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*Three-Dimensional Thermoelastic
Analysis of a Fort St. Vrain
Core Support Block*

Los Alamos

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

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Three-Dimensional Thermoelastic Analysis of a Fort St. Vrain Core Support Block

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THREE-DIMENSIONAL THERMOELASTIC ANALYSIS OF A FORT ST. VRAIN CORE SUPPORT BLOCK

by

T. A. Butler and C. A. Anderson

ABSTRACT

A thermoelastic stress analysis of a graphite core support block in the Fort St. Vrain High-Temperature Gas-Cooled Reactor is described. The support block is subjected to thermal stresses caused by a loss of forced circulation accident of the reactor system. Two- and three-dimensional finite element models of the core support block are analyzed using the ADINAT and ADINA codes, and results are given that verify the integrity of this structural component under the given accident condition.

I. INTRODUCTION

During cooldown following a loss of forced circulation (LOFC) accident in the Fort St. Vrain High-Temperature Gas-Cooled Reactor, thermal stresses in the graphite core support blocks (CSBs) could conceivably exceed the minimum tensile strength of graphite. Potentially large thermal stresses are possible because of the high thermal gradients established across the core support structure during the LOFC phase of the accident and during the following period when the emergency firewater cooldown (FWCD) system partially restores forced circulation. This report summarizes an investigation of the thermal stresses experienced by the CSBs during the cooldown period.

We used the finite element method to calculate the CSB thermal stresses in three successive steps. First, we used two small two-dimensional finite element models to perform scoping studies of the problem. One model was used to determine the temperature distribution and the other to obtain the resulting thermal stresses. Both of these models included the complete hexagonal top surface of the CSB without considering other geometrical details. These same

models were also used to quantify the effect of asymmetries in heat flow on the stresses. As a second step we developed a three-dimensional model of a 60° segment of a CSB. The same mesh was used for both the thermal and stress solutions and included coolant passages but no other structural details, such as the keyways and lugs. Finally, results of the three-dimensional model were used to establish boundary conditions for a detailed two-dimensional model near a keyway in the CSB to determine stress intensities near this geometric discontinuity.

The total analysis included both steady-state and transient computations. Loads and boundary conditions for the thermal solutions were obtained from data supplied by the Oak Ridge National Laboratory (ORNL) based on computations made with the ORECA computer code.¹ Because of uncertainty concerning some of the critical parameters in the thermal solution, several parallel computations were performed to determine the sensitivity of thermal stresses to these parameters.

In the following sections of the report we describe the accident scenario and the resulting potential thermal stress problem. We then discuss our approach for quantifying the CSB thermal stresses and the results of our analyses. Finally, we discuss the uncertainty associated with these computations.

II. CONCLUSION AND RECOMMENDATIONS

Average CSB temperature transients for an LOFC/FWCD accident calculated by ADINAT were in good agreement with ORECA temperature predictions.

The maximum principal stress in the CSB calculated for a 105% power level is 915 psi at the keyway corner on the top of the CSB for an LOFC/FWCD accident. This stress includes factors for asymmetries and stress concentration at the keyway corner. The resulting factor of safety using a minimum tensile strength of 1160 psi is 1.27. This is sufficient for these loading conditions because the thermal stresses are secondary in nature and a local failure would relieve the stresses in the complete CSB. Adding to the confidence level that the CSB will not fail under these conditions is the fact that the stress gradients are very high near the point of maximum stress. That is, most of the CSB remains at a much lower stress level, and any cracks that start in the high-stress region would probably stop before propagating any significant distance. The stress levels for 72% power are approximately 60% of those for 105% power.

This study has not included the effects of graphite oxidation during the life of a CSB. However, other studies² have shown that, as expected, the results of the thinner structure due to oxidation are lower thermal stresses. Major uncertainties in the results of this study are contributed primarily by uncertainty of the loading condition, the geometric modeling approximation that was made to produce a reasonable size model, and uncertainty associated with the magnitude of secondary stress required to cause failure in graphite. Details are given in Appendices A and C.

The stresses reported here are only those secondary stresses arising from thermal gradients due to the FWCD following an LOFC accident. Primary stresses in the CSB result from core weight and pressure differential and are not included; however, these stresses are less than 25 psi.² The effects of primary and secondary stresses on failure of graphite can be significantly different, and it appears that there exists no accepted method for combining them--particularly in this case where thermal stresses have high gradients and localized peaks. Research needs to be conducted to determine whether failures from various classes of secondary stresses in graphite tend to propagate or remain localized. A study in this area could yield consistent methods for combining secondary and primary stresses and for determining the proper factor of safety.

III. ACCIDENT SCENARIO

The Fort St. Vrain reactor is a high-temperature gas-cooled reactor (HTGR) employing helium cooling. The reactor core consists of vertical columns of graphite blocks divided into 37 fuel regions. Each region nominally consists of six fuel columns and one control column and is supported by one CSB. The CSBs are loosely keyed together (Fig. 1) to provide uniform gaps of 0.4 to 1.0 inches between adjacent CSBs depending on the average temperature of the reactor. Figure 1 also shows the alternating lugs and keyways (three of each) on the sides of the CSB, and that each CSB is supported on the bottom by three support posts.

Core heat removal is normally accomplished by the downward flow of the helium coolant through the core. At the base of the active core, the gas is channeled through the lower reflector columns (Fig. 1) into the six coolant channels of the CSB. During an LOFC accident, forced circulation of the helium is interrupted. Before it can be restored, heat is transferred within the core through conduction in the graphite and by free convection in the helium.

Fig. 1. Core support arrangement.

Because the amount of power available in different regions varies significantly, large temperature gradients are established in the support structure. The differences in temperatures between regions is compounded because in low-power regions free convection of the helium is naturally downward, which provides a cooling effect. In high-power regions, the flow is upward, causing the temperatures to rise.

For the accident scenario considered in this report, forced circulation of the helium is partially restored at 90 minutes into the LOFC by the emergency FWCD system. After the forced circulation begins, some of the CSBs initially increase in temperature as heat in the active core is forced toward the bottom of the reactor. During this time, the low-power regions are cooled more rapidly because the helium viscosity is lower and the density greater at lower temperatures. This effect accentuates the already high temperature differences between the CSBs. Figure 2 illustrates the helium temperature as a function of time into the accident for a typical core region.

Fig. 2. Coolant gas temperature history.

Predictions made at ORNL, using the ORECA computer code, indicate that average temperatures of adjacent CSBs may differ by as much as 600⁰F during cooldown following a 90-minute LOFC for the reactor configuration operating at a 105% power level³ (see Fig. 3).

During cooldown heat flow into and out of the CSBs may be put into the following categories.

