.20 Distribution of Source Energy for Neutrons
Contributing to Detector 17
100 cm-Radius Reflected Sphere Case .VS.
Reference Case

neutron, as is evidenced by Figure 5.20. However, for the 20 cm-radius bare sphere case, a clearly different picture is observed. According to Figure 5.19, essentially no neutrons born between 270 keV and 860 keV can ever reach Detector 17. Neutrons born between 270 keV and 20 keV account for about 40% of the total detector response. Even more striking is the almost discrete line structure in the source distribution of neutrons above 860 keV that contributing to Detector 17. There is a sharp and high peak a little below 1 MeV, another about 2 MeV and possibly a third peak near 3 MeV. It can hardly be coincidence that these energies correspond fairly well to the discrete levels inelastically excited in the abundant ^{56}Fe isotope. Neutrons born in the MeV region can reach the 20—27 keV region of Detector 17 in such a small bare sphere only if they inelastically scatter near the threshold of one of the discrete levels and thus slow down to below ~ 100 keV in only one collision. Exactly how inelastic scatterings affect the trajectories of low energy leakage neutrons of a small bare geometry is further explored in the next sub-section.

5.6.2. Role of Inelastic Scattering in the Trajectories for Leakage Neutrons below 500 keV

To investigate the role of inelastic scattering with large lethargy change in neutron trajectories, we shall consider a "conditioned" class of neutrons in this section. The "neutron" or "history" mentioned in the following discussion, if not otherwise specified, will only refer to those with source energies greater than the lowest inelastic excitation threshold

and the "inelastic scattering" will also only refer to an interaction for which a neutron is inelastically scattered from energy above threshold to the energy below the lowest threshold (860 keV).

Four quantities have been estimated from the Monte Carlo histories:

- (a) the fraction of detected neutrons which are inelastically scattered,
- (b) the fraction of detected neutrons which are inelastically scattered by the lowest ^{56}Fe level (1-st level inelastic scattering),
- (c) the mean energy at which the 1-st level inelastic scattering took place,
- (d) the mean energy after the 1-st level inelastic scattering.

For detector energies below 500 keV and for histories contributing to a 20 cm bare, 20 cm reflected and 100 cm reflected sphere, the results are shown in Table 5.6. The numbers for the reference population are also given in this table.

For the 20 cm bare sphere and for detector energy below 500 keV, the fraction of detected neutrons which are inelastically scattered is about 20—25% higher than that for the reference case. For detector energy below 100 keV, this fraction is 1.0, indicating that all the neutrons reach detector as a result of inelastic scattering. Even in the 400—500 keV detector energy region, the fractional contribution to the detector response due to inelastically scattered neutrons is about 95%. Furthermore, of these inelastically scattered neutrons, 80—90% are those scattered by the ^{56}Fe 1-st excited level. For detector energy below 100 keV, neutrons scattered by this level and subsequently contributing to detector response account for 88—90% of the detection. When detector energy is in the 400—500 keV

Table 5.6 Effect of ^{56}Fe 1-st Level Inelastic Scattering to the Neutrons Born above 860 keV and Contributing to Detectors with Response Energy $\leq$ 500 keV

Detector Number	Detector Energy (MeV)	F_i	F_l	E_b (MeV)	E_a (MeV)
Reference Case:					
11	0.500 - 0.400	0.700 ± 0.008	0.564 ± 0.007	1.500 ± 0.003	0.616 ± 0.003
12	0.400 - 0.280	0.755 ± 0.009	0.616 ± 0.008	1.422 ± 0.003	0.541 ± 0.003
13	0.280 - 0.200	0.791 ± 0.007	0.655 ± 0.006	1.359 ± 0.002	0.480 ± 0.002
14	0.200 - 0.120	0.810 ± 0.008	0.677 ± 0.007	1.315 ± 0.003	0.437 ± 0.003
15	0.120 - 0.060	0.820 ± 0.008	0.692 ± 0.007	1.282 ± 0.002	0.405 ± 0.002
16	0.060 - 0.027	0.823 ± 0.007	0.697 ± 0.007	1.273 ± 0.003	0.397 ± 0.003
17	0.027 - 0.020	0.822 ± 0.008	0.695 ± 0.007	1.270 ± 0.003	0.394 ± 0.003
18	0.020 - 0.010	0.822 ± 0.009	0.696 ± 0.008	1.264 ± 0.004	0.388 ± 0.003
20 cm-Radius Bare Sphere:					
11	0.500 - 0.400	0.955 ± 0.010	0.749 ± 0.024	1.435 ± 0.008	0.553 ± 0.008
12	0.400 - 0.280	0.988 ± 0.005	0.772 ± 0.012	1.327 ± 0.003	0.449 ± 0.003
13	0.280 - 0.200	0.994 ± 0.003	0.818 ± 0.015	1.188 ± 0.004	0.316 ± 0.004
14	0.200 - 0.120	0.998 ± 0.005	0.823 ± 0.019	1.102 ± 0.002	0.231 ± 0.003
15	0.120 - 0.060	1.000 ± 0.000	0.885 ± 0.016	1.010 ± 0.006	0.143 ± 0.005
16	0.060 - 0.027	1.000 ± 0.000	0.892 ± 0.030	0.974 ± 0.007	0.107 ± 0.006
17	0.027 - 0.020	1.000 ± 0.000	0.878 ± 0.013	0.946 ± 0.007	0.080 ± 0.007
18	0.020 - 0.010	1.000 ± 0.000	0.925 ± 0.056	0.882 ± 0.005	0.021 ± 0.002
20 cm-Radius Reflected Sphere:					
11	0.500 - 0.400	0.841 ± 0.018	0.683 ± 0.013	1.485 ± 0.005	0.600 ± 0.005
12	0.400 - 0.280	0.892 ± 0.017	0.710 ± 0.015	1.394 ± 0.004	0.513 ± 0.004
13	0.280 - 0.200	0.908 ± 0.019	0.757 ± 0.016	1.284 ± 0.005	0.407 ± 0.005
14	0.200 - 0.120	0.919 ± 0.015	0.760 ± 0.012	1.206 ± 0.006	0.332 ± 0.006
15	0.120 - 0.060	0.923 ± 0.019	0.787 ± 0.015	1.148 ± 0.009	0.276 ± 0.009
16	0.060 - 0.027	0.934 ± 0.033	0.806 ± 0.027	1.092 ± 0.009	0.221 ± 0.009
17	0.027 - 0.020	0.940 ± 0.022	0.815 ± 0.016	1.120 ± 0.012	0.249 ± 0.012
18	0.020 - 0.010	0.917 ± 0.055	0.797 ± 0.041	1.134 ± 0.016	0.261 ± 0.016

18 0.020 - 0.010 0.917 ± 0.055 0.797 ± 0.041 1.134 ± 0.016 0.261 ± 0.016

100 cm-Radius Reflected Sphere:

11	0.500 - 0.400	0.460 ± 0.073	0.371 ± 0.069	1.603 ± 0.022	0.719 ± 0.025
12	0.400 - 0.280	0.598 ± 0.024	0.478 ± 0.021	1.539 ± 0.007	0.653 ± 0.007
13	0.280 - 0.200	0.664 ± 0.041	0.518 ± 0.037	1.467 ± 0.013	0.584 ± 0.013
14	0.200 - 0.120	0.728 ± 0.016	0.591 ± 0.014	1.424 ± 0.012	0.543 ± 0.012
15	0.120 - 0.060	0.683 ± 0.023	0.563 ± 0.016	1.392 ± 0.015	0.512 ± 0.015
16	0.060 - 0.027	0.751 ± 0.039	0.617 ± 0.034	1.360 ± 0.019	0.482 ± 0.019
17	0.027 - 0.020	0.791 ± 0.017	0.656 ± 0.014	1.318 ± 0.008	0.441 ± 0.008
18	0.020 - 0.010	0.810 ± 0.022	0.690 ± 0.018	1.299 ± 0.006	0.422 ± 0.006

Note:

- (a) F_i : Fraction of detected neutrons which are inelastically scattered by all levels from energy above threshold to energy below the lowest inelastic excitation threshold (^{56}Fe 1-st level)
- (b) F_1 : Fraction of detected neutrons which are inelastically scattered by the ^{56}Fe 1-st excited level from energy above the threshold to energy below the threshold
- (c) E_b : Mean energy at which the ^{56}Fe 1-st excited level inelastic scattering took place
- (d) E_a : Mean energy after the ^{56}Fe 1-st excited level inelastic scattering

region, neutrons scattered by this level still account for about 75% of the detector response. Thus an overwhelming proportion of the leakage spectrum below 500 keV for the 20 cm bare sphere is a result of inelastic scattering, especially scattering by excitation the ^{56}Fe 1-st level. For a reflected sphere or larger spheres, the fraction of detected neutrons which are inelastically scattered is in general smaller.

However, it is the energy at which the inelastic scattering takes place that most clearly characterizes the trajectories of the low energy leakage neutrons for the 20 cm radius bare sphere case. For the lowest energy detector (Detector 18), the average initial energy of scattering from the 1-st level is only 21 keV above the threshold (correspondingly, the mean energy after the scattering is only about 21 keV). For Detector 17 which covers the 24 keV minimum, the average initial energy of scattering is about 80 keV above the threshold. When the detector energy is in the 400—500 keV range (Detector 11), this average initial energy of scattering is greater than 1.4 MeV, which is more than 500 keV above the threshold. Thus the favored inelastic collisions are those which bring the neutron to within or just above the detector energy range. For the lower energy detectors this means the inelastic scattering must take place very close to the threshold. In contrast for the reference population in an infinite medium, even at the lowest detector energies the average inelastic scattering occurs about 400 keV above threshold.

