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Glenn R. Elliott, Samuel D. Stearns

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DETECTION AND FREQUENCY TRACKING OF CHIRPING SIGNALS

Glenn R. Elliott, 9212
Samuel D. Stearns, 9311
Sandia National Laboratories
Albuquerque, NM 87185-5800

Abstract - This paper discusses several methods to detect the presence of and track the frequency of a chirping signal in broadband noise. The dynamic behavior of each of the methods is described and tracking error bounds are investigated in terms of the chirp rate. Frequency tracking and behavior in the presence of varying levels of noise are illustrated in examples.

1. INTRODUCTION

In this report we consider the problem of detecting an electro-magnetic pulse that has traveled through a dispersive medium and arrived at a receiver. For our purposes the medium is assumed to be characterized by a previously-described model of the ionosphere [1] in which the transfer function is defined to have unity gain and nonlinear delay. The current version of the nonlinear delay is

$$d = \frac{A}{(\nu + g)^2} + \frac{B}{(\nu + g)^2} \quad (1)$$

d = time delay in μs

ν = frequency in MHz

$A \approx 4.E4$; $B \ll A$; $|g| \approx 1$

(We use ν here for frequency in MHz, and later we use $f = \nu T$, where T = timestep in μs , for normalized frequency in Hz-s).

The chirping transforms that we describe here amount to a piecewise-linear approximation to Eq. 1, so we use only the first term to approximate the delay. Also, frequencies below about 40 MHz are lost by attenuation so, with $\nu > 40$, we neglect the effect of g in Eq. 1. The following simple expressions result for the delay and chirp rate:

$$d \approx \frac{A}{\nu^2} \mu\text{s} \quad (2)$$

$$\dot{d} \approx -\frac{\nu^3}{2A} \text{ MHz}/\mu\text{s}$$

The second expression is obtained by differentiating the first with respect to d .

The waveform in Fig. 1 is an example of an impulse response using Eq. 2 with $A = 4.E4$. The waveform was generated using subroutine IMPULSE [1] with

$$\text{Time Step } T = 0.0025 \mu\text{s} \quad (3)$$

In general, throughout the rest of this paper we will use this value for T in examples and calculations.

The overall time span of the waveform in Fig. 1 is from about 1.0 μs , the delay in Eq. 2 corresponding with the highest frequency of $1/(2T) = 200$ MHz, to about 25 μs , the delay in Eq. 2 corresponding with the lowest frequency of around 40 MHz.

Fig. 1. Top: Chirping impulse response; Center: Segment from 5.00 to 5.64 μs (interpolated); Bottom: Segment from 20.00 to 20.64 μs .

Segments of the impulse response from 5.00 to 5.64 μs (256 samples) and from 20.00 to 20.64 μs (256 samples) are also shown in Fig. 1, the first of these having been interpolated using SPZINT [2] to get a smooth plot. One can observe only a small change in frequency from end to end in each segment but, of course, a large change from one segment to the other.

Fig. 2. Amplitude and delay spectra of the impulse response in Fig. 1.

The amplitude spectrum and group delay of the signal in Fig. 1 are plotted in Fig. 2. The amplitude spectrum, which is the same as the theoretical amplitude response of the medium, is seen to be approximately one from 40 to 200 MHz. The group delay, which is created by differentiating the phase spectrum, is seen to agree with the model in Eq. 2.

We have studied several different approaches for detecting and tracking the frequency of a chirping signal in the presence of noise. In this paper we discuss the behavior of some of the methods which we have used. Included are: The adaptive line enhancer (ALE), a simple two pole Infinite Impulse Response (IIR2P) frequency tracker and enhancer, a method based on the Fast Fourier Transform (FFT), and a more complex method using a special transform called the Matched Chirp Transform (MCT). In each case we have investigated the dynamic tracking characteristics under varying conditions of noise and chirp rate. This paper discusses the dynamic behavior of each of the methods, which was investigated while tracking the chirping signal in the presence of noise.

2. THE ADAPTIVE LINE ENHANCER

2A. STRUCTURE AND ALGORITHM, ALE

The adaptive line enhancer (ALE) is usually in the form of the linear predictor shown in Fig. 3, with the predictor weights

Fig. 3. Adaptive line enhancer in the form of a linear predictor.

$[w_n]$ driven by a normalized version of the well-known least-mean-squares (LMS) algorithm [3,7]. The n^{th} weight, w_n , is adjusted at the k^{th} sample time as follows:

$$w_n(k+1) = w_n(k) + \frac{2u}{NP} e(k)x(k-n); \quad 0 < u < 1 \quad (4)$$

The LMS convergence parameter, u , is normalized by N , the filter size in Fig. 3, and by P , the input signal power, $P = E[x_k^2]$. (The latter may also be adjusted during the adaptive process [6].)

A stationary narrowband signal (spectral line) in the presence of wideband noise is "enhanced" by the ALE if the delay (Δ) is long enough to decorrelate the noise. Then, in order to minimize the mean-squared error (MSE), $E[e_k^2]$, the LMS algorithm will adjust $[w_n]$ such that the filter, $W(z)$, has a narrowband transfer function at the frequency of the line. Then the narrowband signal is predicted, the noise is rejected as well as possible by the filter, the MSE is minimized, and the frequency of the line is at the peak of the inverse predictor gain, i.e.,

$$\text{line frequency at } \max [z^{-\Delta} - W(z)]^{-1} \text{ with } z = e^{j2\pi f} \quad (5)$$

Here f is the frequency in Hz-s. In this paper we are primarily concerned with the ability of the ALE to track the moving line of a chirping signal.