1. Heat transferred to or from the helium flowing through the coolant channels.
2. Heat transferred to the helium in the bypass gas flow through gaps between the CSBs.
3. Heat transferred into or out of the CSBs by conduction through the reflector columns above the CSBs.
4. Heat transferred to adjacent CSBs by conduction at points of contact.
5. Heat transferred to adjacent CSBs by conduction through the helium bypass gas.
6. Heat transferred to adjacent CSBs by radiation.

Fig. 3. Typical CSB temperatures during FWCD.

Less significant heat is transferred through the core support posts and by radiation and convection to the area under the CSBs.

The ORECA computer code considers only categories 1, 3, and 4. The heat transfer path between adjacent blocks is characterized as being solid graphite. Because no gap exists, the mechanisms in categories 2, 5, and 6 are not considered. To get an appreciation of how much difference this simplification makes in the analysis, we can compare the resistances for the two one-dimensional heat transfer networks shown in Fig. 4. We assume that the heat capacity of the CSB is concentrated at its geometric center. For ORECA, the resistance R was found to be $0.105 \text{ h} - \text{ft}^2 - ^\circ\text{F}/\text{BTU}$. For the actual system with the gap included, $R = R_1 + R_2 + R_3$, where R_1 and R_3 are equal to 0.052. R_2 is primarily a function of the radiation across the gap. To obtain the maximum value of R_2 we can assume a linear distribution of temperature between two blocks. Using the values for T_1 and T_4 shown in Fig. 4 we solve the thermal network and find T_3 is 1242.5°F and T_2 is 1057.5°F . The gap resistance R_2 can now be found from the equation

$$R_2 = (1.33) \frac{T_3 - T_2}{\mathcal{F}_{1-2} \epsilon \sigma (T_3^4 - T_2^4)} = 0.0463 \quad (1)$$

if we assume that the shape factor, $\mathcal{F}_{1-2}$, and the emissivity, ϵ , are both equal to 1.0. The factor 1.33 is the ratio of the average cross-sectional area available for conduction to that for radiation. The total resistance in the real case is then a maximum of 0.150, or 1.43 times that used in ORECA. This difference has two effects. During the 90-minute period of the LOFC the ΔT 's between adjacent blocks will not be quite as high as in the real case. However, during the cooldown period, the resistance is lower so the heat flows out faster, giving rise to higher thermal stresses.

The effect of neglecting the bypass gas flow in the ORECA calculations is difficult to quantify. However, we can say qualitatively that during the 90-minute LOFC it has little effect because of low flow rates. During FWCD it will cause faster cooling and, therefore, somewhat higher thermal stresses. Appendix A presents calculations that approximate the additional rate of heat flow from a CSB when bypass flow is considered. This method conservatively estimates the radial heat flow out to be approximately 86% of what it is

Thermal conductivity is 30.0 BTU/h·ft·°F. Distance between CSB geometric centers is 37.8 in.

Fig. 4. Heat transfer between adjacent CSBs.

without considering the bypass flow. Lower radial heat flow from this source is more than offset by the lower radial resistance discussed in the previous paragraph.

IV. METHOD OF ANALYSIS

We analyzed the response of a CSB to the 90-minute LOFC/FWCD accident sequence using several different finite element models. To scope the problem we developed two simple two-dimensional finite element models representing the top surface of a CSB (Fig. 5). One was used to solve the steady-state heat transfer problem and the other to determine the resulting thermal stresses. Appendix B gives detailed derivations of these models. Heat flow rate out of the hot CSB was controlled by a combined heat transfer coefficient that, in one study, was varied as a parameter between 6.0 and 30.0 BTU/ft²·h·°F. Heat input to the CSB was adjusted to maintain an average temperature of 2000°F. Figure 6 presents the results of the two models for a combined heat transfer coefficient of 30.0 BTU/ft²·h·°F. The strong correlation between the maximum principal stress and the combined heat transfer coefficient is shown in Fig. 7.

Fig. 5. Meshes for two-dimensional finite element models.

1	1317
2	1449
3	1580
4	1711
5	1842
6	1974
7	2105
8	2236
9	2368
10	2499

1	-747.7
2	-561.4
3	-375.0
4	-188.6
5	- 2.3
6	184.1
7	370.4
8	556.8
9	743.1
10	929.5

Fig. 6. Temperature and stress distributions from preliminary scoping calculations.

Fig. 7. Maximum principal stress dependence on surface heat transfer coefficient.

A second study was performed to determine the sensitivity of the maximum principal stress to the distribution around the perimeter of the heat transfer rate out of the CSB. This study showed that of the various possibilities for this distribution, based on results of the ORECA code, the most severe is a 2000°F block surrounded by six blocks at an equal and low temperature (500°F). Based on these results, along with model size restrictions, we determined that studies with a three-dimensional model could consider one 60° segment of the CSB.

These models were also used to determine the effects of actual asymmetries predicted by ORECA for the reactor operating at a 105% power level. A complete discussion of these calculations is in Section V.

Because the CSB is such a complex three-dimensional object, we determined that it would be necessary to develop three-dimensional finite element models to adequately represent the complex heat flow patterns and associated stress fields. The finite element computer codes ADINAT⁴ and ADINA⁵ were used to model the heat transfer and thermal stress problems. Before the mesh was generated, the CSB geometry was simplified to make the structure cyclically symmetric and to eliminate discontinuities (such as the keyways and lugs) that could not be adequately represented in a reasonably sized three-dimensional model. A substructure model described in Section IV was used to determine the effect of the keyway. To obtain cyclic symmetry we artificially rotated the coolant hole pattern $10^{\circ} 56.3'$ relative to the boundary of the CSB. The resulting simplified CSB is shown in Fig. 8. The cyclic symmetry of the simplified geometry allowed us to model one 60° segment of the structure. A mesh used for both models is shown in Fig. 9. The mesh is composed of 847 node points with 126 20-node isoparametric elements. The ADINAT mesh includes an additional 50 8-node isoparametric elements to simulate convective boundary heat flow into the coolant channels and lower cavity.

Fig. 8. Schematic of CSB with simplified geometry for three-dimensional model.

Fig. 9. Mesh for three-dimensional finite element model.

The ADINAT heat transfer model was used to determine temperature distributions in the CSB assuming both transient and steady-state conditions. The steady-state model was used for preliminary calculations and for determining the stress concentration factor at the corner of the keyway. For both models, boundary conditions were determined using data from the ORECA computer code.

