RL0-2230TS-204

DOE/ER/70004-204

RL-78-078

T.224

PION STRUCTURE FUNCTION FROM LOW- p_T HADRON PRODUCTION

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Abstract

The recombination model for low- p_T inclusive reactions is extended to the meson fragmentation region. Some basic issues in the model are reexamined, and the differences between the proton and pion initiated reactions are stressed. By applying the model to the $\pi^+p \rightarrow K^+X$ reaction, the fast valence quark distribution in the pion is extracted. Our best fit to the data yields $F_q^\pi(x) \sim (1-x)^{0.8 \pm 0.2}$ for large x .

I. INTRODUCTION

During the past year quantitative understanding of the inclusive distributions of produced particles in hadronic collisions at low p_T in the fragmentation regions has become possible the framework of the parton model. Emphasizing the role of the recombination mechanism⁽¹⁾, a series of investigations have shed light on the meson distributions^(1,2), baryon distributions⁽³⁾, the effect of resonance production^(4,5), and other topics^(6,8). However, so far quantitative comparisons with data have been attempted only in the proton fragmentation region because, on the one hand, the data are better, while on the other the quark distribution functions in the proton have been reliably determined in deep inelastic scattering. The aim of this paper is to apply the parton recombination model to the meson fragmentation region. From the existing (albeit insufficiently accurate) data on the meson initiated reactions we determine the rough behaviour of the meson structure function for $x > 0.75$. Of course, the formalism is amenable to a finer determination of the x dependence when more accurate data become available.

Although the calculational aspect of the application to the meson problem is straightforward, there is a matter of basic importance in the model that deserves further investigation. It has to do with the two-parton joint distribution function which is an essential input in the model. The need for a careful examination of this point becomes especially acute when we consider the difference between the meson and proton structure functions. It will be the main subject of our discussion in Section II.

The data on low- p_T reactions in the fragmentation region are not abundant. From the best data available we are still limited in the range of x_F as well as in accuracy. Thus we shall attempt to determine no more than the exponent of $(1-x)^k$ for the pion structure function for large x . That, however, is of central interest and alone justifies a phenomenological study of this kind. Our analysis for low- p_T meson production in the range $0.75 < x_F < 0.90$ complements the analysis of lepton-pair production in πp collisions⁽⁹⁾, and should jointly yield a pion structure function over the full range of x .

II. THE RECOMBINATION MODEL

In hadron-hadron collisions the number of quark-antiquark pairs in the sea is not the same as that probed in deep inelastic scattering where only a lone proton is involved. This observation has been made in connection with both the low- p_T reactions^(1,2) and the lepton-pair production processes^(10,11). Thus the two-parton distribution function, one of which being a valence quark, the other an antiquark in the turbulent sea, is not what one would associate with a quiescent proton. The origin of the difference is clearly due to the possibility of interactions among gluons from different incident hadrons leading to $q\bar{q}$ productions that are absent in lepton-hadron deep inelastic scattering. Schematically, a diagram that illustrates the point is shown in Fig. 1, where solid lines represent quarks or antiquarks and dashed lines represent gluons. For definiteness we consider protons as incident hadrons. The bubble in the middle depicts many-parton interaction, the details of which are not calculable in any known theory. The recombination model simplifies the problem by making a "cut" along the vertical dotted line and considers the π^+ production, for example, as a two-step question: the availability of a u and $\bar{d}$ in the hadronic system and their likelihood to form a π^+ . More precisely, we write

$$\frac{E}{\sigma_T} \frac{d\sigma}{dp_L} = \frac{x}{\sigma_T} \frac{d\sigma}{dx} = \int F(x_1, x_2) R(x_1, x_2, x) \frac{dx_1}{x_1} \frac{dx_2}{x_2} \quad (2.1)$$

where $F(x_1, x_2)$ is the two-parton joint distribution function and $R(x_1, x_2, x)$ is the recombination function. Clearly, $F(x_1, x_2)$ represents everything to the left of the dotted line, and $R(x_1, x_2, x)$ to the right.

Although there is no large momentum-transfer to justify impulse approximation as in the usual parton model, (2.1) is to be justified on the basis of short-range correlation. For the detected pion in the fragmentation region ($x > 0.5$ say) the value of x_2 (for the $\bar{d}$) should not belong to the central region. This is achieved by $R(x_1, x_2, x)$ vanishing as $x_2 \rightarrow 0$. Note that x_1 and x_2 may be numerically far apart while their separation in rapidity is no more than two units in the fragmentation region. With $\bar{d}$ not being in the central region, the recombination process

may therefore take place independently of the pion production process elsewhere in the phase space. That is why in (2.1) we can ignore the other partons while considering the recombination of the u and $\bar{d}$ at x_1 and x_2 , assuming $x_1 + x_2 = x > 0.5$. In a space-time picture this is equivalent to an impulse approximation. The problem is, of course, to find a reliable form for $F(x_1, x_2)$. The dependence on $R(x_1, x_2, x)$ is not very crucial so long as it vanishes sufficiently fast at $x_2 = 0$ to exclude the $\bar{d}$ in the central region.

