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EQUILIBRIUM TOROIDAL CURRENT IN A TOKAMAK DRIVEN BY
PHASE-SPACE INHOMOGENEITY IN THE BANANA REGIME

By

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MASTER

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ABSTRACT

In the hot core region of a tokamak plasma column, the radial guiding center motion of the charged particles is affected by the existence of the magnetic axis. Identical particles with opposite signs of toroidal velocity drift in opposite radial directions. But, the magnetic axis acts like a rigid wall and imposes a boundary condition on the radial guiding center motions, causing asymmetrical radial particle motions between toroidally oppositely flowing particles, or between radially oppositely drifting particles. Due to this radial asymmetry, it is found that a flat radial pressure profile cannot describe the radial equilibrium state near the magnetic axis, but requires a small amount of toroidal current. The direction of this current is opposite to that of Ohmic current and its magnitude is formally of an order higher in the inverse aspect ratio than that of the bootstrap current.

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I. Introduction

A theoretical understanding of toroidal equilibrium electrical current is essential for studying various aspects of tokamak plasmas. An accurate evaluation of the radial profile of the toroidal electrical current in the hot plasma core can be especially interesting in the sense that it can predict the central q (safety factor)-value and the location of the $q=1$ surface. Previous neoclassical theories^{1,2} calculated two different types of toroidal electrical current: Ohmic and bootstrap currents. Ohmic current is driven by the toroidal electric field and is obtained by subtracting the trapped electron effects from the Spitzer current.³ The bootstrap current^{4,5} is driven by the density and temperature gradients in the radial direction. These neoclassical theories were developed neglecting the effect of finite poloidal gyroradius.

However, by noticing that the electron poloidal gyroradius can become of order one centimeter at the distance one centimeter away from the magnetic axis (ion gyroradius is even much larger) in a typical high pressure tokamak experiment, we can expect that the finite poloidal gyroradius effect may not be neglected in the central core of the plasma column. In the present work, we will study the effect of finite poloidal gyroradius on the radial phase-space variation of the plasma particles. We will show that this effect creates an asymmetry in the radial-drift motions between the particles flowing in the opposite directions toroidally, which thus

gives rise to a net non-Ohmic toroidal current unless this asymmetry effect is compensated by the radial gradient in the plasma pressure.

In order to provide a simple description of such physical phenomena, we will use a tokamak-like geometry, where both the poloidal and toroidal magnetic fields are created by the external currents flowing in a properly arranged set of coils and the flux surfaces are concentric circles. Suppose that, at a minor radial location $r = r_0$ and poloidal angle corresponding to minimum Mod-B ($\theta = 0$), we instantaneously inject a quasineutral set of hot electrons and ions, which forms two nondrifting Maxwellian distribution functions. Due to the inhomogeneity in the magnetic field, instead of staying on the same flux surface defined by $r = r_0$ at $\theta = 0$, the particles drift and spread radially within a time period much faster than the transport time.

The amount and direction of radial spread for each particle is determined by the conservation of canonical angular momentum,

$$\psi - cmv_T R/e = \text{constant}, \quad (1)$$

where ψ is a radial coordinate defined by the poloidal flux, c is the speed of light, m the particle mass, v_T the toroidal component of the particle velocity, R the major radius, and e is the particle's electrical charge. Since $|v_T R|$ will decrease as particles move away from $\theta = 0$, we can immediately see from Eq. (1) that the particles

with negative value for v_T/e spread radially outward (increasing ψ direction) and those with positive v_T/e spread inward (decreasing ψ direction). Thus, exactly half the particles quickly occupy the space defined by $r < r_0$ and the other half occupy the space given by $r > r_0$. The radially inward motion of the particles is bounded by the magnetic axis at $r=0$, and the space at $r < r_0$ is smaller than the almost free space at $r > r_0$. Hence, half of the particles with one sign of v_T are spread into a smaller space than the other half. Because of this effect, we may get some asymmetry in the particle density between $r < r_0$ and $r > r_0$ for the nondrifting Maxwellian particles injected at $r=r_0$.

