SINGLE PASS COLLIDER MEMO

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TITLE: PERMANENT MAGNET SEXTUPOLE PROTOCOL AND TOLERANCES FOR THE DAMPING RINGS *

This note responds to various questions on alignment tolerances for the permanent magnet sextupoles (PMS's) in the damping rings. It should be considered as an addenda to the discussion of Ref. 1 in that I will only discuss questions which are not considered there. The description and characteristics of the sextupoles are discussed in Ref. 2. This will be the first optical use of permanent magnets in storage rings due to a lack of any practical alternative which could provide the same high strength required in the small space available. Even though there are nearly twice as many sextupoles in the DR's than in Spear they are also an order of magnitude stronger.

The Magnets and Conventions

The magnets have strengths comparable to the PEP sextupoles but are small enough to fit inside their bores. They have lengths $l = 12.7$ mm and outer diameters $d = 3.75$ inches (excluding hose clamps). They weigh 12 ounces and should be simple to install and align. They are also about an order of magnitude cheaper to build. Figure 1 defines conventions based on positrons going into the page and electrons coming out. A focusing sextupole augments the bend field or is equivalent to a cutout or hole in the pole boundary. Defocusing quads and sextupoles have reverse polarity etc. Fig. 2 shows an SD and SF made by Mike Baltay for the NW quadrant of the NDR.

Rotational Errors

One can understand concerns about sextupole installation by realizing that ideal sextupoles improperly intalled into an otherwise perfect ring can perturb the various lattice functions e.g. introduce vertical dispersion and also excite additional betatron resonances. For instance, sextupole rotational errors can induce the following resonances:

$$\begin{array}{lll} \nu_y = p & 2\nu_y = p & 4\nu_{x,y} = p \\ 3\nu_y = p & \nu_x \pm \nu_y = p & 3\nu_y \pm \nu_x = p \\ 2\nu_x \pm \nu_y = p & 2\nu_x \pm 2\nu_y = p & 3\nu_x \pm \nu_y = p \end{array}$$

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Only the first and second order resonances are listed, many of which are also excited by unrotated sextupoles¹ or rotated quads such as the linear coupling resonances $\nu_x \pm \nu_y$. One can compute the stop-band widths using the methods of Ref. 3 — as first shown to me by Phil Morton. The result for the first order sum resonance, $2\nu_x + \nu_y$, unique to the rotated sextupoles, is

$$\Gamma = \frac{\beta_x}{2\pi} < k_{2r}^2 >^{1/2} y_m (1 + \frac{\beta_y}{4\beta_x} \frac{x_m^2}{y_m^2})$$

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where $(k_{2r})_i = k_{2i} \sin 3\theta_i$ is the strength of the i^{th} rotated sextupole with angle error θ_i and x_m, y_m are the maximum, transverse excursions at the sextupoles i.e. the beam-stay-clear prescription

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defined in Ref. 4. Using the assumed e^+ emittance for the NDR

$$\epsilon_{inj}^+ = 4.13(\pi) \mu rad m$$

$$\delta = \pm 1\%$$

and Table I, for the strong SD family, one then has:

$$x_m - 2 = (\epsilon^+ \beta_x + \delta^2 \eta_x^2)^{1/2} = 1.3 mm$$

$$y_m - 2 = (\epsilon^+ \beta_y)^{1/2} = 3.6 mm$$

where 2 mm has been added in both directions for orbit distortions⁴.

To compute Γ_{SD} , one needs to specify the rms spread in rotational error of the SD family which is compounded from intrinsic and installation errors. Based on the size and shape of the magnets, one expects little difficulty in achieving a maximum misalignment error of $\Theta_m^A = \pm 0.5^\circ$. Fig. 2 shows the proper orientation of the sextupoles ($SF(10)$ and $SD(11)$) in the ring. It suggests a fairly simple scheme to obtain such rotational alignment errors. The maximum, intrinsic rotational error in a PMS should be no larger than the maximum, transverse easy-axis errors allowed in the PM block sample². However, because we measure these angles and independently cross-check them, the intrinsic rotational error can be made negligible even when all blocks have the maximum error. Because this is $\Theta_m^B \lesssim \pm 1.2^\circ$ for our block sample, the worst case scenario is a maximum, intrinsic rotational error of $\Theta_m^I = \pm 1.2^\circ$. Assuming we don't specify this angle for each magnet, the rms rotational error is then:

$$\sigma_\theta^2 = (\sigma_A^2 + \sigma_I^2) \xrightarrow{box} \frac{1}{3} [(\Theta_m^A)^2 + (\Theta_m^I)^2] = \left(\frac{3}{4}\right)^2.$$