A. Two Parton Distribution $F(x_1, x_2)$

In Ref. 1 a simple form for $F_{ij}(x_1, x_2)$ is assumed

$$F_{ij}(x_1, x_2) = F_i(x_1) F_j(x_2) \rho(x_1, x_2) \quad (2.2)$$

where for proton it is argued that the phase space factor $\rho(x_1, x_2)$ should have the form

$$\rho(x_1, x_2) = \beta(1-x_1-x_2) \quad (2.3)$$

in the limit $x_1 + x_2 = x \rightarrow 1$. For the purpose of satisfying momentum sum rules, it has been suggested^(7,8) that F_{ij} should be specified by the Kutti-Weisskopf formula⁽¹²⁾ for the independent emission model. It is an assumption that renders the calculation of F_{ij} more precise, but not physically more plausible. The $q\bar{q}$ pairs produced by gluon interactions may well be highly correlated. Even the u and $\bar{d}$ that recombine may be correlated, since as we have discussed above they are not far apart in rapidity. Recombination is, after all, a manifestation of correlation. The factorizable form in (2.2) can only be interpreted as a definition of $F_j(x_2)$ since the $\bar{d}$ distribution of the turbulent sea (in both normalization and shape) is unknown anyway. By not identifying $F_j(x_2)$ with the $\bar{d}$ distribution in the quiescent sea as probed by lepton scattering, we are somewhat freed from the assumption of no correlation. Thus $F_j(x_2)$ represents an effective distribution whose form is to be determined on the assumption that the fast valence quark distribution $F_i(x_1)$ is as determined in deep inelastic scattering at "low" Q^2 .

There remains the question of $\rho(x_1, x_2)$ to be examined further. Because $R(x_1, x_2, x)$ is proportional to $\delta(x_1 + x_2 - x)$, $x_1 + x_2$ should not exceed one. The behaviour of $F_{ij}(x_1, x_2)$ in the limit $x = x_1 + x_2 \rightarrow 1$ is controlled only partially by $F_i(x_1)$ and $F_j(x_2)$, but more directly by the probability of finding other partons (especially the other valence quarks) at zero momentum fraction. It is the latter effect that is represented by $\rho(x_1, x_2)$, which we now write as

$$\rho(x_1, x_2) = \rho(x) \theta(1-x), \quad x = x_1 + x_2 \quad (2.4)$$

From Fig. 1 one sees that the limit $x \rightarrow 1$ is complicated by at least two factors over and above what is involved in the hadron wave function. One has to do with the parton interactions in the center-blob, while the other pertains to the separation of the $d\bar{d}$ pair created by a gluon. The former can be overcome kinematically by the assumption that at very high energy the right-movers persist to be right-movers after interaction (similarly for left-movers) and that defectors (from right to left or vice-versa) are negligible due to off-shell suppression. In that case the $x \rightarrow 1$ limit is then relayed to the hadron wave function, a subject to be discussed presently. The other complication due to the dissociation of a gluon into a $d\bar{d}$ pair with the momentum fraction of d approaching zero presumably can introduce an extra kinematical factor. However, such complications are difficult to take into account systematically, and we shall for expediency ignore them here. They certainly deserve some attention for an improvement of the model.

We come now to the hadron wave function and study its implication on the phase space factor $\rho(x)$. It is important first to recognize that $F_{ij}(x_1, x_2)$ refers to the two-parton distribution during the time interval when recombination occurs and the meson is formed. Because the meson is in the fragmentation region, it is produced at a time much later than the meson production in the central region. For x close to one, all other right-moving particles including the baryon are forced to have a small total momentum fraction $\epsilon = 1-x$. For the behaviour of $\rho(x)$ that scales ϵ must not vanish faster than any inverse power of s . Now, for a small but finite value of ϵ , this residual momentum is partitioned predominantly in a way in which pions, if any, are in the wee region

while the baryon carries the major portion of the momentum ϵ . This is because the valence quark distribution vanishes at zero momentum fraction, while the sea quarks do not. Thus the behaviour of $\rho(x)$ at large x is determined mainly by the baryon distribution at ϵ , or more directly by the valence quark distributions. In terms of time scale, they are closest to the recombination time; on the other hand, the wee partons have turned into wee pions long in the past and should not enter into our consideration in any essential way.

