Stability of High Beta Tokamaks to Ballooning Modes

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Ballooning modes are found to possess a second stable regime for high beta. The range of unstable beta values depends on the details of the equilibrium, and in particular, on shear, which can be strongly stabilizing.

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It is desirable that magnetic plasma confinement devices have a large ratio of plasma pressure to magnetic pressure, or beta. Previous work has shown the beta in tokamaks to be limited to a critical value, of order $\beta = \epsilon/q^2$, where $1/\epsilon = R/a$ is the aspect ratio and $1/q$ is the rotational transform. Above this critical value, one finds the onset of pressure driven ballooning modes.

In this letter, we present numerical evidence that in fact there is a second critical beta, above which ballooning modes are stable. The range of unstable beta values depends on the details of the equilibrium, in particular on the shear, which can be strongly stabilizing.

The stabilization of ballooning modes at high beta has been suggested on the basis of simple model calculations assuming circular magnetic flux surfaces.^{1,2,3} In addition, relevant numerical work has been performed by Fielding and Haas,⁴ who considered high mode numbers and nonscalar pressure, and by Freidberg and Goedbloed.⁵ In the present work we concentrate specifically on ballooning modes, which we define as pressure driven internal modes, using hydromagnetic equations of motion with scalar pressure.

Our results are based on two complementary approaches. Long wavelength global modes are studied by solving reduced equations for high beta tokamaks,⁶ which are based on a small ϵ ordering of the hydromagnetic equations of motion. These equations, besides being more tractable numerically than the full set, also provide simple scaling of the results in terms of ϵ and q . The short

wavelength limit was investigated by solving a radially local eigenvalue problem, as described in Refs.^{7,8}

The reduced equations are

$$\frac{dA}{dt} = [U, A] + \frac{B_0}{R_0} \frac{dU}{d\zeta} \quad (1)$$

$$\frac{dW}{dt} = [U, W] - [J, A] - \frac{B_0}{R_0} \frac{dJ}{d\zeta} + \frac{2}{R_0} [R, P] \quad (2)$$

$$\frac{dP}{dt} = [U, P] \quad (3)$$

where A is the poloidal magnetic flux, $[f, g] = \nabla f \times \nabla g \cdot \hat{\zeta}$, ζ is the toroidal angle, $\hat{\zeta}$ is a unit vector along ζ , U is the velocity stream function, $W = \rho \nabla^2 U$ is the vorticity, ρ is the density (assumed constant), $J = -\nabla^2 A$ is the toroidal current, B_0 and R_0 are the leading order constant toroidal field and major radius, and P is the pressure. The plasma boundary is assumed to be a circular conducting wall of radius $r = a$, on which $A = U = 0$.

We have solved these equations numerically, to find equilibria and test their stability, using the HIB computer code.^{9,10} Taking polar coordinates r, θ in the poloidal plane we Fourier analyse the equations in θ and ζ , and use finite differences for radial derivatives. The algorithm takes advantage of the vector processing capability of the MPE CRAY-1 computer. Typically, we use 50 radial mesh points and 15 poloidal harmonics.

To find equilibria, we start with an initial, cylindrically symmetric, nonequilibrium state in which we prescribe profiles for the toroidal current J and pressure P . We have chosen

$$J(r) = \frac{2}{q_0} \left(1 - \frac{r^2}{a^2}\right)^v \frac{B_0 \epsilon}{a} \quad (4)$$

$$P(r) = P_0 \left(1 - \frac{r^2}{a^2}\right)^\mu \quad (5)$$

where q_0 , P_0 are the values of q , P on the magnetic axis. These equations can be solved for $q(A)$ and $P(A)$. We advance Eqs. (1,3) in time, with the constraint $d/d\zeta = 0$, removing kinetic energy until the solution settles into a state of minimum potential energy. During this process, the profiles $q(A)$ and $P(A)$ are constants of the motion.⁶ Given a particular choice of μ , v in (4) and (5), we can find families of equilibria, with the same flux surfaces (contours of A), characterized by a particular value of the poloidal beta, which we here define as

$$\beta_p = 2 \left(\frac{q_0 R_0}{a} \right)^2 \frac{P_0}{B_0^2} \quad (6)$$

A typical computed equilibrium is shown in Fig. 1, in which we plot contours of constant A . Note the substantial shift of the magnetic axis away from the center. This equilibrium is stable, having a value of β_p above the second critical value.