The performance of the ALE in Fig. 3 has been studied extensively, mostly for one or more stationary sinusoids in white Gaussian noise. For this case Griffiths [3] has shown that the convergence factor u in Eq. 4 should be in the range shown and that certain constraints on the input correlation matrix (R_x) must be met for convergence. Treichler [8] has also characterized the convergence in terms of the eigenvalues of R_x , and Rickard and Zeidler [5] have provided a quantitative analysis of the ability of the ALE to enhance a narrowband signal. Regarding the choice of the delay (Δ), Yoganandam et. al. [6] have shown that the ALE performance is significantly improved by choosing an optimal value, Δ_{opt} , for each given situation.

2B. TRACKING CONSIDERATIONS, ALE

In the present case the ALE is supposed to converge or lock on to a nonstationary, chirping narrowband signal which appears suddenly in broadband background noise, and to track the frequency of the signal through time. We are particularly interested in the tracking capabilities and performance of the ALE. Suppose that the ALE is successfully tracking a sinusoid at frequency f (Hz-s) in broadband noise. With a low signal-to-noise ratio (the condition in which we are most interested), the magnitude of the filter transfer function given by $|W(e^{j2\pi f})|$ will ideally be a unit impulse at frequency f . The filter weights, $[w_n]$, will therefore be ideally sinusoidal, in the form

$$w_n = \frac{2}{N} \sin(2\pi n f + \alpha); \quad 0 < n < N-1 \quad (6)$$

where α is phase in radians. If we assume that f is being tracked and that both f and $[w_n]$ are functions of k , the time step, then the weights and their rates of change are given by

$$w(k) = \frac{2}{N} \sin[2\pi n f(k) + \alpha];$$

$$\dot{w}(k) = \frac{4\pi n \dot{f}(k)}{N} \cos[2\pi n f(k) + \alpha]; \quad (7)$$

From this result we see that the weights themselves change sinusoidally with time (k) and that the rate of change is bounded by

$$|\dot{w}_n(k)| \leq \frac{4\pi n}{N} |\dot{f}(k)| \quad \text{Hz-s/sample} \quad (8)$$

Thus, for a given chirp rate, the higher-order weights must change faster. This result is fortuitous for high-signal-to-noise cases where the lower-order weights tend to dominate (only 2 weights are required theoretically in the noise-free case). For low-SNR cases, a further upper bound on the weight rate of change required for tracking is obtained using the final weight (assuming N is large, we use $n=N$ in Eq. 8).

$$|\dot{w}_{N-1}| \leq 4\pi |\dot{f}(k)| \quad \text{Hz-s/sample} \quad (9)$$

In Eqs. 8 and 9 we have essentially established bounds on the required tracking capability of the ALE. On the other hand, the convergence of any gradient-search algorithm like the LMS algorithm in Eq. 4 can be approximated [7] by

$$\dot{w}(k) \equiv w(k+1) - w(k) = 2u [w_{opt} - w(k)] \quad (10)$$

Here the weight tracking error is the difference between the optimal and current weight values, i.e., $w_{opt} - w(k)$. The relationship in Eq. 10 is really good only for a single weight or in the case where R_x has equal eigenvalues; however, the latter is approximately true in low-SNR cases. Since the weights themselves are in general proportional to $2/N$ as seen in Eq. 6, we define a normalized tracking error for weight w_n as follows:

$$E_n(k) = \frac{N}{2} |w_{opt} - w_n(k)| \quad (11)$$

Combining Eqs. 8, 10, and 11, we have

$$E_n(k) \leq \frac{n\pi |\dot{f}(k)|}{u} \quad (12)$$

During tracking, the range from 0 to 0.5 Hz-s is divided into N intervals. We therefore express the chirp rate in terms of $r(k)$, the number of frequency intervals/sample, and obtain

$$E_n(k) \leq \frac{n\pi |r(k)|}{Nu} \quad (13)$$

Using this result, we can at least approximately predict the success of an ALE tracking the frequency of a chirping signal. The filter size (N) must be large enough to allow adequate frequency discrimination. For adequate performance of the LMS algorithm in terms of weight noise [7], the convergence parameter (u) must generally be much smaller than the limit in Eq. 4. Since the weight variance due to noise is proportional to u [7], the actual choice of u must be based on the SNR for which the ALE is designed. In our present study we have SNR's

around -10 dB, and typical values of N and u are $N=32$ and $u=0.01$. The implication in Eq. 13 is that on the order of $\pi/(Nu)$ to π/u , or 10 to 300 samples, may be required for adjusting the weights to switch the ALE response successfully from one frequency increment to the next. Experimentally we have found that the bound in Eq. 13 is reasonable.

Regarding the choice of Δ , the forward delay in Fig. 3, the optimal choice will decorrelate the noise but not the signal at the summing junction. With steady-state signals, Δ can be chosen on the basis of noise characteristics alone, but when the signal is chirping, any value of Δ will tend to decorrelate the signal at the summing junction in Fig. 3. For example, suppose the signal component of $x(k)$ is chirping at a constant rate c Hz-s/sample. Then, in order to cancel the signal at the summing junction, the weights must "tumble" [6] at approximately $c\Delta$ Hz-s, the difference between the frequency of the signal in $x(k)$ and the frequency of the signal in $x(k+\Delta)$. On the other hand, in Eq. 7 with the constant chirp rate assumed here, the time-varying sine argument becomes $2\pi nck$. Therefore, neglecting tumbling, the n^{th} weight must oscillate at the frequency cn in order to track the signal. To summarize,

Constant chirp rate c Hz-s/sample:

Tracking frequency (maximum) of weight $w_N = cN$

Additional tumbling frequency of any weight $= c\Delta$ (14)

From this result we can conjecture that if Δ is a significant fraction of N , the tracking capabilities of the LMS-driven weights will be affected.