The above discussion, for large x but not in the extreme limit $x \rightarrow 1$, provides the reasoning for our adoption of the following formula for $\rho(x)$:

$$\rho(x) = \beta_0 \int F_{v_1}(y_1) F_{v_2}(y_2) \delta(1-x-y_1-y_2) \frac{dy_1}{y_1} \frac{dy_2}{y_2} \quad (2.5)$$

where β_0 is some constant of order unity, and $F_{v_i}(y_i)$ are the distribution functions of the two remaining valence quarks, assuming that it is the proton fragmentation region that we are studying. One could insert under the integral some sea quark distributions, and the convoluted result should not change the character. However, one should not sum (to infinity) over the number of sea quarks, as in the Kuti-Weisskopf model⁽¹²⁾, because that would change the nature of $\rho(x)$ from an emphasis on the valence quarks to the wee partons, a situation which we have argued above to be not physical.

If we assume that the small y behaviour of $F_v(y)$ is

$$F_{v_i}(y_i) \propto y_i^k \quad (2.6)$$

where the value of k is to be discussed below, then (2.5) implies (for a nucleon)

$$\rho_N(x) = \beta_N (1-x)^{2k-1}, \quad x \sim 1, \quad (2.7)$$

where β_N is a constant. This result is stable against the introduction of a wee parton distribution, as in

$$\rho_N(x) = \beta_1 \int F_{v_1}(y_1) F_{v_2}(y_2) F_w(y_3) \delta(1-x-\sum y_i) \prod_{i=1}^3 \frac{dy_i}{y_i} \quad (2.8)$$

or any finite number of them, provided that $F_w(y)$ approaches a constant as $y \rightarrow 0$. The extension of our consideration to meson fragmentation is trivial. Using only one valence quark in (2.5), we obtain for a meson

$$\rho_M(x) = \beta_M (1-x)^{k-1}, \quad x \sim 1 \quad (2.9)$$

Clearly, the fewer valence quarks there are in the incident hadron, the easier it is for the fast meson to approach the kinematic limit $x \rightarrow 1$. The fact that (2.9) differs from (2.7) is crucial for our consideration of the pion structure function in the following, and justifies our rather lengthy discussion of the point.

The exponent k in (2.6) specifies the behaviour of a valence quark at zero momentum fraction with a hadronic system forced to $x \sim 1$. That behaviour should probably be given by the counting rule⁽¹³⁾ $k = 1$, and not by the Regge behaviour $k = 1 - \alpha_R$, because the latter relies on summing over all intermediate states in a unitarity equation for the virtual-photon-hadron scattering amplitude, a circumstance which is far from the problem of our interest. If we substitute $k = 1$ into (2.7) and (2.9) we have

$$\rho_N(x) = \beta_N (1-x), \quad \rho_M(x) = \beta_M, \quad (2.10)$$

for x near 1. The first expression agrees with the one used originally⁽¹⁾ for a proton, while the second expression is new.

We can use (2.10) for the region $0.5 < x < 0.95$. They are approximate expressions that are adequate to the degree of accuracy of the model. Significant departure is expected for x very close to one (say $x > 0.95$) because of, for example, the $d\bar{d}$ correlation in a pair creation by a gluon, mentioned earlier but ignored. In that extreme limit there are also other possible complications⁽⁸⁾. In our phenomenological application we shall have no occasion to go into that extreme region.

B. Recombination Function $R(x_1, x_2, x)$

It has been argued that the function $R(x_1, x_2, x)$ describing the probability of recombination between a quark at x_1 and an antiquark at x_2 to form a meson at x should be given by⁽¹⁾

$$R(x_1, x_2, x) = R(\xi_1, \xi_2) \cdot \delta(\xi_1 + \xi_2 - 1) \quad (2.11)$$

where $\xi_i = x_i/x$, $R(\xi_1, \xi_2)$ is related to the valence part of the meson structure function, and has the form

$$R(\xi_1, \xi_2) = \alpha_M(\xi_1, \xi_2)^k \quad (2.12)$$

where α_M is another constant. The exponent is the same k that appears in (2.6). In the past we have used the counting rule and set $k = 1$. The vanishing of R at $\xi_2 = 0$ is necessary to introduce short-range correlation in the recombination process and to guarantee that the wee partons have no rôle in the production of mesons in the fragmentation region.