In ORECA, each CSB is represented by a single node. Heat flows into and out of this node through seven heat transfer paths, one from the node above, representing a portion of the lower reflector, and six radial heat flow paths allowing heat to exchange with surrounding CSBs. As discussed in Section III, ORECA does not consider heat transferred to adjacent blocks by radiation or to the bypass gas flow by convection. However, the simplification in ORECA that the graphite is continuous between blocks (no gap) lessens the resistance slightly and is a good approximation of the real situation. To obtain the heat input to the top of our model, we assumed that the total heat flow predicted by

ORECA is evenly distributed over the top surface. The radial heat flow was computed by dividing the sum of all the radial heat flows by six (because the model represents a 60° segment). The resulting flux was then evenly distributed over the outside surface of the block. Figure 10 shows the heat flux histories for one particular CSB for a 72% power case. Zero time in the figure is at the start of FWCD. This CSB is in region 35 and was chosen by ORNL for analysis because it has a high temperature and is next to a low-power and, therefore, low-temperature region, maximizing the radial heat flow. Similar heat flux histories for the 105% power case are shown in Fig. 11. The heat fluxes for the 72% power case come from an ORECA run that had a low value for the thermal conductivity.¹ The fluxes for the 105% power case were generated using the higher, updated thermal conductivity discussed later in this section. The higher conductivity considerably increases the heat flux.

Several comments can be made concerning these transients. The heat flow into the top of the CSB is initially positive and increases from the initial value for a short time before gradually decreasing and eventually becoming negative (heat flow out of block). This phenomenon occurs because, during cooldown, cool helium is introduced at the top of the core and cooling proceeds from top to bottom. Also, all of the components above the CSB have an order of magnitude more cooling area per unit mass. This causes its response to lag behind the adjoining component (lower reflector).

Fig. 10. CSB exterior surface heat flux for 72% power.

Fig. 11. CSB exterior surface heat flux for 105% power.

The radial heat flow initially increases because, as explained in Section III, during FWCD, gradients at first become more severe. As time proceeds, these gradients gradually begin to smooth out, decreasing the heat flow rate.

The assumed cyclic symmetry requires that no heat flow through the two side boundaries of the model. The remaining boundary condition is that of the heat exchanged between the helium gas and the CSB in the coolant channels and cavity. In these areas we used the eight-node convection boundary elements in ADINAT. The gas temperature was assumed constant throughout the coolant channels and cavity at each time point, but varies with time as predicted by ORECA (Figs. 12 and 13). Note the sudden change in temperature at 160 minutes into the FWCD for the 72% power case. During the cooldown, the gas flow rate in this region slows down and eventually, at 160 minutes, changes direction. Reversed flow occurs and the much cooler gas from the lower plenum under the CSBs flows upward through this region of the core.

Reversed flow does not occur during the transient for the 105% power case. However, at approximately 70 min into FWCD the gas begins cooling and becomes quite cool near the end of the transient.

Because of current limitations in the ADINAT code, we were forced to use constant values for the surface convection coefficient. In ORECA the surface

Fig. 12. Temperature history of coolant gas for 72% power.

Fig. 13. Temperature history of coolant gas for 105% power.

convection coefficient for the CSB is assumed to be the same as for the coolant holes in the fuel elements. Using the geometry of these coolant holes, the flow remains laminar throughout the LOFC/FWCD. Its magnitude varies with the mass flow rate to the $1/3$ power. Fig. 14 shows the approximate variation of the coefficient with time as used in the ORECA code for the 72% power case. For all of our computations for a 72% power level, except for when we varied this coefficient as a parameter, we used a constant value of $10.0 \text{ BTU/h}\cdot\text{ft}^2\cdot^\circ\text{F}$. For the 105% power level, we used a constant convection coefficient of $16.0 \text{ BTU/h}\cdot\text{ft}^2\cdot^\circ\text{F}$, while in ORECA the value varies from approximately 15.6 to $19.1 \text{ BTU/h}\cdot\text{ft}^2\cdot^\circ\text{F}$.

Whether the problem has to be analyzed as a transient can be determined by comparing the characteristic times of the heat load (flux) functions with the time constant for the CSB. To determine a realistic time constant, we assume that the CSB is cooled by conductive heat flow out each of its six sides. If we assume that all of the CSB heat capacitance is concentrated at its geometric center, then its response is governed by the one-dimensional conduction equation

Fig. 14. Surface convection coefficient for coolant passages (72% power).

$$-c\rho V dT = \frac{kA}{L} (T - T_{\infty}) d\theta, \quad (2)$$

where c is the specific heat of PGX graphite;
 ρV is the mass of the CSB;
 T is the average CSB temperature; T_{∞} is the temperature of an adjacent CSB;
 A is the surface area of the side of the CSB;
 L is the distance between geometric center of adjacent CSBs;
 dT is the temperature change during an increment of time $d\theta$;
 k is the thermal conductivity of the CSB material (PGX graphite).

The solution to Eq. (2) is

$$\frac{T - T_{\infty}}{T_0 - T_{\infty}} = \exp \frac{-kA}{c\rho VL} \theta,$$

from which the time constant is seen to be $c\rho VL/kA$. Using data from the ORECA code we determined that the time constant is approximately 275 min. This time is of the same order as that for the heat flux transients, so the problem is transient in nature.

For initial conditions we assumed that the CSB was at a uniform temperature at the start of FWCD. This is a reasonable approximation because its average temperature changes by less than 50°F for the 72% power case and 110°F for the 105% power case during the last 50 min of the 90-min LOFC. Also, at the start of the FWCD, temperature gradients across the core support structure are relatively small, with the maximum temperature difference between adjacent CSBs being 94°F for a 72% power case and 171°F for a 105% power case. During the 90 minute LOFC, the lower reflector node directly above the CSB that we considered was hotter than the CSB by a temperature of 19°F for the 72% power case and 38°F for the 105% power case. So, little heat was flowing into the CSB during this period.

Mechanical boundary conditions for the ADINA stress model were quite simple. Because of the assumed cyclic symmetry, the sides of the model were restricted to move only vertically and radially. One point on the bottom of the model (Fig. 15) was constrained in the vertical direction, simulating a support post.

Material properties used for the PGX graphite for all of our analyses, except for one final transient run at each power level (72% and 105%), are given in Table I. Using these properties, we were able to perform linear analyses with ADINAT and nonlinear thermoelastic analyses with ADINA. In reality the thermal conductivity is a function of temperature⁷ (Fig. 16). Note that the values of thermal conductivity given in Table I and used for most of our analyses fall below this curve. Because of programming restrictions in ADINAT, we could not use orthotropic and temperature dependent conductivity simultaneously. Therefore, we simplified the two curves in Fig. 17 to the single curve in Fig. 18. This representation of the thermal conductivity was used for one nonlinear run of ADINAT at 72% and one at 105% power level (see Section VI). Two curves showing a temperature dependent coefficient of thermal expansion are presented in Fig. 16. One curve was used for preliminary analyses and the other updated curve for the final analyses at 72% and 105% power levels. The updated curve is based on the formulas presented in Ref. 7.

