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BY

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PLASMA PHYSICS
LABORATORY

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CONVECTIVE CELLS AND TRANSPORT IN TOROIDAL PLASMAS

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ABSTRACT

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The properties of convective cells and the diffusion resulting from such cells are significantly influenced by an inhomogeneity in the external confining magnetic field, such as that in toroidal plasmas. In the first place, the zero-frequency convective cell of the homogeneous medium (vortex) is transformed into the interchange flute mode in the presence of the field inhomogeneity. For stable interchange profiles, this has the effect of reducing the step size of the diffusion process. Secondly, in an inhomogeneous field, convective motions damp by magnetic pumping at a rate typically much faster than perpendicular viscous damping. This reduces the lifetime of the cell. The convective diffusion in the presence of a field inhomogeneity is estimated. For a thermal background, this diffusion is shown to be substantially smaller than

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classical collisional diffusion. For a model nonthermal background, the diffusion is estimated, for typical parameters, to be at most of the order of collisional diffusion. The model background employed is based on spectra observed in numerical simulations of drift-wave-driven convective cells.

I. INTRODUCTION

Convective cells have been shown to contribute significantly to the transport of plasma across a uniform magnetic field.¹⁻⁴ The physical process is that the plasma which makes up the cell is convected across appreciable distances before the cell loses its coherence. The plasma has then effectively stepped across the magnetic field, and the resultant diffusion coefficient may be quite large. In fact, for systems in a strong, homogeneous, magnetic field, this convective diffusion has been shown to predominate over classical collisional diffusion, even in cases where the excitation of convective cells is strictly at a thermal level.^{1,2,3} It is also possible that an enhanced level of convective cells could result from some nonthermal feature in the plasma.⁴ In this case, the resultant convective diffusion may be expected to be even larger.

In this paper, we show that the convective cell, in toroidally confined plasmas, is to be identified with the ordinary, interchange flute mode. Consequently, the theory of transport by convective cells becomes identical, in toroidal geometries, to the theory of transport by the interchange mode. We will show that such transport is, in fact, smaller than classical collisional transport (provided the interchange mode is actually stable).

The similarity between the convective cell and the interchange mode is that both modes involve a convective exchange of neighboring flux tubes. This allows us to exploit some of the techniques of the nonlinear theory of transport by convective cells,

already well developed,¹⁻³ in estimating the transport due to interchange modes. However, the essential difference between the two modes is that the interchange mode has restoring (or destabilizing) forces associated with it. These forces, arising from the magnetic field inhomogeneity, lead to a significant deviation of the inhomogeneous theory from the homogeneous one. In particular, if the plasma is unstable to the interchange mode, a strongly turbulent convection develops; application of the methods of homogeneous theory for this case then grossly underestimates the transport. On the other hand, if the interchange is stable, the associated restoring forces restrict the plasma convection to an oscillation at the interchange frequency. That is to say, the "convective cell" simply "rocks" back and forth instead of freely "turning over." This reduces the step-size of the diffusion process and, correspondingly, the transport. In the present paper, we shall only concern ourselves with situations that are either stable or marginally stable to the interchange.

We also wish to note in this paper a further difference between the convective cell and the interchange mode; namely, that in the presence of an inhomogeneous magnetic field, convective plasma motions are subject to a viscous damping that, in general, is much stronger than the damping resulting from classical perpendicular viscosity. This damping results from the fact that the moving fluid elements, upon encountering varying magnetic field strengths, compress or decompress to conserve magnetic flux. This "magnetic pumping" then leads to a heating of the plasma in turn damping the motion. Thus, the lifetime of convective motions is considerably

shortened, a fact which also significantly affects the transport. We take this into account in the present paper.

The first half of the paper concerns itself with employing simplified models to elucidate the above-mentioned properties of convective motions in nonuniform plasmas; namely, the identification of the convective cell with the interchange mode and the damping of this mode by magnetic pumping. The second half of the paper is devoted to estimating the convective transport in a plasma immersed in an inhomogeneous magnetic field. This transport is estimated by employing simple step-size decorrelation-time arguments, and the calculation is carried out both for the case where the background level of convection is thermal and for a model case of nonthermal convection. For the latter case, a steady-state ambient electric field spectrum is assumed, with the choice of spectral shape obtained from the numerical simulations reported by Cheng and Okuda.⁴ We also evaluate the transport separately for the cases where the plasma is either stable or marginally stable to the interchange. Our conclusions are as follows. For the stable case, the transport due to the interchange mode is, in general, substantially smaller, for both thermal and nonthermal backgrounds, than classical collisional ion heat transport. Moreover, the scaling of the diffusion coefficient with the plasma parameters is quite different from the scaling predicted by the homogeneous theory. For the marginally stable case, it is shown that for parameter regimes of interest and for a plausible choice of convective cell spectra, magnetic pumping damps the convective cell before the cell can perform

one full rotation. Thus, the "self-consistent" decorrelation time employed in the homogeneous theory, which is of the order of the cell turnover time, is usually an overestimate that in turn leads to an overestimate of the convective diffusion.

In Section II we begin by describing semi-quantitatively the properties of convective cells in plasmas immersed in an inhomogeneous field. We show that the convective cell transforms into the interchange mode, and we heuristically estimate the size of the damping produced by magnetic pumping. In Sec. III, we perform a detailed calculation to obtain the characteristics and lifetime of a solitary convective cell in an inhomogeneous slab plasma. In Sec. IV, we assess the effects of field inhomogeneity on convective transport. Our findings are summarized in Sec. V where a comparison with experiments is also made.

We remark that in all of our calculations we disregard the effects of shear in the magnetic field. In general, magnetic shear tends to reduce the convective transport (just as it inhibits the interchange). Several authors have investigated this problem.^{5,6,7} In this paper, we shall simply confine ourselves to shear-free systems.

II. SEMI-QUANTITATIVE DESCRIPTION

In this section, we derive the convective cell of the homogeneous medium and then show its transformation to the interchange mode upon the addition of either a gravitational field or a gradient in the external magnetic field. We also describe in this section, heuristically, how convective motions are damped by magnetic pumping.

