27
2/16/79
2/16/79
NTS

MASTER

UCID-18035

Lawrence Livermore Laboratory

Slowing Down of Ions Due to Coulomb Collisions in a Plasma

Keiichi Nishimura

January 1979

This is an informal report intended primarily for internal or limited external distribution. The opinions and conclusions stated are those of the author and may or may not be those of the laboratory.

Prepared for U. S. Department of Energy under contract No. W-7405-Eng-48.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Slowing Down of Ions Due to
Coulomb Collisions in a Plasma

Keiichi Nishimura

Abstract

Lenard's form of the plasma kinetic equation incorporating dynamic shielding was used to calculate the rate of energy loss by fast ions due to Coulomb collisions inside a plasma. The rate thus calculated is significantly larger than the results obtained on the basis of conventional theories with static shielding especially if the plasma is very dense and the ion speed is not too high.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights.

Work performed under the auspices of the U. S. Department of Energy by the Lawrence Livermore Laboratory under contract No. W-7405-ENG-48.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

829

I. Introduction

A factor commonly referred to as Coulomb logarithm is invariably introduced in the calculation of the slowing down rate of fast ions due to Coulomb collisions inside a plasma if a conventional method based on static shielding is employed. The Coulomb logarithm becomes necessary in order to prevent divergence of an integral due to small-angle collisions and it is a function of density and temperature of the plasma, increasing generally with temperature and decreasing with density. The whole theory based on static shielding, fails, however, when the Coulomb logarithm is small and of the order of unity, a situation not infrequently encountered in laser fusion research in our Laboratory.

The purpose of this paper is to use the plasma kinetic equation in the form developed by Lenard¹ in order to calculate the slowing down rate of fast ions undergoing Coulomb collisions inside a plasma. The kinetic equation incorporates dynamic shielding and this effectively replaces the dielectric constant of the plasma by a function which depends upon the velocity of the colliding particles. A cutoff is still necessary in the calculation but instead of the introduction of a somewhat ambiguous notion of smallest scattering angle which leads to the Coulomb logarithm, the cutoff in the present work will be derived from a more basic principle, the uncertainty principle of quantum mechanics.

Numerical calculations based on this theory yield a much greater slowing down rate than the conventional calculation does for dense plasmas at relatively low temperatures. The rate of energy loss to the plasma electrons

is nearly the same by the both theories. Since a very fast ion loses its energy primarily to the plasma electrons with the loss to the plasma ions increasing only gradually as the ion slows down, the present theory will have little effects on the range of very fast ions but the effects can be significant on the range of somewhat slower ions.

II. Calculation

Consider a situation where monoenergetic beam of ions with mass m_b and particle density n_b travels inside a uniform plasma. The rate of loss of kinetic energy by the beam per unit volume is then

$$\frac{dE_b}{dt} = \frac{m_b}{2n_b} \int d^3v v^2 \partial f_b(\underline{v}, t) / \partial t \quad (1)$$

where f_b is the velocity distribution function of the beam ions normalized to n_b . The variable t in the argument of velocity distribution functions will hereafter be suppressed for simplicity.

According to Lenard,¹ the plasma kinetic equation takes the following form:

$$\begin{aligned} \frac{\partial f_b(\underline{v})}{\partial t} = & - \frac{q_b^2}{2\pi\epsilon_0 m_b} \int \frac{d^3k}{k^4} \sum_j \frac{q_j^2}{4\pi\epsilon_0} \underline{k} \cdot \frac{\partial}{\partial \underline{v}} \int d^3u \\ & \frac{\delta(\underline{k} \cdot \underline{v} - \underline{k} \cdot \underline{u})}{|\epsilon(\underline{k}, -\underline{k} \cdot \underline{v})|^2} k \cdot \left\{ \frac{f_b(\underline{u})}{m_j} \frac{\partial f_j(\underline{u})}{\partial \underline{u}} - \frac{f_j(\underline{u})}{m_b} \frac{\partial f_b(\underline{u})}{\partial \underline{v}} \right\} \end{aligned} \quad (2)$$

where q is the charge, ϵ_0 is the vacuum dielectric constant and the dielectric function ϵ is given by

$$\epsilon(\underline{k}, \omega) = 1 - \sum_j \frac{q_j^2}{m_j \epsilon_0 k^2} \int d^3v \frac{\underline{k} \cdot \partial f_j(\underline{v}) / \partial \underline{v}}{\omega + \underline{k} \cdot \underline{v} - i\delta} \quad (3)$$

If the plasma is assumed to contain only one species of ions for the sake of simplicity, the summation in (2) will be only over i (ions) and e (electrons) and (1) will take the following form after (2) is substituted into it:

$$dF_b/dt = (dF_b/dt)_i + (dF_b/dt)_e \quad (4)$$

The two terms on the right hand side represent the rate of energy transfer due to collisions with the plasma ions and electrons, respectively.