In reality, PGX graphite has orthotropic stiffness properties. However, based on data from Ref. 7, the modulus of elasticity in the radial and axial directions are both within 20% of 10^6 psi. Again, computer code restrictions, this time in ADINA, prevent us from using an orthotropic stiffness

TABLE I
PGX GRAPHITE MATERIALS PROPERTIES USED FOR PRELIMINARY
CALCULATIONS AND PARAMETER STUDIES

Thermal conductivity ⁵	
Radial.	12.3 BTU/h•ft•°F
Axial	13.9 BTU/h•ft•°F
Heat capacity ⁶	0.4 BTU/lb•°F
Young's modulus ⁶	1.0×10^6 psi
Poisson's ratio ⁶	0.15
Minimum tensile strength ⁶	1160 psi
Thermal expansion coefficient	See Fig. 16

Fig. 15. Mechanical boundary conditions for ADINA finite element model.

Fig. 16. Coefficient of expansion for PGX graphite.

Fig. 17. Average PGX graphite thermal conductivity vs temperature.

Fig. 18. Nonlinear PGX graphite thermal conductivity used in ADINAT.

along with a thermoelastic material model. Also, the PGX graphite heat capacity varies between 0.35 and 0.43 BTU/lb·°F for the temperature range we consider. However, computer code restrictions again prevent us from using a temperature dependent value. All of these restrictions have a minimal effect on our final results.

V. STRESS CONCENTRATION AT KEYWAY

Because we did not include the keyway in our three-dimensional model, we must determine the stress concentration effects at this location. This is particularly important because, for all the following analyses, the location with the highest stress is the outer edge of the CSB at the keyway.

To determine the stress concentration at the keyway, we used the results of a preliminary steady-state analysis for a 72% power level (Fig. 19). In the model used for that analysis, only approximate material properties and boundary conditions were used, so the results should not be considered as totally meaningful in an absolute sense. Also, the steady-state solution overpredicts the resulting stresses. However, the results of this analysis can be used in a relative sense to predict a stress concentration factor that can be applied to the results of other analyses using the three-dimensional model.

Fig. 19. Temperature and maximum principal stress distribution in CSB for preliminary steady-state analysis.

For this study, where only thermal stresses are being considered, a fortunate situation exists in that all exterior surfaces of the CSB are in a plane stress condition. This means that we can, given the appropriate boundary conditions and thermal loads, model any portion of the surface with a two-dimensional plane-stress finite element model. Figure 20 shows the three-dimensional finite element mesh with the shaded region that we chose for the keyway study. We will refer to the analytical model of this area as the substructure model. Fig. 21 shows the maximum principal stress distribution for this portion of the three-dimensional model.

Two different substructure models were analyzed. The first, without a keyway (Fig. 22), was used to adjust the boundary convection coefficient to obtain a similar temperature distribution as occurs for this region in the three-dimensional model. It was also used to check the accuracy of our technique by comparing the stress field that it predicts with that predicted by the three-dimensional model. Once the correct surface convection boundary coefficient was obtained, we applied it to the side of the substructure model that included the keyway (Fig. 23).

To obtain the proper temperature and displacement boundary conditions for the substructure models, we mapped the results from the three-dimensional model onto the boundary points of the two-dimensional substructure mesh. The temperature, T , and displacement, (u,v) , fields on the upper surface of the elements on the three-dimensional model can be expressed as

Fig. 20. Region of three-dimensional finite element mesh used for substructuring analysis.

Fig. 21. Maximum principal stress distribution on upper surface of three-dimensional model.

$$\begin{aligned}
 T(x,y) &= \sum_{i=1}^8 H_i(r,s) T_i \\
 \text{and } u(x,y) &= \sum_{i=1}^8 H_i(r,s) u_i, \\
 v(x,y) &= \sum_{i=1}^8 H_i(r,s) v_i,
 \end{aligned} \tag{3}$$

where $H(r,s)$ are the cubic interpolation functions for eight-node isoparametric finite elements.⁸ T_i and (u_i, v_i) , $i = 1, \dots, 8$, are the nodal temperatures and displacements for the eight nodes describing the top of each three-dimensional element. The parameters r and s are the local coordinates for any global point x, y . The proper temperature and displacement for any

Fig. 22. Mesh and boundary conditions for two-dimensional model without keyway.

point x, y can then be determined once its local coordinates r, s are known. To determine r, s we must solve the nonlinear equation set

$$\begin{aligned} x(r, s) &= \sum_{i=1}^8 H(r, s)_i x_i \\ y(r, s) &= \sum_{i=1}^8 H(r, s)_i y_i \end{aligned} \quad (4)$$

Here $x(r, s)$ and $y(r, s)$ are global coordinates of any point within the finite element boundary. x_i and y_i are global coordinates of the element's node

Fig. 23. Mesh and boundary conditions for two-dimensional model with keyway

points. We know $x(r,s)$, $y(r,s)$, x_i , $i = 1 \dots 8$, and y_i , $i = 1 \dots 8$. We use an iterative solution scheme with the Newton-Raphson equations to determine r and s . The resulting recursive relationship can be expressed as

$$\begin{Bmatrix} r \\ s \end{Bmatrix}^{n+1} = \begin{Bmatrix} r \\ s \end{Bmatrix}^n - [J^n]^{-1} \begin{Bmatrix} f_x(r,s) \\ f_y(r,s) \end{Bmatrix}^n, \quad (5)$$

where $[J^n]^{-1}$ is the inverse of the Jacobian evaluated for the n th iterative values of r and s and

$$\begin{aligned} f_x(r,s) &= x_n(r,s) - x(r,s) \\ f_y(r,s) &= y_n(r,s) - y(r,s) \end{aligned} \quad (6)$$

A computer program was written to determine r and s for each boundary point of the substructure model. After r and s were determined, we used Eq. (3) to obtain boundary conditions for boundary mesh points in the substructure model.

Once the substructure mesh and boundary conditions were determined, the TSAAS computer code⁹ was used to formulate and solve the thermoelastic problem. Results for the mesh without the keyway included are shown in Fig. 24. To obtain a temperature field similar to that in the three-dimensional model, we used a boundary convection coefficient of $10.4 \text{ BTU/h}\cdot\text{ft}^2\cdot^\circ\text{F}$ instead of $12.5 \text{ BTU/h}\cdot\text{ft}^2\cdot^\circ\text{F}$, which was used in the three-dimensional model. The resulting stress field is very similar to that for the three-dimensional model. Compare Fig. 24 with Fig. 21 and note especially the saddle point and the values for the maximum principal stress.

As a final step we applied the same boundary conditions, including the boundary convection coefficient, to the substructure model that includes the keyway. The resulting temperature and stress fields are shown in Fig. 25. A more detailed representation of the maximum principal stresses near the corner of the keyway is shown in Fig. 26. One particularly noteworthy characteristic of this stress field is the high gradient near the corner of the keyway. The maximum principal stress here is 1574 psi, compared to 867 psi for the model without the keyway. Based on these values, we use a stress concentration factor of 1.8 for the remainder of this report.