One can show that $\langle k_{2r}^2 \rangle^{1/2} = 3 \langle k_2 \rangle \sqrt{N} \sigma_\theta$, so the full width, Γ_{SD} , becomes:

$$\Gamma_{SD}(2\nu_x + \nu_y) = \frac{\sqrt{N} \beta_x}{2\pi} \langle k_2 \rangle (3\sigma_\theta) y_m \left(1 + \frac{\beta_y}{4\beta_x} \frac{x_m^2}{y_m^2}\right) \xrightarrow{box} 0.0014.$$

This worst case estimate is comparable to the expected tune spread in the DR and implies we don't need to specify the intrinsic rotational errors with a maximum allowed installation error of $\pm 0.5^\circ$. For later use, the corresponding unrotated sextupole sum resonance, $2\nu_y + \nu_x$, should be broader by roughly $\sqrt{N}/3\sigma_\theta$. The actual numbers give

$$\Gamma_{SD}(2\nu_y + \nu_x) = 0.123,$$

which restricts the fully coupled tunes to $\nu_{x,y} \lesssim 0.31$ during injection which isn't a problem.

Strength/Longitudinal Placement Errors

The strengths of the two sextupole families (SD & SF) were based on their longitudinal positions as determined by Jean Pierre Delahaye and Dick Jeong^F. Interleaved sextupoles for chromatic correction of quadrupoles require strengths $nk_2 \approx (2k_1)/\eta_z$ where n is the number of sextupoles per quadrupole of strength k_1 . The strength definition throughout this report^{1,2} is that of PATRICIA⁶. Although $n = 2$ for the damping rings, one can see from Table I why the DR sextupoles are still much stronger than in rings like SPEAR which have large η -functions in the arcs. Strength errors, which can result from intrinsic magnet errors and longitudinal position errors, then become of more concern.

The variation of chromaticity with systematic, longitudinal offsets, δz_S , in mm, was determined by Delahaye to be:

$$\begin{aligned}\delta\xi_x &\approx 0.0667\delta z_{SF} + 0.0093\delta z_{SD} \xrightarrow{\delta z_{SF}=\pm 2.0mm} \pm 0.226 \\ \delta\xi_y &\approx 0.0105\delta z_{SF} + 0.0173\delta z_{SD} \xrightarrow{\delta z_{SD}=\pm 10mm} \pm 0.194\end{aligned}$$

The sign convention is such that a positive displacement in δz , for any sextupole type, is always towards its associated quadrupole. A $\delta\xi_{x,y} \lesssim 0.1$ due to longitudinal position errors is desirable which implies a maximum longitudinal position error of $\delta z_m \lesssim \pm 2mm$. The magnets are only 12.7mm long with front/back surfaces that are flat and parallel to better than $\pm 0.1mm$. Clearly, variations in the strengths of individual sextupoles can be corrected in this way as well as a variable chromaticity achieved. The important question for installation is whether we should correct the strengths in this way.

Intrinsic strength variations in PMM's come primarily from radius errors and variations in the remanent field, B_{ri} , of the individual blocks². For negligible radius variations, the worst strength error possible is just the worst error allowed for B_r in our block sample. Since the maximum, measured field error² is $(\delta B_r/B_r)_m \leq \pm 2.0\%$, the worst relative strength error is just $\pm 2.0\%$ about the mean. For the strong SD sextupoles when all $B_{ri} = 0.93T$, one finds from Fig. 3 that $S_D = 111.0T/m$ and

$$\frac{\partial S_D}{\partial B_r} = 1.11T/m/\%,$$

$$\frac{\partial S_D}{\partial R} = -0.64T/m/mil$$

for the worst case scenario. In fact, one can show that it is possible to use a set of blocks with large field errors and still make a magnet with the correct strength. Figure 4 shows results for eighteen sextupoles intended for the NW quadrant of the NDR. The error bars are standard deviations of a set of measurements involving multiple assembly and measurement. The rms radius error is less than 1mil which is small compared to the possible field contribution. Lenny Rivkin, using PATRICIA and a box distribution for the strengths with a maximum error of $\pm 2.0\%$, showed there was no significant increase in emittance or decrease in dynamic aperture so that specific strength corrections via magnet radius, block matching or magnet location should not be necessary!