A. Convective Cells in Homogeneous Media

The convective cell is a normal mode of a homogeneous fluid or plasma. This mode has zero frequency and a small damping due to viscosity. We obtain the mode most easily by taking the curl of the linearized equation of motion. This yields

$$\begin{aligned} -i\omega n M \vec{\nabla} \times \vec{v} &= \vec{\nabla} \times (-\vec{\nabla} p + \vec{j} \times \vec{B} + n M \mu \nabla^2 \vec{v}) \\ &= \vec{B} \cdot \vec{\nabla} \vec{j} - \vec{j} \cdot \vec{\nabla} \vec{B} + n M \mu \nabla^2 \vec{\nabla} \times \vec{v}, \end{aligned} \quad (1)$$

where all perturbed quantities are taken to vary as $e^{-i\omega t}$, and the quantities n , p , $\vec{j}$, $\vec{B}$, $\vec{v}$, M , and μ denote, respectively, the density, pressure, current density, magnetic field, flow field, ion mass, and kinematic viscosity. For an ordinary fluid, $\vec{B} \approx 0$; Fourier analysing in space, we obtain the convective cell

$$(-i\omega + \mu k^2) \vec{\nabla} \times \vec{v} = 0 \quad (2)$$

where $\vec{k}$ is the wave vector. A nonzero curl in the flow thus decays only on the viscous time scale. Such a cell is also referred to as a vortex or an eddy.

In a plasma in a strong $\vec{B}$, if there is no equilibrium spatial variation of $|\vec{B}|$ and if $\vec{B} \cdot \vec{\nabla} + i\vec{k} \cdot \vec{B} \approx 0$, the first two terms on the

right-hand side of (1) vanish and we obtain the same dispersion as above. Here, the magnetic field lines act as "charged rods" of charge density $\vec{B} \cdot (\vec{\nabla} \times \vec{v})$; when neighboring rods acquire unequal charge, the rods move relative to each other setting up a curl in the flow pattern. The convective cell in a plasma is thus made up of circulating flux tubes which carry plasma with them.

B. Inhomogeneous Media and the Interchange Mode

Consider next the effects of a gravitational field on this convective mode. For an ordinary unmagnetized fluid in equilibrium in a gravitational field $\vec{g}$, with $\vec{g} \cdot \vec{\nabla} n \neq 0$, the curl of the linearized equation of motion gives

$$-i\omega(n\vec{\nabla} \times \vec{v} + \vec{\nabla} n \times \vec{v}) = \vec{\nabla} n \times \vec{g}, \quad (3)$$

where $\vec{n}$ is the density perturbation and we have omitted the viscosity for brevity.

Note the presence of additional terms influences the time evolution of $\vec{\nabla} \times \vec{v}$. Thus the convective cell no longer has zero frequency. Employing the standard cartesian coordinate system, with $\vec{g}$ in the $(-\hat{x})$ direction and $\vec{k}$ in the $\hat{y}$ direction, the z component of (3) yields (upon neglecting some terms based on the assumptions that $k^{-1} |d \ln n / dx| < 1$, and that the effective wavelength along $\hat{x}$ is very long compared to k^{-1}).

$$i\omega n v_x = \vec{n} \cdot \vec{g}.$$

Assume an incompressible fluid for simplicity. Then, from the equation of continuity

$$-i\omega \vec{n} + \vec{v} \cdot \vec{\nabla} n = 0,$$

where primes denote differentiation with respect to x . Combining the last two equations, we obtain the dispersion relation

$$\omega^2 = -g(n'/n) . \quad (4)$$

We first emphasize that this mode is the same branch as the zero-frequency convective cell of the homogeneous medium. [That this is true may also be deduced by first identifying all the possible normal modes with $\vec{k} \cdot \vec{B} = 0$ in a homogeneous plasma, one of which is the convective cell (2), and then following each of these modes in adiabatic fashion as g (or $|\vec{v}B|$) is added. This approach is considered in the Appendix.] Thus, in inhomogeneous media, the convective cell has a finite real frequency. We further observe that mode (4) is simply the well-known Rayleigh-Taylor interchange mode, stable for $n' < 0$. For a stable equilibrium, then, a convective cell generated in the plane containing $\vec{g}$ simply "rocks" back and forth at the interchange frequency $|gn'/n|^{1/2}$. In this sense, the cell does not "turn over." The above calculation is, however, a linearized one. Evidently for large enough rotational energy, the cell will turn over. The initial speed v_0 required for turnover of a circular cell of size r_0 will be roughly given by

$$v_0/r_0 \gtrsim |gn'/n|^{1/2} . \quad (5)$$

The cell thus behaves as a pendulum, performing small oscillations for small energies and turning over for large enough energies.

A directly analogous situation holds for convective cells in a low β plasma immersed in an externally imposed, inhomogeneous magnetic field (with $g=0$). The required calculation parallels the

foregoing one.^{8,9} For a collisionless plasma, the stability criterion is $p_{\perp}'/p_{\perp} < (B^2)'/B^2$, where $p_{\perp}$ is the perpendicular pressure, and the dispersion relation is given by

$$\omega^2 = (B'/B)[2(B'/B) - (p_{\perp}'/p_{\perp})]c_s^2, \quad (6)$$

where $c_s^2 = p_{\perp}/nM$. The result (6) may be understood in a simple manner as follows.

Consider the slab plasma with $n' > 0$ and $B' < 0$. If we now let the plasma "flute" in the positive $\hat{x}$ direction, then the fluted flux tube undergoes a transverse dilation of relative amount $\delta x |B'/B|$ where δx is the distance moved by the tube. The density in the flux tube decreases by this relative amount as does the perpendicular temperature (in the absence of collisions), again by the same relative amount. The net pressure then goes down by $2\delta x(B'/B)$. If the new, lower pressure $p_{\perp} - 2\delta x |B'/B|$ in the flux tube is even lower than the ambient pressure, $p_{\perp} + \delta x p_{\perp}'$, the fluted flux tube tends to be pushed back and the perturbation is stable. Thus the stability criterion is $2B'/B > p_{\perp}'/p_{\perp}$. If the perturbed pressure is higher than the ambient pressure, the flute is unstable and an interchange instability ensues. The oscillation frequency [Eq. (6)] (or growth rate) may also be estimated by similar arguments.