Let us consider the flux-surface-averaged radial distribution function, $f(r)$, after a few bounce periods from the injection time and try to determine if there should be an asymmetry in $f(r)$ between $r < r_0$ and $r > r_0$. If the r_0 we have chosen is much greater than the radial banana width of most of the higher energy particles, the finiteness of the spatial volume for $r < 0$ may hardly show up in $f(r)$, at all. However, this question of asymmetry can be more clearly answered by making the r_0 not much greater than the particles' banana widths. In the case when r_0 is equal to only a few banana widths of our particles, we can easily see that all the particles

spread radially inward are confined to a small space whose radial width is only a few banana widths. On the other hand, particles that drift radially outward can spread without any restriction over a larger volume. Thus, $f(r)$ will be smaller for $r > r_0$ than for $r < r_0$.

The above argument shows that the vertical drift motions created by the magnetic inhomogeneity cause asymmetric spatial-density behavior between the particles flowing in opposite toroidal directions. This requires $f(r)$ to be larger at $r > r_0$ if the net toroidal current at $r = r_0$ is to be zero. This statement is identical to saying that a symmetric $f(r)$ between $r > r_0$ and $r < r_0$ cannot yield a nondrifting Maxwellian distribution at $r=r_0$ and instead causes a net toroidal current at $r=r_0$. In other words, a radially flat density profile should produce a net toroidal current.

This feature of particle distribution function near the magnetic axis serves as a boundary condition to the radial solution for a Fokker-Planck distribution function in tokamaks. An interesting consequence of this observation is that the simplest solution of the Fokker-Planck equation,

$$f(x,v) = C_1 \exp(-\epsilon/C_2),$$

cannot satisfy the boundary condition near the magnetic axis, where ϵ is the particle energy, and C_1 and C_2 are constants independent of position and velocity. In the next section, we obtain a solution for

$f(r)$ which satisfies the boundary condition discussed here. For simplicity, we first assume that the lowest order distribution function is still a spatially homogeneous Maxwellian and then calculate the perturbed inhomogeneous part whose presence is required by the spatial inhomogeneity created by the magnetic axis.

In the particles' kinetic motions, this asymmetric behavior shows up in the radial dependence of phase-space volume. The radial spread width for the particles put in at $r=r_0$ and $\theta=0$ is defined by the maximum banana width experienced by the marginally trapped particles. As the minor radius decreases toward the magnetic axis, the maximum banana width of the inward-drifting particles experiences radial contraction. The inward-shifted particles occupy less volume, and thus the volume-averaged density increases. The kinetic analyses presented in Section II explicitly show this kind of particle behavior.

One immediate consequence of the present findings is that the unphysically sharp negative radial-gradient of Ohmic current away from the magnetic axis in a high temperature plasma,⁶ owing to the sharp increase in the trapped electron population, is relaxed through the mechanism described above. Usually, flat density and temperature profiles are used to describe the plasma equilibria near the magnetic axis. Since the bootstrap current is proportional to the plasma density and temperature gradients, it cannot modify the sharp peaking nature of the neoclassical electrical current near the center of the plasma column in the usual analyses. However,

according to the present findings, the flat density and temperature profiles generate a toroidal current near the magnetic axis in the opposite direction to the Ohmic current. This negative current near the magnetic axis cuts away the peak in the Ohmic current, making the total current profile smoother.

We note here that the effect of magnetic axis discussed here is purely kinetic and only exists in a collisionless plasma. In the cases when the plasma is collisional, the banana orbits are not well defined and the plasma equilibrium is completely described by fluid-type thermodynamic forces.

