LEGIBILITY NOTICE

A major purpose of the Technical Information Center is to provide the broadest dissemination possible of information contained in DOE's Research and Development Reports to business, industry, the academic community, and federal, state and local governments.

Although a small portion of this report is not reproducible, it is being made available to expedite the availability of information on the research discussed herein.

LA-UR--89-205

DE89 006525

AUTHORS: Richard S. Silver

SUBMITTED TO: Plenum Press

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or device by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

1. **Identify the main purpose of the document.** (1 point)
 2. **Summarize the key findings of the study.** (2 points)
 3. **Discuss the implications of the findings for future research.** (3 points)
 4. **Provide a conclusion based on the evidence presented.** (2 points)

[illegible]

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

TABLE I

MASTER

FINAL STATE EFFECTS IN QUANTUM FLUIDS

Richard N. Silver

Theoretical Division and Los Alamos Neutron Scattering Center
MS B262 Los Alamos National Laboratory
Los Alamos, New Mexico 87545

INTRODUCTION

The extraction of momentum distributions from high energy scattering experiments depends on the validity of the impulse approximation (IA), which has been extensively discussed in the workshop overview [1]. In the IA

$$F_{IA}(Y) = \frac{Q}{M} S_{IA}(Q, \omega) = n_0 \delta(Y) + \frac{1}{4\pi^2 \rho} \int_{|Y|}^{\infty} k n_k dk \quad (1)$$

Here $Y = M\omega/Q = Q/2$ is the West [2] scaling variable, n_k is the momentum distribution, n_0 is the Bose condensate fraction in the case of superfluid ^4He , and ρ is the density. The IA assumes that in the scattering process the kinetic energy imparted to a recoiling particle is large compared to the potential energy due to neighboring particles. While this may be true for x-ray Compton scattering from electronic systems, it is less true for quasielastic electron-nucleus scattering, and it is invalid for deep inelastic neutron scattering (DINS) from quantum fluids and solids such as helium. In the latter case, the interatomic potential [3] has a steeply repulsive core which is never negligible compared to the kinetic energies which can be imparted in feasible neutron scattering experiments. The corrections to the IA due to interactions of the recoiling atom with neighboring atoms are termed "final state effects" (FSE). This paper presents the theory of FSE for the case of deep inelastic neutron scattering from ^4He quantum fluids. The lessons should also be applicable to momentum distribution experiments in many other systems.

The simplest theory for FSE was presented in the original application of the IA to quantum fluids by Hohenberg and Platzman [4] (HP) in 1966. They postulated that the IA was lifetime broadened due to collisions of the recoiling atom with neighboring atoms at rate $1/\tau = \rho \sigma_{\text{tot}}(Q)Q/M$, where $\sigma_{\text{tot}}(Q)$ is the He-He total cross-section. Then the neutron scattering law is a convolution

$$F(Y) \equiv \frac{Q}{M} S(Q, \omega) = \int_{-\infty}^{\infty} dY'' R(Y - Y'') F_{IA}(Y'') \quad , \quad (2)$$

where the broadening predicted by HP is Lorentzian

$$R_{HP}(Y) = \frac{1}{\pi} \frac{\Gamma}{Y^2 + \Gamma^2} \quad ; \quad \Gamma(Q) \equiv \rho \sigma_{tot}(Q) \quad . \quad (3)$$

The FSE broadening satisfies $\Delta Y_{FWHM} = 2\rho\sigma_{tot}(Q)$ and Y -scales to the extent that $\sigma_{tot}(Q)$ is independent of Q . In fact, $\sigma_{tot}(Q)$ for ^4He [2] decreases as $\log Q$ with “glory” oscillations due to Bose statistics which decrease as $1/Q$. The observation of oscillations in $S(Q, \omega)$ for $Q \lesssim 10 \text{\AA}^{-1}$ [5] has been thought to provide confirmation of the HP theory. However, the ω^2 -sum rule on $S(Q, \omega)$ requires

$$\int_{-\infty}^{\infty} F(Y) Y^2 dY = \frac{2M}{3} \cdot K F \quad . \quad \Rightarrow \quad \int_{-\infty}^{\infty} R(Y) Y^2 dY = 0 \quad . \quad (4)$$