In our phenomenological application of the model in this paper, we shall leave k unspecified in both (2.9) and (2.12). Moreover, since we shall consider pion initiated reactions, we shall need the valence quark distribution of the pion, at least in the region $x > 0.5$. It is to be used as an input in (2.2). Having no electron production data on a pion, one must resort to theoretical conjectures on $F^\pi(x)$, about which there have been a few suggestions. The aim of this paper is to determine the large x behaviour of $F^\pi(x)$ from the low p_T data. Hence, to be consistent with the foregoing, we parametrize again in the form

$$F^\pi(x) = C_\pi(1-x)^k \quad (2.13)$$

where k is to be identified with the same appearing in the exponents in (2.9) and (2.12). We shall vary k to obtain the best fit to the data. We do not expect (2.13) to be true for all x , but we shall use it in the entire range $0 < x < 1$ for the sake of definiteness in calculations. Since the data on pion fragmentation is for $x > 0.75$, the smaller x part of (2.13) is irrelevant and untested.

III. PION FRAGMENTATION

As demonstrated in Refs. (1), (2), (3), the recombination model has been successfully applied to data on proton fragmentation, e.g. $pp \rightarrow \pi^\pm X, K^\pm X$. The approach was to take the quark distributions in proton as revealed in deep inelastic scattering and to feed these into (2.1) and (2.2). By comparing with the data, one could then learn what antiquark distributions were required. In this paper, we reverse the procedure using the same sea-quark distributions as found before and compare with data on $\pi p \rightarrow \text{meson} + X$ to get information on the pion structure function.

A. Experimental situation for pion fragmentation

There is a great scarcity of data on the inclusive process meson + proton $\rightarrow$ fast meson + X at high energies. Most of such data which does exist corresponds to "exotic exchange" processes, e.g. $\pi^+ p \rightarrow \text{fast } \pi^- + X$ which is not best suited for the application of the recombination model since the π^- is not a direct consequence of either valence quark in the π^+ . Indeed such data is consistent with ρ^0 production where the ρ^0 subsequently decays to give the observed π^- . The only reasonable data of direct utility to us appears to be the so-called FSASF experiment⁽¹⁴⁾ at Fermilab. They have measured $\pi^\pm p \rightarrow \text{fast } K^\pm X$ for p_{lab} up to 175 GeV/c in the region $0.75 < x < 0.95$ and also measure the corresponding proton fragmentation reactions $pp \rightarrow K^\pm X$. They parametrise their data in this region in the form

$$\frac{d^2\sigma}{dt d(\frac{M^2}{s})} \sim \left(\frac{M^2}{s}\right)^{1-2\alpha_{eff}} \exp [b_{eff} t] \quad (3.1)$$

$$\begin{aligned} \text{where } \alpha_{eff} &= 0.20 \text{ for } \pi^+ p \rightarrow K^+ X \\ &= -0.65 \text{ for } pp \rightarrow K^+ X \end{aligned}$$

Actually we need $(E/\sigma_T) d\sigma/dp_L$ so we have to transform from $t, M^2/s$ to x and p_T^2 and then integrate w.r.t. p_T^2 . After this we find that the data on $\pi^+ p \rightarrow K^+ X$ appears to be very flat with x ; see fig. 2. At this stage we might ask how much faith we have in this data. We note that the FSASF data on $pp \rightarrow K^+ X$ agrees very closely with the recent data of the Fermilab-

Illinois group⁽¹⁵⁾. Nevertheless we are inclined to treat the extremely flat nature of the FSASF $\pi^+p \rightarrow K^+X$ distribution with a little suspicion. Their analysis indicates $\alpha_{\text{eff}} = 0.2$ in the triple-Regge formula - which is somewhat high when one considers that this trajectory represents K^* exchange, not at $t = 0$, but over a range $0.1 < |t| < 0.65 \text{ GeV}^2$. If α_{eff} were lower, the resulting x distribution would be less flat. So we try to bear this suspicion in mind when presenting our conclusion later on.

B. Results of fit to data

Inserting (2.2), (2.9), (2.12), (2.13) into equation (2.1) we get

$$\frac{E}{\sigma_T} \frac{d\sigma}{dp_L} = \alpha_M \beta_M \frac{(1-x)^{k-1}}{x} \int_0^x C_\pi (1-x_1)^k F_S^-(x-x_1) \left[\frac{x_1(x-x_1)}{x} \right]^{k-1} dx_1 \quad (3.2)$$

for the process $\pi^+p \rightarrow K^+X$ and using (2.7)

$$\frac{E}{\sigma_T} \frac{d\sigma}{dp_L} = \alpha_N \beta_N \frac{(1-x)^{2k-1}}{x} \int_0^x F_u^p(x_1) F_S^-(x-x_1) \left[\frac{x_1(x-x_1)}{x} \right]^{k-1} dx_1 \quad (3.3)$$

for the process $pp \rightarrow K^+X$.