There are also significant differences in the results for bare and reflected spheres and for spheres of various sizes as well. For example,

for Detector 17 and a bare 20 cm sphere, the average initial energy at scattering from the 1-st level is about 80 keV above the threshold, but with a 20 cm reflected sphere, the mean energy at scattering from the 1-st level is 250 keV above the threshold. Furthermore, with a 100 cm bare sphere, this initial energy of scattering has increased to about 400 keV above the threshold (not shown in the table) and to about 450 keV for a 100 cm reflected sphere.

While the inelastic scatterings from levels higher than the first contribute a much smaller fraction to the exit current, the initial energies of scattering of those leakage neutrons at low energies (≤ 100 keV) are also close to the threshold for small bare systems and increase rapidly with large or reflected systems.

It may be noted that neutrons inelastically scattering in small bare systems seldom land into detector energy range directly by inelastic scattering. On the average, they are carried by inelastic scattering to an energy region not too far above detector response energy, and then suffer further elastic collisions before leaking out of the system. The number of collisions suffered by neutrons after the inelastic scattering depends on the characteristics of the cross section immediately above the detector energy limit. This is evidenced by the results shown in Table 5.7 in which the average number of collisions are given for neutrons contributing to detectors with response energy below 500 keV (Notice that the data shown in Table 5.7 are only for those neutrons which are slowed down to energies below the lowest inelastic excitation threshold by inelastic scattering).

Table 5.7 Average Number of Collisions for Neutrons Having Initial Energy above 860 keV and Contributing to Detectors with Response Energy below 500 keV*

Detector No.	$\frac{1}{n_1}$	$\frac{2}{n_{ei}}$	$\frac{3}{n_t}$
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Reference Case:

11	1.49 ± 0.01	3.37 ± 0.05	16.41 ± 0.19
12	1.53 ± 0.01	3.59 ± 0.07	21.34 ± 0.16
13	1.55 ± 0.01	3.83 ± 0.07	27.08 ± 0.17
14	1.57 ± 0.01	4.00 ± 0.07	36.06 ± 0.16
15	1.58 ± 0.01	4.13 ± 0.06	49.61 ± 0.14
16	1.58 ± 0.01	4.16 ± 0.07	68.67 ± 0.21
17	1.57 ± 0.01	4.15 ± 0.06	83.37 ± 0.28
18	1.57 ± 0.01	4.17 ± 0.06	96.07 ± 0.32

20 cm-Radius Bare Sphere:

11	1.47 ± 0.04	2.79 ± 0.20	10.71 ± 0.35
12	1.47 ± 0.02	2.75 ± 0.11	12.94 ± 0.22
13	1.54 ± 0.03	3.13 ± 0.11	11.91 ± 0.25
14	1.52 ± 0.03	2.94 ± 0.13	15.13 ± 0.26
15	1.55 ± 0.03	3.24 ± 0.12	17.32 ± 0.91
16	1.41 ± 0.11	3.93 ± 0.46	26.60 ± 1.29
17	1.45 ± 0.05	2.79 ± 0.16	32.80 ± 1.50
18	1.32 ± 0.15	3.94 ± 0.96	12.32 ± 2.36

20 cm-Radius Reflected Sphere:

11	1.52 ± 0.03	3.13 ± 0.10	14.23 ± 0.24
12	1.48 ± 0.01	3.24 ± 0.07	17.97 ± 0.26
13	1.53 ± 0.03	3.64 ± 0.13	20.31 ± 0.30
14	1.57 ± 0.02	3.76 ± 0.13	26.34 ± 0.58
15	1.65 ± 0.03	4.07 ± 0.14	35.64 ± 0.59
16	1.66 ± 0.06	4.43 ± 0.30	47.82 ± 1.28
17	1.60 ± 0.03	4.19 ± 0.21	65.44 ± 1.06
18	1.63 ± 0.06	4.35 ± 0.23	74.49 ± 1.98

100 cm-Radius Reflected Sphere:

11	1.55 ± 0.12	4.47 ± 1.00	27.85 ± 2.14
12	1.56 ± 0.06	3.36 ± 0.19	29.57 ± 0.70
13	1.51 ± 0.04	3.81 ± 0.27	34.93 ± 0.57
14	1.51 ± 0.03	3.84 ± 0.20	45.74 ± 0.70
15	1.57 ± 0.05	3.91 ± 0.17	59.69 ± 1.10
16	1.55 ± 0.07	3.48 ± 0.43	74.82 ± 1.55
17	1.57 ± 0.02	4.03 ± 0.11	88.46 ± 0.70
18	1.58 ± 0.03	4.23 ± 0.22	98.76 ± 1.02

Note:

- * The results are only for those detected neutrons which were scattered inelastically in their life histories from an energy above threshold to an energy below 860 keV
- 1. Average number of inelastic collisions
- 2. Average number of elastic collisions before last inelastic scattering taking place
- 3. Average total number of collisions

It is seen from this table that the mean number of collisions for neutrons contributing to detectors for the case of 20 cm bare sphere is considerably less than the corresponding result for the reference case, particularly for detectors with response energy below about 100 keV. For instance, those contributing to Detector 17 for a 20 cm-radius bare sphere suffer a total of about 32 collisions, whereas for the reference case a "normal" neutron suffers a total of about 83 collisions. Furthermore, for a 20 cm-radius bare sphere, neutrons exiting in Detector 17 were carried by inelastic scattering into the high cross section resonance region above the 24 keV minimum, consequently these neutrons suffered about 28 further collisions before finally leaking. For Detector 18 covering 10—20 keV region, the final energy after the inelastic scattering was in the vicinity of the 24 keV dip and it took only about 8 further collisions for neutrons to be lost to the system. For neutrons contributing to a reflected sphere or to larger spheres, the energy after inelastic scattering is well above detector energy limit, consequently for a given detector response, they suffer many more collisions than do neutrons contributing to a 20 cm bare sphere. In particular, histories contributing both to Detector 17 and to Detector 18 were scattered by inelastic scattering to the energy region above the 27 keV resonance, it took about 9 more collisions for neutrons to be detected in Detector 18 than in Detector 17.

5.7. Conclusion

In this chapter, Monte Carlo analysis has been used to unravel the transport mechanisms of fission neutrons in various geometrical configurations of iron. Special emphasis has been on "exceptional" neutrons, i.e. situations in which the detected neutrons must have histories that differ markedly from those of the average neutron reaching a given energy region. One such exceptional situation has been the spectrum of neutrons below ~ 500 keV leaking from bare spheres with radii of the order of 20 cm. Here the analysis has been largely successful in isolating the distinctive features of the contributing neutron histories. In particular, it has been shown that the leakage spectrum at these energies is very sensitive to the values of the inelastic cross section near the threshold of the lowest excited levels. Further the elastic scattering processes play paradoxical roles compared to what occurs at normal or deep penetrations. Scattering in the high cross section, resonance peak, regions is important in suppressing leakage as the neutrons slow down. The minima, on the other hand, function only to promote leakage and therefore reduce the detected spectrum below them. This exceptional behavior is of considerable practical importance, because it has become fashionable to test the validity of evaluated sets of iron data with integral experiments on geometries comparable to the small bare spheres studied here. The analysis presented above emphasizes the danger of using such experiments to assess the correctness of data sets for use in deep penetration calculations.

The analysis has not been as successful in dissecting the other exceptional situation—that of deep penetration to distances well beyond that reached by the average neutron at the given energy. A number of significant conclusions have emerged, however, some of them rather unexpected. The point that has to be made most strongly of all is that what constitutes "exceptional" behavior depends strongly on the energy region being considered. Thus for neutrons detected in the low MeV range, transport to 1 meter constitutes quite deep penetration; for neutrons in the 10—20 keV range 1 meter is a penetration rather close to the average. Most practical shielding situations around reactors do not involve layers of iron more than 1 meter thick. The overwhelming proportion of neutrons from a fission source observed at that distance are at energies below 500 keV. For these neutrons the penetration histories at 1 meter do not deviate very far from the mean, nor do the minima, at least in the data set employed, play any overwhelming role. This suggests that a modest number of energy groups, chosen with a reasonably intelligent correlation to the cross section features, should suffice for the calculation of the low energy spectrum to 1 meter. At larger distances, up to 2.0 or even 2.6 meters, the deviations from average behavior even at energies below the 24 keV minimum are more marked. Distances large compared to the mean-free-path length are traveled in the region of the minima. Still, no one minimum, not even the notorious dip at 24 keV, can be said to "dominate" the transport. The histories of those neutrons that reach distances of ~ 2 meters are "stretched" compared to the average history,

but the stretching in the 100—800 keV region seems to occur in intervals of high cross sections as well as in the minima. Another surprising piece of evidence that the minima don't "steal the whole show" is provided by the count of the number of collisions. At these distances, the increase in the number of collisions above that of the "average" neutron is the same, per lethargy interval, in the regions of high cross section as where the cross section is less than 1 or 2 barns.

It appears that the deep penetration histories of fission neutrons in iron more closely correspond to the "featureless" situation of Chapter 3 than the small iron problem of Chapter 4. The complexity of the iron cross section phenomena, evidenced by Figure 5.1, could in no way be called "featureless". But perhaps there is a similarity in that any sizable lethargy interval in the keV region is likely to contain both resonance peaks and resonance minima—the same kinds of features are present all the way up to 500 keV and beyond. That is not true in the region near the lowest S-wave resonance and its associated interference minimum at 24 keV, but by then most of the transport has already taken place.

Preeg's earlier work⁽²⁹⁾ suggested that for practical shield thicknesses of the order of 1 meter, transport of neutron detected in the neighborhood of 1 MeV would indeed be dominated by the minima. Unfortunately with our sample there are just too few histories at this distance to analyze with significance. Here, clearly is the biggest gap in the present study.