In evaluating $(dF_b/dt)_j$ where $j = i$ and e , it will be assumed that f_i and f_e are Gaussian distribution functions characterized by different temperatures T_i and T_e while

$$f_b(\underline{v}) = n_b \delta^3(\underline{v} - \underline{v}_b) \quad (5)$$

where $\underline{v}_b$ is the uniform velocity of the beam ions. It will be assumed furthermore that $n_b \ll n_i, n_e$ so that the summation in (3) will also be over i and e only.

The 9-dimensional integration, which (1) turns out to require, is considerably simplified because of symmetry and the existence of delta functions. After a few integrations by parts, it can be reduced to a 2-dimensional integral with respect to k ($=|\underline{k}|$) and μ ($=$ the cosine of the angle between $\underline{k}$ and $\underline{v}_b$). The k -integral diverges at its upper limit and requires a cutoff. Let k_j be the upper limit of the k -integration required for the calculation of $(dF_b/dt)_j$. Since k in Lenard's formulation represents the momentum in units of $\hbar$ transferred by the collision, k_j

will be chosen as the inverse of the smallest measurable distance within which the colliding particles can find each other in view of the uncertainty principle. The uncertainty in the canonically conjugate variable, i.e. momentum, will be taken equal to the center-of-mass momentum of the colliding particles approximated by

$$m_j m_b (m_j + m_b)^{-1} (v_j^2 + v_b^2)^{1/2} \quad (6)$$

As can be expected, the final result is not sensitively dependent on the exact expression of this momentum.

With the introduction of cutoffs k_j , the k -integration can be performed analytically, leaving only the μ -integration to be carried out numerically.

III. Results and Discussion

A computer code has been created for the purpose of performing the final μ -integration by the Gaussian integration method where this method is practicable and by the straight integration method elsewhere. It was found that the Gaussian method can be used in almost all practical cases.

The code has been made usable for cases involving a plasma with a mixture of more than one species of ions by an appropriate use of averages. Thus, the input data to be supplied are: n_i (= the total particle density of plasma ions), A_b (= atomic number of the beam ions), $\langle A_i \rangle$ (= the weighted average of the atomic numbers of plasma ions), Z_b (= the charge number of the beam ions), $\langle Z_i \rangle$ (= the weighted average of the charge numbers of plasma ions), $\langle Z_i^2 \rangle$ (= the weighted average of the squares of the charge numbers of plasma ions), T_e (= temperature of the plasma electrons), T_i (= temperature of the plasma ions), and E_b (= kinetic energy of the beam ions).

For the purpose of comparison, cases of an equally mixed DT plasma were considered so that $\langle A_i \rangle = 2.5$ and $\langle Z_i \rangle = \langle Z_i^2 \rangle = 1$. For $T_e = T_i = 5$ keV, 25 keV and 250 keV and for various values of E_b the rates $(dE_b/dt)_i$, $(dE_b/dt)_e$ and dE_b/dt as defined following (4) were calculated. The results for the case $Z_b = 40$, $A_b = 98$ and the mass density of the plasma = 10^3 gm/cc are shown in Figs. 1 ~ 3. In all these figures, solid lines denote the present calculation and dotted lines denote the calculation carried out by Perkins² by the conventional method involving Coulomb logarithm.³

It is to be noted that these are cases of extremely dense plasmas. Where the plasma temperature is very high and/or the beam ions are moving very fast, however, the conventionally defined Coulomb logarithms can be substantially larger than unity and this is why the solid and dotted curves converge toward each other in this region. If the plasma temperature is not so high and/or the beam ions are not moving very fast, the Coulomb logarithm becomes smaller so as to cast doubts on the validity of the conventional method of calculation itself and this is the region where the difference between the two curves is most significant, the present calculation giving a much larger value of $(dE_b/dt)_i$.

It is to be noted that $(dE_b/dt)_e$ is nearly the same by either method of calculation. It is not surprising because electrons, being much lighter, move much faster, giving rise to larger Coulomb logarithm values. Since $(dE_b/dt)_e \gg (dE_b/dt)_i$ only in the region of very high ion speed, the difference in the total rate is significant only in the region of somewhat slower speed and when the plasma temperature is low.

This means that the present calculation will not yield a significantly different result in the calculation of range for very fast ions such as fission fragments with very high kinetic energy because a large portion of the range will be travelled while the ions lose their energy primarily to the plasma electrons. The range of ions with somewhat lower initial kinetic energy, however, will be significantly affected by the present calculation, the faster energy transfer rate resulting in shorter ranges.

Fig. 1

Fig. 2

Fig. 3

References:

1. Lenard, A., Ann. Phys., 10, 390 (1960).
2. Perkins, S. T., private communication.
3. Perkins, S. T., Lawrence Livermore Laboratory Report UCRL-80770 (1978).

DISTRIBUTION

E. G. Corman, L-71

R. E. Kidder, L-71

G. D. Kramer, L-71

K. Nishimura, L-71 (5 copies)

C. B. Tarter, L-71

J. C. Weisheit, L-71

G. B. Zimmerman, L-477

T-Division (1 copy)

TIC (15 copies)

TIC (27 copies)