Our procedure is to determine k , and thus the shape of the u quark distribution in π^+ at large x , by comparing (3.2) with the data on $\pi^+p \rightarrow K^+X$. For consistency we simultaneously compare (3.3) with the data on $pp \rightarrow K^+X$ from the same experiment since this reaction also depends on k .

As before^(1,2,5) we take a parametrisation of $F_u^p(x_1)$ from the fit of Field and Feynman⁽¹⁶⁾ to the deep inelastic data. As far as the anti-quark distribution $F_S^-(x_2)$ is concerned, we have argued in Sec. II that it is an effective sea quark distribution associated mainly with the gluon interactions in hadron-hadron collisions. Since the produced pions in the central regions are similar in pp and πp collisions, it is reasonable to assume a universality in such gluon interactions. For that reason we set F_S^- to be the same in (3.2) and (3.3). It has been taken in previous fits as^(1,5)

$$F_{\bar{q}}(x_2) = c_{\bar{q}}(1-x_2)^{n_{\bar{q}}} \quad (3.4)$$

with $n_{\bar{q}} = 6 \rightarrow 8$.

In Fig. 2 we show the result of fitting both $\pi^+p \rightarrow K^+X$ and $pp \rightarrow K^+X$ data from the FSASF experiment⁽¹⁴⁾. The best fit corresponds to a value $k = 0.8$. In this fit $n_{\bar{s}} = 8$ was used but the curves are only slightly different for $n_{\bar{s}} = 6$. The errors on the parametrisations of the data are such that if k is varied by more than 0.2 from the best value of 0.8, the fits worsen significantly. Although the fit to $\pi^+p \rightarrow K^+X$ data lies within the errors of the parametrisation of the data over most of the x range 0.75 to 0.95 the detailed shape is not in spectacular agreement. As pointed out in the previous subsection we have some doubt as to whether the data should really be so flat between 0.75 and 0.85. Further data on meson fragmentation processes, e.g. $\pi^+p \rightarrow \text{fast } \pi^0 + X$ at Fermilab will be of considerable interest in resolving these doubts.

Bearing in mind these questions, we would conclude that the data on meson fragmentation (and indeed also the data on proton fragmentation since (3.3) involves k) is consistent with a u quark distribution in π^+ :

$$F_u^{\pi^+}(x) \sim (1-x)^{0.8 \pm 0.2} \quad (3.5)$$

for large x .

IV. CONCLUSIONS

The phenomenological result obtained in (3.5) for the large x behaviour of the pion structure function is in rough agreement with the counting rule⁽¹³⁾, but in distinct disagreement with the conjecture that it behaves as a constant at large x ⁽¹⁶⁾. Clearly, better data on hadron production in the pion fragmentation region are needed. It will be interesting to compare our result with what emerges from an analysis of the lepton-pair production at large x in pion initiated reactions⁽⁹⁾.

In this paper we have also considered further some of the basic issues in the recombination model. We have found that the two-parton distribution function for the pion differs significantly from that for the proton. The fact that a self-consistent value of k exists and fits the pion data, yielding a sensible large x behaviour for the pion structure function, are indications that the model can be effective in extracting information on the constituent structure of hadrons, unavailable to leptonproduction.

ACKNOWLEDGEMENT

We would like to thank C S Lam and M J Teper for helpful discussions. One of us (RCH) also wants to acknowledge the warm atmosphere of the Theory Division at the Rutherford Laboratory where he has enjoyed his visit for the past year.

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FIGURE CAPTIONS

Fig. 1 Schematic description of the recombination model for $pp \rightarrow \pi^+ X$. Solid lines denote quarks or antiquarks, dashed lines represent gluons. The dotted line indicates where the recombination picture is described.

Fig. 2 Comparison of parametrisation of the data on
(a) $\pi^+ p \rightarrow K^+ X$,
(b) $pp \rightarrow K^+ X$
from the FSASF collaboration⁽¹⁴⁾ with the recombination expressions given by eqs. (3.2), (3.3). The fit shown corresponds to $k = 0.8$. That is $F_q^\pi(x) \sim (1-x)^{0.8}$
Note that the normalisations of the curves are independent and quite arbitrary.

Fig.1

Fig. 2