In the present work, for simplicity, we discuss the magnetic axis effect on electrical current only. But it is possible that this new effect can change other coefficients in the transport matrix. It is also possible that the difference in the banana widths between the electrons and ions can cause a radial electrostatic sheath structure near the magnetic axis. The extension of the present work to other transport coefficients and to the study of a possible new radial sheath will be addressed in subsequent works.

A rigorous electron-kinetic derivation is presented in Section II with electron-ion collisions, and the effects of electron-electron collisions and the ion flow term are added in Section III. The results are discussed in Section IV with some examples of current profiles to demonstrate the impact of the present effect on the current profiles.

II. Kinetic Calculation

The kinetic approach chosen here will be similar to the one that appeared in Ref. 1. In order to analyze the kinetic behavior more accurately and easily in the radially dependent phase space, we use two separate steady-state drift kinetic equations for trapped and passing electrons:

$$\mathbf{v}_{||} \cdot \nabla f_j + \mathbf{v}_d \cdot \nabla f_j = C^l(f_j) + (eE_{||}/M) \partial f_j / \partial v_{||} \quad , \quad j=t \text{ or } p, \quad (2)$$

where $\mathbf{v}_{||}$ is the particle velocity vector parallel to $\mathbf{B}$; f_t and f_p are the distribution functions for trapped and passing particles, respectively; $C^l(f_j)$ is the linearized collision operator on f_j ; e is the unit charge; $E_{||}$ is the equilibrium parallel electric field, and M is the electron mass.

The distribution function f_t and f_p are related to each other by collisional interactions. To the lowest order in the usual poloidal gyroradius ordering, f_{t0} and f_{p0} form a Maxwellian distribution together,

$$f_{t0} + f_{p0} = f_m. \quad (3)$$

This distinction between f_{t0} and f_{p0} is merely for convenience in

calculating the next order perturbation in the particle distribution. As will be shown later, the individual contribution to our current by f_{t0} and f_{p0} will cancel out each other exactly as can be expected from Eq. (3). Only the next order distribution function will contain the information on the asymmetric phase-space and thus contribute to our current.

Assuming small poloidal gyroradius ordering and neglecting the parallel electric field term, which will give us the Ohmic current, the first order version of Eq. (2) becomes

$$\mathbf{v}_{||} \cdot \nabla f_{j1} + \mathbf{v}_d \cdot \nabla f_{j0} = C^l(f_{j1}) \quad , \quad j=t \text{ of } p, \quad (4)$$

where the subscripts 0 and 1 represent expansions in poloidal gyroradius ordering. For the linearized collision operator, we use the electron-ion version¹

$$C^l(f_{j1}) = \nu [L f_{j1} + 2 v_{||} (u_i / v_e^2) f_m] ,$$

where ν is the electron-ion collision frequency, L is the Lorentz pitch-angle scattering operator, u_i is the ion toroidal flow speed, and v_e is the electron thermal speed.

It is more convenient to work with a function H defined as

$$f_{j1} = (2v_{||}u_i/v_e^2) f_{j0} + H_j ,$$

to cancel the ion flow term in the collision operator. Equation (4) is then transformed into

$$v_{||} \cdot \nabla [H_j + (2v_{||}u_i/v_e^2) f_{j0}] - vL(H_j) = -v_d \cdot \nabla f_{j0} .$$

The terms proportional to f_{j0} represent the free energy sources: The term in the right-hand side contains the free energy sources contained in the inhomogeneity of the plasma. They are the radial density and temperature gradients, and the radial dependence in the particle phase-space as discussed in the previous section. The term proportional to u_i in the square bracket in the left-hand side represents the electron free energy source from ion flow motion.