We have thus shown that the convective mode in a plasma in an inhomogeneous B field is exactly the interchange mode. For small flute perturbations in a stable plasma, the convective cell does not turn over. "Turn-over" may be effected only for large enough rotational kinetic energy. With $|p_{\perp}'/p_{\perp}| \approx a^{-1} > |B'/B| \approx R^{-1}$, the

turnover criterion for convective cells in a plasma with non-uniform $|\vec{B}|$, analogous to Eq. (5), is $v_o/r_o > c_s/R \epsilon_n^{1/2}$, where $\epsilon_n \equiv a/R$.

C. Magnetic Pumping of Convective Motions

In a homogeneous plasma, a convective cell of size k^{-1} damps due to shear in the flow velocity at the rate $k^2 \rho^2 \nu_i$, where ρ is the ion gyroradius and ν_i is the ion-ion collision frequency. In a plasma in an inhomogeneous magnetic field, the convective mode (or interchange mode) damps at a much faster rate due to magnetic pumping. The latter damping results in the following manner. As the cell turns, each element of the cell alternately compresses and decompresses in step with the magnetic field strength that it encounters. During the compression phase, the rotational energy of the cell is fed into the perpendicular temperature, $T_\perp$, with this input energy being recovered upon decompression provided that there are either no collisions or an infinite number of collisions in this period. However, for a finite collision frequency, a portion of the input energy that was shared with the parallel temperature, $T_\parallel$, is not recoverable. This leads to an irreversible transformation of kinetic energy into heat and the cell damps. The rate at which the cell plasma heats has been heuristically estimated in Ref. 10 [see Sec. IX therein] for the case where the collision frequency, ν , is greater than the rate of compression, f , i.e., $f < \nu$. The rate in the opposite limiting case, $\nu < f$, may be estimated by similar means. These limiting rates are:

$$\partial \ln T / \partial t \sim (\delta B / B)^2 \begin{cases} f^2 / \nu & ; \quad f < \nu \\ \nu & ; \quad \nu < f \end{cases} \quad (7)$$

where $\delta B/B$ is the fractional compression of the flux tubes.

Equating this heating rate with the rate of change of rotational kinetic energy gives the damping rate. The kinetic energy may be easily estimated both for the case where the cell simply "rocks," in which case $f \rightarrow c_s/R \epsilon_n^{1/2}$, and for the case where the cell turns over, for which $f \rightarrow (v_o/r_o) \equiv \omega_o$. The resulting damping rates (up to numerical factors) for both "rocking" cells and cells which turn over are then

$$\gamma \sim \begin{cases} \Omega^2/\nu & ; \quad f < \nu \\ (\Omega/f)^2 & ; \quad \nu < f \end{cases} \quad (8)$$

with

$$f \rightarrow \begin{cases} \omega_o & ; \quad \Omega < \epsilon_n^{1/2} \omega_o \text{ (turnover)} \\ \Omega/\epsilon_n^{1/2} & ; \quad \epsilon_n^{1/2} \omega_o < \text{(rocking)} \end{cases}$$

where $\Omega \equiv c_s/R$. These rates will be obtained more precisely, for a particular situation, in the next section.

III. DETAILED CALCULATION

In this section, we perform a detailed calculation to determine the oscillatory properties and damping due to magnetic pumping of a solitary convective cell in a low β plasma immersed in a strong, inhomogeneous magnetic field. We employ the slab model shown in Fig. 1. The magnetic field $B = B_z \hat{z}$ is externally imposed and we assume $(\ln B)' \neq 0$ and $(\ln n)' \neq 0$. We shall assume that there is no variation along $\hat{z}$ and so $\vec{B} \cdot \vec{\nabla} = 0$. The convective motion of the cell is in the x-y plane.

Since we are interested in investigating a situation where a closed magnetic Flux tube is exchanged with a neighboring tube in such a way that the net magnetic energy remains unaltered, the foregoing slab model is an adequate representation of this interchange. In a shearless tokamak, for example, such an interchange may be carried out between two closed flux tubes on neighboring magnetic surfaces. (Although at any given point on the field line the density and magnetic field gradients are not unidirectional, in contrast to the slab model, nevertheless the average effect is one of "favourable curvature"; the gradient in B in our slab model then represents some sort of average.) Likewise, in multipoles without a toroidal field, a similar interchange between neighboring poloidal field lines may be imagined.

We employ the following equations to describe the time evolution of the convective cell in Fig. 1.

$$\partial n / \partial t + \vec{\nabla} \cdot (n \vec{v}) = 0, \quad (9)$$

$$\vec{\nabla} \cdot \vec{j} = 0, \quad (10)$$

$$\partial \vec{v} / \partial t + \vec{v} \cdot \nabla \vec{v} = -(\nabla p_{\perp}) / nM + \vec{j} \times \vec{B} / nM, p_{\perp} \equiv nT_{\perp}, \quad (11)$$

$$\vec{\nabla} \phi = \vec{v} \times \vec{B}, \quad (12)$$

$$(\partial / \partial t + \vec{v} \cdot \vec{\nabla})(T_{\perp} / B) = -(\nu / 3)(T_{\perp} - T_{\parallel}) / B, \quad (13)$$

$$(\partial / \partial t + \vec{v} \cdot \vec{\nabla})T_{\parallel} = (2\nu / 3)(T_{\perp} - T_{\parallel}). \quad (14)$$