which is violated by the Lorentzian tails predicted by the HP theory

The solution of this problem was first discussed by Gersch and Rodriguez [6] (GR) in 1973. They developed a many body theory for FSE using novel time-ordered cumulant expansions for $S(Q, \omega)$. Due to a difference in the description of FSE as dephasing rather than lifetime broadening, $R(Y)$ at small $|Y|$ should be positive but satisfy $\Delta Y_{FWHM} \approx \rho\sigma_{tot}(Q)$, which is a factor of 2 smaller than the HP theory. $R(Y)$ should be negative at large $|Y|$ in order to satisfy the ω^2 -sum rule. This is achieved by incorporating the physics of the real space correlations in the ground state wave function, which implies that the scattering rate at short recoil distances should be zero. In hindsight, we believe that the GR solution was qualitatively correct, and it is compared to the latest experiments in ref. [7]. However, it was largely ignored in the more than twenty subsequent papers on FSE [8], which can be traced to several problems: the cumulant methods are novel and have not been widely applied elsewhere; the paper predates the concept of Y -scaling and reliable many body calculations of n_k , and, the numerical results were contained in an experimental paper [9] which is most notable for reporting an erroneously low 2% Bose condensate fraction in superfluid ^4He .

In the past two years, the Quasiclassical Approximation (QCA) of Silver and Ketter [10] and the Hard Core Perturbation Theory (HCPT) of Silver [11] have provided new approaches to FSE which confirm and extend the original GR theory. The HCPT predictions for FSE have been successfully compared to recent pulsed neutron scattering experiments [12] in normal and superfluid ^4He , which is further discussed by Sosnick, et al. [13]. The present paper outlines the physical ideas, the new theoretical approaches and the numerical predictions for FSE, while referring the reader to other papers for more detail.

QUASICLASSICAL APPROXIMATION

The simplest theory for FSE which includes the effect of real space correlations in the ground state is the QCA theory [10]. We begin with the standard expression for the incoherent part of the neutron scattering law, assuming the coherent part is

$$S(Q, \omega) = \frac{1}{\pi} \int_0^\infty dt e^{i\omega t} \langle e^{-i\hat{Q} \cdot \hat{r}(t)} e^{i\hat{Q} \cdot \hat{r}(0)} \rangle, \quad (5)$$

negligible at high Q . Here, $\langle \dots \rangle$ denotes the ground state expectation value. Since the wavelengths are short, one can hope to apply semiclassical methods to evaluate Eq. (5). A quasiclassical approximation based on the Wigner distribution function [14] suggests that at high Q we can replace the quantum mechanical operator $\hat{r}(t)$ by a classical trajectory for particles of momentum $\hat{Q}/2 + \hat{p}$

$$\hat{r}(t) \rightarrow \frac{\hat{Q}t}{2M} + \frac{\hat{p}t}{M} + \delta\hat{r}(t), \quad (6)$$

where $\delta\hat{r}(t)$ is the difference between collisional and collisionless classical trajectories. Then

$$S(Q, \omega) \approx \frac{1}{\pi} \text{Re} \int_0^\infty dt \int \frac{d^3p}{(2\pi)^3} n(\vec{p}) e^{i\left(\omega - \frac{Q^2}{2M} - \frac{\vec{Q} \cdot \vec{p}}{M}\right)} \langle e^{i\delta\hat{r}(t) \cdot \vec{Q}} \rangle. \quad (7)$$

Here $\langle \dots \rangle$ denotes an average over all such trajectories. If we ignore collisions, the IA, Eq. (1), is recovered from the $\delta\hat{r}(t) \rightarrow 0$ limit of Eq. (7). The QCA is also exact for neutron scattering from an harmonic oscillator. Further applications of the QCA are discussed by Reiter [15].

Consider then the effect of collisions for a single trajectory at $\vec{Q}$'s large compared to $\vec{p}$. In this case the phase factor, $\delta\hat{r}(t) \cdot \vec{Q}$, is zero for times less than the collision time, t_c , and it is very large for times greater than the collision time. Then,

$$e^{i\delta\hat{r}(t) \cdot \vec{Q}} \approx \Theta(t_c - t), \text{ which implies } \langle e^{i\delta\hat{r}(t) \cdot \vec{Q}} \rangle \approx P_{\text{NC}}(t) \quad (8)$$

when one averages over trajectories. Here, $\Theta(t)$ is a step function and $P_{\text{NC}}(t)$ is the probability of no collisions in time t . It is convenient to use scaled variables, $\chi = Qt/M$, which is the twice the distance along the trajectory for a particle of momentum $Q/2$. $P_{\text{NC}}(t)$ can be calculated from the collisions per unit distance along the trajectory, $\Gamma(\chi)$, which depends on the spatial distribution of particles as expressed by the radial distribution function, $g(r)$. Thus

$$P_{\text{NC}}(t) = \exp\left(-\int_0^{\chi_c} \Gamma(\chi) d\chi\right), \quad \Gamma(\chi) = \rho/\pi \int_0^{r_0} db^2 g(r(\chi, b)) \quad (9)$$