We then add the banana collisionality ordering, $vqR/v \ll 1$ (q is the safety factor), and use the superscripts $H = H^0 + H^1 + H^2 + \dots$ to obtain, to the zeroth order,

$$v_{||} \cdot \nabla [H_j^0 + (2v_{||}u_i/v_e^2) f_{j0}] + v_d \cdot \nabla f_{j0} = 0 , \quad (5)$$

and to the first order,

$$v_{||} \cdot \nabla H_j^1 = vL(H_j^0). \quad (6)$$

By using the following expression for the radial drift

$$\mathbf{v}_d \cdot \nabla \psi = -I \mathbf{v}_{||} \cdot \nabla (v_{||} / \Omega),$$

we find a solution for H_j^0 from Eq. (5)

$$H_j^0 + (2v_{||} u_j / v_e^2) f_{j0} = -I (v_{||} / \Omega) \partial f_{j0} / \partial \psi + G, \quad j=p \text{ or } t, \quad (7)$$

where ψ is the poloidal flux, $I=RB_t$, and $\Omega=eB/Mc$ is the gyroradius. Here the function G is constant in the poloidal direction and is determined by the solubility condition obtained from the bounce average over the next order equation (6):

$$\oint d\theta (J_c B / v_{||}) L[-I (v_{||} / \Omega) \partial f_j / \partial \psi - (2v_{||} \mu_j / v_e^2) f_{j0} + G] = 0, \quad (11)$$

where θ is a poloidal angle, J_c is the Jacobian, and the integral represents the usual bounce average integral over the particle motion; the average is taken over the angle 2π for the passing particles and over a banana orbit for the trapped particles.

It is well known¹ that G should be zero for the trapped particles, but retains a non-zero value for the passing particles in order to satisfy the steady-state condition imposed on them during their unidirectional orbital motions: Trapped particles spend the same amount of time traveling in a backward direction as in a forward

direction and the collisional effects (which conserve parity in the sign of $v_{||}$) cancel out, yielding $G=0$ from Eq. (8). However, the passing particles will build up collisional distortion of their distribution function H_p , leading to a non-steady-state situation, unless H_p itself is small by having an appropriate G -function, according to Eq. (8). We will find that the solubility condition Eq. (8) will lead us to a function G highly localized at the boundary between trapped and passing regions in velocity space. The first order solution, in the collision frequency expansion, is then obtained from Eq. (6).

The zeroth order distribution functions f_{t0} and f_{p0} , which satisfy Eq. (3), can be easily expressed in terms of a local pitch-angle parameter β defined as

$$\beta = \mu B/w = v_{\perp}^2 / v^2 ,$$

where μ is the magnetic moment density per unit mass, w is the particle kinetic energy density, and $v_{\perp}$ is the perpendicular (to B) component of particle velocity v . In terms of β , our f_{j0} becomes

$$f_{t0} = f_m \Theta(\beta, \beta_c, 1), \text{ and } f_{p0} = f_m \Theta(\beta, 0, \beta_c), \quad (9)$$

where β_c is the value of β at the trapped-passing boundary, and Θ is a step function defined as

$$\begin{aligned}\Theta(\beta,a,b) &= 1 && \text{for } a < \beta < b, \text{ and} \\ &= 0 && \text{otherwise.}\end{aligned}$$

The radial gradient in Eq. (7) will then be operating on the Maxwellian distribution f_m and on the step functions. The gradient on f_m will yield the usual density and temperature gradients, which lead to the bootstrap current, and the gradient on the step function will represent the change in the trapped particle fraction as the minor radius changes.

In order to focus our attention more efficiently on the analyses of the radial phase-space variation effect on toroidal current as discussed in the previous sections, we will further assume that the temperature, density, and the electrostatic potential are constant in radius. This assumption may prevent us from finding higher order bootstrap current terms, if any, formally of the same order as present current terms. But these higher order bootstrap current terms are proportional to the total density and temperature gradients, which will be linearly independent to the driving term of our concern, and can be neglected for simplicity since they will be dominated by the usual bootstrap current term of leading order. In this case we obtain

$$\partial f_{t0}/\partial\psi = -f_m \partial\beta_c/\partial\psi \delta(\beta - \beta_c), \quad (10)$$

$$\partial f_{p0}/\partial\psi = f_m \partial\beta_c/\partial\psi \delta(\beta - \beta_c), \quad (11)$$

and H_j^0 is then determined from Eq. (7).