Chapter 6

Summary and Conclusions

The life histories of fast neutrons created by the straight Monte Carlo method in various attenuation media have been examined. The medium studied ranges from the one with simple, featureless properties to iron with very complicated cross section structure. The life histories of exceptional neutrons, i. e. those staying very close to the source, or those going very far from the source, were compared with that of the general population. By general population is meant those neutrons which contributed to a given detector response irrespective of the geometric location of detection. When the exceptional neutrons exploited a particular collision property in a narrow energy band in order to reach a given detector, the method of analyzing Monte Carlo histories was able to provide a clear physical picture and single out the influence of that property on the macroscopic behavior of the neutrons. Two such phenomena have been demonstrated by using this technique, each of which is of special importance in practical application. One is a particular situation in which transport in a cross section minimum dominates the deep penetration of the neutrons. It is shown that in such a circumstance, the neutrons inevitably have collisions in the region of the minimum with a path length distribution very much biased so as to favor long flights. Further, the neutron tends to have more collisions than is normal in the region of the minimum. It is these characteristics which result in having most of the spatial transport accomplished by the traveling at energies in and near

the minimum, while little transport occurs at any other energies. The second example involves the effect of inelastic scattering on the low energy leakage spectra for small bare assemblies. It is shown that for a small bare iron sphere and for a fission source, the exit current spectrum below 100 keV is extremely sensitive to the details of the inelastic scattering near threshold. At greater distances or with reflected systems, this region of the inelastic scattering is no longer singled out for emphasis. The conclusions drawn from the measurements of leakage spectra with small bare assemblies, such as those which have been made at RPI⁽³⁰⁾ and LLL⁽³¹⁾, or in the current "shielding benchmark" series⁽³²⁾ may not be appropriate for the study of cross section sensitivities or transport mechanisms for deep penetrations.

When cross section properties are much the same throughout a broad slowing down range, such as those for a featureless medium or for iron spanning from MeV to keV region, the life histories of neutrons with moderately deep penetrations exhibit no startling differences from the general population. Rather the unusual transport occurs as the result of cumulative effects of small modifications in all collisions. Under such circumstances, while the history analysis technique could show the deviations in collision parameters from normal situation, no particular significance in any collision property can be singled out for emphasis. Moreover, the deviations are sometimes small and subtle and therefore difficult to detect them completely. Thus the Monte Carlo history analysis may be less illuminating as to the transport mechanisms.

It is important to notice, however, that whether a penetration is deep or not should be judged on the basis of comparison with the penetration distance of the general population at the same energy. In iron, an average neutron travels about 90 cm when it slows down from a fission source to below 24 keV. Hence, there is no particularly out-of-the ordinary behavior in the life history of the neutrons penetrating through a layer of iron of 1 meter or so in thickness and contributing to a detector responding to energy at 24 keV. Indeed, the study showed that no one cross section minimum in iron can be said to dominate the transport of the neutrons contributing to keV detectors within the thickness of practical interest.

In this study, we have often encountered the difficulty that in some exceptional situations, the number of histories available for the analysis was too few to give statistically significant results. Since we were interested in the examination of the life history of neutrons over a wide range of source-detector configurations, biased sampling was not and could not be used. It may be worthwhile to carry the present technique one step further to include the analysis of a biased set of histories in some case where a particular source-detector structure is of interest, e.g. the MeV detector in iron.

Above all, the most important conclusion to be drawn here is that the analysis of Monte Carlo histories can indeed provide information on the details of transport mechanisms that is not available through forward or even adjoint deterministic transport calculations. It might therefore be useful in many instances to record the details of the particle histories

in any given Monte Carlo calculation. Whenever a deeper understanding of the relation between microscopic properties and macroscopic quantities would be valuable, the analysis of the histories could then provide some directly relevant information.

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Appendix A

Mean Square Radial Displacement for Neutron Slowing Down

(I) Constant Cross Section, Isotropic Distribution for Scattering Azimuth

Consider a neutron emitted from a point source. The source position is defined as the origin. The radius vector, $\vec{r}_n$, of the neutron at its n-th collision can be written as

$$\vec{r}_n = \sum_{i=1}^n \vec{x}_i,$$

where $\vec{x}_i$ is the vector of the i-th random flight. If we let

$$r_n^2 = \vec{r}_n \cdot \vec{r}_n,$$

and

$$x_i^2 = \vec{x}_i \cdot \vec{x}_i$$

for $i=1, \dots, n$, then the square of the radial displacement from the origin at the n-th collision can be written as

$$r_n^2 = \sum_{i=1}^n x_i^2 + \sum_{i=1}^n \sum_{j=i+1}^n 2 \cdot (\vec{x}_i \cdot \vec{x}_j),$$

and the mean square radial displacement, $\overline{r_n^2}$, is

$$\overline{r_n^2} = \sum_{i=1}^n \overline{x_i^2} + 2 \cdot \sum_{i=1}^n \sum_{j=i+1}^n (\overline{\vec{x}_i \cdot \vec{x}_j}),$$

where the bars represent the averages. If we assume that each step in random walk sequence is independent and that there is no correlation between path length and scattering angle (Assumption 1), then

$$\overline{r_n^2} = \sum_{i=1}^n \overline{x_i^2} + 2 \cdot \sum_{i=1}^n \sum_{j=i+1}^n \overline{|\vec{x}_i| |\vec{x}_j| \omega_{ij}},$$

where ω_{ij} is the cosine of the scattering angle between vector $\vec{x}_i$ and $\vec{x}_j$, and $|\vec{x}_i|$ is the length of vector $\vec{x}_i$, $|\vec{x}_j|$ is the length of vector $\vec{x}_j$.

$\omega_{i,j}$ can be expressed in terms of polar and azimuthal angles of scattering as follows

$$\omega_{i,j} = \omega_{i,j-1} \omega_{j-1,j} + \xi_{i,j-1} \xi_{j-1,j} \cos \Phi_{i,j-1,j}$$

where $\xi_{\ell,k} = \sqrt{1 - \omega_{\ell,k}^2}$, and $\Phi_{i,j-1,j}$ is the scattering azimuth of the j -th step on the plane normal to the $(j-1)$ th vector and with respect to the plane formed by the vectors of step $j-1$ and step i . According to Assumption 1,

$$\overline{\omega_{i,j}} = \overline{\omega_{i,j-1}} \overline{\omega_{j-1,j}} + \overline{\xi_{i,j-1}} \overline{\xi_{j-1,j}} \overline{\cos \Phi_{i,j-1,j}}$$

If we assume here that the azimuthal angle of scattering has a uniform distribution (Assumption 2), then

$$\begin{aligned} \overline{\omega_{i,j}} &= \overline{\omega_{i,j-1}} \overline{\omega_{j-1,j}} \\ &= \overline{\omega_{i,j-2}} \overline{\omega_{j-2,j-1}} \overline{\omega_{j-1,j}} \\ &\vdots \\ &= \overline{\omega_{i,i+1}} \dots \overline{\omega_{j-1,j}} \\ &= \overline{\cos \theta_i} \dots \overline{\cos \theta_{j-1}}, \end{aligned}$$

where θ_k is the polar angle of scattering between k -th and $(k+1)$ th collision in the LAB system.

Suppose that the distribution of polar angle of scattering is the same throughout the whole collision sequence (Assumption 3). Then

$$\overline{\cos \theta_i} = \dots = \overline{\cos \theta_{j-1}} = \overline{\mu}$$

and

$$\overline{\omega_{i,j}} = \overline{\mu}^{j-i}$$

where $\overline{\mu}$ is the average cosine of the polar angle of scattering. So

$$\overline{\tau_n^2} = \sum_{i=1}^n \overline{x_i^2} + 2 \cdot \sum_{i=1}^n \sum_{j=i+1}^n |\overline{x_i}| |\overline{x_j}| \overline{\mu}^{j-i}$$

Under the assumption of constant cross section (Assumption 4), the path length distribution $p(x_i)$ has a common mean λ and standard deviation σ for all i . Thus $\overline{r_n^2}$ can be written as

$$\begin{aligned} \overline{r_n^2} &= n(\lambda^2 + \sigma^2) + 2 \cdot \lambda^2 \sum_{i=1}^n \sum_{j=i+1}^n \overline{\mu}^{j-i} \\ &= \frac{2\lambda^2}{1-\overline{\mu}} \left\{ n+1 - \frac{1-\overline{\mu}^{n+1}}{1-\overline{\mu}} \right\} + n(\sigma^2 - \lambda^2) \end{aligned}$$

For sufficiently large n , namely

$$\frac{\mu}{n} \frac{1-\overline{\mu}^n}{1-\overline{\mu}} \ll 1,$$

$\overline{r_n^2}$ can be reduced to

$$\overline{r_n^2} = \frac{2n\lambda^2}{1-\overline{\mu}} + n(\sigma^2 - \lambda^2)$$

Since λ , σ and $\overline{\mu}$ are all independent of n , the average radial displacement from the origin for a set of neutrons suffering on the average $\overline{n}$ collisions is

$$\overline{r^2} = \frac{2\overline{n}\lambda^2}{1-\overline{\mu}} + \overline{n}(\sigma^2 - \lambda^2) \quad (A.1)$$

In the above derivation for $\overline{r^2}$, we have assumed that the mean and standard deviation of the distribution of path length are the same for every collision. There is no particular form of the distribution of the path length required however. In general, the distribution of path length is a simple exponential function. In such a case, since $\sigma = \lambda$, $\overline{r^2}$ can be reduced to

$$\overline{r^2} = \frac{2\overline{n}\lambda^2}{1-\overline{\mu}} \quad (A.2)$$

(II) Effect of Biased Distribution of Scattering Azimuth on the Neutron Transport

Instead of assuming uniform distribution for the scattering azimuth in Part I (other conditions being the same), we now assume that the distribution of scattering azimuth deviates slightly from uniform distribution and that

$$\overline{\cos \phi_{i,j-1,j}} = \epsilon$$

for all i and j , where ϵ is a small but non-zero quantity. The mean cosine angle of scattering between i -th and j -th random walks, $\overline{\omega_{i,j}}$, can be written as following

$$\begin{aligned}\overline{\omega_{i,i+1}} &= \overline{\mu} \\ \overline{\omega_{i,i+2}} &= \overline{\mu} \overline{\omega_{i,i+1}} + \overline{\epsilon_{i,i+1}} \overline{\epsilon_{i+1,i+2}} \epsilon \\ &\vdots \\ \overline{\omega_{i,j}} &= \overline{\mu} \overline{\omega_{i,j-1}} + \overline{\epsilon_{i,j-1}} \overline{\epsilon_{j-1,j}} \epsilon\end{aligned}$$

where the bars again represent the expectations, and $\overline{\mu}$ is the average cosine of the polar angle of scattering, as above in Part I.

