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TWO-PHASE-FLOW MODELS AND THEIR LIMITATIONS

M. Ishii and G. Kocamustafaogullari

Reactor Analysis and Safety Division University of Wisconsin at
Argonne National Laboratory Milwaukee
Argonne, Illinois Milwaukee, Wisconsin

ABSTRACT

An accurate prediction of transient two-phase flow is essential to safety analyses of nuclear reactors under accident conditions. The fluid flow and heat transfer encountered are often extremely complex due to the reactor geometry and occurrence of transient two-phase flow. Recently considerable progresses in understanding and predicting these phenomena have been made by a combination of rigorous model development, advanced computational techniques, and a number of small and large scale supporting experiments. In view of their essential importance, the foundation of various two-phase-flow models and their limitations are discussed in this paper.

INTRODUCTION

In view of the great importance of two-phase flow analyses, a number of publications on two-phase flow formulation, constitutive relations and experimental correlations appeared in the past. A general review of these works can be found in standard text books by Tong (1), Brodkey (2), Wallis (3), Hewitts and Hall-Taylor (4), Collier (5), Govier and Aziz (6), Hsu and Graham (7), Layey and Moody (8) and others. In particular, recent developments in two-phase flow analyses related to the light water reactor safety have been reported at the yearly Water Reactor Research Information Meeting sponsored by NRC and various specialist meetings. Some of the new results are reviewed and summarized in detail in symposium proceedings (9-10). In spite of these developments, available two-phase flow models have a number of shortcomings and limitations as pointed out by Wallis (11).

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These difficulties mainly arise from the fundamental nature of internal structures of two-phase flow, i.e., flow regimes. From the earliest time it has been recognized that the various transfer mechanisms between two-phase mixture and structures as well as between two phases depend on the flow regimes. Yet, experimental determinations of the flow regimes (6, 12-16) have been quite subjective until recently. More objective methods based on statistical analyses of data from pressure transducers or void measurements have been proposed (17-19). However, the concept of the flow regimes is based on a global phenomena and, therefore, their identification based on one parameter such as the void fraction is still subjective. This is particularly true in a complicated geometrical system such as nuclear reactors. Often the flow regimes depend on the size of the volume used to define them. Another problem arises from the fact that the conventional parameters used to identify the flow regimes such as superficial velocities may not be sufficient to characterize a geometrical configuration of two-phase flow, particularly under transient and entrance conditions (20).

In general, physical problems of two-phase flow are represented by macroscopic field and constitutive relations using continuum formulation (21-23). Two-phase flow models can be developed from several different methods; however, the greatest difficulty is associated with the establishment of the constitutive relations. In many cases little data base exists to extend existing correlations to a new situation or to develop new correlations. This implies that the prediction of two-phase flow in new or hypothetical situations is always very difficult. It is important, therefore, that a researcher knows the limitations of a model used and the range of the data base for the constitutive relations applied.

FOUNDATION OF TWO-PHASE FLOW MODELS

The most important characteristic of two-phase flow systems is the presence of interfaces separating the phases. Depending upon the geometry of the interfaces, the flow of two-phase mixture has been classified into several flow regimes. The transport mechanisms can be quite different between two different flow regimes; however, two or more flow regimes can occur simultaneously in a single system. This leads to further complications in two-phase flow modeling.

Locally, a two-phase flow system consists of a number of single phase regions bounded by moving interfaces. Therefore, at least in theory, the problem could be formulated in terms of the standard balance equations applicable to single phase flow with appropriate matching boundary conditions at an interface. However, in general, a formulation based on this local instant variables and moving interfaces results in a multi-boundary problem with positions of the interfaces being unknown. Therefore, obtaining a solution from this

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formulation is impossible or unpractical for most cases due to these mathematical difficulties.

In order to overcome these difficulties, a certain macroscopic view of two-phase flow should be taken. In the past, mainly three approaches have been used to develop a two-phase flow model:

- 1) Interacting Continua Assumption $\rightarrow$ Diffusion Model
- 2) Control Volume $\rightarrow$ Postulated Balance Equations
- 3) Averaging Method $\rightarrow$ Averaged Balance Equations.