Transverse Placement Errors

Transverse errors relate to errors in the equilibrium closed orbit and the ability to measure and correct them. Measurement quality is supposed to be $\pm 2-4 \text{ mils}$ while the orbit correction capability is about an order of magnitude worse or $\sigma_{x,y}^{CO} \approx 1 \text{ mm}$. Dan Wright has said that the relative alignment between quadrupole and sextupole beam pipes is not better than $\pm 10 \text{ mils}$ or twenty five percent of the present rms closed orbit capability. With improved representations of the perturbed lattice, better orbit correction will be possible so alignment tolerances for sextupoles should not become a limitation. The calculations here consider the implications of this.

Because transverse orbit errors in sextupoles make these magnets look like skew quadrupoles, we can use some of Courant and Snyder's results⁷. For instance, if the tune perturbation around the ring due to horizontal orbit errors in sextupoles is $\Delta\nu_x = \frac{1}{4\pi} \sum \beta_{xi} k_{2i} x_i^{CO}$ then one expects rms tune shifts per mm of rms position error to be

$$\langle \Delta\nu_x^2 \rangle^{1/2} = \frac{\sqrt{N}}{4\pi} [(\beta_x k_2)_D^2 + (\beta_x k_2)_F^2]^{1/2} \sigma_x = 0.018/\text{mm}$$

$$\langle \Delta\nu_y^2 \rangle^{1/2} = \frac{\sqrt{N}}{4\pi} [(\beta_y k_2)_D^2 + (\beta_y k_2)_F^2]^{1/2} \sigma_x = 0.042/\text{mm}$$

assuming uncorrelated errors. If σ_x is the closed orbit error then there will be correlations so this expression underestimates the shift by at least $\sqrt{2}$ but should be valid for sextupole placement errors in the damping rings. These numbers are large but consistent with modelling errors being worse in the vertical than horizontal⁸. An rms placement error of $\sigma_x = 10 \text{ mils}$ implies an rms tune shift $\langle \Delta\nu_y \rangle \approx 0.010$ which is marginally acceptable. The beta-function errors related to these tune shifts are

$$\frac{\langle \Delta\beta_x^2 \rangle^{1/2}}{\beta_x} = \sqrt{2\pi} \frac{\langle \Delta\nu_x^2 \rangle^{1/2}}{|\sin 2\pi\nu_x|} = 0.085/\text{mm}$$

$$\frac{\langle \Delta\beta_y^2 \rangle^{1/2}}{\beta_y} = \sqrt{2\pi} \frac{\langle \Delta\nu_y^2 \rangle^{1/2}}{|\sin 2\pi\nu_y|} = 0.196/\text{mm}$$

for fractional tunes of $\nu_x = \nu_y = 0.30$ which implies rms beta variations of 5% due to sextupole placement errors. Apart from modeling errors this also reduces the acceptance by

$$A = \frac{(\text{Aperture})^2}{\beta} \rightarrow A_0 \left(1 - \frac{\langle \Delta\beta^2 \rangle^{1/2}}{\beta} \right)$$

i.e. by up to 5% for the sextupole errors assumed above which is again marginally acceptable because it increases with decreasing tunes. These offsets should be measured so that errors significantly greater than $\pm 10 \text{ mils}$ are either corrected or included in the model.

While horizontal errors lead to tune and beta changes like normal quad errors, vertical errors produce coupling between horizontal and vertical beta and eta functions i.e. lead to the skewness in the quad analogy. Because closed orbit errors $x_C, y_C > \sigma_y \approx \sigma_x$ and $\beta_{x,y} \gg \eta_x$ as well as $\beta_x y_C \gg \eta_x \sigma_y (\gg \eta_y \sigma_x \text{ or } \sigma_x \sigma_y)$ one is usually more interested in the beta coupling especially since the horizontal eta function has been suppressed in the damping rings. Of course, this depends on many things such as the tunes and whether one wants a ring for damping or luminosity. Because vertical eta effects are hard to correct even when due to closed orbit errors, they can be depended on to increase emittance and reduce acceptance for off momentum particles and are therefore of most concern here. The vertical eta perturbation, at a location with β_y , due to vertical position errors is:

$$\langle \eta_y^2 \rangle^{1/2} = \sqrt{\frac{N}{8}} \frac{\beta_y^{1/2}}{\sin \pi \nu_y} [(\beta_y \eta_x^2 k_2^2)_D + (\beta_y \eta_x^2 k_2^2)_F]^{1/2} \sigma_y = 9.2 \frac{\sqrt{\beta_y(m)}}{\sin \pi \nu_y} \sigma_y(m) \quad [m]$$