If the distribution of the cosine of polar angle of scattering is uniform in $(-1, 1)$ in the CM system, and if the mass number of the medium is much larger than 1, the distribution of the cosine of polar angle of scattering in the LAB system will be nearly uniform in $(-1, 1)$. We may assume therefore that the distribution of $\omega_{i,j}$ is also nearly uniform in $(-1, 1)$, i.e. $p(\omega_{i,j}) \sim 1/2$, for all i and j . Using this

assumption, $\bar{\xi}_{2,k}$ can be evaluated,

$$\bar{\xi}_{i,i+1} = \bar{\xi}_{i+1,i+2} = \dots = \bar{\xi}_{i,j-1} = \bar{\xi}_{j-1,j} \approx \frac{\pi}{4}.$$

Hence

$$\omega_{i,i+1} \approx \bar{\mu}$$

$$\omega_{i,i+2} \approx \bar{\mu}^2 + c\varepsilon$$

$$\vdots$$

$$\omega_{i,j} \approx \bar{\mu}^{j-i} + c\varepsilon \bar{\mu}^{j-i-2} + \dots + c\varepsilon$$

where $c = (\pi/4)^2$.

The mean square displacement from the origin for a neutron at

its n -th collision in an infinite medium can be written as

$$\begin{aligned} \bar{r}_n^2 = & \frac{2n\lambda^2}{1-\bar{\mu}} \left\{ 1 + \frac{1}{n} - \frac{1}{n} \frac{1-\bar{\mu}^{n+1}}{1-\bar{\mu}} \right\} + n(\sigma^2 - \lambda^2) \\ & + \frac{2c\varepsilon\lambda^2}{1-\bar{\mu}} \left\{ \frac{(n-1)(n-2)}{2} - \frac{\bar{\mu}(n-2)}{1-\bar{\mu}} + \frac{\bar{\mu}^2(1-\bar{\mu}^{n-2})}{(1-\bar{\mu})^2} \right\}. \end{aligned}$$

For $n \gg 1$, and $\bar{\mu} \sim 0$

$$\bar{r}_n^2 = \frac{2n\lambda^2}{1-\bar{\mu}} + \frac{2c\varepsilon\lambda^2}{1-\bar{\mu}} \cdot \frac{n^2}{2} + n(\sigma^2 - \lambda^2).$$

As in Part I, averaging over a set of neutrons gives

$$\begin{aligned} \bar{r}^2 = & \frac{2\bar{n}\lambda^2}{1-\bar{\mu}} + \frac{2c\varepsilon\lambda^2}{1-\bar{\mu}} \frac{\bar{n}^2}{2} + \bar{n}(\sigma^2 - \lambda^2) \\ = & \frac{2\bar{n}\lambda^2}{1-\bar{\mu}} \left\{ 1 + \frac{c\varepsilon\bar{n}}{2} \right\} + \frac{2c\varepsilon\lambda^2}{1-\bar{\mu}} \frac{\sigma'^2}{2} + \bar{n}(\sigma^2 - \lambda^2), \end{aligned}$$

where σ' is the standard deviation of the distribution of n and $\bar{n}$ is the

mean. If we denote $\frac{2\bar{n}\lambda^2}{1-\bar{\mu}}$ by $\bar{r}_0^2$ (the mean square distance of slowing down in the case of uniform distribution for the scattering azimuth), and for $\bar{n}^2 \gg \sigma'^2$,

$$\bar{r}^2 = \bar{r}_0^2 \left\{ 1 + \frac{\bar{n}c\varepsilon}{2} \right\} + \bar{n}(\sigma^2 - \lambda^2) \quad (\text{A.3})$$

Again if the path length distribution is exponential, then since $\sigma = \lambda$,
hence

$$\overline{r^2} = r_0^2 \left\{ 1 + \frac{\overline{n} c \varepsilon}{2} \right\} \quad (A.4)$$

Thus it is seen that the correction due to the non-uniformity in the
distribution of scattering azimuth is proportional to $\overline{n}^2$.

Appendix B

Computation of The Scattering Azimuth

In this thesis, the distribution of the scattering azimuth relative to the direction which gives the maximum chance of detection has been studied. For spherical geometry, this reference direction is that of the projection of the radius vector on the plane normal to the incident vector for the scattering (cf. Chapter 3, Section 3.4.3). The mathematical computation of the scattering azimuth relative to the reference direction defined above is shown below.

It will be remembered that on the history file, the geometrical information concerning a neutron motion in a medium is the three spatial coordinates (in a universal coordinate system). Since the scattering azimuth is measured on the plane normal to the incident vector for a scattering, we thus transform the representation of the scattering vector after the collision from the universal coordinate system to the coordinate system defined by the incident vector for the collision and the reference direction for the measurement of the scattering azimuth.

Firstly, let us assume the rotation convention as following:

- (a) rotation is counterclockwise
- (b) azimuthal angle is measured relative to the X-axis .

For two coordinate systems XYZ , $X'Y'Z'$, a set of Eulerian angle (θ, ϕ, ψ) can be defined*. The transformation from the XYZ system to the $X'Y'Z'$ system then can be written, in a matrix form, as

* Goldstein, H., "Classical Mechanics", Addison-Wesley, 1962, p109.

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\psi \cos\phi - \cos\theta \sin\phi \sin\psi & \cos\psi \sin\phi + \cos\theta \cos\phi \sin\psi & \sin\psi \sin\theta \\ -\sin\psi \cos\phi - \cos\theta \sin\phi \cos\psi & -\sin\psi \sin\phi + \cos\theta \cos\phi \cos\psi & \cos\psi \sin\theta \\ \sin\theta \sin\phi & -\sin\theta \cos\phi & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

For a collision at c' , we define the following quantities:

$\vec{c'c''}$: the incident vector for the collision at c'

$\vec{c'c'}$: the scattering vector for the collision at c'

$\vec{c'o}$: the radius vector

$\vec{c't}$: the reference vector for measuring the scattering azimuth

i, j, k : unit vectors in X, Y, Z directions respectively

i', j', k' : unit vectors in X', Y', Z' directions respectively.

According to the transformation, we can write

$$k' = \sin\theta \sin\phi i - \sin\theta \cos\phi j + \cos\theta k.$$

But since k' is now in the direction of $\vec{c'c''}$, we can also write

$$k' = \alpha i + \beta j + \gamma k,$$

where α, β, γ are direction cosines of $\vec{c'c''}$, i.e.

$$\alpha = \frac{x_{c'} - x_{c''}}{l_{c'c''}}$$

$$\beta = \frac{y_{c'} - y_{c''}}{l_{c'c''}}$$

$$\gamma = \frac{z_{c'} - z_{c''}}{l_{c'c''}} \quad l_{c'c''} = \sqrt{(x_{c'} - x_{c''})^2 + (y_{c'} - y_{c''})^2 + (z_{c'} - z_{c''})^2}$$

Since α, β, γ are known, θ and ϕ can be determined from above relations:

$$\sin \theta = \sqrt{1 - \gamma^2} \quad , \quad \cos \theta = \gamma \quad ,$$

$$\sin \phi = \frac{\alpha}{\sqrt{1 - \gamma^2}} \quad , \quad \cos \phi = \frac{-\beta}{\sqrt{1 - \gamma^2}} \quad .$$

We now have to determine ψ . Since, by definition, the reference vector for measuring the scattering azimuth is on the X' -axis, therefore a unit vector in the direction $\vec{c}'_0$ must have zero projection on the Y' -axis. So

$$S \sin \psi = R \cos \psi \quad ,$$

where

$$S = u \cos \phi + v \sin \phi$$

$$R = -u \cos \theta \sin \phi + v \cos \theta \cos \phi + w \sin \theta$$

$$u = \frac{x_{c'}}{r_{c'}}$$

$$v = \frac{y_{c'}}{r_{c'}}$$

$$w = \frac{z_{c'}}{r_{c'}}$$

$$r_{c'} = \sqrt{x_{c'}^2 + y_{c'}^2 + z_{c'}^2} \quad .$$

The rotation angle ψ can thus be determined by solving the above equation.

$$(1) \quad \cos \psi = \frac{|S|}{\sqrt{S^2 + R^2}} \quad , \quad \sin \psi = \frac{|R|}{\sqrt{S^2 + R^2}} \quad ,$$

$$(2) \quad \cos \psi = \frac{-|S|}{\sqrt{S^2 + R^2}} \quad , \quad \sin \psi = \frac{-|R|}{\sqrt{S^2 + R^2}} \quad ,$$

where $|S|$, $|R|$ represent the absolute values for S and R respectively.

Among the two sets of $\cos \psi$ and $\sin \psi$ values, there is only one satisfactory solution. This can be determined by either one of the following conditions:

$$(a) \quad S \cos \psi > -R \sin \psi \quad ,$$

$$(b) \quad S \cos \psi < -R \sin \psi \quad .$$

Condition (a) says that the radius vector is taken to be outward, and is used when the scattering takes place inside the detector surface. Condition (b) says that the radius vector is taken to be inward (towards the origin) and is used when the scattering takes place outside the detector surface.