In all three approaches the details of the local instant formulation are partially eliminated. Instead of considering local instant transfers at interfaces, collective interaction of fluid particles and the dynamics of the interfaces are modeled in a macroscopic two-phase flow formulation. Therefore it is necessary to understand fundamental mechanisms of local transfers and then to obtain a macroscopic description of these transfers. For dispersed flows, for example, it may be necessary to determine the average particle size, collective drag law for a multiparticle system, as well as rates of nucleation, evaporation or condensation.

Three distinct methods of obtaining a two-phase flow formulation are listed above. The first and second approaches are mainly based on hypotheses, physical intuitions, and assumed similarity with a single phase flow system. On the other hand, the averaging method is mathematically rigorous but requires lengthy manipulation of equations:

In the first approach it is assumed that each point in the mixture is occupied simultaneously by two phases, and the thermodynamic and transport properties of a mixture depend on those of each phase and concentration. Each phase moves with its own center of mass velocity, thus interdiffusion of phases is included in the model. This approach was first used for gas mixtures without interfaces. Later it was extended to more general mixtures by Truesdell (24) and others. The continuum approach applied to dispersed two-phase flow was summarized by Soo (25) in his book. The model is based on three hypotheses concerning the mixture properties, mixture balances, and transfers between phases or components. It is recalled that a two-phase flow system consists of finite single phase regions bounded by deforming interfaces. Therefore, except in the case of highly dispersed flows, there is a conceptual difficulty over the continuum assumption in this approach. The most fundamental characteristic of two-phase flow, which is the existence of singular surfaces and associated discontinuities of various variables, is bypassed by a sweeping assumption of continuity everywhere for both phases. Generally, interfacial transfer is lumped into a single term in a phasic balance equation. Thus, some difficulties can arise in interpreting or splitting this term into physically understandable or measurable

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quantities. For example, a controversy on the form of the pressure gradient term in the phasic momentum equation arose due to different interpretations.

The second approach is to write a balance equation for mass, momentum, and energy for a control volume. It can be done either for a mixture as a whole or for individual phases. In most cases the flow is considered to be one-dimensional or quasi-one-dimensional. Thus the model can be visualized as two separated phases flowing parallel with an imaginary interface separating the phases. The transfers between phases are permitted through this interface. Because of the simplicity of the method and practicality of the one-dimensional formulation, a number of models have been developed from this approach (2, 3, 26-29) and used extensively in the past. This model can be quite sufficient and accurate for a separated two-phase flow, such as annular two-phase flow, provided the interfacial shear and heat transfer correlations are satisfactory. However, in dispersed flow or mixed flow regimes, various effects of distribution of phases, velocity and temperature in the transverse direction can become very important. In such a case, a more sophisticated model based on the averaging method is recommended.

The rational approach to obtain the macroscopic two-phase flow formulation is the application of proper averaging. The averaging procedure is basically low pass filtering which eliminates unwanted high frequency signals from local instant fluctuations of variables. However, the statistical properties of these fluctuations and collective contribution of interfaces which affect the macroscopic processes should be taken into account in the formulation through various constitutive relations.

Averaging procedures which can be applied to two-phase systems can be classified into three main groups based on the basic physical concepts used to formulate dynamical problems (23). These are the Eulerian, Lagrangian, and Boltzmann statistical averages. They can be further divided into sub-groups based on a variable with which averaging is defined. The most important and widely used group is the Eulerian averaging. This can be explained by the traditional and effective use of the Eulerian description of thermo-fluid mechanics and its close relation to experimental observations and instrumentations. In particular, the Eulerian time (or statistical) and Eulerian area averaging procedures form the theoretical foundation of rigorous formulation of two-phase flow problems.