Equations (9) and (10) are equations of mass and charge continuity, respectively. Eq. (11) is the equation of motion where we have assumed that the field is strong enough so that the pressure tensor is primarily diagonal¹¹ and equal to $p_{\parallel} \hat{b} \hat{b} + (\hat{1} - \hat{b} \hat{b}) p_{\perp}$, with $\hat{b} \equiv \vec{B} / B$. Since $\hat{b} \cdot \vec{\nabla} = 0$, the divergence of this tensor yields $\vec{\nabla} p_{\perp}$. We assume the plasma has low β and look only for time rates of change slower than magnetoacoustic frequencies. In that case, we may neglect any variation in $\vec{B}$. The electric field is then derivable from a scalar and Ohm's law is written as in Eq. (12). Finally, Eqs. (13) and (14) are phenomenological equations of state describing the time evolution of $T_{\perp}$ and $T_{\parallel}$. With the right-hand sides omitted, these equations are simply the double-adiabatic Chew-Goldberger-Low¹¹ (CGL) equations (with $\hat{b} \cdot \vec{\nabla} = 0$); the right-hand sides simulate a relaxation between unequal $T_{\perp}$ and $T_{\parallel}$ at the collision frequency ν . [These equations apply to either species. A more detailed discussion of these model CGL-type equations may be found in Refs. 12 and 13. For the "collisional" part of the ensuing calculation, the heat transport equations as given by Braginskii¹⁴ are equally adequate. We utilize the phenomenological equations (13) and (14) for greater generality here.] All finite Larmor radius effects (occurring, for example, in the pressure tensor and in Ohm's law) have been neglected in the

foregoing set of equations. This is justified so long as the interchange frequency $\Omega/\epsilon_n^{1/2}$ is larger than the ion diamagnetic drift frequency (see Refs. 15 and 16). This puts a lower bound on the cell size, r_0 , namely,

$$\rho < \epsilon_n^{1/2} r_0.$$

where ρ is the ion gyroradius.

We now make the additional assumptions that $(\ln B)' = (\ln n)' \equiv R^{-1}$, and $(\ln T_\perp)' = 0$. These assumptions are made for the sake of simplicity and do not detract from the physics. Note that in applying this theory to a tokamak, $(\ln n)'$ and $(\ln T)'$ are effective averages so that they may indeed be comparable to $(\ln B)'$ even if the aspect ratio is not of order unity. (We comment on arbitrary profiles at the end of this section.) The foregoing set of equations is then solved by an expansion in (r_0/R) . We look for a solution which in the limit $R \rightarrow \infty$ reduces to a stationary convective flow in the x-y plane. For simplicity, we take this flow to be circular and center it about a polar coordinate system as shown in Fig. 1. The solution

$$\vec{v} = \hat{\theta} r \omega_0(r), \quad r \omega_0(r) \equiv (\partial \phi / \partial r) / B, \quad n = \text{const.},$$

$$T_\perp = T_\parallel = T = \text{const.}, \quad (15)$$

$$n M r \omega_0^2 = -j_\theta B,$$

then satisfied Eqs. (9) through (14) to lowest order in (r_0/R) for arbitrary $\omega_0(r)$. (We

assume the ω_0 profile decays in a distance r_0 .) The j_0 current indicates that the centrifugal forces are balanced by a slight dip in the magnetic field strength assuming $nMr_0^2 \ll B^2$.

Before going to next order, we annihilate the leading order terms in the equation of motion (11). This is done by integrating over a closed streamline. With this integration, the $\vec{v} \cdot \vec{\nabla} \vec{v}$ term vanishes identically viz. $\oint (d\ell/v) \vec{v} \cdot \vec{\nabla} \vec{v} = \oint (d\ell/v) \vec{v} \cdot \vec{\nabla} v^2/2 = 0$; while the $\vec{j} \times \vec{B}/nM$ term vanishes as a result of the simplifying assumption $(\ell n n)' = (\ell n B)'$ as follows: $\oint (d\ell/nv) \vec{v} \cdot \vec{j} \times \vec{B} = -\oint (d\ell/nv) \vec{j} \cdot \vec{\nabla} \phi = -\oint d\ell |\vec{\nabla} \phi|^{-1} \vec{\nabla} \phi \cdot \vec{j} (B/n)$, where we have used Eq. (12).

From the low β assumption, $\partial n/\partial t = 0$. Thus (B/n) is a constant and may be extracted from the integral. This leaves $\oint d\ell \vec{j} \cdot \vec{\nabla} \phi / |\vec{\nabla} \phi|$ which is simply the net outflux of charge from inside the closed streamline and must vanish since $\vec{\nabla} \cdot \vec{j} = 0$.

The annihilated version of (11) is now given by

$$(\partial/\partial t) \oint d\ell \cdot \vec{v} = -\oint d\ell \cdot \vec{\nabla} p_\perp (nM)^{-1}. \quad (16)$$

Evaluating both sides of (16) to lowest order in (r/R) , we obtain

$$r^2 \partial \omega_0 / \partial t \approx (r/R) c_g^2 \oint (d\theta/2\pi) (T_\perp/T) \sin \theta \quad (17)$$

where we have assumed a profile of the form $\ell n n = x/R$. Thus, to obtain $\partial \omega_0 / \partial t$ (to second order), we need $T_\perp$ to first order in (r/R) . This is obtainable from Eqs. (13) and (14) taken to first order. Taking appropriate linear combinations of (13) and (14), we obtain

$$(\partial/\partial t + \omega_0 \partial/\partial \theta) U = -2\omega_0 \epsilon \sin \theta \quad (18)$$

$$(\partial/\partial t + \omega_0 \partial/\partial \theta + \nu)\Delta = -\omega_0 \epsilon \sin \theta \quad (19)$$

where $\Delta \equiv (T_{\perp} - T_{\parallel})/T$ is the anisotropy, $U \equiv (2T_{\perp} + T_{\parallel})/T$ is the perturbed internal energy, and $\epsilon \equiv \tau/R$. With these definitions, (17) is rewritten

$$\partial \omega_0 / \partial t = (\Omega^2 / 3\epsilon) \oint (d\theta / 2\pi) (U + \Delta) \sin \theta. \quad (20)$$

Equations (18), (19), and (20) now constitute a complete set of nonlinear differential equations in the unknowns U , Δ , and ω_0 , from which the time evolution of $\omega_0(r, t)$ is obtained. Since r enters only as a parameter, these are partial differential equations in θ and t . We examine the solutions separately in the two limits $\omega_0 < \Omega$ and $\Omega < \omega_0$.