The tracking characteristics discussed in this section are demonstrated in the following examples.

2C. TRACKING EXAMPLES, ALE

In this section we demonstrate the results above using the type of signal described in Section 1. First, a signal is illustrated with relatively low noise (average SNR=0dB) in Fig. 4. The

Fig. 4. Signal with frequency described in Eq 15, $k_0=5000$ and SNR=0 dB.

waveform begins at $k_0=5000$ samples and has the frequency in accordance with Eq. 1, $d=(k-k_0)T$, $T=0.0025$,

$$f(k) = \frac{10}{\sqrt{k-k_0}} \quad \text{Hz-s} \quad (15)$$

The additive noise is Gaussian and has uniform power density from 0.05 to 0.45 Hz-s. The signal first appears at $k=5400$ samples where, in accordance with Eq. 15, $f(k)$ comes within the range (0,0.5) Hz-s. From Eq. 15, the chirp rate amplitude is

$$|f(k)| = 5(k - k_0)^{-3/2} \text{ Hz-s/sample} \quad (16)$$

This rate has a maximum of 6.25×10^{-4} Hz-s/sample at $k=5400$ where the signal first appears. Using $u=0.01$, Eq. 12 gives a reasonable tracking error for the low-order weights. Thus we would expect the LMS algorithm to begin tracking the signal near its onset, and of course the amplitude in Eq. 16 decreases rapidly with k , so the tracking may or may not improve as the chirp rate decreases with k .

A tracking example for the waveform in Fig. 4 is shown in Fig. 5. In the upper plot, the ALE is seen to improve the SNR

Fig. 5. ALE tracking under low noise conditions. Input waveform $x(k)$ as in Fig 4 with $k_0=5000$, $\Delta=1$, $N=32$, $u=0.01$, and avg. SNR=0 dB.

of the waveform in Fig. 4, and the frequency estimation in Eq. 5 (boxes) in the lower plot matches the actual frequency (smooth curve) until the instantaneous SNR, which decreases during the signal, decreases to a low value. For this example we used $N=32$ weights, $\Delta=1$, and $u=0.01$. The input power, P in Eq. 4 was tracked using a single-pole smoothing filter described in [8] with forgetting factor $\alpha=0.001$.

Plots of the weights $w_8(k)$ and $w_{16}(k)$ vs k , which exhibit the relationship in Eq. 7, are shown in Fig. 6. The amplitude

Fig. 6. Plots of weights w_8 and w_{16} , illustrating the behavior predicted in Eq. 7.

variation of w_8 is close to the value 0.06 predicted by Eq. 7. The amplitude variation of w_{16} is less because, with a high SNR, not all of the weights are required for signal cancellation. The frequency of weight variation is found by taking the derivative of the phase $\theta_n(k)$, then substituting Eq. 16 for the present example:

$$\text{phase } \theta_n(k) = 2\pi n f(k) + \alpha$$

$$\begin{aligned} \text{frequency } f_w(k) &= \frac{1}{2\pi} \left[\frac{d\theta_n(k)}{dk} \right] = n |f(k)| \text{ Hz-s} \quad (17) \\ &= 5n(k - k_0)^{-3/2} \text{ Hz-s} \end{aligned}$$

For example, at $k=7500$, this result gives $f_w(k)=(4.5-5)n$ and hence periods of 3125 and 1562 samples for weights w_8 and w_{16} respectively, which agree with the plots in Fig. 6.

The degradation in tracking caused by a larger value of Δ , which in turn causes the weight tumbling expressed in Eq. 14, is shown in Fig. 7 where Δ is increased from 1 to 5 samples. The

Fig. 7. Loss of tracking from Fig. 5 caused by increasing Δ from 1 to 5.

Gaussian noise was made white (with the same SNR) for this case so that no other correlated components were present in the waveform. As predicted, the tracking capability is affected by making Δ a significant fraction of N .

A waveform similar to Fig. 4 but with SNR decreased to -10 dB is plotted in Fig. 8 and tracked using the same ALE

Fig. 8. Tracking as in Fig. 5 but with avg. SNR=-10 dB.

used for Fig. 5. Two effects can be noted in the frequency track in the lower plot. First, the tracking error, $E_n(k)$, in Eq. 12 has not changed and, as expected, tracking is still accurate except where the weights are misadjusted due to the increased noise in the weight solutions [5,11] due to the lower SNR. Note, however, that lowering the convergence parameter u in order to lower the misadjustment would in this case produce a proportional increase in $E_n(k)$, and produce an adverse effect on tracking. Secondly, as we would expect, the overall frequency track is noisier due to the increased misadjustment over the case in Fig. 5.

3. A TWO-POLE IIR LINE ENHANCER, IIR2P

3A. STRUCTURE AND ALGORITHM, IIR2P

Next we look at the two-pole (one complex pair) IIR adaptive line enhancer and frequency tracker [9,10]. This form is simple and requires a minimal of computation; although both multiplication and division are required, which can be a problem for implementation in simple processing hardware. The structure is shown in Fig 9.

Fig. 9 Structure of the two-pole IIR adaptive line enhancer and tracker.