VI. ANALYTICAL RESULTS

We used the radial and axial heat flow rates shown in Fig. 10 for the transient analysis of the 72% power case. These curves were derived directly from ORECA predictions for radial region 35, axial node 10 (see Section IV). The coolant gas temperature history shown in Fig. 12 is also taken directly from the ORECA predictions. The 72% power calculations were, for the most part, performed using the material property data presented in Table I. One final computer run was made using the nonlinear thermal conductivity (Fig. 18).

Figure 27 shows how the CSB temperature predicted by ADINAT compares with the temperature calculated by ORECA. The comparison is very good until reversed flow occurs starting at about 150 min. After that time, ADINAT predicts a faster cooldown than ORECA. This is because, in ADINAT, the convection coefficient remains constant while in ORECA it goes to zero when reversed

Temperature

Maximum principal stress

* 1493
1 1490
2 1520
3 1551
4 1581
5 1611
6 1642
7 1672
8 1702
9 1733
10 1763
* 1763

* 1.3
1 1.4
2 89.5
3 177.5
4 265.6
5 353.6
6 438.9
7 524.3
8 609.6
9 694.9
10 780.3
* 867.0

Fig. 24. Response of substructured region without keyway.

Temperature

Maximum principal stress

* 1492
1 1493
2 1529
3 1566
4 1602
5 1639
6 1664
7 1688
8 1713
9 1738
10 1762
* 1762

* -4.2
1 -4.2
2 90.7
3 185.5
4 280.3
5 375.1
6 614.9
7 854.6
8 1094.3
9 1334.0
10 1573.7
* 1573.7

Fig. 25. Response of substructured region with keyway.

Fig. 26. Maximum principal stress distribution at keyway corner.

flow occurs and then slowly increases. Therefore, too much heat is transferred to the coolant flow in ADINAT. This difference has a small effect on the maximum thermal stress, which occurs at the top of the CSB at the keyway location. The time history of this stress* is presented in Fig. 28. It gradually increases for approximately 150 minutes. At this time the reversal of the coolant gas flow, along with a decrease in the radial heat flow, cause the magnitude of this stress to begin dropping rapidly.

When reversed flow occurs, the stresses at the intersection of the coolant holes with the cavity in the CSB are predicted by ADINA to become quite large (see Fig. 29). Maximum values of these stresses are considerably overpredicted because of the constant convection coefficient used in ADINAT (i.e. when they attain their peak values, ADINAT is overpredicting the cooling rate).

*Note that all stresses presented in this section are directly from the three-dimensional analysis (no factors applied) unless otherwise specified.

Fig. 27. CSB temperature histories for 72% power.

Even so, the values are acceptable when compared to the capability of PGX graphite (Table I).

Figure 30 shows the constant temperature lines and the maximum principal stress distribution for this transient at $t = 150$ min. The maximum stress is at the keyway location, with high stresses also occurring near the coolant holes at the top of the block. The constant temperature and stress lines are similar to those for the steady-state analysis described in the previous section. However, the gradients have not had sufficient time to become fully established. This confirms that, because of the long time constant exhibited by the CSB, the problem must be analyzed as being fully transient. Also, a steady-state analysis considerably overpredicts the thermal stresses. The establishment of the thermal gradients and resulting thermal stresses are given in more detail in the following discussion of the CSB response during FWCD after an LOFC during operation at 105% power.

Fig. 28. Time history of maximum principal stress at keyway corner for 72% power.

Fig. 29. Maximum principal stress distribution at 200 min after start of cooldown for 72% power.

Fig. 30. Response of CSB to 72% power transient at 150 min after start of cooldown.

To estimate the effects of the radial heat flux and the coolant channel convection coefficient on the final predicted maximum principal stresses, we performed a brief parameter study. While holding the convection coefficient at its original value, we ran the complete transient separately for both double ($2Q_r$) and one-half ($0.5Q_r$) the original radial heat flow rate. Similarly, we held the radial heat flow constant and ran with the convection coefficient at double ($2h_c$) and one-half ($0.5h_c$) its original values. The results are summarized in Table II. As expected, increased radial heat flow increases the keyway stresses, while increased coolant channel flow has the largest effect on stresses at the coolant channel entry to the CSB cavity region.

We also investigated the effects of changing the axial heat flow into the top of the CSB. The 105% power case, discussed later in this section, was analyzed with data directly from ORECA and with the axial heat flow cut by one-half. Differences were minor, with the stress at the keyway location being 2.5% lower for the lower flow rate.

Table III presents the numerical values of the maximum principal stresses for 72% power. Data are presented for both the original thermal conductivity values (Table I) and the updated, temperature dependent properties.⁶ The higher average conductivity values, represented by the updated curve, lower the thermal stresses considerably. However, these lower values must be judged on the basis that the same input data was used from the ORECA code. Therefore, the effect of the higher average conductivity is not accounted for in the heat flow rates into and out of the CSB. Its only effect is to lower the thermal gradients with constant heat flow. In reality, the higher average conductivity increases the radial heat flow from the CSB, which in turn causes the stresses to become somewhat higher. ORECA was rerun for both the 72% power case and the 105% power case using the updated thermal conductivity data. As expected, heat flow from CSB to CSB is increased.

For the 105% power case, we analyzed radial region 19. Figure 11 shows the radial and axial heat flow rates used for this analysis. Note that the

TABLE II
RESULTS OF PARAMETRIC STUDIES FOR 72% POWER

	Original Q_r and h_c	$2 \times Q_r$	$0.5 \times Q_r$	$2 \times h_c$	$0.5 \times h_c$
Normalized keyway stress	1.0	1.72	0.58	0.85	1.06
Normalized cavity stress	1.0	0.59	1.29	1.56	0.56

TABLE III
SUMMARY OF MAXIMUM PRINCIPAL STRESSES FOR 72% POWER

	Original K ⁽⁵⁾	UPDATED K ⁽⁶⁾
ADINA Prediction	240 psi	130 psi
With Stress Concentration Factor (1.8)	432 psi	234 psi

values are considerably higher than those used for the 72% power case previously discussed. This is attributed to the higher average thermal conductivity used for this later analysis. The coolant gas temperature history is shown in Fig. 13. Unlike the 72% power case, coolant gas flow does not reverse. It does, however, become considerably cooler than the average CSB temperature after approximately 140 min. This causes similar temperature and stress fields to develop as for the 72% power case. Figure 31 shows the constant stress lines at 240 min. The higher stress region where the coolant hole enters the cavity is caused by the cool gas flowing at a relatively high rate. For this case the gas flow rate varies less than for the 72% power case (from 36.0 to 66.2 lb/min), so our use of a constant convection coefficient is much more accurate, especially because it varies with the flow rate to the 1/3 power. Except for the thermal conductivity, the material properties from Table I were used. We used the thermal conductivity given in Fig. 18.