Since θ , ϕ and ψ have been uniquely determined, the linear transformation is also uniquely determined. The scattering azimuth relative to the direction defined above can thus be obtained. Suppose that the direction cosines of $\vec{cc'}$ with respect to the X, Y and Z coordinates are ω_x , ω_y , ω_z respectively, then the scattering azimuth relative to the reference direction, γ , is

$$\cos \gamma = \frac{l\omega_x + m\omega_y + n\omega_z}{\sqrt{1 - \mu^2}}$$

where

$$\mu = \alpha \cdot \omega_x + \beta \cdot \omega_y + \gamma \cdot \omega_z$$

$$l = \cos \psi \cos \phi - \cos \theta \sin \phi \sin \psi$$

$$m = \cos \psi \sin \phi + \cos \theta \cos \phi \sin \psi$$

$$n = \sin \psi \sin \theta$$

Appendix C

Fission Spectrum Used in SAM-F

ENERGY	-LOG(INTEGR SPECTR)	INTEGR SPECTRUM
,1851648540E+08	,1508956244E+02	,2796960274E+06
,1781418650E+08	,1385472140E+02	,9615479530E+06
,1554372350E+08	,1151802593E+02	,9949125203E+05
,1289304190E+08	,9236735566E+01	,9739501143E+04
,1007018540E+08	,6903992280E+01	,9987637538E+03
,8718370900E+07	,5823166536E+01	,2958222979E+02
,7169276240E+07	,4606986991E+01	,9981848444E+02
,5956441340E+07	,3680914205E+01	,2519992640E+01
,5318626410E+07	,3205281868E+01	,4054747099E+01
,4934977160E+07	,2923616753E+01	,5373897480E+01
,4562217680E+07	,2653510000E+01	,7040366201E+01
,4273185220E+07	,2446717500E+01	,8657731060E+01
,4055200090E+07	,2292412807E+01	,1010224200E+00
,3899040230E+07	,2182805377E+01	,1127248509E+00
,3700141110E+07	,2044394789E+01	,1294585160E+00
,3557654060E+07	,1946107946E+01	,1428288889E+00
,3376169840E+07	,1822031505E+01	,1616969289E+00
,3246158450E+07	,1733951352E+01	,1765852790E+00
,3121153610E+07	,1649930727E+01	,1920632130E+00
,3040502940E+07	,1596084251E+01	,2026886440E+00
,2923417600E+07	,1518438470E+01	,2190536789E+00
,2810841030E+07	,1444394839E+01	,2358887841E+00
,2738208850E+07	,1396955029E+01	,2473489889E+00
,2632764410E+07	,1328565856E+01	,2648568319E+00
,2531380480E+07	,1263370535E+01	,2826995701E+00
,2465969560E+07	,1221611227E+01	,2947548671E+00
,2371008660E+07	,1161427492E+01	,3130390011E+00
,2309741760E+07	,1122884905E+01	,3253398549E+00
,2250057990E+07	,1085562155E+01	,3377118861E+00
,2163411530E+07	,1031785479E+01	,3563701009E+00
,2107508940E+07	,9973545263E+00	,3688539450E+00
,2026351830E+07	,9477532389E+00	,3876109150E+00
,1971437710E+07	,9144580023E+00	,4007337640E+00
,1915531000E+07	,8807902596E+00	,4144552550E+00
,1854528780E+07	,8443267711E+00	,4298466460E+00
,1795469240E+07	,8093052112E+00	,4451672560E+00
,1738290520E+07	,7756715354E+00	,4603945060E+00
,1682932720E+07	,7433736169E+00	,4755070280E+00
,1629337850E+07	,7123611728E+00	,4904847100E+00
,1577449770E+07	,6825857117E+00	,5053087190E+00
,1527214120E+07	,6540004644E+00	,5199615230E+00
,1477365100E+07	,6258788961E+00	,5347911960E+00
,1425869740E+07	,5970927269E+00	,5504095030E+00

,1376169300E+07	,5695740174E+00	,5657663940E+00
,1328201240E+07	,5432698430E+00	,5808458690E+00
,1281905160E+07	,5181293629E+00	,5956337210E+00
,1237222790E+07	,4941037404E+00	,6101174860E+00
,1194097880E+07	,4711460704E+00	,6242863820E+00
,1146649800E+07	,4461587312E+00	,6400821630E+00
,1101087100E+07	,4224431061E+00	,6554435440E+00
,1057334850E+07	,3999377333E+00	,6703617860E+00
,1015321130E+07	,3705839444E+00	,6848304810E+00
,9736040080E+06	,3576407802E+00	,6993242410E+00
,9270076830E+06	,3345678591E+00	,7156472800E+00
,8865854270E+06	,3148378556E+00	,7299072150E+00
,8441537790E+06	,2944262084E+00	,7449589160E+00
,8001773890E+06	,2736104635E+00	,7606283100E+00
,7584919590E+06	,2542139301E+00	,7755258760E+00
,7189781420E+06	,2361444606E+00	,7896665900E+00
,6768387840E+06	,2172318027E+00	,8047434060E+00
,6327900320E+06	,1978793741E+00	,8204688170E+00
,5916079790E+06	,1801957084E+00	,8351067580E+00
,5484729510E+06	,1621251870E+00	,8503347470E+00
,5063488250E+06	,1449546208E+00	,8650615480E+00
,4632940310E+06	,1279270566E+00	,8799175510E+00
,4175439600E+06	,1104589160E+00	,8954231170E+00
,3737123120E+06	,9438104792E+01	,9099359670E+00
,3275983510E+06	,7822952986E+01	,9247521440E+00
,2716837150E+06	,5982052923E+01	,9419335680E+00
,2060235950E+06	,4013020161E+01	,9606643510E+00
,1697828860E+06	,3030255380E+01	,9701519560E+00
,1153049140E+06	,1721284364E+01	,9829344510E+00
,7830720440E+05	,9737933189E+02	,9903093270E+00
,5318089270E+05	,5492363485E+02	,9945226920E+00
,9900000000E+03	0,	,1000000000E+01

Appendix D

Study of Fusion Neutron Penetration in Iron

In this Appendix, we examine briefly the behavior of penetrating neutrons which are emitted from a D-T fusion source and slowed down in an iron medium. The source spectrum is assumed to have the form*

$$f(E) = A \exp \left\{ -C (\sqrt{E} - \sqrt{E_0})^2 \right\} ,$$

where energy E is in unit of MeV and

$$E_0 = 14.07 \text{ MeV}$$

$$C = M/mkT$$

$$M = \text{mass of reacting ions} = M_d + M_t$$

$$m = \text{mass of neutron}$$

$$kT = \text{plasma temperature}$$

$$A = \text{normalization constant}$$

Typically the plasma temperature is taken to be 20 keV. Hence

$$C = 250.$$

The normalization condition

$$\int_0^{\infty} f(E) dE = 1$$

gives $A = 1.18909903$.

The integrated source spectrum which is required for the SAM-F input is given in Appendix E.

The energy spectrum of outgoing current is sampled in 18 energy

* Muir, D.W., "Sensitivity of Neutron Multigroup Cross Sections to Thermal Broadening of the Fusion Peak", Los Alamos Scientific Lab., (LA-UR-74-613)

bins from 15 MeV to 10 keV. These bins act as energy selective detectors. The energy boundaries for these detectors are the same as those used in Chapter 5. Some graphic comparisons of exit current spectra for neutrons emitted from a D-T source are illustrated in Figure D.1, D.2, D.3 and D.4.

Some preliminary analyses are presented in the following:

(a) For a given geometric situation, the neutron exit current spectrum due to a fusion source is very much alike to that due to a fission source, except that portion of the spectrum where the energy is greater than 5 MeV for small spheres. Similar results have been reported by H. Ing and W. G. Cross^{*}.

(b) The current spectra again show that high energy neutrons disappear rapidly as the source-detector distance increases. The main reason for this rapid disappearance of high energy neutrons is the high probability of suffering inelastic scattering at high energies. Even for a 20 cm-radius sphere, the peak of the exit current below 10 MeV is around 500-600 keV.

(c) The number of neutrons exiting from a small iron sphere in the energy bin 10-15 MeV is relatively large compared to that in lower energy bins. These high energy exiting neutrons would not suffer many collisions in their life history because the probability of scattering by

* Ing, I., & W. G. Cross, "Spectra and Dosimetry of Neutrons from Various Sources Shielded by Iron," Nucl. Sci. Eng., 58, 420 (1975).

Figure D.1 Comparison of Exit Current Spectra for 20, 60, 100 cm-Radius Reflected Iron Spheres, D-T Neutron Source

Figure D.2 Comparison of Exit Current Spectra for 20 cm Radius Bare and Reflected Iron Spheres, D-T Neutron Source

Figure D.3 Comparison of Exit Current Spectra for 60 cm Radius Bare and Reflected Iron Spheres, D-T Neutron Source

Figure D.4 Comparison of Exit Current Spectra for 100 cm radius Bare and Reflected Iron Spheres, D-T Neutron Source

the $(n, 2n)$ reaction or by an inelastic collision which excites the continuum is very high ($\sim 40\%$ /collision) and such a collision would remove the neutrons out of this energy region. Thus the mean number of collisions for the neutrons detected in this region for a 20 cm-radius sphere is only about 2. For a 40 cm-radius sphere the detected neutrons on the average suffer about 4 collisions. Then why are there still sizable neutrons contributing to the detector in this bin for a 20 cm or even a 40 cm-radius sphere? Some reasons for this phenomenon may be noted here by examining the cross section properties. First, the elastic scattering is highly anisotropic, peaking forward in this region. Second, the total cross section goes down to an average of about 2.6 barns in this region, which is relatively low compared to about 4 barns at lower energies. Both of these help neutrons to increase spatial transport. The mean path length between collisions is about 7 cm for neutrons contributing to a 20 cm-radius sphere while it is about 10 cm for neutrons contributing to a 40 cm-radius sphere.