The transfer function $H(z)$ is

$$H(z) = \frac{(1-r^2)}{z^2 - wz + r^2} \left[\frac{wz}{(1+r^2)} - 1 \right] \quad (18)$$

where

$$0 < r < 1, |w| < 2r$$

With these constraints on r and w the poles of $H(z)$ are complex and inside the unit circle on the z -plane; therefore $H(z)$ is stable.

For weight (w) update, a Recursive Least Mean Squares (RLMS) algorithm which has been modified to include a normalization factor is used:

$$w(k+1) = w(k) + \frac{u\epsilon(k)\alpha(k)}{\psi(k)} \quad (19)$$

where u is the convergence parameter -- used here similarly as in the ALE. The other two terms, $\alpha(k)$ and $\psi(k)$ are computed as follows:

$$\alpha(k) = \frac{(1-r^2)}{(1+r^2)} x(k-1) + w\alpha(k-1) + y(k-1) - r^2\alpha(k-2) \quad (20)$$

and,

$$\psi(k) = \gamma\psi(k-1) + (1-\gamma)\alpha^2(k) \quad (21)$$

$$0 < \gamma < 1$$

The term $\alpha(k)$ is an estimate of the derivative of $y(k)$ with respect to $w(k)$, $\delta y(k)/\delta w(k)$. $\psi(k)$ is a special normalizing factor based upon an instantaneous estimate of the power in the gradient search vector [9]. The "forgetting factor", γ , controls the time constant of this estimate and, in effect, controls the performance of the algorithm to changes in signal power level.

In our application the frequency of the signal being tracked is of importance and can be calculated from

$$f(k) = \frac{1}{\pi} \arccos \left(\frac{w(k)}{1+r^2} \right) \text{ Hz-s} \quad (22)$$

where $f(k)$ is the estimate of the tracked frequency in Hz-s at time k .

3B. TRACKING CONSIDERATIONS, IIR2P

The IIR2P functions remarkably well considering its simplicity and small computational demands -- eight multiplies and one division per sample. Providing the input signal has only minor variations in variance, the system can track a single frequency in difficult noise conditions. This is shown in Fig. 10 where a

Fig. 10. The two pole IIR tracker/enhancer tracking a 100 MHz sinusoid followed by a 75 MHz sinusoid embedded in white noise with SNR = -5 dB.

100 MHz sinusoid lasts for 10 μ s and then is followed by a 75 MHz sinusoid. Both equal amplitude sinusoids are embedded in white noise with SNR = -5 dB. One can readily see that the simple algorithm can track frequencies where the signal is hardly discernible in the background noise. For this example the parameters were set as follows:

$$\begin{aligned} r &= .91 \\ u &= .012 \\ \gamma &= .95 \end{aligned}$$

Practical experience with the algorithm has shown that rarely can r exceed 0.95 without stability problems. With the limitation placed on w in Eq. 18, and the frequency relationship as expressed in Eq. 22, tracked frequencies are limited to a range of about 20 to 80% of Nyquist. Although the algorithm does con-

tain the normalizing factor, $\psi(k)$, its interaction with the adaptive coefficient, u , makes adaptation rather slow when the input signal has an abrupt change in variance accompanied by a rapid change in frequency.

Fig. 11 shows the result of a hypothetical signal where both

Fig. 11. Tracking characteristics of the IIR2P where the frequency and power change rapidly.

frequency and amplitude change rapidly. The upper trace shows first a dispersed signal where A of Eq. 1 is $1.E4$ followed by a dispersed signal where A is $4.E4$. It is not possible for dispersion constants to change this rapidly; but this input serves to illustrate the difficulty of implementing the algorithm in a system where adaptation parameters cannot be changed. A small amount of random background noise was added to contribute a degree of realism. The adaptation parameters were set as follows:

$$\begin{aligned} r &= .91 \\ u &= .0052 \\ \gamma &= .90 \end{aligned}$$

Although the IIR2P has limitations for use in our application, it does have some remarkable capabilities which we discovered during our investigation of its adaptation characteristics. We do not have expressions for the effects of the time rate of change of the input frequency, f , as well as the effects of time variations in the power/gradient estimates as we had for the ALE. Consequently, we developed several test signals to show the adaptation characteristics of the IIR2P. The first series of test signals was intended to show the delay in adaptation in cases where the frequency of the input signal changes rapidly. A linearly changing frequency was used to make the estimate of delay more straightforward. Also, the variance was allowed to change linearly and at varying rates.

Two of these signals are shown in Fig. 12. Frequency varies linearly from 90% to 10% of Nyquist over an interval of $10 \mu s$ in one case and $5 \mu s$ in the second case.

Fig. 13 shows the output of the IIR2P tracker in both cases. We can see that after the initial adaptation time, the algorithm tracks both cases very well. It fails near the very end of the faster changing signal. Using these results, we can compare the time differences in the two cases to get a feel for the ability of the IIR2P to compensate for the changes in input signal rate of frequency change and rate of amplitude change and arrive at an estimate of the tracking error.

Fig. 12. A set of input signals for testing the ability of the IIR2P to track when frequency and amplitude change at varying rates with no noise.

Fig. 13. Results of the IIR2P tracking the signals of Fig. 12.

Fig. 14 shows a portion of each curve in Fig. 13 centered around 142 MHz on the Y axis. The time delay in each case is shown by the lines, D1 and D2. The lines, representing the tracking error, are essentially equal in length. This is indeed

Fig. 14. IIR2P tracking performance with differing rates of change in frequency and amplitude.

confirmed when one subtracts the tracked frequency from the actual in each case; the results are virtually equal. In this case the IIR2P compensates very well for amplitude and frequency changes.