Here, r_0 is the effective hard sphere radius of a helium atom for the collision determined by the turning point of the classical trajectory, and b is the impact parameter for the collision. The $r(\chi, b)$, given by $r(\chi, b)^2 = b^2 + [\chi + \chi_c - b^2]'^2$, is appropriate when χ is defined as the distance between the edges of the particles, as illustrated in Fig. 1. Substituting Eq. (9) into Eq. (7) results in a physically appealing interpretation of $R(t)$ as the Fourier transform of the probability of no collisions as a function of recoil distance, with the West scaling variable, Y , as the conjugate momentum, i.e.

$$R_{\text{QC4}}(Y) = \frac{\text{Re}}{\pi} \int_0^x dx e^{iYx} P_{\text{SC}}(x) \quad (10)$$

To the extent that r_0 is independent of Q , QCA predicts a Y -scaling FSE broadening. In the limit of ignoring real space correlations, i.e. $g(r) \rightarrow 1$, the HP theory, Eq. (3), is recovered with $\Delta Y_{FWHM} \approx \rho \sigma_{tot}$ and $\sigma_{tot} = \pi r_0^2$ for classical collisions. The inclusion of real space correlations has a dramatic effect, as illustrated in Fig. 2. When a particle recoiling from a neutron collision begins its classical trajectory, its initial position is far away from the hard cores of the potential due to neighboring atoms which are responsible for final state effects. It must travel some distance before it encounters a hard core, so there are no collisions at small x . Thus $P_{\text{SC}}(x)$ is one for small x , and all of its derivatives are zero as $x \rightarrow 0$. This implies that it satisfies the ω^2 -sum rule, Eq. (4), by letting $R_{\text{QC4}}(Y)$ be positive at small $|Y|$ and negative at larger $|Y|$. The QCA provides a very simple way to derive the essential qualitative physics of FSE.

However, the QCA can not be expected to be quantitatively correct. It appears to be plagued by a number of factors of 2. One difficulty can be traced to the neglect of forward diffractive scattering in the QCA. The quantum mechanical σ_{tot} for a hard sphere is twice the classical value of πr_0^2 due to forward diffractive scattering. For example, the QCA prediction, $\Delta Y_{FWHM} \approx \rho \pi r_0^2$, is a factor of 2 smaller than expected. QCA is also incapable of describing glory oscillations which are due to interference between forward and backward diffractive scattering for identical Bose particles. Another obvious factor of 2 comes from giving the classical trajectory a momentum of $Q/2$ rather than Q . Further, one expects that x in $r(x,b)$ should be the distance between the centers of the particles rather than the distance between the edges as in QCA. These difficulties will be overcome in the fully quantum theory to be discussed in the next section, but the mathematical form and physical consequences of the final answer will resemble Eqs. (9-10).

THE QUANTUM THEORY OF FSE

Perturbation theory about the non-interacting state is notoriously slowly convergent for almost hard core systems such as helium. However, excellent variational and Monte Carlo methods exist to determine ground state properties such as n_k and $g(r)$. The n_k determines the FA prediction, and the QCA leads to the expectation that $g(r)$ is essential to the correct physics of FSE. This motivates an approach to calculate $S(Q, \omega)$ by perturbative expansion about the strongly interacting ground state wave function using projection techniques. In the following, I shall outline the steps in the derivation of a quantum theory of FSE. I shall emphasize the approach and identify where approximations have been made. I shall refer the reader to other papers where details may be found.