By writing the pitch-angle scattering operator L in the following form

$$L(f) = 2\xi h (\partial/\partial\lambda) \lambda \xi (\partial f/\partial\lambda), \quad (12)$$

where $\xi = v_{||}/v$, and $\lambda = \beta h = \mu B_0/w$ is a constant of motion, we can change the solubility condition, Eq. (8), into

$$\begin{aligned} & \langle \xi (\partial/\partial\lambda) [I v (\xi/\Omega) (\partial\beta_c/\partial\psi) \delta(\beta - \beta_c) + 2v \xi u_i/v_e^2 - \hat{G}] \rangle \\ & = \langle \xi (\partial/\partial\lambda) [I v (\xi/\Omega) h (\partial\beta_c/\partial\psi) \delta(\lambda - \lambda_c) + 2v \xi u_i/v_e^2 - \hat{G}] \rangle = 0, \end{aligned}$$

where $\lambda_c = \beta_c h$ is the value of λ at the trapped-passing boundary and $\hat{G} = G/f_m$. We can easily see that the function $\hat{G}$ has to be in the form of δ -function in order to satisfy the above equation. By neglecting the relatively weak dependence of ξ on λ , we obtain, to the lowest order,

$$\hat{G} = \langle \xi \rangle^{-1} (I v / \Omega_0) \langle \xi^2 (\partial\beta_c/\partial\psi) \rangle \delta(\lambda - \lambda_c). \quad (13)$$

We now calculate the electric current,

$$\langle J / h \rangle = -e \langle \int d^3v \, H v_{||} / h \rangle, \quad (14)$$

where $h = B_0/B$. Equation (14) can be changed into a more convenient form by using the Spitzer function f_S defined as³

$$C^{\dagger}(v_{||} f_S) = (e/T) v_{||} f_m.$$

For the Lorentz collision model we can easily solve for f_S :

$$f_S = -(e/T) f_m / v.$$

We then obtain

$$\begin{aligned} \langle J / h \rangle &= -e \langle \int d^3v \, H f_m v_{||} / h \rangle \\ &= -T \langle \int d^3v \, (H / h) v L(v_{||} f_S) \rangle \\ &= -T \langle \int d^3v \, (v_{||} f_S / h) v L(H) \rangle, \end{aligned} \quad (15)$$

where, in the last equality relation, the self-adjointness of the linearized collision operator has been used. This form of electrical

current, used in Ref. 1 with H approximated by H^0 , is more advantageous than that given by Eq. (14) in that it can include the effect of collisions on the function G more naturally through the use of solubility condition, Eq. (8).

Substituting the pitch-angle scattering operator, Eq. (12), for C^1 in Eq. (15), setting $H = H^0$ to the lowest order in collision frequency in Eq. (15), and integrating by parts twice in the pitch-angle variable λ , we obtain

$$\langle J / h \rangle = 4\pi \int dw w v f_s \langle \int d\lambda H^0 / (f_m h^2) \rangle. \quad (16)$$

The λ -integral is then evaluated by using Eqs. (7) and (8) as follows:

$$\begin{aligned} Y &= \langle \int_{\text{trap}} d\lambda H_t^0 / (f_m h^2) \rangle + \langle \int_{\text{pass}} d\lambda H_p^0 / (f_m h^2) \rangle \\ &= \langle \int_{\text{trap}} d\lambda \text{Iv}(\xi/\Omega_0) (\partial \beta_c / \partial \psi) \delta(\lambda - \lambda_c) \rangle \\ &\quad - \langle \int_{\text{pass}} d\lambda \text{Iv}(\xi/\Omega_0) (\partial \beta_c / \partial \psi) \delta(\lambda - \lambda_c) \rangle \\ &\quad + \langle \int_{\text{pass}} d\lambda \hat{G} / h^2 \rangle + \langle \int d\lambda (2v_{||} u_i / v_e^2 h^2) \rangle. \end{aligned}$$