Figure 32 shows a comparison of the CSB temperature predicted by ADINAT with that predicted by ORECA. This close comparison is attributed to the good approximation of the convection coefficient throughout the analysis. The time history of the predicted maximum principal stress at the keyway location is shown in Fig. 33. In this case the stress increases for approximately 100 min.

Fig. 31. Maximum principal stress distribution at 240 min after start of cooldown for 105% power.

Figure 34 shows constant temperature and stress lines at 100 min, which is the time that the maximum stress at the keyway occurs. If we multiply this stress by the stress concentration factor found in the preliminary steady state analysis (Section V), the resulting maximum principal stress is 857 psi. Figure 35 shows the stress variation as a function of height at the keyway location. Note that the stress values on the curve have not had the stress concentration factor applied to them. Radial stress variation moving in from the keyway on the top of the block can be seen in a relative sense in the steady-state results (Fig. 25).

At this point we have accounted for the stress concentration at the keyway but have not taken into account any asymmetries in the heat distribution in the CSB. Recall that the radial heat flow used in our analyses has been the average of the flow from the six exterior side surfaces as predicted by ORECA. To approximate the effects of realistic asymmetries, we used the following procedure. The two-dimensional model of the top of the CSB was used to perform steady-state analyses of three different conditions representing the radial heat flow at 140 min into the transient. These three cases are shown schematically in Fig. 36. The first represents the three-dimensional case described above where flow out of each of the six sides is equal to the average of the six values predicted by ORECA at 140 min. A further adjustment was

Fig. 32. CBS temperature histories for 105% power.

Fig. 33. Time history of maximum principal stress at keyway location for 105% power.

Fig. 34. Response of CSB to 105% power transient at 100 min after start of cooldown.

Fig. 35. Maximum principal stress at the keyway location shown as a function of height from base of CSB.

made to this and the other radial flows to account for the difference in graphite volumes represented by the two-dimensional model and an actual CSB. Because relatively little heat is flowing into the top of the CSB at this time (Fig. 11), the axial heat flow is neglected. The middle node point in the two-dimensional thermal model was fixed at the temperature predicted in the three-dimensional analysis. A second case was run in a similar manner except that the maximum of the six radial heat flows from ORECA was used for all sides of the two-dimensional model. To determine the appropriate temperature at which to set the middle node point, we reran the three-dimensional model using the time history of the maximum radial heat flow predicted by ORECA. This three-dimensional model was also used to obtain stresses for this condition. As a last case, the two-dimensional model was run with the actual radial heat flows predicted by ORECA at 140 min, with the adjustment for volume being used again.

Results of all of these analyses are summarized in Table IV. Several comments should be made concerning the results. First, we can show that the ratio of the results of the first two two-dimensional analyses is very close to the ratio of the results for the two three-dimensional runs. This indicates that the two-dimensional results can be used in a relative sense to predict results for a similar three-dimensional analysis. Therefore, taking the ratio of two-dimensional cases one and three (1.07) and applying the resulting factor to the first three-dimensional analysis should give a good approximation to the actual stresses in the CSB, with asymmetries included.

Fig. 36. Thermal boundary conditions used to study the effects of asymmetries on thermal stresses.

TABLE IV
SUMMARY OF MAXIMUM PRINCIPAL STRESSES FOR
105% POWER LOFC/FWCD

	Three- Dimensional Model	Two- Dimensional Model	Corrected for Asymmetries	With Stress Concentration Factor
Maximum radial flux on all sides	856	496	-	-
Average radial flux on all sides	475	273	-	-
Actual flux	-	293	508	915

The final result is 915 psi for the maximum principal stress in the CSB for a 105% power level FWCD/LOFC accident.

With ORECA predictions that used the updated thermal conductivity data for PGX graphite⁶ we partially reanalyzed the 72% power case. Data from ORECA were used as input to our three-dimensional ADINAT and ADINA models. The results are that the maximum thermal stress at the keyway location is approximately 66% of the value for 105% power. Lower stresses occur even though the radial heat flow is greater for 72% power than for 105% power. The principal reasons for the lower stress are the effects of a lower average CSB temperature on thermal conductivity and the thermal coefficient of expansion. Heat flow into the top of the CSB is also considerably lower.

APPENDIX A RADIAL HEAT TRANSFER FROM A CORE SUPPORT BLOCK

Consider a one-dimensional heat transfer network as shown in Fig. A-1 with the heat capacity of each block concentrated at its geometric center. Heat flows from the center of the hot CSB ($T = 2000^{\circ}\text{F}$) to its edge ($T = T_1$) by conducting through the PGX graphite. From the CSB edge, heat is transferred to an adjacent cold CSB ($T = 500^{\circ}\text{F}$) by radiation and conduction through the coolant gas. Heat is also transferred by convection to the bypass gas flow. The method for determining the bypass gas temperature is detailed later in this Appendix.

We solve the network by first neglecting q_3 and determining the temperatures at the CSB edges, T_1 and T_2 . This allows us to determine R_3 , which is dependent on those temperatures. The complete network is then solved, including determination of new values for T_1 and T_2 . R_3 is then recomputed and compared with its previous value. If there is not a significant change, the current solution is correct. If R_3 changes, the computations are repeated until convergence occurs.

Thermal resistances R_1 , R_2 , and R_5 can be computed directly from geometry and the assumed temperature independent material properties. To compute R_1 and R_5 we use the average cross-sectional area of a CSB

Fig. A-1. Thermal network.

(assuming that it is solid) between a side and a parallel vertical plane passing through the CSB center. Using a value for thermal conductivity for PGX graphite of 30 BTU/ft·h·°F and CSB geometrical parameters gives a value for R_1 and R_5 of 0.013 h·°F/BTU. The thermal resistance across the gap from the helium gas, R_2 , is 0.039 h·°F/BTU using a value of 0.25 BTU/ft·h·°F for helium thermal conductivity and a 0.4 in. gap width.

To obtain an initial approximation for the radiation resistance across the gap, R_3 , we assume, from symmetry, that the temperature at the center of the gap is 1250°F and the temperature drop across the gap is 200°F. This gives temperatures of 1350°F and 1150°F for T_1 and T_2 . We use the area of one side face and assume that the emissivity and shape factor are both unity. Using these values, the initial estimate for R_3 is 0.0080 h·°F/BTU. Combining R_2 and R_3 gives a total gap resistance, R_{GAP} , of 0.007 h·°F/BTU.

Neglecting q_3 and solving the network gives values of 1330°F and 1170°F for T_1 and T_2 . Thermal resistance R_3 becomes 0.0085 h·°F/BTU. Based on the value for the thermal convection coefficient, h_c , which is calculated later in this appendix, we find that $R_4 = 0.021$ h·°F/BTU. The following equations simulating the complete network can now be solved. The equations are

$$q_1 = q_2 + q_3, \quad q_1 = \frac{2000 - T_1}{R_1},$$

$$q_2 = \frac{T_1 - 500}{R_c + R_5}, \quad \text{and} \quad q_3 = \frac{T_1 - 1200}{R_4}.$$

The solution to these gives values of 1352°F and 1054°F for T_1 and T_2 . We alternately recompute a new value for R_3 and solve the equations until convergence occurs (that is, R_3 does not change significantly). The final system parameters are shown in Table A-1.