The Eulerian time (or statistical) averaging, which has been used extensively for analyzing a single-phase turbulent flow, is the foundation of a fully three-dimensional two-phase flow model (21, 23, 30-35). The model is useful where a quasi-one-dimensional approach is too inaccurate; for example, flows in an entrance region or in a vessel. Except these special cases, the Eulerian area

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averaging over a cross section of a flow area gives the most practical two-phase flow formulation (21-23, 36-38). By area averaging, the resulting field equations become quasi-one-dimensional. Therefore, the information on changes of variables in the direction normal to the main flow within a flow channel is basically lost. Thus the transfer of momentum and energy between the wall and fluid should be specified by empirical correlations or by simplified models. When the area averaging is applied to a subchannel within a rod bundle, a cross flow momentum equation (37-40) should be included in the formulation. Furthermore, it is noted that generally the average of a product of variables is different from a product of the averages. This difference is defined as a covariance and it plays a very important role in the averaged formulation. For example, in the time averaged equations (23) the well-known turbulent transfer terms, such as the Reynolds stress, correspond to the covariance arising from the convective fluxes. In the area averaged formulation, the important effect of phase and velocity distributions on the relative velocity was pointed out and several efforts to model this effect have been made (36, 41, 42).

VARIOUS TWO-PHASE FLOW MODELS

Besides these different approaches for developing a model, there are two fundamentally different formulations of the macroscopic field equations for two-phase flow systems: namely, the two-fluid model and the mixture model. The two-fluid model (2, 3, 21-23) is formulated by considering each phase separately in terms of two sets of conservation equations governing the balance of mass, momentum, and energy of each phase. Since the macroscopic fields of one phase are not independent of the other phase, the interaction terms which couple the transport of mass, momentum, and energy of each phase across the interphases appear in the field equations.

Previous studies have indicated that unless phasic momentum interaction terms are accurately modeled, the advantage of the two-fluid model over the mixture model disappears and in certain cases numerical instabilities result. At least two transient forces, i.e., the virtual mass and Basset forces, exist in addition to the drag and interfacial shear forces (20, 34, 43, 44). However, these transient momentum interaction terms are not firmly established. Because of their importance under rapid transient conditions and for numerical-stability problems, further research in this area is required. Another approach to achieving numerical stability is the inclusion of "artificial viscosity" in the numerical algorithm to damp out high frequency oscillations occurring possibly due to imprecise modelling. This approach is currently being followed by Amsden and Harlow (39) in their two-fluid digital computer codes. In spite of these shortcomings of two-fluid models, there is, however, no substitute available for modeling accurately two-phase

phenomena where two phases are weakly coupled.

A three-dimensional two-fluid model has been obtained by using temporal or statistical averaging (23). The model is expressed in terms of two sets of conservation equations governing the balance of mass, momentum, and energy in each phase. However, since the averaged fields of one phase are not independent of the other phase, the interaction terms appear in the field equations as source terms. For most practical applications, the model developed by Ishii (1) can be simplified to the following forms:

Continuity Equation

$$\frac{\partial \alpha_k \rho_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \vec{v}_k) = \Gamma_k \quad (1)$$

Momentum Equation

$$\begin{aligned} \frac{\partial \alpha_k \rho_k \vec{v}_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \vec{v}_k \vec{v}_k) = & -\alpha_k \nabla p_k + \nabla \cdot \alpha_k (\bar{\tau}_k + \tau_k^t) \\ & + \alpha_k \rho_k \vec{g} + \vec{v}_{ki} \Gamma_k + \vec{M}_{ik} - \nabla \alpha_k \cdot \tau_i \end{aligned} \quad (2)$$

Enthalpy Energy Equation

$$\begin{aligned} \frac{\partial \alpha_k \rho_k H_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k H_k \vec{v}_k) = & -\nabla \cdot \alpha_k (\bar{q}_k + q_k^t) + \alpha_k \frac{D_k}{Dt} p_k \\ & + H_{ki} \Gamma_k + \frac{q_{ki}''}{L_s} + \phi_k \end{aligned} \quad (3)$$

Here Γ_k , $\vec{M}_{ik}$, τ_i , q_{ki}'' , and ϕ_k are the mass generation, generalized interfacial drag, interfacial shear stress, interfacial heat flux, and dissipation, respectively. The subscript k denotes k phase, and i stands for the value at the interface. L_s denotes the length scale at the interface, and $1/L_s$ has the physical meaning of the interfacial area per unit volume (20, 23). Thus,

$$\frac{1}{L_s} = a_i = \frac{\text{Interfacial Area}}{\text{Mixture Volume}} \quad (4)$$

Note that the corresponding subchannel model has been obtained from area averaging (38).