A. Low Energy Cells: $\omega_0 < \Omega$

This limit corresponds to linearizing (18)-(20) for small ω_0 . Linearizing and taking all perturbed quantities to vary as $e^{-i\omega t}$, we easily obtain the dispersion relation

$$\omega^2_{(w+iv)} = \Omega^2_{(w+2iv/3)}. \quad (21)$$

In the two limits $\nu < \Omega$ and $\Omega < \nu$, the roots corresponding to the interchange mode are

$$\omega = \begin{cases} \pm \Omega - i\nu/6 & ; \nu < \Omega \\ \pm (2/3)^{1/2} \Omega - i\Omega^2/6\nu & ; \Omega < \nu. \end{cases} \quad (22)$$

(The third root corresponds to a relaxation mode between $T_{\perp}$ and $T_{\parallel}$; this need not concern us here.) We note that the interchange modes have a real frequency, corresponding to a "rocking" of the convective cell, and an imaginary part which gives the damping due to magnetic pumping. These damping rates agree with our heuristic estimates of the previous section. [In the collisional limit, $\Omega < \nu$, the oscillation frequency is lowered by $(2/3)^{1/2}$ because collisions keep the temperature isotropic and distribute the work done over 3 degrees of freedom rather than 2.]

We now consider the limit in which the cell turns over, namely, $\Omega < \omega_0$.

B. High Energy Cells: $\Omega < \omega_0$

The equations, in this limit, are best solved by making the ansatz $\partial/\partial t \ll \omega_0$ in Eqs. (18) and (19). The sinusoidal parts of U and Δ may then be easily solved for in lowest order. Using these in (20) yields

$$\frac{\partial}{\partial t} \ln \omega_0 = - \frac{\Omega^2}{6\nu} \frac{1}{1 + (\omega_0/\nu)^2} \quad (23)$$

This solution is valid provided $\Omega < \omega_0$. Note that (23) reduces to our heuristic estimate (8) for the two limits $\omega_0 < \nu$ and $\nu < \omega_0$.

C. More General Profiles

The above calculation has been carried out for the particular profile $(\ln n)' = (\ln B)'$ with $(\ln T)' = 0$. This profile is interchange

stable. It is straightforward to extend the calculation to other stable profiles [or to the marginally stable profile $(\ln p)' = (\ln B^2)$, cf. Eq. (6)]. In this more general case, it is no longer possible to set $\partial n / \partial t = 0$. However, the final results, up to factors of $\epsilon_n^{1/2}$, where $\epsilon_n = (\ln B)' / (\ln n)'$, are essentially the same as those obtained above. More particularly, for stable profiles the results for cells which do not turn over ($\omega_0 < \Omega$) are the same as the corresponding results obtained above if we made the substitution $\Omega \rightarrow c_s / R \epsilon_n^{1/2}$ [cf. Eq. (6)]. For cells which do turn over ($\Omega < \omega_0$), the appropriate substitution is $\Omega \rightarrow c_s / R$. Finally, for marginally stable profiles, there is no "rocking" (no real frequency). However, the cells still damp by magnetic pumping at the rate given by Eq. (23), wherein $\Omega \rightarrow c_s / R$.

IV. CONVECTIVE TRANSPORT IN PLASMAS IN INHOMOGENEOUS $\vec{B}$

The convective cell in plasmas immersed in a nonuniform magnetic field is, as we have shown, identical to the interchange mode. If the plasma is stable to the interchange, the "convective cell" simply rocks back and forth at the interchange frequency (unless the cell is extremely energetic). This effectively reduces the step size of the convective diffusion. Furthermore, the cells damp by magnetic pumping at a rate much faster than classical perpendicular viscosity. This property shortens the lifetime of the cell and so also influences the transport. We examine these implications in this section employing simple, physically-motivated, arguments to estimate the diffusion in inhomogeneous systems. Our arguments will be of a tentative nature, and restricted to obtaining order of magnitude estimates for upper bounds to the diffusion.

The diffusion due to convective cells in a homogeneous plasma has been estimated as follows.¹⁻³ A cell of size k^{-1} makes a step $v_k \tau_k$ before losing correlation; here $v_k = k\phi_k/B$ is the $\vec{E} \times \vec{B}$ drift, and τ_k is the decorrelation time (assuming $v_k \tau_k \leq k^{-1}$). From this, we may obtain test particle diffusion due to cells of size k^{-1} . Then, summing over all $\vec{k}$'s gives the net diffusion coefficient viz.

$$D = \sum_{\vec{k}} D_k \equiv \sum_{\vec{k}} v_k^2 \tau_k \quad (24)$$

For the decorrelation time, τ_k , the self-consistent viscous damping time $(k^2 D)^{-1}$ is employed (assuming that D is larger than the collisional viscosity $\rho^2 \nu_i$, which is the case for a sufficiently strong magnetic field). Next, v_k^2 is estimated from the electric field

spectrum of convective cells. This spectrum may be either thermal or nonthermal in origin. In thermal equilibrium, each convective mode has energy $T/2$ with most of this energy being rotational kinetic energy (for $nMc^2/B^2 \gg 1$). For a plasma contained in a box of parallel length R and perpendicular width a , this implies the thermal spectrum $(1/2)nMv_k^2 a^2 R = (1/2)T$, which may be rewritten

$$v_k = \alpha c_s \quad ; \quad \alpha \equiv (na^2 R)^{-1/2} . \quad (25)$$

Using this in (24), and converting the summation to an integral as usual, we obtain the convective diffusion due to thermal cells in a homogeneous plasma¹⁻³

$$D = \left[\frac{T}{2\pi nMR} \ln(k_{\max}/k_{\min}) \right]^{1/2} . \quad (26)$$

Consider now the effects of an inhomogeneous magnetic field on this theory. We shall be mainly interested in estimating the importance of these effects for tokamaks. As stated before, we shall ignore all effects of shear and envision a shearless tokamak in which a convective cell is set up simply by interchanging two flux tubes on neighboring surfaces. Since the tokamak has "average minimum-B," the interchange is stable and the "rocking" frequency of such a cell is of order $\Omega = c_s/R \propto n^{1/2}$. (We shall, however, also examine the diffusion on those surfaces that are marginally stable.) Finally, we shall use the fact that $v_i < \Omega$ for present-day tokamaks.

We first examine the diffusion from a thermal level of convective cells.