(d) Neutrons contributing to detector responding to energies 1-5 MeV for a 20 cm-radius sphere suffered on the average 5-6 collisions. Since neutrons are born in the vicinity of 14 MeV, the favored trajectories leading neutrons to contribute to the detector response must be those involving segment in energy axis which allows for large lethargy change upon scattering. Such a "channel" is provided by nonelastic collision leading to the $(n, 2n)$ reaction or to an inelastic scattering which excites

the continuum. More detailed analyses on these high energy neutrons have been reported by Cramer* and Ching**.

(e) The mean number of collisions for neutrons contributing to a 20 cm-radius bare sphere is much less than that for neutrons contributing to either a reflected sphere or larger spheres for detector response energies below 500 keV. For instance, for neutrons contributing to Detector 17 (covering the 24 keV minimum), the mean number of collisions is about 27 for a 20 cm bare sphere, but it is about 65 for a 20 cm reflected sphere. When the detector radius increases to 100 cm, neutrons contributing to the same detector response suffer on the average more than 90 collisions. Thus the nonelastic scattering which results in very large fractional energy loss in the slowing down process is expected to play predominant role in determining the flux spectrum at low energies for a small bare geometry. In particular, since the neutrons are born in the vicinity of 14 MeV, it seems that, for the low energy leakage neutrons of a small bare sphere, there must be at least one nonelastic collision taking place sometime in the neutron life history that leads to either the $(n, 2n)$ reaction or the inelastic scattering which excites the continuum.

In Table D.1, the statistical information concerning some effect of inelastic scattering is shown. This table is similar to that given in

* Cramer, S.N., et. al., "Monte Carlo Calculations and Sensitivity Studies of the LLL Pulsed Sphere Program," Oak Ridge National Lab., 1973 (ORNL-TM-4072).

**Ching, J, et. al., "Application of a Discrete-Energy, Discrete-Ordinates Technique to the Study of Neutron Transport in Iron," Oak Ridge National Lab., 1974 (ORNL-TM-4235).

Section 5.6.2, Chapter 5, for the fission neutron problem. Similar to the fission neutron problem, again an overwhelming proportion ($>97\%$) of the leakage neutrons below 500 keV for a 20 cm-radius bare sphere have the feature that a nonelastic collision took place sometime in the life history which resulted in such a large fractional energy loss that the scattering carried the neutrons all the way down to an energy well below the lowest threshold for inelastic scattering (the threshold for ^{56}Fe 1-st level at 860 keV). For neutrons contributing to detectors responding to energies below 100 keV for a small bare sphere, such a scattering, if it excites a discrete level, takes place at energy very near threshold. For example, for neutrons contributing to a 20 cm-radius bare sphere in Detector 17, the inelastic scattering which excites ^{56}Fe 1-st level takes place at a mean energy only about 70 keV above the threshold. On the other hand, however, one may notice that about 40—50% of the time the nonelastic scattering which results in very large fractional energy loss is due to ^{56}Fe 1-st excited state for detector response energy below 500 keV and for a 20 cm-radius bare sphere. Under the same geometric and energy conditions, it is about 75—90% for the fission neutron problem. As mentioned earlier, almost all the detected neutrons are nonelastically scattered. Thus, the nonelastic scattering by levels higher than the ^{56}Fe 1-st excited state plays equally important role in neutron slowing down process for a fusion source.

In Table D.2, the following data are given: (1) the fraction of the

13	0.260 ± 0.000	0.950 ± 0.010	0.548 ± 0.010	1.377 ± 0.008	0.402 ± 0.008
14	0.200 ± 0.120	0.952 ± 0.018	0.557 ± 0.013	1.210 ± 0.004	0.337 ± 0.004
15	0.120 ± 0.060	0.963 ± 0.020	0.577 ± 0.013	1.135 ± 0.008	0.262 ± 0.008

Table D.1 Effect of ^{56}Fe 1-st Level Inelastic Scattering to the Neutrons Emitted from a D-T Fusion Source and Detected at Energies below 500 keV

Detector Number	Detector Energy (MeV)	F_i	F_1	E_b (MeV)	E_a (MeV)
Reference Case:					
11	0.500 - 0.400	0.772 ± 0.010	0.391 ± 0.008	1.496 ± 0.003	0.611 ± 0.003
12	0.400 - 0.280	0.816 ± 0.009	0.435 ± 0.008	1.416 ± 0.003	0.535 ± 0.003
13	0.280 - 0.200	0.840 ± 0.009	0.471 ± 0.007	1.357 ± 0.003	0.478 ± 0.003
14	0.200 - 0.120	0.856 ± 0.009	0.490 ± 0.008	1.313 ± 0.004	0.436 ± 0.004
15	0.120 - 0.060	0.865 ± 0.009	0.504 ± 0.008	1.277 ± 0.004	0.401 ± 0.004
16	0.060 - 0.027	0.866 ± 0.009	0.505 ± 0.008	1.265 ± 0.004	0.390 ± 0.004
17	0.027 - 0.020	0.868 ± 0.009	0.507 ± 0.007	1.263 ± 0.003	0.387 ± 0.003
18	0.020 - 0.010	0.870 ± 0.009	0.513 ± 0.007	1.256 ± 0.003	0.381 ± 0.003

20 cm-Radius Bare Sphere:

11	0.500 - 0.400	0.975 ± 0.033	0.457 ± 0.025	1.430 ± 0.005	0.546 ± 0.005
12	0.400 - 0.280	0.992 ± 0.021	0.452 ± 0.014	1.317 ± 0.006	0.439 ± 0.005
13	0.280 - 0.200	0.997 ± 0.022	0.560 ± 0.014	1.185 ± 0.003	0.313 ± 0.003
14	0.200 - 0.120	0.999 ± 0.022	0.537 ± 0.016	1.100 ± 0.003	0.229 ± 0.003
15	0.120 - 0.060	1.000 ± 0.000	0.558 ± 0.017	0.994 ± 0.005	0.128 ± 0.004
16	0.060 - 0.027	1.000 ± 0.000	0.566 ± 0.052	0.966 ± 0.009	0.101 ± 0.009
17	0.027 - 0.020	1.000 ± 0.000	0.404 ± 0.023	0.933 ± 0.006	0.069 ± 0.005
18	0.020 - 0.010	1.000 ± 0.000	0.492 ± 0.109	0.884 ± 0.004	0.022 ± 0.004

20 cm-Radius Reflected Sphere:

11	0.500 - 0.400	0.874 ± 0.026	0.432 ± 0.019	1.474 ± 0.004	0.592 ± 0.004
12	0.400 - 0.280	0.919 ± 0.014	0.467 ± 0.010	1.382 ± 0.003	0.502 ± 0.003
13	0.280 - 0.200	0.930 ± 0.015	0.548 ± 0.013	1.279 ± 0.008	0.402 ± 0.008
14	0.200 - 0.120	0.952 ± 0.018	0.557 ± 0.013	1.210 ± 0.004	0.337 ± 0.004
15	0.120 - 0.060	0.963 ± 0.020	0.557 ± 0.013	1.135 ± 0.008	0.262 ± 0.008
16	0.060 - 0.027	0.969 ± 0.024	0.616 ± 0.019	1.105 ± 0.009	0.236 ± 0.009
17	0.027 - 0.020	0.954 ± 0.019	0.579 ± 0.012	1.118 ± 0.008	0.249 ± 0.003
18	0.020 - 0.010	0.960 ± 0.024	0.570 ± 0.019	1.135 ± 0.020	0.265 ± 0.020

100 cm-Radius Reflected Sphere:

17	0.027 - 0.020	0.954 ± 0.019	0.579 ± 0.012	1.118 ± 0.008	0.249 ± 0.008
18	0.020 - 0.010	0.960 ± 0.024	0.570 ± 0.019	1.135 ± 0.020	0.265 ± 0.020

100 cm-Radius Reflected Sphere:

11	0.500 - 0.400	0.542 ± 0.066	0.379 ± 0.053	1.619 ± 0.024	0.741 ± 0.023
12	0.400 - 0.280	0.671 ± 0.025	0.351 ± 0.016	1.536 ± 0.013	0.651 ± 0.012
13	0.280 - 0.200	0.735 ± 0.026	0.382 ± 0.014	1.504 ± 0.012	0.622 ± 0.012
14	0.200 - 0.120	0.762 ± 0.027	0.426 ± 0.020	1.414 ± 0.008	0.534 ± 0.007
15	0.120 - 0.060	0.790 ± 0.021	0.426 ± 0.017	1.405 ± 0.008	0.525 ± 0.008
16	0.060 - 0.027	0.815 ± 0.048	0.481 ± 0.041	1.384 ± 0.016	0.500 ± 0.016
17	0.027 - 0.020	0.845 ± 0.014	0.497 ± 0.012	1.302 ± 0.004	0.425 ± 0.004
18	0.020 - 0.010	0.869 ± 0.023	0.490 ± 0.020	1.299 ± 0.009	0.422 ± 0.009

Note:

- (a) F_i : Fraction of detected neutrons which are inelastically scattered by all levels from energy above threshold to energy below the lowest inelastic excitation threshold (^{56}Fe 1-st level)
- (b) F_1 : Fraction of detected neutrons which are inelastically scattered by the ^{56}Fe 1-st excited level from energy above the threshold to energy below the threshold
- (c) E_b : Mean energy at which the ^{56}Fe 1-st excited level inelastic scattering took place
- (d) E_a : Mean energy after the ^{56}Fe 1-st level inelastic scattering

Table D.2 Effect of Non-elastic Scatterings with Threshold Energies greater than 2.5 MeV to the Neutrons Emitted from a D-T Fusion Source and Detected at Energies below 500 keV