The above field equations indicate that several interfacial transfer terms appear on the right-hand sides of the equations. Since these interfacial transfer terms also should obey the balance laws at the interface, interfacial transfer conditions could be obtained from an average of the local jump conditions (1). They are given by

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$$\sum_k \Gamma_k = 0$$

$$\sum_k \dot{M}_{ik} = 0 \quad (5)$$

$$\sum_k (\Gamma_k H_{ki} + q_{ki}''/L_s) = 0$$

Therefore, constitutive equations for $\dot{M}_{ik}$, q_{ki}''/L_s , and q_{fi}''/L_s are necessary for the interfacial transfer terms. The enthalpy interfacial transfer condition indicates that specifying the heat flux at the interface for both phases is equivalent to the constitutive relation for Γ_k if the mechanical-energy transfer terms can be neglected (23). This aspect greatly simplifies the development of the constitutive relations for interfacial transfer terms.

By introducing the mean mass transfer per unit area defined by

$$\Gamma_k \equiv a_i \dot{m}_k, \quad (6)$$

we can rewrite the interfacial energy-transfer term in Eq. (3) as

$$\Gamma_k H_{ki} + \frac{q_{ki}''}{L_s} = a_i (\dot{m}_k H_{ki} + q_{ki}'') \quad (7)$$

The heat flux at the interface should be modeled using the driving force or the potential for an energy transfer. Thus,

$$q_{ki}'' = h_{ki} (T_i - T_k), \quad (8)$$

where T_i and T_k are the interfacial and bulk temperatures based on the mean enthalpy. In view of the above, the importance of the interfacial area, a_i , in developing constitutive relation for this term is evident. The interfacial transfer terms are now expressed as a product of the interfacial area and the driving force. It is essential to make a conceptual distinction between the effects of these two parameters. The interfacial transfer of mass, momentum, and energy increases with an interfacial-area concentration toward the mechanical and thermal equilibrium.

By neglecting the lift force due to the rotations of particles and the diffusion force due to the concentration gradient, the generalized drag force for a dispersed phase can be modeled by a simple form (43) such as

$$\dot{M}_{id} = \alpha_d \vec{F}_D/B_d + \alpha_d \vec{F}_V/B_d + \frac{9}{2} \frac{\alpha_d}{r_d} \sqrt{\frac{\rho_c \nu_m}{\pi}} \int_t^{D_d} \frac{D_d}{D\xi} (\vec{v}_c - \vec{v}_d) \frac{d\xi}{\sqrt{t - \xi}}, \quad (9)$$

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where $\vec{F}_D$, B_d , $\vec{F}_V$, and μ_m are the standard drag force, volume of a typical particle, virtual mass force, and mixture viscosity, respectively. The last term is the Basset force. The standard drag force acting on the particle under steady-state conditions can be given in terms of the drag coefficient C_D based on the relative velocity as

$$\vec{F}_D = -\frac{1}{2} C_D \rho_c \vec{v}_r |v_r| A_d \quad (10)$$

where A_d is the projected area of a typical particle and $\vec{v}_r$ is the relative velocity given by $\vec{v}_r = \vec{v}_d - \vec{v}_c$. Hence, the portion of M_{id} represented by the drag force becomes

$$\alpha_d \vec{F}_D / B_d = - \left(\alpha_d \frac{A_d}{B_d} \right) \frac{C_D}{2} \rho_c \vec{v}_r |v_r| \quad (11)$$

For spherical particles it can be shown that

$$\alpha_d \vec{F}_D / B_d = -a_i \left[\frac{C_D}{4} \frac{\rho_c \vec{v}_r |v_r|}{2} \right] \quad (12)$$

As can be seen from Eqs. (7) and (12), the interfacial area plays a very important role in determining the phase interactions. Basically, the first order geometrical effects on the interfacial transfer are taken into account by the interfacial area concentration a_i . An extensive literature survey on experimental data and a preliminary modeling effort for a_i has been carried out (20, 45, 46).