Notice here that the first and second terms cancel out each other and the only contribution is from the highly localized function G and the ion flow term. This cancellation between the first two terms is

the result of thermal equilibrium (Maxwellian distribution) to the lowest order as mentioned after Eq. (3). Without the kinetic effect, our G will be zero and our new current will not exist. Using Eq. (13) for G , we find

$$\begin{aligned} Y = & - \langle \int_{\text{pass}} d\lambda \hat{G} / h^2 \rangle + \langle \int d\lambda (2v_{||}u_j / v_e^2 h^2) \rangle \\ = & - (I v / \Omega_0) \langle \xi_C \rangle^{-1} \langle \xi_C^2 (\partial \beta_C / \partial \psi) \rangle + 4v u_j / 3v_e^2. \end{aligned} \quad (17)$$

For the large aspect case, we can easily find β_C to be

$$\beta_C = (1 - r/R_0) / [1 + (r/R_0) \cos\theta],$$

thus

$$\partial \beta_C / \partial \psi = - (1 + \cos\theta) / (R_0^2 B_{p0}) \quad (18)$$

$$\xi_C = (1 - \beta_C)^{1/2} = (r/R_0)^{1/2} (1 + \cos\theta)^{1/2}, \quad (19)$$

where $\partial \psi / \partial r = R_0 B_{p0}$ has been used. By substituting Eqs. (17)–(19) into Eq. (16), and integrating over the kinetic energy w , it is straightforward to find

$$\langle J / h \rangle = -1.25 (r/R_0)^{1/2} (cnT / R_0 B_{p0}) + eu_j. \quad (20)$$

For the special case considered here, where the radial gradients of n , T , and electrostatic potential are zero, the first term in Eq. (20) represents the current driven by the trapped electrons'

inhomogeneous phase-space, and the second term represents the electron current driven by the free energy coming from the friction with the ion flow motion whose source in this case is the trapped ions' phase-space inhomogeneity.

III. Effects of Electron-Electron Collisions and Ion Flow

The effect of electron-electron collisions can be studied following the procedure of Ref. 7. Assuming a large aspect ratio, Coulomb collisions near the trapped-passing boundary can be approximated by the Lorentz operator. Then the modification to Eq.(16) will simply be the replacement of ν by $\nu_{ee} + \nu_{ei}$, where ν_{ee} is the electron-electron collision frequency and ν_{ei} is the electron-ion collision frequency. The Spitzer function f_s in this case should not be obtained from the Lorentz model only because the non-Lorentz-type scattering existing in electron-electron collisions is as important as the Lorentz-type scattering in determining f_s . By using the tabulated values of f_s given in Ref. 3 for the case $Z_{eff}=1$, and numerically integrating in w , we obtain

$$\langle J / h \rangle = - 2.09 (r/R_0)^{1/2} (cnT / R_0 B_{p0}) + eu_i. \quad (21)$$

If we notice that in the limit of large z_{eff} the effect of electron-electron collisions will be negligible compared to the electron-ion collisions, as given by the relation

$$\nu_{ee} + \nu_{ei} \propto 1 + \phi(v)/Z_{eff},$$

where $\phi(v)$ is a function of velocity appearing in v_{ee} , we can approximate the Z_{eff} dependence as follows:

$$\langle J / h \rangle = -1.25(1+0.67/Z_{eff}) (r/R_0)^{1/2} (cnT/ R_0 B_{p0}) + eu_i. \quad (22)$$