The neutron scattering law, $S(Q, \omega)$, is related to the dynamic susceptibility, $\chi(Q, \omega)$, by

$$\chi(Q, \omega) = \frac{1}{\pi N} \cdot [S^2(\omega, \rho_{\text{SC}})] \cdot \dots \quad \text{Im} \chi(Q, \omega) = S(Q, \omega) - S(Q, -\omega) \quad (11)$$

where

$$S^Q(\omega) \equiv i \int_0^\infty dt e^{i\omega t} e^{iH_0 t} Q_Q e^{-iH_0 t} ; \quad Q_Q \equiv \sum_k a_{k+Q}^\dagger a_k \quad (12)$$

The k th component of the dynamic susceptibility, χ_k , may be defined by

$$\chi_k \equiv \frac{1}{\pi N} \times [S^Q(\omega), a_{k+Q}^\dagger a_k] \quad (13)$$

Then diagrammatic perturbation theory can prove that the χ_k satisfy an exact Dyson equation

$$\chi_k = \chi_k^0 \left[1 + \sum_{k'} I(k, k') \chi_{k'} \right] ; \quad \chi_k^0 = \frac{1}{\pi N} \frac{n_{k+Q} - n_k}{\omega - \epsilon_{k+Q} + \epsilon_k + i\epsilon} \quad (14)$$

Ignoring collisions corresponds to setting the kernel, $I(k, k')$, to zero, in which case the IA is recovered from Eq. (14). The kernel is calculated [11,16] by truncating a projection superoperator expansion of $\chi(Q, \omega)$ at lowest order in the two-body t-matrix and keeping only the non-zero terms in the "asymptotic limit" of high Q and hard core interactions,

$$I(k, k') = - \frac{1}{n_k n_{k'}} \sum_{k_1} t_{k_1, k+Q-k-k'}^H \Phi(k_1, k, k-k') \quad (15)$$

A diagrammatic representation of the Dyson equation for FSE is shown in Fig. 3. The dashed line in Fig. 3 represents the Bose symmetrized two-body t-matrix

$$t_{k_1, k_2, q}^H = - \frac{4\pi}{2mk_1 M} \sum_{l, \text{even}} (2l+1) (e^{i\theta} + 1) P_l(\cos \theta) \Rightarrow t^H(Q, q) \quad (16)$$

The shaded box in Fig. 3 represents the two-particle density matrix

$$\Phi(k_1, k_2, q) = \langle a_{k_1+q}^\dagger a_{k_2+q}^\dagger a_{k_2} a_{k_1} \rangle \quad (17)$$

In the limit of keeping only the $q=0$ part of Φ , the Lorentzian broadening theory of Hohenberg and Platzman, and more specifically Platzman and Tzoar [17], is recovered. The more general expression, Eq. (17), retains all the information about the spatial correlations in the ground state wave function which are required to get the correct physics of FSE. In particular, Φ is related by a sum rule to $g(r)$:

$$\frac{1}{N} \sum_{k_1, k_2} \Phi(k_1, k_2, q) = \rho \int d^3r e^{iq \cdot r} g(r) \quad (18)$$

Also,

$$\lim_{q \rightarrow 0} \frac{1}{N} \sum_{k_1, k_2} \Phi(k_1, k_2, q) = n_{k_1} \quad (19)$$

I take the two-body t-matrix to be on-shell and I make semiclassical approximations [18] for eq. (16), which should certainly be valid at high Q . I also approximate Φ to satisfy Eqs. (18-19), i.e.

$$\sum_{k_1} \Phi(k_1, k, q) = n_k \rho \int d^3r e^{iq \cdot r} g(r) \quad (20)$$

Equation (20) amounts to a separable kernel approximation for the Dyson equation for χ_k , which results in the form, Eq. (2), for the final state broadening as a convolution of an $R(\epsilon)$ with the IA. Then an exact analytic solution of the Dyson equation for

CLASSICAL TRAJECTORIES

FIG. 1 - Schematic representation of classical trajectories which contribute to the GR, QCA, and HCPT theories for FSE. Shown is the initial configuration of particles 1 and 2 immediately after a neutron imparts momentum Q to 1, which moves to the right to collide with 2 with impact parameter b . The circle represents the range of the steeply repulsive core about 1. In each theory, x is the distance between particle 2 and the point with the corresponding theory label.

$G(R), V(R)$ FOR ^4He

FIG. 2 - Plot of the radial distribution function, $g(r)$, for ^4He and the He-He potential, $V(r)$. After a neutron collision, the recoiling particle starts its trajectory in the attractive well of the potential at some distance from the steeply repulsive core responsible for final state effects. The scaling variable, Y , is conjugate momentum to distance, x , along the classical trajectory.