Detector Number	Detector Energy (MeV)	$F_i(>2.5)$	E_b (MeV)	E_a (MeV)
Reference Case:				
11	0.500 - 0.400	0.394 ± 0.007	11.546 ± 0.125	0.636 ± 0.005
12	0.400 - 0.280	0.348 ± 0.006	11.512 ± 0.125	0.580 ± 0.005
13	0.280 - 0.200	0.336 ± 0.005	11.454 ± 0.133	0.547 ± 0.005
14	0.200 - 0.120	0.334 ± 0.006	11.454 ± 0.110	0.514 ± 0.004
15	0.120 - 0.060	0.331 ± 0.006	11.456 ± 0.126	0.489 ± 0.004
16	0.060 - 0.027	0.333 ± 0.006	11.508 ± 0.119	0.474 ± 0.004
17	0.027 - 0.020	0.332 ± 0.005	11.512 ± 0.114	0.472 ± 0.004
18	0.020 - 0.010	0.329 ± 0.006	11.535 ± 0.102	0.466 ± 0.004
10 cm-Radius Bare Sphere:				
11	0.500 - 0.400	0.604 ± 0.030	12.030 ± 0.303	0.508 ± 0.008
12	0.400 - 0.280	0.656 ± 0.021	11.940 ± 0.220	0.384 ± 0.005
13	0.280 - 0.200	0.592 ± 0.028	11.850 ± 0.410	0.268 ± 0.005
14	0.200 - 0.120	0.610 ± 0.031	11.990 ± 0.270	0.178 ± 0.004
15	0.120 - 0.060	0.597 ± 0.029	12.263 ± 0.440	0.105 ± 0.003
16	0.060 - 0.027	0.691 ± 0.057	12.465 ± 0.787	0.067 ± 0.006
17	0.027 - 0.020	0.709 ± 0.055	12.668 ± 0.525	0.041 ± 0.002
18	0.020 - 0.010	0.714 ± 0.083	13.804 ± 0.138	0.016 ± 0.001
20 cm-Radius Bare Sphere:				
11	0.500 - 0.400	0.475 ± 0.022	11.596 ± 0.272	0.579 ± 0.006
12	0.400 - 0.280	0.501 ± 0.016	11.568 ± 0.102	0.461 ± 0.006
13	0.280 - 0.200	0.401 ± 0.017	10.969 ± 0.353	0.349 ± 0.009
14	0.200 - 0.120	0.439 ± 0.018	11.498 ± 0.222	0.238 ± 0.005
15	0.120 - 0.060	0.415 ± 0.017	11.815 ± 0.200	0.149 ± 0.006
16	0.060 - 0.027	0.405 ± 0.062	11.990 ± 0.707	0.093 ± 0.006
17	0.027 - 0.020	0.578 ± 0.022	12.292 ± 0.297	0.063 ± 0.003
18	0.020 - 0.010	0.508 ± 0.110	11.738 ± 0.985	0.036 ± 0.005
20 cm-Radius Reflected Sphere:				
11	0.500 - 0.400	0.407 ± 0.018	11.577 ± 0.261	0.627 ± 0.004
12	0.400 - 0.280	0.421 ± 0.010	11.562 ± 0.237	0.543 ± 0.007
13	0.280 - 0.200	0.354 ± 0.009	11.235 ± 0.223	0.460 ± 0.008
14	0.200 - 0.120	0.366 ± 0.013	11.310 ± 0.144	0.381 ± 0.008
15	0.120 - 0.060	0.363 ± 0.014	11.282 ± 0.219	0.313 ± 0.015
16	0.060 - 0.027	0.337 ± 0.016	11.441 ± 0.387	0.286 ± 0.011
17	0.027 - 0.020	0.351 ± 0.015	11.769 ± 0.143	0.265 ± 0.012
18	0.020 - 0.010	0.380 ± 0.014	11.373 ± 0.485	0.360 ± 0.028
100 cm-Radius Reflected Sphere:				
11	0.500 - 0.400	0.163 ± 0.039	11.212 ± 1.102	0.729 ± 0.024
12	0.400 - 0.280	0.293 ± 0.019	11.763 ± 0.320	0.664 ± 0.009
13	0.280 - 0.200	0.331 ± 0.022	11.251 ± 0.358	0.640 ± 0.015
14	0.200 - 0.120	0.303 ± 0.018	11.212 ± 0.215	0.609 ± 0.009
15	0.120 - 0.060	0.323 ± 0.013	11.033 ± 0.262	0.586 ± 0.012
16	0.060 - 0.027	0.308 ± 0.028	11.268 ± 0.240	0.572 ± 0.014
17	0.027 - 0.020	0.320 ± 0.008	11.434 ± 0.215	0.500 ± 0.008
18	0.020 - 0.010	0.350 ± 0.012	11.401 ± 0.191	0.507 ± 0.008

Note:

- (a) $F_i(>2.5)$: Fraction of detected neutrons which are scattered by neutron-producing, non-elastic reactions with threshold energy greater than 2.5 MeV from energy above threshold to energy below 860 keV
- (b) E_b : Average energy at which the non-elastic scattering took place
- (c) E_a : Average energy after the non-elastic scattering

detected neutrons below 500 keV which are scattered by nonelastic collision with a threshold energy greater 2.5 MeV and from energy at scattering to energy below the lowest threshold for inelastic scattering, (2) the mean energy at which such collision event took place, (3) the mean energy after such collision event. The geometries considered are: 10 cm bare sphere, 20 cm bare sphere, 20 cm reflected sphere, 100 cm reflected sphere and the reference population. It is seen that for a 20 cm bare sphere, about 40—50% of the detector response below 500 keV is due to neutrons scattered by the nonelastic collision with threshold energy higher than 2.5 MeV. Further, the averages for quantities in Table D.2 are for neutrons scattered by inelastic collision with threshold energy greater than 2.5 MeV or by the $(n, 2n)$ reaction. However, the mean energies at which the scattering took place are about 11-12 MeV for detectors responding to energies below 500 keV and for a 20 cm-radius bare sphere, while the mean energies after the scattering are very near the detector response. Thus the most possible interaction for the collision event would be the inelastic scattering which excites the continuum or the $(n, 2n)$ reaction at energy not too far from the source. Such interactions would be more important for a bare iron sphere with still smaller radius. For a 10 cm-radius bare sphere, for instance, neutrons scattered by nonelastic collision that leads to the $(n, 2n)$ reaction or to an inelastic scattering which excites the continuum and that carries the neutrons to an energy just above the detector response account for about 60—70% of

the detector response at energies below 500 keV. It may be noted that, for reflected spheres or large spheres, while the mean energies at scattering are still about 11 MeV, the mean energies after scattering are well above detector response limits. Also the fractional contribution to the detector responses by neutrons slowed down to below 860 keV by nonelastic scattering has decreased considerably.

The above observation thus indicates that, for a fusion source, the favored trajectories for the neutrons contributing to low energy fluxes of a small, bare iron assembly include many nonelastic scattering segments. The fluxes, therefore, may be relatively insensitive to changes affecting only one of the segments.

(f) Similar to the fission problem, the spatial channel provided by the cross section minima below 500 keV insures that the overwhelming proportion of the exit spectrum at large distance is below 500 keV. Noting the similarities of the current spectra between a fission and a fusion problem, one would expect that the effect of cross section minima on neutron transport for a fusion source is also similar to that for a fission source.

Appendix E

Fusion Spectrum Used in SAM-F

ENERGY	-LOG(INTEGR SPECTR)	INTEGR SPECTRUM
,1800000000E+08	,2700000000E+02	,1279528817E-11
,1790000000E+08	,2700000000E+02	,1879528817E-11
,1670000000E+08	,2700000000E+02	,1879528817E-11
,1612000000E+08	,2007173810E+02	,1918469500E-08
,1602000000E+08	,1842157740E+02	,9991037500E-08
,1601000000E+08	,1826045254E+02	,1173778700E-07
,1600000000E+08	,1810003719E+02	,1378014300E-07
,1564000000E+08	,1280151176E+02	,2756602100E-05
,1537000000E+08	,9443723229E+01	,7918503500E-04
,1517000000E+08	,7301373292E+01	,6746117000E-03
,1501000000E+08	,5799948523E+01	,3027710600E-02
,1488000000E+08	,4719050299E+01	,8923649400E-02
,1477000000E+08	,3901351301E+01	,2021457700E-01
,1467000000E+08	,3234459040E+01	,3938150300E-01
,1459000000E+08	,2752816417E+01	,6374806700E-01
,1452000000E+08	,2368706936E+01	,9360168100E-01
,1446000000E+08	,2066799865E+01	,1265902400E+00
,1440000000E+08	,1789708654E+01	,1670088200E+00
,1435000000E+08	,1577420339E+01	,2065071300E+00
,1430000000E+08	,1381725176E+01	,2511449100E+00
,1426000000E+08	,1236874718E+01	,2902900400E+00
,1422000000E+08	,1102205547E+01	,3321377300E+00
,1418000000E+08	,9774879879E+00	,3762550700E+00
,1415000000E+08	,8903295828E+00	,4105204300E+00
,1412000000E+08	,8085119959E+00	,4455205100E+00
,1409000000E+08	,7319119385E+00	,4809884900E+00
,1406000000E+08	,6603980747E+00	,5166456300E+00
,1403000000E+08	,5938308724E+00	,5522077900E+00
,1402000000E+08	,5727157439E+00	,5639917000E+00
,1401000000E+08	,5521282199E+00	,5757232400E+00
,1400000000E+08	,5320625051E+00	,5873922200E+00
,1398000000E+08	,4934729068E+00	,6105024900E+00
,1396000000E+08	,4568987229E+00	,6332444700E+00
,1394000000E+08	,4222901435E+00	,6555438100E+00
,1392000000E+08	,3895958092E+00	,6773305900E+00
,1390000000E+08	,3587628793E+00	,6985399700E+00
,1388000000E+08	,3297371046E+00	,7191127600E+00
,1386000000E+08	,3024628655E+00	,7389959300E+00
,1384000000E+08	,2768833095E+00	,7581429600E+00
,1382000000E+08	,2529403880E+00	,7765141700E+00
,1380000000E+08	,2305750340E+00	,7940768500E+00
,1378000000E+08	,2097272537E+00	,8108053600E+00
,1376000000E+08	,1903362932E+00	,8266810800E+00
,1374000000E+08	,1723407591E+00	,8416923100E+00