The ion toroidal flow due to the ion phase-space inhomogeneity can be calculated in the same way as for the electron current presented here. The assumption of large aspect ratio is needed here again to justify the use of the Lorentz pitch-angle scattering operator for ion-ion collisions. To treat the ion-impurity collisions the same way as for the electron-ion case, we assume that the impurity ions are heavy. Then the effective impurity charge seen by the main ions is $\alpha_i = \sum_k n_k Z_k^2 / n_i Z_k^2$, where the summation is over the impurity species k and the following relation is used:

$$\sum_k (v_{i i} + v_{i k}) \propto 1 + \phi(v)/\alpha_i.$$

The term eu_i is then calculated in the same way leading to Eq. (16),

$$eu_i = 4 \pi \sum_k \int_0^\infty dw w (v_{i i} + v_{i k}) f_s < \int_0^{\lambda_m} d\lambda H_{ij}^0 / (f_{mi} h^2) >.$$

Here $j=t$ or p , and H_{ij}^0 is the trapped or passing ion distribution, to

the zeroth order in collision frequency, given by [see Eq.(7)],

$$H_{ij}^0 = I (v_{ij}/\Omega) \partial f_{j,m}/\partial \psi - G_i, \quad j=p \text{ or } t.$$

Here the function G_i is given by Eq. (13) with Ω_0 defined as the ion gyrofrequency at the magnetic axis.

It can be easily seen that eu_i thus obtained will be identical to the first term in Eq. (22) with $n_e T_e$ replaced by $n_i T_i$. Therefore, the final expression for the new current term becomes, with an approximate fit for the effective impurity contribution,

$$\langle J / h \rangle = - 1.25 (r/R_0)^{1/2} (c/ R_0 B_{p0}) (S_e n_e T_e + S_i n_i T_i), \quad (23)$$

$$S_e = 1 + 0.67/Z_{\text{eff}}, \quad S_i = 1 + 1.34 / (1 + \alpha_i).$$

Here the form $1/(1+\alpha_i)$ has been chosen in α_i -fitting in S_i in order to avoid the unphysical singularity we would get as α_i goes to zero if we used $1/\alpha_i$, and $n_e(n_i)$ denotes the electron(ion) density. Equation (23) is then the total electron current driven by the phase-space inhomogeneity. Notice that Eq. (22) or (23) is formally of one order higher in inverse aspect ratio than the usual bootstrap current term and represents current flowing in the opposite direction.

IV. Results and Discussions

We have derived a new higher order equilibrium toroidal current generated by the boundary conditions imposed at the magnetic axis in a tokamak geometry. The new current flows in the opposite direction to the Ohmic current and flattens the current shape in the central region of the plasma. Since its magnitude is proportional to the plasma pressure, the effect of this new current term is greater for higher pressure tokamaks.

Equation (23) shows a diverging behavior as r approaches zero. This unphysical feature represents a breakdown of our analyses near the magnetic axis due to the small banana width ordering we have taken. Furthermore, as r approaches zero, the trapped particles are in the plateau regime of collisionality. This also limits the validity of our weakly collisional theory.

Figure 1 shows a comparison of radial profiles between the usual bootstrap and present new currents for a typical set of tokamak parameters. As can be seen from the figure, our new current stands out at the central part of the plasma column and flows in the opposite direction of the usual bootstrap current. The parameters taken here are for a moderate pressure L-mode discharge, but the relative comparison given in Fig. 2 is quite representative of a wide variety of tokamak parameters, including much higher pressures.

Modification to the toroidal current by this new current for an Ohmically powered, moderate pressure plasma with $T_e = T_i$ is minimal, as shown in Fig. 3. But as the plasma pressure becomes

high, the contribution from the usual bootstrap and present new currents becomes significant and the modification to the Ohmic current becomes distinctive and important. To illustrate this point, a case of higher T_i profile is taken while keeping the T_e profile and total current (1.4 MA) the same as in Fig. 2. This allows the term proportional to $n_i T_i$ to be large while keeping the Ohmic conductivity unchanged, and the result is shown in Fig. 4. As can be seen from the figure, raised ion pressure makes the contribution from our new current term (and bootstrap term) larger and modifies the current profile to be flatter. Comparison of the q -profile between the two cases is presented in Fig. 5, which shows the rise of central q -values by the flattening of current profile as a result of our new current term. We note here that the total negative toroidal current due to our new term is about 18% of the usual bootstrap current.