FIG. 3 - Diagrammatic representation of the Dyson equation, Eqs. (14-15), for the HCPT of FSE. The shaded box is the two-particle density matrix, Φ . The dashed line is the two-particle t-matrix. Arrows denote the direction of the flow of momentum. Only the lines with Q carry high momentum.

$R_{\text{HCPT}}(Y)$

FIG. 4 - The final state effects (FSE) broadening function, $R(Y)$, predicted by HCPT for ^4He .

the χ_k exists, details of which may be found in ref. [11].

The final result for $R(Y)$ is identical to Eq. (10) obtained by the QCA, with

$$P_{\text{NC}}(x) = \exp \left(- \int_0^x \Gamma(x') dx' \right) ; \quad \Gamma(x) = \rho \pi \int_0^\infty db^2 f_b g(r(x, b)) \quad (21)$$

This is similar to Eq. (9) but differs in three details. First, there is no factor of 2 in the limit of the integral. Second,

$$f_b = e^{2i\delta_b} - 1 + e^{2i\delta_b} \frac{i\pi Q b}{2} \quad (22)$$

Here, δ_b is the semiclassical (JWKB) phase shift for He-He scattering at impact parameter b . The third term in Eq. (22) is due to Bose symmetrization and gives rise to the hard sphere glories in the He-He total cross section. The real part of $\Gamma(x)$ is again the collisions per unit distance. With $\text{Re } \Gamma(\infty) = \rho \sigma_{\text{tot}}/2$ by the optical theorem, there is an interesting cancellation of the 2 in $\sigma_{\text{tot}} = 2\pi r_0^2$, so that $\text{Re } \Gamma(\infty)$ is simply the classical result, $\rho \pi r_0^2$. Finally, $r(x, b) = \sqrt{x^2 + b^2}$ for HCPT. The x 's appropriate for the HCPT, QCA and GR theories are shown in Fig. 1.

HCPT PREDICTIONS FOR EXPERIMENT

The HCPT predictions [19] for deep inelastic neutron scattering on ^4He can be evaluated using as input the variational and Monte Carlo n_k , the experimental $g(r)$ measured by neutron diffraction [20], and the semiclassical predictions for the He-He phase shifts. The semiclassical methods are accurate at high Q , especially since the He-He potential at short distances is inferred from measured σ_{tot} by inverting the semiclassical formulae [3].

Figure 4 shows the $R(Y)$ predicted at $Q = 30 \text{ \AA}^{-1}$ which is achievable at pulsed neutron sources, but is much larger than reactor experiments for which $Q \leq 12 \text{ \AA}^{-1}$. While $\Delta Y_{FWHM} \approx \rho \sigma_{\text{tot}}(Q)$ as expected, $R(Y)$ is negative at larger $|Y|$ as required to satisfy the ω^2 -sum rule. Figure 5 shows specifically how this is accomplished by displaying the integrand of the sum rule, $Y^2 R(Y)$. The sum rule requires the area under this curve to be zero. The scale of the oscillations in $Y^2 R(Y)$ is determined by the scale of the spatial structure in $g(r)$, i.e. the period is approximately $2\pi/2.5 \text{ \AA}^{-1}$.

The n_k predicted for superfluid ^4He by Greens' Function Monte Carlo (GFMC) [21] is shown as the solid line in Fig. 6. GFMC predicts a 9.2% Bose condensate fraction, n_0 . The n_k predicted for normal fluid ^4He by Path Integral Monte Carlo (PIMC) [22] at $T = 3.3 \text{ K}$ is shown as the dashed line. The normal fluid n_k is smooth and approximately Gaussian.

The predictions for $F(Y)$ for normal fluid ^4He are shown in Fig. 7. The pluses are the 1A prediction for $F(Y)$ calculated by inserting the PIMC n_k into Eq. (1). The solid line is the HCPT prediction including FSE, which is calculated using the $R(Y)$ in Fig. 4 assuming that $g(r)$ is relatively temperature independent. FSE are predicted to

FIG. 5 - $Y^2 R(Y)$ predicted by HCPT. The ω^2 sum rule requires the area under this curve to be zero. The scale of the structure is governed by Fourier transform of the radial distribution function, $g(r)$.