We also explored the limits to investigate where the IIR2P might not be capable of following frequency and amplitude. Fig. 15 shows the result of a case where the frequency and ampli-

Fig. 15. IIR2P tracking characteristic shown a failure to track when the frequency and amplitude of the input signal change too rapidly.

tude vary more rapidly than in Fig. 13 -- the complete sweep lasts only 1 μ s. We can see that the IIR2P fails to keep up with this rate of change. Probably one could make the algorithm perform better by selecting different adaptation coefficients for each case. We cannot do this in our application; but it could be done in off-line processing. For the tests shown in Figs. 12 - 15 the parameters were fixed at:

$$\begin{aligned} r &= .91 \\ u &= .012 \\ \gamma &= .90 \end{aligned}$$

3C. TRACKING EXAMPLES, IIR2P

In order to compare the IIR2P methods, we now will show the results of using the IIR2P algorithm with a set of typical input signals. In Fig. 16, where the SNR = -6 dB, the IIR2P

Fig. 16. Frequency detection using the IIR2P with SNR = -5 dB. The detection is successful for the lower frequency values.

method successfully detects the decreasing frequency over a wide range. We note that the part of the signal above approximately 150 MHz is missed. This is probably due to the factors mentioned in the preceding section.

With the SNR = -12 dB the IIR2P still is able to track the frequency fairly well. As shown in Fig 17, the algorithm detects

Fig. 17. Frequency detection using the IIR2P method with SNR = -12 dB.

the dispersed signal for at least part of its duration. In fact, its performance is possibly better than some of the more complex methods discussed in this paper. It is subject to the limitations expressed in the preceding section. In both Fig. 16 and 17 the adaptation parameters were as follows:

$$\begin{aligned} r &= .91 \\ u &= .012 \\ \gamma &= .90 \end{aligned}$$

3D. CONCLUSIONS, IIR2P

The IIR2P appears to be an attractive and capable algorithm for tracking the varying frequency and amplitude signal. Its advantages are that it performs well on a wide variety of signals and has very low computational requirements. If some restrictions can be placed on the input signal parameters, the algorithm functions as well as, and in some cases better than, the much more complicated Matched Chirp Transform algorithm which is described in a Section 6.

The IIR2P should be an area for more study. If one could arrive at methods for improving its ability to compensate for a wider range of changes in the input signal, and could do so without adding significantly to the computational requirements, the result might be an exceptionally good line tracking algorithm.

4. FAST FOURIER TRANSFORM METHOD

4A. STRUCTURE AND ALGORITHM, FFT

The simplest approach to the measurement of the frequency of a chirping sinusoid is to estimate the frequency periodically in short segments using the Fast Fourier Transform (FFT) algorithm. At regular intervals, we take the FFT of a short segment of the signal and assume that the frequency of that segment is the frequency corresponding with the FFT component having the maximum magnitude. We use the Blackman data window on each segment in order to obtain spectral smoothing and hence a more reliable spectral estimate. Fig. 18 shows an example with overlapping segments of length 256 samples. The length 256 gives 128 FFT components from 0 to 200 MHz, that is, a frequency increment of

$$\Delta\nu = \frac{1}{256T} = 1.5625 \text{ MHz} \quad (23)$$

The degree of frequency definition given by this $\Delta\nu$ is considered about right for the present application..

Fig. 18. Overlapping segments of length 256 samples. Overlap factor is 3/4.

The overlap factor of 3/4 shown in Fig. 18 means that each segment overlaps its predecessor by this amount, that is, the delay from one FFT to the next is 1/4 segment, or 64 samples, or 0.16 μ s. (This rate (6.25M/s) of computing 256-point FFT's in real time will require some rather sophisticated hardware.)

The FFT method is demonstrated in Figs. 19 and 20. Broadband Gaussian noise was added to the signal in Fig. 1 to produce the sequences $[x(k)]$ in each figure. The signal shown in Fig. 1 actually begins at $t = 15 \mu$ s in Figs. 19 and 20 and lasts 25 μ s, until $t = 40 \mu$ s. In Fig. 19 we see that the FFT method successfully detects the correct frequency over most of the signal. In Fig. 20, where the noise power is four times that in Fig. 19, the FFT method detects the correct signal frequency only occasionally and is marginally successful in this case.

Fig. 19. Frequency detection using the FFT method with SNR = -5 dB. In this case, the method is successful.

Fig. 20. Frequency detection using the FFT method with SNR = -11 dB.

4B. EFFECT OF CHIRPING ON THE FFT

When we take the FFT of a segment of a signal whose frequency is varying, we expect the FFT to have nonzero components at all frequencies that appear during the segment. Thus, there is the possibility that the chirping signal used with the FFT method just described could distort or modify the frequency-versus-time data produced by this method.

To see more exactly the effect of chirping on the FFT, suppose we have a complex sinusoidal sequence given by

$$x(k) = e^{j\theta(k)}; \quad \theta(k) = 2\pi k \left(f_0 + \frac{kf}{2} \right) \quad (24)$$

In this sequence k is the sample number and the radian frequency of $x(k)$ is $d\theta(k)/dk$. Thus, f_0 is the frequency in Hz-s at $k = 0$, and f is the chirp rate in Hz-s/sample. We can write $x(k)$ as a product,

$$x(k) = [e^{j2\pi k f_0}] [e^{j\pi k^2 f}] \quad (25)$$

The DFT of a product is the convolution of the individual DFT's [2], that is,

$$\text{DFT}[x(k)] = \frac{1}{N} \text{DFT}[e^{j2\pi k f_0}] * \text{DFT}[e^{j\pi k^2 f}] \quad (26)$$

If the segment $[x(k)]$ is sampled over an even number of periods of f_0 , the first DFT in Eq. 26 has only a single nonzero component at m_0 , the index corresponding with f_0 ; that is,

$$\text{DFT}[e^{j2\pi k f_0}] = N\delta_{m-m_0} \quad (27)$$

where δ_{m-m_0} is a unit impulse at $m = m_0$.