In general, the interfacial momentum interaction term may be split into four groups, i.e., standard drag force, virtual mass force, Basset force, and effect of the average interfacial stress, in addition to the momentum transfer due to phase change. The second and third terms are the transient effects, whereas the last term represents the effects of a shear flow and concentration gradient. The first term representing the standard drag force is the most important term which determines the equilibrium relative velocity between phases. Recently local drag correlations for dispersed two-phase flows were developed from simple similarity criteria and a mixture viscosity model (34, 35). The comparison of the theoretical results with over 1000 experimental data indicated satisfactory agreements at wide ranges of the concentration and Reynolds number. These comparisons indicated that the postulated drag similarity law based on the mixture viscosity concept was appropriate. Therefore, the drag law governing the motion of bubbles, drops, and particles in various dispersed two-phase flows can be explained by a unified and consistent model.

The local drag force is related to the local relative velocity through the drag coefficient which has been discussed above. When

a one-dimensional model is used, a careful area averaging of the local drag law is necessary. Particular attention should be paid to the effect of a covariance which represents the difference between the average of a product and the product of the averages for two variables.

It was found that the interfacial drag term could be given approximately by

$$\langle M_{Id} \rangle = - \frac{3}{8} \frac{C_D}{r_d} \langle \alpha_d \rangle \rho_c \langle v_r \rangle |\langle v_r \rangle|. \quad (13)$$

Here $\langle v_r \rangle$ is the simple area average of the local relative velocity. However, in the two-fluid model, the center of mass velocity of each phase is used in the formulation. The true relative velocity, $\bar{v}_r = \langle \langle v_d \rangle \rangle - \langle \langle v_c \rangle \rangle$, which is based on these center of mass velocities, is quite different from $\langle v_r \rangle$. As a first approximation it is recommended to use a relation

$$\langle v_r \rangle = \frac{1 - \langle \alpha_d \rangle C_0}{1 - \langle \alpha_d \rangle} \langle \langle v_g \rangle \rangle - C_0 \langle \langle v_f \rangle \rangle \quad (14)$$

where C_0 is the distribution parameter (36, 42) used in the drift flux model. For example, C_0 for a flow in a round tube under adiabatic condition is given approximately by $C_0 = 1.2 - 0.2 \sqrt{\rho_g / \rho_f}$ in a bubbly or churn flow. This expression for $\langle v_r \rangle$ is compensated for the slip due to distribution of phases and it is not equal to $\langle \langle v_g \rangle \rangle - \langle \langle v_f \rangle \rangle$.

This difference between $\bar{v}_r$ and $\langle v_r \rangle$ has never been taken into account in the conventional two-fluid model. In most two-phase flow systems, the slip due to the distribution of phases is much greater than the local slip between phases. Therefore, neglecting the above mentioned effect will lead to large errors in the predictions of the void fraction and velocities in bubbly, slug, and churn-turbulent flow regimes. As a result, even the steady-state prediction from two-fluid model has never been as good as that from a drift-flux model in these flow regimes. This was indeed the most significant shortcoming of the conventional two-fluid model and it should be corrected in all future analyses. There are some other important aspects in the one-dimensional formulation such as the covariance terms and the effect of the interfacial shear stress and void gradient. These are discussed in detail in Ref. (46).

It can be said that the weakest link in the two-fluid model formulation is the constitutive equations for the interfacial transfer terms, particularly the energy transfer terms. The interfacial transfer terms are strongly related to the interfacial area concentration and to the local transfer mechanisms such as the degree of turbulence near interfaces (20). Basically, the interfacial

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transport of mass, momentum, and energy is proportional to the interfacial area concentration, a_i , and to a driving force. This parameter, a_i , having the dimension of the reciprocal of length, characterizes the geometrical effects on the interfacial transfers. On the other hand, the driving forces for the interfacial transports depend on the local turbulence, transport properties, driving potentials and some length scale at the interfaces. This length scale may be related to a transient time such as the particle residence time or to the interfacial area concentration and void fraction α .