A. Thermal Level of Convective Cells

The electric field spectrum is given by (25). For surfaces that are interchange stable, we first need to know whether the cells "turn over." This is determined by comparing ω_0 to Ω (cf. Sec. III). With $\omega_0 \equiv kv_k$ being the rotation rate, we have

$$\omega_0/\Omega = kR(v_k/c_s) . \quad (27)$$

Using (25) to estimate v_k , we have $\omega_0/\Omega \approx \alpha kR$. For the parameters $n \sim 10^{13} \text{ cm}^{-3}$, $a \sim 30 \text{ cm}$, $R \sim 10^2 \text{ cm}$, α is extremely small, of order 10^{-9} . Thus, even the smallest cells, $k^{-1} \approx \rho$, do not turn over. Instead the cells "rock" at frequency Ω and damp due to magnetic pumping at the rate $v_i/6$ [cf. Eq. (22)].

We now estimate the diffusion due to these rocking cells in the following manner. For the k^{th} cell, the step is taken to be v_k/Ω , the amplitude of the oscillation. The decorrelation time is simply the magnetic pumping damping time $(v_i/6)^{-1}$. (This model overestimates the diffusion since the step size is actually much less than v_k/Ω .) These estimates yield the diffusion coefficient

$$D = \sum_{\vec{k}} v_k^2 v_i / 6\Omega^2 . \quad (28)$$

Using (25) to estimate v_k and converting the sum to an integral, we obtain

$$D/\rho^2 v_i = (\alpha^2/24\pi\epsilon_n) (k_{\text{max}} a^2/\rho)^2 , \quad (29)$$

where $\epsilon_n \equiv a/R$. Taking $k_{\text{max}} \sim \rho^{-1}$, $\epsilon_n \sim 1/3$, we get $D/\rho^2 v_i \sim 10^{-11}$.

Thus the convective diffusion is much smaller than classical collisional diffusion. Furthermore, the scaling of D with the plasma parameters is quite different from that predicted by Eq. (26). [Note also that the interchange frequency is so large and the thermal energy is so small that it is physically unreasonable to even envision any convection; the amplitude of the interchange mode, v_k/Ω , is, for the above parameters, of order 10^{-7} cm.]

Consider now the diffusion in the neighborhood of those surfaces that are marginally stable to the interchange. In this case, there is no "rocking" and the convective transport should be identical to that calculated from homogeneous theory provided that the damping rate due to magnetic pumping, which manifests itself in the inhomogeneous system, is smaller than the turnover rate ω_0 . This is because the latter rate is taken to be the decorrelation rate in the homogeneous theory. For typical tokamak parameters, we may estimate that $\omega_0 < v_i$ [cf. Eq. (27) with Ω/v_i not too large]. In that case, the damping rate from magnetic pumping is given by Eq. (23) to be $\Omega^2/6v_i$. Since $v_i < \Omega$, this rate is larger than v_i which in turn exceeds ω_0 . Thus homogeneous theory again overestimates the transport by the large factor $\Omega^2/v_i\omega_0$ and predicts an incorrect scaling.

We now turn to the case where the convective cells are driven by some nonthermal plasma feature to a level much above the thermal level given by (25).

B. Nonthermal Level of Convective Cells

Consider a level of convective cell turbulence which is driven by some nonthermal feature, say, unstable drift modes (as in Ref. 4). We

do not propose to inquire here how such a spectrum of convective cells may be generated. Rather, we simply assume that there exists a steady-state background spectrum of convective cells of the form

$$(ka)(e\phi_k/T) = 10(\rho/a)(ka)^{-\delta} ; \rho < k^{-1} < a, \quad (30)$$

where δ may be equal to zero or some positive quantity. This says that $(e\phi_k/T)$ is roughly 10% (for large cells) and drops off like $k^{-(1+\delta)}$. We are motivated in this choice by the spectra observed from the numerical simulation of Cheng and Okuda.⁴ These authors' simulations in fact indicate that δ is positive. We shall however allow for the possibility that $\delta = 0$ which corresponds to assigning a large effective temperature assuming that each mode has $\frac{1}{2} T_{\text{eff}}$. [Taking $(ka)\hat{\phi}_k = 10^{-1}$ is equivalent to taking $T_{\text{eff}} \sim 10^{12}$ T.] In the following, we shall calculate the nonthermal diffusion in inhomogeneous plasmas for the two cases $\delta = 1$ and $\delta = 0$.

We begin by considering the case of stable interchange. With the spectrum (30), we first inquire whether the cells with turn over.

Using (30) in (27) with $v_k = k \phi_k / B$, we get

$$\omega_0 / \Omega = (ka)^{1-\delta} (\rho/a)^2 10 / \sqrt{\epsilon_n} . \quad (31)$$

Assuming $\rho \approx 3 \times 10^{-1}$ cm, we find, for both $\delta = 0$ and $\delta = 1$, that $\omega_0 < \Omega$ as long as $k\rho < 1$. Thus, cells larger than an ion gyroradius do not turn over. In this case, the diffusion is the same as that given by Eq. (28). Using the spectrum (30) in (28) and converting the sum to an integral, we then obtain

$$\frac{D}{2v_i} = \frac{1}{6\epsilon_n} \left(\frac{10\rho}{a} \right)^2 \frac{1}{2\pi} \left\{ \begin{array}{ll} \ln(k_{\text{max}}/k_{\text{min}}) & ; \quad \delta = 1 \\ \frac{1}{2} (k_{\text{max}} a)^2 & ; \quad \delta = 0 . \end{array} \right. \quad (32a)$$

$$\quad (32b)$$

We note that for $\delta = 1$, the diffusion is smaller than the classical collisional diffusion $\rho^2 v_i$. For $\delta = 0$, taking $k_{\max} \sim \rho^{-1}$, we get $D \sim 4\rho^2 v_i$. Thus in this case the diffusion is of the order of (cylindrical) collisional diffusion (although smaller than the "neoclassical" diffusion).