Therefore, the convolution in Eq. 26 produces a frequency-shifted version of the second DFT; that is,

$$\text{DFT}[x(k)] = \text{DFT}\left[e^{j\pi k^2 f}\right]_{m \leftarrow m - m_0} \quad (28)$$

Thus, the DFT is smeared around the frequency f_0 in proportion to f .

Some examples with $N = 256$ and $f_0 = 0.25$ are shown in Fig. 21. In these examples the chirp rate, f , is zero or negative.

Fig. 21. Examples of the spectrum of a chirping sinusoidal sequence with $N = 256$ samples. The initial frequency is $f_0 = 0.25$ Hz-s; that is, $m = 20$. The chirp rate, f , varies from plot to plot as shown.

When $f = 0$, we have an ordinary DFT with a single component of amplitude N . When $f = -0.0001$ Hz-s/sample, we see that the spectrum is smeared down to about $f_0 + Nf = 0.224$ Hz-s. Similarly, with $f = -0.0005$ Hz-s/sample, we have $f_0 + Nf = 0.122$ Hz-s. Finally, with $f = -0.001$, the chirp rate is such that $f_0 + Nf$ is approximately zero.

In short, chirping has the effect of smearing the FFT over the frequency range covered during the time of the signal, with a resulting decrease in the signal-to-noise ratio.

5. FAST CHIRP TRANSFORM METHOD

We now describe a fast chirp transform (FCT) method that is similar to the FFT method just discussed, but uses a chirping transform. Consider each component of an ordinary FFT as the cross correlation of a signal segment, $[x(k)]$, with a complex sinusoid, $[e^{j\pi km/N}]$, at the fixed frequency $f_m = 2\pi m/N$ Hz-s. In a chirp transform we cross correlate instead a chirping complex sinusoid with $[x(k)]$. In the case where the chirp rate is constant, as described by Li [11], the frequency of the m^{th} complex sinusoid may be expressed as

$$f_m(k) = f_{0m} + \left(k - \frac{N}{2}\right)f \quad \text{Hz-s} \quad (29)$$

Here the chirp rate, f , has units of Hz-s/sample. As k goes from 0 to N , f_m is seen to vary linearly around the center frequency, f_{0m} . The phase corresponding with f_m is

$$\begin{aligned} \theta_m(k) &= 2\pi \int f_m(k) dk \\ &= 2\pi \left[f_{0m}k + \frac{1}{2}(k^2 - Nk)f \right] \text{ rad} \\ k &= 0, \dots, N-1 \\ m &= M_0, \dots, M_1 \end{aligned} \quad (30)$$

We give the center frequency, f_{0m} , the same increment as in the ordinary DFT, so that $f_{0m} = m/N$. We restrict the frequency index (m) so that the frequency in Eq. 29 is in the range (0.0, 0.5). The corresponding range (M_0, M_1) of m is determined as follows, assuming f is negative or zero:

$$\begin{aligned} m = M_0: f_m(N) &= \frac{M_0}{N} + \left(N - \frac{N}{2}\right)f = 0; \\ M_0 &= -\frac{N^2 f}{2} \\ m = M_1: f_m(0) &= \frac{M_1}{N} + \left(0 - \frac{N}{2}\right)f = 0.5; \\ M_1 &= \frac{N}{2}(1 + Nf) \end{aligned} \quad (31)$$

These limits are taken to be integers within the range given by the real values (M_0, M_1). Using Eq. 31 in Eqs. 29 and 30, we have

$$\begin{aligned} f_m(k) &= \frac{m}{N} + \left(k - \frac{N}{2}\right)f \\ \theta_m(k) &= 2\pi \left[\frac{mk}{N} + \frac{1}{2}(k^2 - Nk)f \right]; \quad k = 0, \dots, N-1; \\ m &= M_0, \dots, M_1 \end{aligned} \quad (32)$$

The corresponding DFT is

$$\begin{aligned} X_m &= \sum_{k=0}^{N-1} x(k) e^{-j\theta_m(k)} \\ &= \text{DFT}\left[x(k) e^{-j\pi(k^2 - Nk)f}\right] \end{aligned} \quad (33)$$

This DFT becomes the fast chirp transform, that is,

$$\text{FCT}[x(k)] = \text{FFT}\left[x(k) e^{-j\pi(k^2 - Nk)f}\right] \quad (34)$$

The FFT in Eq. 34 could be implemented as a convolution of FFT's, but there is no advantage in doing so. The preferred implementation of the FCT is shown in Fig. 22.