FIG. 6 - Momentum distributions for liquid ^4He . The solid line is the Greens' Function Monte Carlo (GPMC) prediction for the superfluid at $T=0\text{K}$, which has a delta function at $k=0$ with a 9.2 % Bose condensate fraction. The dashed line is the Path integral Monte Carlo (PIMC) prediction for the normal fluid at $T=3.33\text{K}$.

FIG. 7 - The neutron scattering law, $F(Y)$, for normal ^4He at $T=3.33\text{K}$ using the PIMC momentum distribution. The pluses are the Impulse approximation (IA) prediction. The solid line is the HCPT prediction which includes broadening due to final state effects (FSE). The dashed line is the Lorentzian (LZ) broadening prediction obtained by ignoring ground state spatial correlations and setting $g(r)$ to one.

FIG. 8 - The neutron scattering law, $F(Y)$, for superfluid ^4He at $T=0\text{K}$ using the GPMC momentum distribution. The dashed line is the Impulse approximation (IA) prediction, which has a delta function at $Y=0$ due to the Bose condensate with 9.2% of the weight. The solid line is the prediction of HCPT which includes broadening due to final state effects (FSE).

be almost negligible for the normal fluid at $Q = 30 \text{ \AA}^{-1}$, and the IA should be an excellent description. For comparison, the dashed line is the prediction of the Lorentzian broadening theory obtained by setting $g(r)$ to one. The FSE predicted by HCPT are much smaller than those predicted by theories which ignore spatial correlations. This is expected since the effect of spatial correlations is to reduce collisions at short recoil distances.

The corresponding IA and HCPT predictions of $F(Y)$ for the superfluid are shown in Fig. 8. The IA is calculated by inserting the GFMC n_k in Fig. 6 into Eq. (1). The dashed line is the IA prediction which has 9.2 % of the weight in a delta function at $Y=0$. FSE are included in the HCPT prediction shown by the solid line, which is calculated by convoluting the IA prediction with the $R(Y)$ in Fig. 4, according to Eq. (2). The Bose condensate delta function in the IA is completely wiped out by FSE broadening, so that it is no longer a distinct peak. However, the Gaussian width of the overall distribution is unchanged by FSE as is expected from the ω^2 -sum rule, Eq. (4). Intensity is robbed from high $|Y|$ in order to preserve the sum rule.

Thus, the Bose condensate peak will not become resolvable by going to Q 's much larger than reactor experiments. However, such Q 's permit a theory of FSE, which allows tests of calculations of n_k using DINS data. This prediction does not imply that values of n_0 can not be inferred directly from DINS data, but the problem has become ill-conditioned as is discussed by Sivia and Silver [23] in these proceedings.

Since this theory of FSE predicts that the Bose condensate is unobservable as a distinct peak at $Q = 30 \text{ \AA}^{-1}$, a critical question becomes at what Q 's the Bose condensate may become observable as a distinct peak. As in the original HP theory, the broadening scales with $\sigma_{tot}(Q)$ for 4He, which is very slowly decreasing with increasing Q . Figure 9 shows the classical turning point, r_o , of the He-He potential [3] as a function of Q on a linear-log scale, which shows that the decrease is approximately logarithmic. Figure 10 shows the slow approach to the IA predicted by HCPT by plotting $F(Y)$ at several Q . HCPT FSE vary as $O(Q^0)$ for hard core potentials, and approximately as $O(\log Q)$ for the He-He potential. Experimentalists should certainly try to test this prediction by experiments at much larger Q . However, Q 's greater than $100 \text{ \AA}^{-1}$ with adequate resolution are unfeasible with present spectrometers.

The behavior of FSE should be similar for DINS experiments on 3He Fermi liquid, where a primary goal is to observe the Fermi surface discontinuity predicted in n_k . The solid line in Fig. 11 is the Fermi HyperNetted Chain (FHNC) prediction [24] for n_k which has a discontinuity, and the dashed line is the prediction of the Lhullier and Bouchaud wave function [25] which does not. The FSE predicted by HCPT and the $F(Y)$ at $Q = 30 \text{ \AA}^{-1}$ for 3He using the FHNC n_k as input have been calculated. An apparent n_k could be inferred from a measured $F(Y)$ by assuming the IA was correct and differentiating the data with respect to Y according to Eq. (1). The apparent n_k is shown as the dot-dashed line in Fig. 11, in which the Fermi surface discontinuity is no longer visible due to FSE. It resembles the Lhullier and Bouchaud prediction. Again, the Q dependence of the FSE broadening would be similar to 4He, so that the Fermi surface discontinuity in 3He should not be expected to become directly observable by performing experiments at much higher Q .