Turning now to surfaces which are marginally stable to the interchange, we need to compare ω_0 with the magnetic pumping rate to ascertain the range of applicability of the homogeneous theory. We first estimate ω_0/v_i for $\delta = 1$ and find $\omega_0/v_i \sim 10 \epsilon_n^{-1} (\rho/a)^2 (\ell/R)$, where ℓ is the mean free path defined by $2v_i \ell \approx c_s$. Using the previously given values for ρ etc., we have $\omega_0/v_i \sim 3 \times 10^{-3} (\ell/R)$, which is smaller than or at most of order unity for present-day devices. In this case, the magnetic pumping rate is $\Omega^2/6v_i$ [cf. Eq. (23)], which is faster than the turnover rate ω_0 . Thus homogeneous theory is inapplicable, overestimating the transport and predicting an incorrect scaling.

For the case $\delta = 0$, ω_0/v_i may be estimated, in general, to be larger than unity for cells of interest. The magnetic pumping is not as effective now, damping the cells at the rate $(\Omega/\omega_0)^2 v_i$ [cf. Eq. (23)]. The ratio of this rate to ω_0 is, upon using (30), $\epsilon_n^3 (R/\ell) (a/10\rho)^3 (k\rho)^{-3} \sim 30(R/\ell) (k\rho)^{-3}$. Thus homogeneous theory is applicable to the smaller cells for which $(k\rho)^3 \geq 30(R/\ell)$. This is the only case for which homogeneous theory is applicable and for which convective transport may exceed the classical collisional transport. It assumes that the plasma is marginally stable and that the convective cell electric field spectrum is flat with the rather large effective temperature of order 10^{12} T.

V. SUMMARY AND DISCUSSION

It has been shown previously that the zero frequency convective cell of the homogeneous medium enhances the cross-field diffusion in a magnetized plasma. Since many laboratory devices in which cross-field plasma diffusion is of interest confine the plasma in an inhomogeneous magnetic field, it is necessary to examine the properties of convective cells in an inhomogeneous field. In this paper we have investigated the characteristics and lifetime of this convective mode in an inhomogeneous field. We conclude the following:

1. The zero-frequency convective cell of the homogeneous medium is transformed into the interchange mode in systems with an inhomogeneous magnetic field. Consequently, for profiles which are interchange stable, the convective motion of plasma due to this mode is limited to an oscillation which occurs at the relatively high (interchange) frequency of order c_s/R , R being the gradient scale length of the field. Only in the case where the energy of this oscillation is quite large may the cell "turn over," i.e., an "eddy" be formed.

2. In an inhomogeneous field, the convective mode is strongly damped by magnetic pumping. For tokamak-like parameters, this damping is much stronger than that due to classical perpendicular viscosity.

The above two properties of the convective cell greatly reduce convective plasma diffusion. In particular, the first property leads to an effective reduction in the step size of the diffusion process, while the second property reduces the lifetime of the cell. Using simple step-size decorrelation-time arguments, we have estimated the

convective diffusion, in light of these two properties, for systems with tokamak-like parameters. Our estimates suggest the following:

3. For a thermal level of convective cells, the resultant cross-field diffusion, both on stable and marginally stable magnetic surfaces, is extremely small compared to classical collisional diffusion.

4. The diffusion from a nonthermal level of convective cells is also reduced, in general. More specifically, we have estimated the diffusion resulting from a model steady-state background of convective cells whose electric field energy spectrum either falls off with increasing wavenumber as $|E_k|^2 \approx k^{-2}$, or is constant all the way out to $k \approx \rho^{-1}$. The maximum value of $|E_k|$ for both cases is assumed to correspond with a potential fluctuation of ten percent, i.e., $(e\phi_k/T)_{\max} \approx 10\%$. The diffusion resulting from either of these spectra, for systems in an inhomogeneous field, is smaller than, or at most of the order of, the collisional diffusion except for one case. The one exception is for a flat electric field spectrum in the case of an equilibrium with profiles that are marginally stable to the interchange. In this case, the diffusion may be of the order predicted by the homogeneous theory. It is to be noted, though, that the drift-wave-driven convective cell spectrum, observed in the numerical simulations of Ref. 4, falls off rather quickly with increasing k ; on this basis, the choice of a flat spectrum may not be too realistic.

In general, the homogeneous theory is only applicable for the case where the turnover rate of a cell ω_0 is greater than both the

interchange frequency Ω and the magnetic pumping damping rate γ . Since $\Omega \sim c_s/R$ is a hydrodynamic type frequency, the requirement $\Omega < \omega_0$ is quite stringent.

Comparison with Experiment

Several experiments have been conducted to test the theory of convective diffusion as formulated in Refs. 1 through 3. The scaling of the diffusion coefficient, D , with the plasma parameters was measured, for example, on the GA¹⁷ and Wisconsin Octupoles.¹⁸ A scaling consistent with that predicted by Eq. (26), i.e., $D \propto (T/nM)^{1/2}$, independent of B , was found on both machines. In addition, the absolute value of D was also measured. In the case of the GA Octupole, agreement with theory was found up to a factor of 3; for the Wisconsin Octupole, the diffusion was larger, consistent with an effective temperature of order 10^4 T.

Octupole plasmas are confined by an inhomogeneous magnetic field, and the plasma, at least inside the separatrix, is stable to the interchange flute mode. In this case, as we have shown in this paper, the theory of convective transport in homogeneous plasmas is not applicable. The interchange properties of the plasma, along with the effects due to magnetic pumping, must be taken into account. (Without a toroidal field, octupoles have a shear-free configuration and so the present work is directly applicable.) For the typical parameters of the octupoles it may be verified that thermal cells do not turn over. In fact, even nonthermal cells with $T_{\text{eff}} \sim 10^4$ T do not turn over.

Given the above facts, it is surprising that good agreement was

found between the homogeneous theory of convective transport and experiment. Clearly, some entirely new (non-MHD) physics needs to be introduced in order to resolve this discrepancy. This physics may or may not include some sort of convective mode as an ingredient, and it might indeed be quite fortuitous if this new mode predicted a diffusion scaling that was the same as the scaling predicted from the zero frequency convective cell. What is quite evident, though, is that the mode which is conventionally understood to be the convective cell, namely a hydrodynamic vortex with no variation along the field lines, cannot be considered to be responsible for the transport losses.