In the present application, we noted previously that the frequency range of interest is $\nu = 40$ to 200 MHz. Thus, in Eq. 2 with $A = 4.44$, the range of $\dot{\nu}$ is -100 to -0.8 MHz/ μ s, which we extend to $-100 < \dot{\nu} < 0$ MHz/ μ s. Thus, in the units of Eq. 29,

$$\dot{f} = \dot{\nu}T^2 \quad \text{Hz-s/sample}; \quad -0.25E-4 \leq \dot{f} \leq 0 \quad (35)$$

To use the FCT method, we execute the FCT in Fig. 22 for a se-

Fig. 22. The fast chirp transform. Values beyond the range shown are not used.

quence of values of $\dot{f}$, forming a two-dimensional array of FCT components. We specify the dimensions as

$$\begin{aligned} \text{row}(i) &\leftrightarrow \text{fixed value of } \dot{f} \\ \text{column}(m) &\leftrightarrow \text{fixed value } m \end{aligned} \quad (36)$$

The restriction on m in Eq. 32 means that not all columns in a given row of the array are useful. An illustration of the useful portion of the array for $N = 256$ is presented in Fig. 23. The col-

Fig. 23. Useful points in the array of FFT values produced by Fig. 22. Segment size is $N = 256$, with 101 rate values spanning the range of Eq. 35.

columns correspond with the frequency indices (m) and the rows correspond with 100 $\dot{f}$ indices (i) over the range given in Eq. 35. Each FFT in Fig. 22 gives $N/2$ transform values, but only the values indicated in Fig. 23 are useful.

An example of the FCT of a single segment with $N = 256$ is illustrated in Fig. 24. The data segment, $[x(k)]$, was the segment for $k = 100$ to 1255 taken from the waveform in Fig. 1. At the beginning of this segment we have.

$$d = 1000T = 2.5\mu\text{s} \quad (37)$$

Fig. 24. The fast chirp transform (Eq. 34) of the segment $[x(k)]$ starting at $k = 1000$ with $N = 256$. The peak amplitude is at $m = 81$, $i = 23$.

For this delay value, Eq. 2 gives

$$\nu = \left[\frac{A}{d} \right]^{1/2} = 126 \text{ MHz} \quad (38)$$

$$\dot{\nu} = -\frac{f^3}{2.4} = -25 \text{ MHz}/\mu\text{s} \quad (39)$$

Using the array shown in Fig. 23 with 129 frequency intervals from 0 to 0.5 Hz-s, and with 101 chirp intervals from 0 to -6.25E-4 Hz-s/sample, the indexes for f and $\dot{f}$ in Eqs. 38 and 39 are

$$\text{frequency index } m = \frac{\nu T(N/2)}{0.5} \quad (40)$$

$$= 126T(128)/0.5 = 81$$

$$\text{chirp index } i = \dot{\nu}T^2k/(-6.25E-4) \quad (41)$$

$$= 25T^2(100)/(-6.25E-4) = 25$$

These indexes are close to those of the peak FCT amplitude in Fig. 24. The reason that the FCT gives the frequency and chirp rate in Eqs. 38 and 39 at the beginning of the segment rather than in the midst of the segment is that, as seen in Fig. 1, the overall signal power is decreasing during the signal. The initial part of the segment has the most power.

In Fig. 24 we note that the FCT gives a reasonably sharp indication of both frequency and chirp rate; however, the surface is more sharply peaked in frequency than in chirp rate. We might expect from Fig. 24 that the FCT method would encounter some difficulty when applied to noisy electromagnetic signals such as those in Figs. 19 and 20.

Results similar to Figs. 19 and 20 using the FCT instead of the FFT are shown in Figs. 25 and 26. The segment size was,

as before, 256 samples, and all other factors including the Blackman window were likewise unchanged. Each FCT had 129 frequency values and 101 chirp values as illustrated in Fig. 23. As in the FFT method, the frequency of a segment is defined to correspond with the peak FCT amplitude.

Fig. 25. Frequency detection using the FCT method with SNR = -5 dB; comparable to Fig. 19.

Fig. 26. Frequency detection using the FCT method with SNR = -11 dB; comparable to Fig. 20.

Considering the FCT method as it is described here along with the experimental results in Figs. 25 and 26, we arrive at two main conclusions. First, the FCT requires 100 times the computation of the FFT. However, the real-time computation problem could be solved with parallel processing, and the FCT should be useful in cases where the chirp rate is linear and unknown. Secondly, the FCT has, in the example used here, 100 times the number of degrees of freedom of the FFT. Consequently, its behavior under noisy conditions is questionable. Comparing Fig. 25 with Fig. 19 and Fig. 26 with Fig. 20, we would say that the FCT method offers some improvement over the FFT method.

6. MATCHED CHIRP TRANSFORM METHOD

The fast chirp transform method described above is useful primarily in cases where the chirp rate is unknown and varies from one segment to the next. In the case of the dispersive

electromagnetic pulse considered here, however, we have a model (Eq. 2) that renders the chirp rate known at least approximately at each frequency. Thus, in this case, we do not require all of the degrees of freedom illustrated in Fig. 23.

Let us examine the present situation in more detail. Beginning with Eq. 2, we convert to Hz-s units by letting $f = \sqrt{t}$ Hz-s, $f = \sqrt{t}^2$ Hz-s/sample and $d/T = \delta$ samples to obtain

$$\text{delay } \delta \approx \frac{AT}{f^2} \text{ samples}$$

$$\text{chirp rate } \dot{f} \approx \frac{f^3}{2AT} \text{ Hz-s/sample} \quad (42)$$

Here we have frequency, f , varying inversely with the square root of δ , the number of samples (delay) from the start of the signal. Thus, when we partition f into a set of intervals as we did in the FFT and FCT methods, in this case we also partition δ into a corresponding set of intervals. The resulting equivalence between the frequency and time domains is illustrated in Fig. 27. The time domain is shown partitioned into four non-

Fig. 27. Equivalence of the time and frequency domains under the transformation $\delta = at/f^2$ with $AT = 100$. The time domain in this example is partitioned into $M = 4$ non-overlapping segments.

overlapping equal segments, with the result that the frequency domain is partitioned into four unequal segments. We choose to partition the time (delay) domain evenly, instead of the frequency domain, in the matched chirp transform (MCT) which we now describe.