To investigate stability, we linearize the equations of

motion and assume a ζ dependence of the linear perturbation $\exp(in\zeta)$. It can be readily shown that the frequency of a linear mode depends on n only in the product nq . Thus for a given set of equilibrium profiles, or a particular choice of μ and ν in (4), (5), we can express the growth rate of unstable modes, in units of the shear Alfvén frequency, as a function of only two variables, nq_0 and $\beta_p \epsilon$:

$$\gamma = \left(\frac{C_A}{q_0 R_0} \right) \hat{\gamma}(\beta_p \epsilon, nq_0) \quad (7)$$

In Fig. 2 we plot $\hat{\gamma}^2$ as a function of nq_0 and $\beta_p \epsilon$ for equilibria with $\nu = 1$, $\mu = 4$. It is clear that one has instability in only a finite range $0.25 < \beta_p \epsilon < 1.75$. The maximum growth rate, $\hat{\gamma} = 0.2$, occurs for $\beta_p \epsilon = 1$. We see that as nq_0 increases, the contours appear to tend to asymptotic values independent of nq_0 , as predicted by the large mode number ballooning mode theory.⁸ The lower threshold is $\beta_p \epsilon = 0.25$.

Shear can be strongly stabilizing. Current profiles given by Eq. (4) give a ratio of q at the wall to q at the magnetic axis of $q_a/q_0 = \nu + 1$. In Fig. 3 we compare the stability of equilibria with $\nu = 1$ and $\nu = 3$, both with $\mu = 4$. The $\nu = 3$ case has essentially twice the shear. Shown is $\hat{\gamma}^2$ for $nq_0 = 2$, as a function of $\beta_p \epsilon$. We see the relative width of the instability window; i.e., the ratio of the two critical beta, has dropped about a factor of two for the higher shear case.

The existence of a second critical beta has been confirmed

by using HIB equilibria as input to a slightly modified ballooning code.⁷ We note that our ordering gives $\beta_{crit} \sim \epsilon/q^2$ exactly if stability is limited by the high n modes, as seems to be the case.

Our numerical results have also been checked against those of the PEST code. Taking $\epsilon = 0.1$, good agreement is found between PEST and HIB. The high beta equilibria for PEST were obtained by using a two dimensional transport code operating in a flux conserving mode.¹¹ These equilibria are solutions to the full set of ideal equations and very good agreement has been obtained for critical betas.

A notable feature of our results is that there appears to be no minimum q_0 for stability. In particular the $n = 1$ internal kink is stable down to $q_0 = 0.75$ with β_p above or below the unstable regime of Fig. 2.¹² This is in contrast to earlier results obtained for equilibria with noncircular boundary.⁶ However, we have not considered free boundary kink modes, but in Ref. 5 it appears that these modes can be suppressed if a conducting shell is sufficiently close to the plasma, or if the shear is sufficiently large. An important difference between our results and those of Freidberg and Goedbloed is that they did not find an asymptotic stability condition for large n , as we did. We attribute this to the presence of large mode number surface kink modes. We expect that these modes will saturate at low amplitude nonlinearly, on the basis of our previous study of nonlinear kink modes.¹³

Although we would like to claim that tokamaks might be stably operated at high beta, we must emphasize the limitations of our calculation. The accessibility of the high beta regime with

various equilibria is being explored. Perhaps more important than the absence of a free boundary is the omission of resistive effects. When we include a simple scalar resistivity η in the equations of motion, we find that resistive instabilities occur in the high beta stable regime, with a growth rate with a tearing-like scaling,¹⁴ $\gamma \sim 20 \eta^{3/5} C_A / (q_0 R_0)$ for the profiles we have considered. A study of the high n limit with finite compressibility shows, however, that such modes are stabilized up to high values of n by favorable average curvature.¹⁵ Since this stabilization increases with β , we expect it to be substantial in the high beta equilibria discussed here. The importance of these tearing modes depends also on their nonlinear effects, which are under investigation.

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FIGURE CAPTIONS

- Fig. 1 Flux surfaces $A = \text{const.}$ for a typical high β ballooning stable equilibrium, for $\beta_p \epsilon \approx 2$.
- Fig. 2 Contours of $\hat{\gamma}^2 = \text{const.}$ as a function of nq_0 and $\beta_p \epsilon$. The unstable region is shaded. Surfaces shown are (a) $\hat{\gamma}^2 = 0$, (b) $\hat{\gamma}^2 = 0.03$, (c) $\hat{\gamma}^2 = 0.04$, and (d) $\hat{\gamma}^2 = -0.02$.
- Fig. 3 Dependence of ballooning growth rate on $\beta_p \epsilon$, for low ($\nu = 1$) and high ($\nu = 3$) shear. Doubling the shear practically halves the width of the unstable region.

Fig. 1. 792047

Fig. 2. 792054

Fig. 3. 792052