FIG. 9 - Classical turning points of the He-He potential versus Q , plotted on linear-log scale. This shows that the width of the FSE broadening of the IA, $R(Y)$, should decrease approximately as $O(\log Q)$ with increasing Q .

FIG. 10 - Approach to the impulse approximation (IA) with increasing Q of the neutron scattering law, $F(Y)$, for superfluid ^4He at $T=0\text{K}$ as predicted by HCPT. Only the region near $Y=0$ relevant to the Bose condensate is shown. The Bose condensate does not become a distinct peak at any experimentally feasible Q .

FIG. 11 - Momentum distributions for normal ^3He Fermi liquid at $T=0\text{K}$. The solid line is the Fermi Hypernetted Chain (FHNC) prediction. The dashed line is the prediction using the Lhullier & Bouchaud (LB) wave function. The dot-dash line is the apparent momentum distribution after broadening the FHNC-IA neutron scattering law by the HCPT prediction for FSE. The Fermi surface discontinuity is smoothed out by FSE.

FIG. 12 - Full Width Half Maximum (FWHM) of the neutron scattering law, $F(Y)$, for superfluid ^4He at $T=0\text{K}$ including FSE broadening. The HCPT is at the bottom. The Lorentzian broadening prediction (LZ) at the top is obtained by setting $g(r)$ to one. The FWHM and the glory oscillations are much smaller for HCPT than for LZ. They are close to the impulse approximation (IA) prediction.

DISCUSSION AND EXTENSIONS

The direct comparison of the HCPT predictions for $F(Y)$ with pulsed neutron experiments at $Q = 23 \text{ \AA}^{-1}$ on 4He is discussed in ref. [12], and in the paper by Sosnick, et al. [13] in these proceedings. The agreement is excellent with no adjustable parameters. A discussion of the formal relation of HCPT to other theories may be found in ref. [8], and a comparison between the data and these theories is presented in ref. [7]. HCPT has the best agreement with experiment. Given this success, one may question the need for improvements. However, in addition to further experimental tests and applications to other systems, there remain many outstanding problems in the theory of FSE.

First, the approximation of the two-particle density matrix (Φ) by Eq. (20) satisfies Eqs. (18-19), but it does not satisfy other known properties of Φ . Recently Ristig and Clark [26] have calculated a general structure for Φ which remedies this defect. By substituting their expression for Φ into Eqs. (14-15), one can formally show that the convolution form of the FSE, Eq. (2), and the relative temperature independence of FSF should be regarded as approximations. The critical issue remaining is whether the excellent agreement between theory and experiment [12] becomes fortuitous after the corrections to the IA using better Φ are calculated.

The approximations of HCPT, such as the semiclassical treatment of the t -matrix, begin to fail at the lower Q 's of reactor experiments. While HCPT has kept the leading $O(\log Q)$ FSE, Sears [27] has proposed that the $O(Q^{-1})$ FSE corrections to the IA are approximately antisymmetric in Y . They can be the dominant corrections in cases where the HCPT FSE are small, e.g. the almost Gaussian momentum distributions such as for normal 4He as shown in Fig. 7. There appears such a slight asymmetric component even in the $Q = 23 \text{ \AA}^{-1}$ experiments [12], and such an asymmetry is pronounced in reactor experiments on 4He and other noble gases. The glory oscillations are also an $O(Q^{-1})$ correction, which may be estimated by retaining the third term in Eq. (22). Figure 12 shows that the glory oscillations should be much smaller than predicted by the Lorentzian broadening theories which ignore ground state spatial correlations. However, a complete theory for FSE which includes all the $O(Q^{-1})$ terms remains to be developed and compared to experiment, and the connection to the Sears [27] expansion of FSE in powers of Q needs to be clarified.

The theory should be extended to the problem of FSE in quasielastic electron-nucleus scattering (QENS). As is discussed in more detail in ref. [28], QENS can be compared to DINS on 4He by appropriate scaling of the physical variables. The nuclear repulsive core is softer by three orders of magnitude and the nuclear density is comparatively low, which suggest that FSE might be smaller in QENS. However, QENS requires much smaller Q 's in order to avoid inelastic processes and much larger Y 's in order to obtain information on short range correlations in nuclei, which suggests that FSE should be more important. Some naive applications of HCPT to QENS are discussed in ref. [28], but the HCPT approximations are less valid for QENS. For example, the semiclassical methods and neglect of the self-energy of the initial particle can hardly be justified at low Q 's, and the on-shell approximation for the two-body t -matrix becomes less valid at large $|Y|$. Hopefully, the superoperator projection methods will provide a starting point for a more complete theory of FSE in QENS.