so $\langle \eta_y^2 \rangle_D^{1/2} \approx 0.007m$ at the SD's and $0.005m$ at the SF's for the sextupole errors and tunes used above. For lower tunes, $\langle \eta_y \rangle$ easily exceeds $1cm$. At the bends, $\beta_{x,y}$ varies between the SD and SF values so $\langle \eta_y \rangle \approx 0.006m$ compared to $\langle \eta_x \rangle \approx 0.09m$. Ignoring betatron coupling and assuming equal damping times $\tau_x = \tau_y$ gives a fractional increase in emittance of

$$\langle \frac{\delta \epsilon_y}{\epsilon_x} \rangle \approx \frac{N}{8} \frac{\langle \beta_x \rangle_B}{\sin^2 \pi \nu_y} [(\beta_y k_2^2)_D + (\beta_y k_2^2)_F] \sigma_y^2 = 1.24 \frac{\langle \beta_x(m) \rangle_B}{\sin^2 \pi \nu_y} \sigma_y^2(mm) \quad [\%]$$

or $\lesssim 1.0\%$ for the sextupole errors and high fractional tunes assumed above. This is probably acceptable compared to other sources such as rotational errors in bends or quads or vertical errors in quads.

Such effects should only be directly observable for tunes $\nu_y \lesssim 3.05$ or for correlated errors like the closed orbit. The later are a concern at the $1mm$ level and are only correctable with sufficiently good orbit measurements and sufficiently small lattice perturbations away from model assumptions. Alex Chao studied this problem for the PEP ring⁹ by simulating such errors and found that the uncorrectable η_y perturbations, $\Delta \eta_y$, were most sensitive to $\Delta \nu_y$. One would expect such a result from the expressions above for random transverse sextupole errors:

$$\Delta \eta_y = \frac{\partial \eta_y}{\partial \nu_y} \Delta \nu_y \approx \langle \eta_y \rangle > \frac{\pi \langle \Delta \nu_y \rangle}{\tan \pi \nu_y}$$

which is quite sensitive to the fractional tune and directly proportional to both σ_x and σ_y . Inability to correct η_y is therefore directly proportional to both x, y placement errors whose magnitudes constrain the vertical tune space available. A detailed simulation with all sextupole errors (and other errors) is required for a more accurate estimate but together with the other implications of transverse errors this seems sufficient to set a goal of $\sigma_{x,y} \lesssim 10mils$.

Pitch/Yaw or Tilt Errors

These are rotational errors about the x and y axes respectively which induce harmonic strength perturbations and transverse-longitudinal couplings e.g. in lowest order a sextupole's strength perturbation from a y -rotational error is:

$$\frac{\delta S}{S} \approx \frac{1 - \cos \theta_y}{\cos \theta_y} \approx \frac{\theta_y^2}{2}$$

which can be at least 1% as already discussed so $(\theta_y)_m \lesssim 8^\circ$ based solely on the fundamental harmonic. There will also be an associated, horizontal dipole kick error $k_0^S \approx \frac{1}{6} k_2 (\frac{1}{2})^2 \theta_y^2$ which should be small compared to bending magnet errors $\delta k_0^B \gtrsim 9^\circ \times 10^{-4}$ due to the small lengths, l . Matching this to the dipole kicks from transverse errors using $\langle k_0^S \rangle = (\frac{\sigma_x}{2})^2 (k_{2F} + k_{2D})$, with $\sigma_x = 10 \text{ mils}$, then implies $\sigma_{\theta_y} = 2\sqrt{3}(\sigma_F) \lesssim 4^\circ$.

Harmonic Field Errors

Besides harmonic errors resulting from installation or closed orbit errors, there are always intrinsic errors to be minimized and measured. The higher the fundamental harmonic, the more difficult this becomes - especially with no active knobs for tuning. Using PEP for analogy, a rather arbitrary limit of 1% on all normalized multipoles was set at a normalization radius, outside the maximum beam-stay-clear around the ring, of $R_N = 8 \text{ mm}$. Because the PEP sextupoles¹⁰ have length to radius ratios $3.5 \lesssim \frac{l}{R} \lesssim 6.5$ and the DR sextupoles have $0.9 \lesssim \frac{l}{R} \lesssim 1.0$, it should be much harder to get good harmonic characteristics in the DR sextupoles. The first symmetry allowed harmonic for symmetric, 12-block PMS's is $n = 15$ while it is $n = 9$ for conventional, iron dominated sextupoles. Some low order error harmonics for the magnets tabulated in Fig. 4 are:

$$\langle \frac{A_0}{A_2} \rangle_{SD} = 0.38 \pm 0.08 \%$$

$$\langle \frac{A_0}{A_2} \rangle_{SF} = 0.41 \pm 0.20 \%$$

$$\langle \frac{A_1}{A_2} \rangle_{SF} = 0.73 \pm 0.18 \%$$

$$\langle \frac{A_3}{A_2} \rangle_{SF} = 0.47 \pm 0.24 \%$$

These amplitudes are related to the strengths used here by $\frac{A_{n-1}}{A_2} = \frac{2}{(n-1)!} \left(\frac{k_{n-1}}{k_2} \right) \frac{R_N^{n-1}}{R_2^2}$ which can be translated into machine effects to set harmonic tolerances.

Of the possible approaches such as the Hamiltonian method used above or various tracking techniques, a first-order perturbation approximation is especially useful e.g. one can check Rivkin's tracking results on the effects of sextupole strength variations (see Fig. 4) quoted above. The change in emittance in an otherwise perfect ring, $\delta\epsilon_y$, at location z_0 with $\alpha_0 = 0$, for $M + 1$ turns is:

$$\frac{\delta\epsilon_y}{\epsilon_y} \simeq \sqrt{\epsilon_x} \sum_{n=1}^{2N} \sum_{m=0}^M (\delta k_2)_n \beta_{yn} \sqrt{\beta_{zn}} \sin 2(2\pi m\nu_y + \Delta\phi_{yn}) \cos (2\pi m\nu_z + \Delta\phi_{zn})$$

where $\Delta\phi_{yn} = \phi_{yn} - \phi_{y0}$ etc. Near the $2\nu_y \pm \nu_z$ resonances this sum reduces to a single term with a standard deviation:

$$\left\langle \frac{\delta\epsilon_y}{\epsilon_y} \right\rangle_{n=3} \approx \sqrt{\frac{N}{8}} \frac{\sqrt{\epsilon_x}}{\sin \pi(2\nu_y \pm \nu_z)} \left(\frac{\sigma_{k_2}}{k_2} \right) [(\beta_x \beta_y^2 k_2^2)_F + (\beta_x \beta_y^2 k_2^2)_D]^{\frac{1}{2}} \xrightarrow{\epsilon_{mj}^*} 5 \times 10^{-3}$$

for fractional tunes of 0.2 as in the simulations¹. The corresponding expression for on-energy particles in the median plane is only important for ν_x or $3\nu_x$ near an integer. More general expressions, based on sums over all harmonics and magnets, can be written which appear ideal for interactive computing and display¹¹. The quadrupole harmonic which appears strongest here is equivalent to a measuring coil misalignment error of only 1mil.

Summary and Conclusions

A reasonably consistent set of error limits result from the above which can be specified in terms of the maximum errors allowed in the distribution, as opposed to the standard deviations, since this seems preferable from the standpoint of the people actually doing the installation. The main results are:

$$x_m \lesssim \pm 15 \text{ mils}$$

$$y_m \lesssim \pm 15 \text{ mils}$$

$$z_m \lesssim \pm 2 \text{ mm}$$

$$(\theta_z)_m \lesssim \pm \frac{1}{2}^\circ$$

Needless to point out, there are many interesting machine physics experiments to be done to verify all the work that has gone into this. Hopefully such experiments will justify more extensive use of REPMM's in storage rings and colliders. This application will also provide some necessary knowledge on radiation and thermal lifetimes in typical working environments. It also suggests some interesting beam diagnostic devices.

Table I: Sextupole Damping Ring Parameters from PATRICIA.

Sex Type & Number	$k_2(m^{-2})$	$\beta_x(m)$	$\beta_y(m)$	$\eta_x(m)$
$S_F(N = 36)$	19.3	1.93	1.16	0.13
$S_D(N = 36)$	-27.3	0.28	3.10	0.07

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Fig. 1: Nomenclature for the North/South Damping Rings (e^+/e^-). Definition of magnetic midplane ($y=0$) for dipole, quadrupole, and sextupole elements. Magnetic polarities indicate multipole elements that are positive with respect to one another. The upper figure is borrowed from SLAC-75 and the lower one from ref. 8.

SD #11

SF #10

Fig. 2: "Xerox" copies of Sextupoles SF#10 & SD#11 for positrons in the North Damping Ring.