In tokamaks with strong direct ion heating, plasma ions can have much higher pressure than the electrons, and the central q -values may be raised above unity by the above mechanism. Connection between this observation and the absence of sawtooth activity in TFTR supershots⁸ can be conjectured, but a more extensive numerical analysis is needed before any conclusion can be reached.

The new current term described in the present work may also play an important role in the study of ignition-oriented tokamaks such as CIT and ITER. For example, a possible strong modification to the q -profile in the central plasma region can have a critical impact on the design and analyses of these high performance tokamaks. In

some unfavorable plasma conditions, a hollow j-profile may even be possible due to the negative directional property of this new current.

It is interesting to notice here that in a weakly collisional plasma the radial forces from the two major free energy sources $\partial p / \partial r$ and $(r/R) \partial \eta_t / \partial r$ can cancel each other, producing no net toroidal electrical current other than Ohmic current. This happens when $\partial p / \partial r = -(r/R) p \partial \eta_t / \partial r$ is satisfied. Simply taking $\eta_t = (r/R)^{1/2}$, we have a very small radial pressure gradient given by the relation

$$\partial p / \partial r = -(r/R)^{3/2} p / r, \quad (24)$$

for a condition to have no toroidal electric current other than the Ohmic current in a central tokamak core with pure hydrogenic plasma. This means that a radial equilibrium is achieved by allowing for a slight radial pressure gradient, rather than a flat pressure profile, if we neglect the effect of the Ohmic electric field. It is due to the fact that the spatial homogeneity is destroyed by the existence of the magnetic axis. Coulomb collisions will try to destroy such spatial nonuniformity by phase randomization and, thus, by destroying the banana orbits. The collisionless equilibrium radial gradient given by Eq. (24) will be collisionally relaxed and become smaller in the plateau regime of collisionality. And, in a collisional plasma, the dissipationless equilibrium plasma pressure profile will be flat. The effect of the magnetic inhomogeneity is

purely kinetic. It is absent in the fluid description of the plasma.

Equation (24) also implies a small, radially inward particle pinch effect driven by the spatial inhomogeneity, which can be rigorously calculated following similar mathematical procedures as given here. This inward pinch term satisfies the Onsager symmetry relation together with our negative current term.

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Figure Captions

Fig. 1 Comparison between the usual bootstrap term and our new current term as a function of inverse aspect ratio. Dotted line represents the bootstrap current and solid line represents the new current.

The plasma conditions are given as follows:

$B_{t0} = 4.8 \text{ T}$, Total plasma current = 1.4 MA,

$n(0) = 7 \times 10^{13} \text{ cm}^{-3}$, $T_e(0) = T_i(0) = 7 \text{ keV}$,

$n(r) = n(0) [1 - (r/a)^2]$, $T(r) = T(0) [1 - (r/a)^2]$,

where a is the limiter radius.

Fig. 2 Plot of current profiles as function of inverse aspect ratio with plasma parameters typical of L-mode discharges.

Solid line is the result with the new term included and the dotted line represents the result with previous theories only.

Not much difference can be noticed between the two currents.

The plasma conditions are the same as in Fig. 1.

Fig. 3 Comparison between the currents generated by previous theories only(dotted line) and by including the new term(solid line).

All the plasma parameters are identical to those for Fig. 1 except that the ion temperature is raised to $T_i(0) = 30 \text{ keV}$.

Fig. 4 Change of q-profiles obtained by raising T_i . Solid line is with $T_i = 30$ keV and dotted line is with $T_i = 7$ keV. All other parameters are as given in Fig. 1. Without including the new term the solid line will be very close to the dotted line.

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Fig. 1

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Fig. 2

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Fig. 3

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Fig. 4

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