I believe there remains little doubt about the general properties of the FSE for DINS in helium. However, the quasiclassical approximation and the superoperator expansions I have used in their derivation are no more familiar to the condensed matter physics community than the time-ordered cumulant expansions originally used by Gersch and Rodriguez [6]. All of these methods deserve further development, as well as applications to other problems involving the dynamical response of systems with strong ground state correlations. In addition, a treatment of FSE using diagrammatic perturbation theory starting with the non-interacting ground state remains to be provided.

ACKNOWLEDGEMENT

Research supported by the Office of Basic Energy Sciences of the U.S. Dept. of Energy. I thank K. Bedell, J. W. Clark, G. Reiter, and P. E. Sokol for many helpful discussions.

REFERENCES

- [1] J. W. Clark, R. N. Silver, P. E. Sokol, this volume.
- [2] G. B. West, *Physics Reports* **18**, 263 (1975).
- [3] R. Felten, H. Kirst, K. A. Koehler, F. Torello, *J. Chem. Phys.* **26**, 2360 (1982).
- [4] P. C. Hohenberg, P. M. Platzman, *Phys. Rev.* **152**, 198 (1966).
- [5] P. Martel, E. C. Svensson, A. D. B. Woods, V. F. Sears, R. A. Cowley, *J. Low Temp. Phys.* **23**, 285 (1976).
- [6] H. A. Gersch, L. J. Rodriguez, *Phys. Rev. A* **8**, 905 (1973).
- [7] T. R. Sosnick, W. M. Snow, P. E. Sokol, R. N. Silver, this volume.
- [8] See R. N. Silver, *Phys. Rev. B* **38**, 2283 (1988) for a summary of prior theories.
- [9] L. J. Rodriguez, H. A. Gersch, H. A. Mook, *Phys. Rev. A* **2**, 2085 (1974).
- [10] R. N. Silver, G. Reiter, *Rap. Comm. Phys. Rev. B* **35**, 3647 (1987).
- [11] R. N. Silver, *Rap. Comm. Phys. Rev. B* **37**, 3794 (1988), *ibid.* **B38**, 2283 (1988).
- [12] T. R. Sosnick, W. M. Snow, P. E. Sokol, R. N. Silver, to be published.
- [13] T. R. Sosnick, W. M. Snow, P. E. Sokol, this volume.
- [14] G. Reiter, R. N. Silver, *Phys. Rev. Lett.* **54**, 1047 (1985).
- [15] G. Reiter, this volume.
- [16] R. N. Silver, *Condensed Matter Theories*, V. **4**, Plenum Press (1989), to be published.
- [17] P. M. Platzman, N. Tzoar, *Phys. Rev. B* **30**, 6397 (1984).
- [18] R. K. B. Helbing, *J. Chem. Phys.* **50**, 493 (1969).
- [19] R. N. Silver, to be published.
- [20] E. C. Svensson, V. F. Sears, A. D. Woods, P. Martel, *Phys. Rev. B* **21**, 3638 (1980).
- [21] P. Whitlock, R. Panoff, *Can. J. of Phys.* **62**, 1409 (1987).
- [22] D. M. Ceperley, E. F. Pollock, *Can. J. of Phys.* **65**, 1416 (1987).
- [23] D. S. Sivia, R. N. Silver, this volume.
- [24] F. Fabrocini, S. Fantoni, V. R. Pandharipande, to be published.
- [25] J. P. Bouchaud, C. Lhuillier, *Europhys. Lett.* **3**, 1273 (1987).
- [26] M. T. Ristig, J. W. Clark, this volume.
- [27] V. F. Sears, *Phys. Rev. B* **30**, 44 (1984).
- [28] J. W. Clark, R. N. Silver, *Proceedings of the Third International Conference on Nuclear Reaction Mechanisms*, Varenna, Italy, June 13-18, 1988, E. Gadiali, ed. Supplemento **66**, p. 531-540 (1988).