The mixture model is formulated by considering the motion of a two-phase mixture as a whole in terms of the mixture momentum equation (21-29). The relative motion between phases is taken into account by a constitutive equation for the relative velocity. The most important assumption associated with the mixture model is that a strong coupling exists between the motions of two phases. This implies that the mixture model is an accurate approximation to the two-fluid model whenever there is a sufficient interaction time or length for two phases. Certain two-phase problems involving a sudden acceleration of one phase, with respect to the other or a flow in a very short tube or vessel, may not be appropriately described by this model. In these cases inertia terms of each phase should be considered separately by use of a two-fluid model.

Depending on the form of the constitutive equation for the relative velocity and on the treatment of the thermal non-equilibrium between phases, a number of different mixture models have been proposed, i.e., the homogeneous flow, slip flow, and drift-flux models. When the effect of gravity is important as in the flooding, flow reversal, and counter-current flow, only the drift-flux model (3, 36, 37, 42), is satisfactory. The use of slip flow model based on a slip ratio correlation or void-quality correlation is limited to a high speed uni-directional flow. The effect of thermal non-equilibrium can be taken into account by the phenomenological model in which the vapor generation rate Γ_g is specified, or by the mechanistic model in which energy equation for each phase is used and the energy transfers to the interfaces are specified by constitutive relations.

The void fraction and the interfacial area concentration characterize a geometrical configuration of a two-phase flow. In classical two-phase flow analyses, the concept of the interfacial area concentration has not been explicitly introduced. Instead, two-phase flow regime criteria and regime-dependent constitutive equations have been used extensively. Traditionally, flow regimes are identified from a flow regime map based on the liquid and gas volumetric fluxes (6, 12-16). This approach may be suitable for slow transient and near fully developed conditions where a mixture model such as the drift flux model is sufficient.

However, in view of the practical applications of the two-fluid model to transient analysis of nuclear reactors under various accident conditions, several observations can be made. First, the flow regime criteria, based on the volumetric fluxes of liquid and vapor, may not be consistent with the two-fluid model formulation. This can be explained as follows. From the basic definitions of variables, the void fraction can be uniquely determined from volumetric fluxes j_g and j_f and relative velocity v_r . In a two-fluid model the relative velocity is an unknown to be solved from the field equations. Therefore, the void fraction, which is the most important geometrical parameter, cannot be determined uniquely from volumetric fluxes j_g and j_f . Consequently, a flow regime map based on j_g and j_f alone is unsuitable to the two-fluid model formulation. This difficulty does not arise in the drift flux model because the constitutive relation for the relative velocity can be used to determine the void fraction. However, for a two-fluid model a direct geometrical parameter such as the void fraction should be used in flow regime criteria. Some efforts in this direction have already been made (20).

Secondly, two-phase flows encountered under accident conditions occur almost always under transient conditions; however, more importantly, many flows should be considered as entrance flows due to complicated geometries of the reactor systems. It is well known that a flow regime in an entrance region can be quite different from that in a developed flow. However, only very limited studies have been made for a quantitative description of these effects. The flow regimes as well as the interfacial area concentration can be very sensitive to initial conditions. In this case, phase changes, coalescences, and disintegrations of fluid particles become very important. The most general method to include these effects in the two-fluid model formulation would be to introduce a transport equation for the interfacial area concentration (20, 23). This equation should have source terms which take account for bubble or droplet expansions or collapses, coalescences, disintegrations, and interfacial instabilities. This approach is highly complicated, however, it is expected that the inclusion of this equation makes the identification of two-phase flow regimes more mechanistic.

SUMMARY

In summary, physical problems of two-phase flows are represented by idealized boundary geometries, boundary conditions, initial conditions, field, and constitutive relations using the continuum formulation. Several different foundations of two-phase flow models, as well as their limitations, have been discussed. Although a number of differences exist among various models, the correct forms of balance equations have emerged after the

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introduction of proper averaging procedures. The greatest difficulty is associated with the development of the constitutive relations. However, experimental data can be taken by accepting the model which indicates the possibility of measurements. This feedback from the data to the model improves the constitutive relations. The validity of the complicated two-phase flow formulation can be shown only by solving a number of physical problems and comparing with data.

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