Several conclusions can be made from these calculations. One is that heat transferred to the bypass gas flow is relatively insignificant in that it

TABLE A-1
SYSTEM PARAMETERS FOR TWO-CSB HEAT TRANSFER NETWORK

R_1	=	0.013 h-°F/BTU	T_1	=	1357°F
R_2	=	0.039 h-°F/BTU	T_2	=	1046°F
R_3	=	0.0092 h-°F/BTU	q_1	=	49 380 BTU/h
R_4	=	0.021 h-°F/BTU	q_2	=	42 010 BTU/h
R_5	=	0.013 h-°F/BTU	q_3	=	7370 BTU/h
R_{GAP}	=	0.0074 h-°F/BTU			

represents less than 15% of the total. A second is that, because of neglecting the gap, the total radial resistance between two adjacent CSBs used in ORECA is approximately 70% of its actual value. This will cause the temperature gradient predicted by ORECA to be less severe than would be expected for the real case, but for a given temperature difference between adjacent BSVs, the heat flow, which is one of the most critical parameters affecting thermal stresses, will be higher.

In the following paragraphs we discuss the evaluation of the bypass gas temperature and the heat transfer correction coefficient used for estimating the resistance R_4 .

We assume that the gas flow rate for the region in question is 50 lb/min. The bypass flow percentage is then computed based on the ratio of the gap area between adjacent CSBs to the coolant hole area. Using this percentage, we obtain a bypass gas flow rate of 322 lb/h. A heat balance on the flowing bypass gas between axial nodes in the ORECA model is then used to determine the bypass gas temperature when it reaches the CSBs. The heat balance equation is $MC_p(T_{in} - T_{out}) = h_c A \Delta T$, where M is the gas mass flow rate, C_p is the specific heat at constant pressure, A is the available area for heat transfer, T_{in} is the gas temperature entering an axial region, T_{out} is the exit temperature, and ΔT is the average temperature difference between the gas and core in the region. Core temperatures are obtained by averaging the temperature of the radial region in question with the six surrounding regions. Then, given the surface correction coefficient, we iterate within each region to determine the exit gas temperature from that region.

The surface convection coefficient is determined by considering flow between two flat parallel plates. Using the mass flow rate for the gas that was

obtained previously, we evaluate the Reynolds number, $Re = \rho V \delta / \mu$, where μ is the viscosity, which for helium at 700° is 2.13×10^{-5} lb/ft·s. The distance between surfaces is taken to be 0.5 in., and the flow rate per unit area, ρ/V , is 1434 lb/h·ft². The Reynolds number is then equal to 779, which means that the flow is laminar.

To determine whether entrance effects are important, we calculate the Graetz number, $Gr = Re P_r / L / \delta$. For this case, again assuming $\delta = 0.5$, in., and using p_r for helium at $700^\circ F$ to be equal to 0.72, Gr is equal to 23. Here we have used a minimum characteristic flow length of 2 ft. We now use a plot of the mean Nusselt number, Nu_m , versus the Graetz number (Ref. 10) to obtain an approximate value for Nu_m of 8.5.

The relationship, $Nu_m = 2 h_c \delta / k$, is then used to obtain the connection coefficient. The thermal conductivity, k , for helium at 700° is 0.135 BTU/ft·°F. The connection coefficient, h_c , is then equal to 13.67 BTU/ft³·h·°F.

APPENDIX B

FINITE ELEMENT DERIVATION FOR TWO-DIMENSIONAL THERMAL ANALYSIS

The top of the CSB can be conveniently divided into six equilateral triangular finite elements (Fig. 5). If we use six node points to represent each element, the temperature field can be represented quite accurately in quadratic form. Consider the fundamental element shown in Fig. B-1. The temperature field can be represented in local coordinates ξ and η as

$$T(\xi, \eta) = a_0 + a_1\xi + a_2\eta + a_3\xi^2 + a_4\xi\eta + a_5\eta^2 \quad . \quad (B1)$$

Substituting the appropriate local coordinates and nodal temperatures $T_1 \dots T_6$ into Eq. (B1) and solving for $a_0 \dots a_5$, the temperature field can be represented by

$$T(\xi, \eta) = \sum_{i=1}^6 N_i(\xi, \eta) T_i \quad , \quad (B2)$$

where

$$N_1(\xi, \eta) = 1 - 3\xi + 2\xi^2$$

$$N_2(\xi, \eta) = 2\xi - 2\sqrt{3}\eta - 2\xi^2 + 2\sqrt{3}\xi\eta$$

$$N_3(\xi, \eta) = -1/2\xi + \sqrt{3}/2\eta + 1/2\xi^2 - \sqrt{3}\xi\eta + 3/2\eta^2 \quad (B3)$$

$$N_4(\xi, \eta) = \xi^2 - 3\eta^2$$

$$N_5(\xi, \eta) = -1/2\xi - \sqrt{3}/2\eta + 1/2\xi^2 + \sqrt{3}\xi\eta + 3/2\eta^2$$

$$N_6(\xi, \eta) = 2\xi + 2\sqrt{3}\eta - 2\xi^2 - 2\sqrt{3}\xi\eta.$$

To calculate the conductivity matrix we evaluate

$$K_{ij} = k \iint_S \left[\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right] dx dy. \quad (B4)$$

Here the conductivity is assumed to be constant and isotropic. Now,

$$\frac{\partial N_i}{\partial x} = \frac{\partial N_i}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial N_i}{\partial \eta} \frac{\partial \eta}{\partial x}$$

$$\frac{\partial N_i}{\partial y} = \frac{\partial N_i}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial N_i}{\partial \eta} \frac{\partial \eta}{\partial y}. \quad (B5)$$

But, from Fig. B-2,

$$\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix}, \quad (B6)$$

where λ is a magnification factor. Therefore, taking derivatives and substituting into (B5) gives

$$\frac{\partial N_i}{\partial x} = \frac{1}{\lambda} \cos\theta \frac{\partial N_i}{\partial \xi} - \frac{1}{\lambda} \sin\theta \frac{\partial N_i}{\partial \eta} \quad (B7)$$

$$\frac{\partial N_i}{\partial y} = \frac{1}{\lambda} \sin\theta \frac{\partial N_i}{\partial \xi} + \frac{1}{\lambda} \cos\theta \frac{\partial N_i}{\partial \eta}.$$

Fig. B-1. Fundamental finite element.