,1372000000E+08	,1556788235E+00	,8558340200E+00
,1371000000E+08	,1478285517E+00	,8625789900E+00
,1370000000E+08	,1402883754E+00	,8691075700E+00
,1369000000E+08	,1330505140E+00	,8754208700E+00
,1368000000E+08	,1261072144E+00	,8815203300E+00
,1367000000E+08	,1194506954E+00	,8874077600E+00
,1366000000E+08	,1130732491E+00	,8930852400E+00
,1365000000E+08	,1069671500E+00	,8985551900E+00
,1364000000E+08	,1011247105E+00	,9038203100E+00
,1363000000E+08	,9553831182E-01	,9088835400E+00
,1362000000E+08	,9020033632E-01	,9137481100E+00
,1361000000E+08	,8510326405E-01	,9184174400E+00
,1360000000E+08	,8023960456E-01	,9228951900E+00
,1359000000E+08	,7560192755E-01	,9271852200E+00
,1358000000E+08	,7118289283E-01	,9312915500E+00
,1357000000E+08	,6697522616E-01	,9352183700E+00
,1356000000E+08	,6297173852E-01	,9389700100E+00
,1355000000E+08	,5916533459E-01	,9425509200E+00
,1354000000E+08	,5554901080E-01	,9459656600E+00
,1353000000E+08	,5211586418E-01	,9492188800E+00
,1352000000E+08	,4885909104E-01	,9523153100E+00
,1351000000E+08	,4577201736E-01	,9552597200E+00
,1350000000E+08	,4284807689E-01	,9580569300E+00
,1349000000E+08	,4008080052E-01	,9607118100E+00
,1348000000E+08	,3746388927E-01	,9632292000E+00
,1347000000E+08	,3499114161E-01	,9656139700E+00
,1346000000E+08	,3265650638E-01	,9678709600E+00
,1345000000E+08	,3045405286E-01	,9700050000E+00
,1344000000E+08	,2837800356E-01	,9720208700E+00
,1343000000E+08	,2642272587E-01	,9739233000E+00
,1342000000E+08	,2458272438E-01	,9757169700E+00
,1341000000E+08	,2285267456E-01	,9774064700E+00
,1340000000E+08	,2122735454E-01	,9789963600E+00
,1339000000E+08	,1970175118E-01	,9804910600E+00
,1338000000E+08	,1827096209E-01	,9818949400E+00
,1337000000E+08	,1693026149E-01	,9832122500E+00
,1336000000E+08	,1567507457E-01	,9844471400E+00
,1335000000E+08	,1450096259E-01	,9856036700E+00
,1334000000E+08	,1340366917E-01	,9866857600E+00
,1333000000E+08	,1237906544E-01	,9876972400E+00
,1332000000E+08	,1142320710E-01	,9886417900E+00
,1331000000E+08	,1053226004E-01	,9895230100E+00
,1330000000E+08	,9702578146E-02	,9903443400E+00
,1329000000E+08	,8930649805E-02	,9911091100E+00
,1328000000E+08	,8213105403E-02	,9918205300E+00
,1327000000E+08	,7546714954E-02	,9924816900E+00
,1326000000E+08	,6928416140E-02	,9930955300E+00
,1325000000E+08	,6355261964E-02	,9936640900E+00
,1324000000E+08	,5824449221E-02	,9941924800E+00

.1323000000E+08	.5333296778E-02	.9946809000E+00
.1322000000E+08	.4879284371E-02	.9951326000E+00
.1321000000E+08	.4459980946E-02	.9955499500E+00
.1320000000E+08	.4073113877E-02	.9959351700E+00
.1319000000E+08	.3716497630E-02	.9962904000E+00
.1318000000E+08	.3388083077E-02	.9966176500E+00
.1317000000E+08	.3085936608E-02	.9969188200E+00
.1316000000E+08	.2808209331E-02	.9971957300E+00
.1315000000E+08	.2553176583E-02	.9974500800E+00
.1314000000E+08	.2319217307E-02	.9976834700E+00
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.1306000000E+08	.1040290915E-02	.9989602500E+00
.1305000000E+08	.9371990338E-03	.9990632400E+00
.1304000000E+08	.8435356762E-03	.9991568200E+00
.1303000000E+08	.7585276093E-03	.9992417600E+00
.1302000000E+08	.6814521358E-03	.9993187800E+00
.1301000000E+08	.6116370118E-03	.9993885500E+00
.1300000000E+08	.5484603769E-03	.9994516900E+00
.1299000000E+08	.4913406881E-03	.9995087800E+00
.1298000000E+08	.4397566788E-03	.9995603400E+00
.1297000000E+08	.3932273037E-03	.9996068500E+00
.1296000000E+08	.3135091389E-03	.9996865400E+00
.1295000000E+08	.2490009982E-03	.9997510300E+00
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.1293000000E+08	.1552920572E-03	.9998447200E+00
.1292000000E+08	.9536454705E-04	.9999046400E+00
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.1290000000E+08	.1008005080E-04	.9999899200E+00
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Appendix F

Calculation of Mean and Standard Deviation

In the analysis, the statistical results were presented as $\bar{x} \pm \sigma_{\bar{x}}$, where $\bar{x}$ is the sample mean and $\sigma_{\bar{x}}$ is the standard deviation of $\bar{x}$.

The mean and the standard deviation were computed by breaking the total sample of size n into statistical groups or batches with equal number of samples in each group. Let N be the number of groups and X_i be the estimate for the i -th group, then the sample mean is given by

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N X_i$$

and the variance for the aggregate estimates, S^2 , is given by

$$S^2 = \frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{x})^2$$

The variance for the sample mean, $\sigma_{\bar{x}}^2$, is therefore

$$\sigma_{\bar{x}}^2 = \frac{S^2}{N}$$

The standard deviation for the sample mean then can be written as

$$\sigma_{\bar{x}} = \sqrt{\frac{1}{N-1} \left\{ \sum_{i=1}^N \frac{X_i^2}{N} - \left(\sum_{i=1}^N \frac{X_i}{N} \right)^2 \right\}}$$

In the analysis, N was taken to be 10.

Appendix G

Chi-Square Statistics

G.1. Test of Goodness of Fit

Suppose we divide the range of observation of a random variable x into J cells, and let n_j ($j = 1, 2, \dots, J$) be the independent observation in cell j on the variable x with probability density function $p(x)$. Also let p_j ($j = 1, 2, \dots, J$) be the expected frequency in cell j for the underlying (or the expected) probability density function $p_0(x)$. The hypothesis that the two distributions $p(x)$ and $p_0(x)$ are the same can be written as

$$H_0: p(x) = p_0(x) \quad .$$

To use chi-square test for the hypothesis, we first note that the S^2 defined as

$$S^2 = \sum_{j=1}^J \frac{(n_j - np_j)^2}{np_j} \quad (G.1)$$

has approximately the χ^2 -distribution with $(J-1)$ degree of freedom,

where n is the total number of samples, namely $n = \sum_{j=1}^J n_j$. If we let

$$\hat{p}_j = \frac{n_j}{n}$$

then S^2 can be written as

$$S^2 = n \left\{ \sum_{j=1}^J \frac{\hat{p}_j^2}{p_j} - 1 \right\}.$$

The error for the approximation involved in using (G.1) as a χ^2_{J-1}

variable is particularly small when the np_j are equal or nearly so. We

take this advantage therefore by using cells of equal probability for the

underlying distribution in all the analyses.

The chi-square test for the hypothesis $H_0: p(x) = p_0(x)$ says that H_0 should be rejected if S^2 is greater than the value $\chi^2(\alpha)$ for the critical region size α . The values of $\chi^2(\alpha)$ for various α can be found in standard statistical books.

G. 2. Test of Independence

Considered a general contingency table containing r rows and c column. Let $p_{i,j}$ be the probability that an individual selected at random from the population under consideration will be a member of the cell in the i -th row and j -th column of the contingency table. Let p_i be the probability that the individual will be a member of the i -th row and let p_j be the probability that the individual will be a member of the j -th column. The hypothesis that the two variables are independent can be written as

$$H_0: p_{i,j} = p_i \cdot p_j$$

for $i=1, \dots, r, j=1, \dots, c$.

Suppose that a sample of n individuals is selected and $n_{i,j}$ of them are found in the cell in the i -th row and j -th column. We define the quantity

S^2 as

$$S^2 = \sum_{i=1}^r \sum_{j=1}^c \frac{(n_{i,j} - np_{i,j})^2}{np_{i,j}}$$

Under the hypothesis H_0 , this expression will become

$$S^2 = \sum_{i=1}^r \sum_{j=1}^c \frac{(n_{ij} - np_i p_j)^2}{np_i p_j}.$$

The quantity S^2 has a χ^2 -distribution with $(r-1)(c-1)$ degree of freedom, provided that n is sufficiently large and H_0 is true. Notice that in the expression for S^2 , p_i and p_j are unknown. They can be estimated, however, from the sample. Using the maximum likelihood estimate, we have

$$\hat{p}_i = \frac{n_{i.}}{n}$$

and

$$\hat{p}_j = \frac{n_{.j}}{n}$$

where

$$n_{i.} = \sum_{j=1}^c n_{ij}, \quad n_{.j} = \sum_{i=1}^r n_{ij}.$$

If p_i and p_j are replaced by the estimates given above, S^2 will be

$$S^2 = n \left\{ \sum_{i=1}^r \sum_{j=1}^c \frac{p_{ij}^2}{\hat{p}_i \cdot \hat{p}_j} - 1 \right\}.$$

The chi-square test for the hypothesis of independence says that H_0 should be rejected if the S^2 computed is greater than $\chi^2(\alpha)$ for the critical region size α .

Detector 13 (200-280 keV)

Detector 15 (60-120 keV)