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APPENDIX, FLUTE MODES IN UNIFORM AND NONUNIFORM MEDIA

Consider a plasma embedded in an external field $\vec{B} = B_z \hat{z}$ bounded by infinitely conducting rigid walls at $x = \pm a$. We wish to enumerate the normal modes of oscillation with wavenumber k_y [so that all quantities behave as $f(\exp(ik_y y))$ where f vanishes at the walls]. We consider two cases: Case I, no gravitational field and uniform B_z ; and Case II, $\vec{g} = g\hat{x} \neq 0$ and/or ρ and B nonuniform in x . We will show that there are seven normal modes in each case and establish a one-to-one correspondence between the two sets of modes. For six of the modes the correspondence will be natural and obvious. The seventh mode in Case I will be the convective cell while that in Case II will be the interchange mode. This will establish the identification of the convective cell with the interchange mode.

That there must be seven modes of oscillation is evident since there are seven independent perturbed quantities, δv_x , δv_y , δv_z , δn , δp , δB_z , and one of δB_x and δB_y , which may be specified initially and whose subsequent time behaviour is to be determined by analysis in terms of normal modes. (The condition $\vec{\nabla} \cdot \vec{\delta B} = 0$ ties together the quantities δB_x and δB_y leaving only one independent.)

Consider Case I. There are five zero-frequency normal modes. These are specified by: (1) δB_x an arbitrary function of x with δB_y chosen to satisfy $\vec{\nabla} \cdot \vec{\delta B} = 0$, and all other quantities vanishing; (2) δv_z the only non-zero quantity; (3) δn the only non-zero quantity (entropy mode); (4) δp and δB_z the only non-zero quantities chosen arbitrarily subject to $\delta p + \frac{1}{2} \delta B_z = 0$; (5) δv_x and arbitrary function of x with δv_y chosen to make $\vec{\nabla} \cdot \vec{\delta v} = 0$, all others vanishing (the

convective cell); (6) and (7) δv_y , δp , δB_z , δn the non-vanishing quantities, giving two magneto-acoustic modes propagating in the directions $\pm y$. Modes (1)-(5) are zero-frequency modes corresponding essentially to small shifts to neighboring equilibria.

It is clear that any arbitrary initial conditions may be expressed as linear combinations of these normal modes. For example, to obtain the initial condition: $\delta p = f$ with all other quantities zero, we begin by choosing linear combinations of the magneto-acoustic modes such that $\delta p + B_z \delta B_z = \delta p_1 + B_z \delta B_{z1} = f$. We now add to this the mode (4) with an amplitude $\delta p = \delta p_2$ (i.e., $B_z \delta B_z = -\delta p_2$) such that $\delta p_2 = f - \delta p_1$. Finally adding the correct amount of mode (3) eliminates δn . This gives us the desired initial condition.

We now enumerate the corresponding modes for Case II. For simplicity, we take $\vec{v}_p = n\vec{M}g$ and $\vec{v}_B = 0$. Modes (2) and (4) are identical in structure and are again zero-frequency modes. If $|\vec{v}_n|$ and g are small, the magneto-acoustic modes (6) and (7) retain nearly the same structure and frequency spectrum. Modes (1) and (3) are changed slightly since associated with δn is a small force $Mg\delta n$. This may be balanced by properly choosing $\delta p + B_z \delta B_z$ [i.e., mixing in a small portion of modes (4), (6), and (7)] giving, still, a zero-frequency mode. Likewise, mode (1) is changed slightly due to the small force $n\vec{M}g \cdot \delta B/B$; this force, too, may be balanced by adding proper portions of (4), (6), and (7). Finally, mode (5), the convective cell, now has a density perturbation $v_x |\vec{v}_n|$ associated with it. This clearly interacts with g to produce a perturbed force. The resulting mode is the interchange mode. (The foregoing description is similar to degenerate perturbation theory in quantum mechanics.)

Thus, an identification of two complete sets of modes for the two cases has been made in which the interchange mode must correspond to the convective cell. We are thus justified in identifying these two modes.

REFERENCES

1. J. B. Taylor and B. McNamara, Phys. Fluids 14, 1492 (1971).
2. J. M. Dawson, H. Okuda, and R. N. Carlile, Phys. Rev. Lett. 27, 491 (1971); H. Okuda and J. M. Dawson, Phys. Fluids 16, 408 (1973).
3. J. A. Krommes and C. Oberman, J. Plasma Phys. 16, 229 (1976).
4. C. Z. Cheng and H. Okuda, Nucl. Fusion 18, 587 (1978).
5. M. N. Rosenbluth and C. S. Liu, in Proceedings of the Fifth European Conference on Controlled Fusion and Plasma Physics, Grenoble, France (1972).
6. H. Okuda and J. M. Dawson, Phys. Fluids 16, 1456 (1973).
7. T. Kamimura and J. M. Dawson, UCLA Physics Department Report PPG-228.
8. M. N. Rosenbluth and C. L. Longmire, Ann. Phys. 1, 120 (1957).
9. A. B. Hassam, Ph.D. Dissertation (Princeton University, 1978), p. 88.
10. A. B. Hassam and R. M. Kulsrud, Phys. Fluids (to be published).
11. G. F. Chew, M. L. Goldberger, and F. E. Low, Proc. Roy. Soc. Lond. A236, 17 (1958).
12. J. M. Berger, W. A. Newcomb, J. M. Dawson, E. A. Frieman, R. M. Kulsrud, and A. Lenard, Phys. Fluids 1, 301 (1958).
13. A. H. Boozer, Princeton Plasma Physics Laboratory Report EPRI-4 (1976).
14. S. I. Braginskii, in Reviews of Plasma Physics, edited by M. A. Leontovich (Consultants Bureau, New York, 1965), Vol. I, p. 214.
15. M. N. Rosenbluth, N. A. Krall, and N. Rostoker, Nucl. Fusion

- Suppl., Pt. 1, 145 (1962).
16. K. V. Roberts and J. B. Taylor, Phys. Rev. Lett. 8, 197 (1962).
 17. T. Tamano, R. Prater, and T. Ohkawa, Phys. Rev. Lett. 30, 431 (1973).
 18. J. R. Drake, J. R. Greenwood, G. A. Navratil, and R. S. Post, Phys. Fluids 20, 148 (1977).

Fig. 1. Model slab geometry, with the convective cell set up in the x - y plane. (University of Maryland)

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