Taking the increment in area $dxdy = \det J \, d\xi d\eta$, where

$$\det J = \det \begin{bmatrix} \partial x / \partial \xi & \partial x / \partial \eta \\ \partial y / \partial \xi & \partial y / \partial \eta \end{bmatrix} = \lambda^2, \quad (B8)$$

the elements of the conductivity matrix are

$$K_{ij} \equiv k \iint_{\text{Area}} \left[\left(\cos \theta \frac{\partial N_i}{\partial \xi} - \sin \theta \frac{\partial N_i}{\partial \eta} \right) \left(\cos \theta \frac{\partial N_j}{\partial \xi} - \sin \theta \frac{\partial N_j}{\partial \eta} \right) + \left(\sin \theta \frac{\partial N_i}{\partial \xi} + \cos \theta \frac{\partial N_i}{\partial \eta} \right) \left(\sin \theta \frac{\partial N_j}{\partial \xi} + \cos \theta \frac{\partial N_j}{\partial \eta} \right) \right] d\xi d\eta, \quad (B9)$$

where the partial derivatives

$\frac{\partial N_i}{\partial \xi}$, $\frac{\partial N_i}{\partial \eta}$, $\frac{\partial N_j}{\partial \xi}$, and $\frac{\partial N_j}{\partial \eta}$ can be determined from Eq. (B4).

To numerically integrate Eq. (B9), we use three integration points, one at the center of each side of the element. Because the equation is quadratic, the integration is exact if weights of 1/3 are used for each point.

The nodal loads for a uniform normal heat flux are evaluated with the following integral.

$$F_i = \iint_{\text{Area}} N_i(s,y) \, dx dy .$$

Carrying out this integration gives

$$F = -\frac{QA}{3} \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{Bmatrix} .$$

The convection boundary condition

$$-k \frac{\partial T}{\partial \xi} = h (T - T_\infty)$$

is now considered. Referring to Fig. B-2 and realizing that $\xi = 1$ and $-\sqrt{3}/3 \leq \eta \leq \sqrt{3}/3$, we can transform coordinates along the boundary using $\eta = \sqrt{3}/3 (2s/\ell - 1)$. The shape functions on the boundary then become

$$\begin{aligned} N_3(s) &= (1 - 2s/\ell) (1 - s/\ell) \\ N_4(s) &= 4s/\ell (1 - s/\ell) \\ N_5(s) &= -s/\ell (1 - 2s/\ell) . \end{aligned}$$

The contribution to the load vector from the boundary condition is calculated from

$$F_i = -h T \int_0^\ell N_i(s) \, ds .$$

$$N_1(s) = \left(1 - \frac{2s}{\ell}\right) \left(1 - \frac{s}{\ell}\right)$$

$$N_2(s) = 4 \frac{s}{\ell} \left(1 - \frac{s}{\ell}\right)$$

$$N_3(s) = -\frac{s}{\ell} \left(1 - \frac{2s}{\ell}\right)$$

Fig. B-2. Finite element boundary representation.

Carrying out this integration for each of the three nodal boundaries, we get

$$F_3 = -hT_\infty / 6$$

$$F_4 = -2hT_\infty / 3$$

$$F_5 = -hT_\infty / 6 .$$

The adjustment to the conductivity matrix based on the convective boundary condition is evaluated from

$$h_{ij} = h \int_0^\ell N_i(s) N_j(s) ds .$$

This gives the following.

$$h_{11} = 2h\ell/15$$

$$h_{12} = h_{21} = h\ell/15$$

$$h_{13} = h_{31} = -h\ell/30$$

$$h_{22} = 8h\ell/15$$

$$h_{23} = h_{32} = h\ell/15$$

$$h_{33} = 2h\ell/15 .$$

APPENDIX C

QUANTIFICATION OF UNCERTAINTY OF CALCULATED STRESSES

Error may be introduced into the calculations presented in this report through the following.

- (1) Uncertainties in loads predicted by ORECA,
- (2) uncertainties in material properties,
- (3) modeling approximations, and
- (4) inaccuracies in finite element model solutions.

We are not in a position to evaluate the error in the ORECA predictions except for the calculations presented in Appendix A. There we find that by neglecting the gap between adjacent CSBs, the radial resistance in ORECA is up to 30% low. The effect of this is discussed in Appendix A. Because ORECA probably predicts larger than actual radial heat flows, our results are conservative. When we used data from ORECA runs that have different thermal conductivities for PGX graphite, the thermal stresses were, as expected, higher for the higher conductivities (lower resistance).

We have used the most recent material properties available for PGX graphite (Ref. 6). Small variations in any of the properties, other than thermal conductivity, affect the stress predictions by small amounts. However, the stresses are somewhat sensitive to changes in thermal conductivity. Table IV in Section VI of the main body of this report indicates that by increasing the thermal conductivity by a factor of three in our analyses, the thermal stress is reduced by approximately one-half. The dominant uncertainty, we feel, is the magnitude of thermal stresses required to cause graphite failure.

Several modeling approximations were used in developing the models for these studies. The most significant was the geometrical simplifications imposed to force the CSB to be cyclically symmetric. Our models placed the keyway midway between the coolant holes. In reality one coolant hole is located much closer to the keyway. However, we could not expect major differences in results because the maximum stresses occur very near the keyway corner and decay rapidly away from the keyway. Other modeling approximations include assumptions of constant correction coefficients on a coolant channel surface, constant radial heat flow over the complete side of the CSB, and evenly distributed heat flow over the entire top surface.

Numerical errors from the models used in this study can be only partially quantified. To obtain a true error estimate because of mesh density, for instance, we would have to redo the analyses with successively finer meshes to show how the results converge. Because of the model size this would be very expensive. We can, however, get some idea of accuracy from other references where model results are compared with tests and closed form solutions. Also, several of the studies reported here lend credence to model accuracy.

The temperature field predicted with ADINAT is, neglecting errors in loads and material properties, probably within 5% of the exact solution. Several problems are solved with ADINAT in Ref. 3 and compared with closed form solutions. These include steady-state and transient applications where conduction and convection are considered for several different load conditions. For all of the problems the ADINAT predictions are nearly exact.

The transient temperatures of the CSB predicted by ADINAT also compare very well with the predictions made by ORECA (Figs. 27 and 30). The substructure model developed with computer code TSAAS, which essentially gave a very refined mesh for a portion of the CSB, predicted isotherms that compare closely with the ADINAT predictions.

We also checked for temporal convergence of our transient calculations. This was accomplished by reducing the integration time step until convergence occurred and then using this time step for the remainder of our analyses.

The ADINA calculations are also probably accurate to within 5% of the exact solution. Reference 4 compares ADINA predictions to solutions obtained using other techniques for two problems involving thermal gradients. The results are very close.

The substructure model of the keyway region gives an indication of the effect of mesh refinement. The maximum principal stresses predicted by the three-dimensional ADINA model are within 2% of those predicted by TSAAS with a much finer mesh (see Figs. 21 and 24).

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