To specify the time domain spacing for the MCT, we define δ_0 as the minimum delay corresponding with the highest detectable frequency, 0.5 Hz-s, and we define δ_1 as the maximum delay corresponding with the lowest frequency of interest, f_{min} , with $f_{min} > 0$. (In the present study, the range of ν from 40 to 200 MHz corresponds with the range from $f_{min} = 40T = 0.1$ to $200T = 0.5$ Hz-s.) From Eq. 42, we have

$$\delta_0 = AT(0.5)^{-2} ; \quad \delta_1 = AT(f_{min})^{-2} \text{ samples} \quad (43)$$

The "time domain" for the MCT is now defined to be the domain from δ_0 to δ_1 samples, and we divide this domain into M equal intervals specified by

$$\delta_m = \delta_0 + m(\delta_1 - \delta_0)/M \text{ samples; } m = 0, 1, \dots, M \quad (44)$$

(We assume that $(\delta_1 - \delta_0)$ is a multiple of M here.) The frequency of a signal initially at the frequency corresponding with δ_m , and chirping down as the time index (k) increases in accor-

dance with Eq. 42, is

$$f_m(k) = \left[\frac{AT}{\delta_m + k} \right]^{1/2}$$

$$= \left[\frac{MAT}{(N-m)\delta_0 + m\delta_1 + Mk} \right]^{1/2} \text{ Hz-s;} \quad (45)$$

$$m = 0, 1, \dots, M; \quad k \geq 0$$

We define the m^{th} MCT component to be the correlation of the N -sample sequence $[x(k)]$, defined above, with a complex sinusoid chirping in accordance with Eq. 45. The phase of the complex sinusoid is given by

$$\theta_m(k) = 2\pi \int f_m(k) dk$$

$$= 4\pi \left[\frac{AT}{M} \right]^{1/2} [(M-n)\delta_0 + m\delta_1 + Mk]^{1/2}; \quad (46)$$

$$m = 0, 1, \dots, M;$$

$$k = 0, 1, \dots, N-1$$

The MCT of $[x(k)]$ has the form of the other transforms above, that is,

$$\text{MCT}[x(k)] = \sum_{k=0}^{N-1} x(k) e^{-j\theta_m(k)}; \quad m = 0, 1, \dots, M \quad (47)$$

Unlike the FCT in Eq. 34, the MCT cannot be written as the FFT of a dechirped sequence, because of the form of $\theta_m(k)$ in Eq. 46. However, if all $N(M+1)$ values of $\exp[-j\theta_m(k)]$ in Eq. 47 are tabulated, the MCT does not require the excessive computing of the FCT, and its computation in real time is feasible in our applications.

6A. APPLICATION OF THE MCT

An example of the MCT of a single segment is shown along with the corresponding FFT in Fig. 28. The segment has length $N=256$ samples and is taken from the waveform in Fig. 1 beginning at 1000 samples or 2.5 μs , where the signal frequency is, in accordance with Eq. 42, 0.3162 Hz-s or 126.5 MHz. Like the FFT, the MCT is computed over $M=128$ delay intervals so that it has the number of degrees of freedom of the waveform segment. As expected, the MCT amplitude is concentrated into a band that is narrow compared with the FFT band.

We apply the MCT in essentially the manner described at the beginning of Section 2 above. The incoming waveform, containing a signal similar to Fig. 1 in noise, is partitioned into segments. The segments may be overlapping. (Currently we are using an overlap of 75%, as illustrated in Fig. 18.) The MCT of each segment is taken without windowing as the segment arrives, and the delay (δ) value of the segment is taken to be the delay corresponding with the largest MCT magnitude

An example comparing the MCT and FFT is shown in Fig. 29. Both transforms were applied to the detection of the waveform of Fig. 1 added to the same broadband (20-180 MHz) Gaussian noise used for Figs. 20 and 26. The signal begins at 15.0 μs . The MCT parameters were the same as in Fig. 28. Segment overlap was 75% for both transforms. The FFT gives the frequency of each segment, while the MCT gives the delay.

The MCT produces a somewhat clearer detection of the signal in this example, as it does generally.

The linearity of the MCT delay plot in Fig. 29 is an advantage in detection for several reasons. First, a simple detection

Fig 28. Comparison of the FFT and the MCT of the segment $[x_k]$ starting at $k = 1000$ with $N = 256$ -- the segment used also for Fig. 24. The MCT size is $M = 128$, and

Fig. 29. Detection using the FFT and MCT methods on the signal in Fig. 1 plus noise. The signal begins at 15.0 μs .

criterion can be established by fitting a least-squares line to the delay plot and placing a threshold on the squared error. Also, by extrapolating the line back to its intercept with the time axis, one can establish an accurate estimate of the signal arrival (15.0 μ s in Fig. 29).

Furthermore, suppose that the constant A used in the MCT is not chosen correctly, and that $\hat{A}$ is used instead. In Eq. 42 we see that this produces a scaling of the delay (δ) by the factor $\hat{A}/A$, which is indicated by the slope of the delay-vs-time line. Thus we can establish A from

$$A = \frac{\hat{A}}{\text{slope of delay - vs - time line}} \quad (48)$$

This result, at least under low-noise conditions, indicates the possibility of determining the actual A value from the assumed A value.

7. SUMMARY, TRANSFORM METHODS

The matched chirp transform provides the best of the transform schemes for detecting the type of signal described in the introduction. When the chirp rate is nonlinear as it is in this type of signal (Eq. 2), the MCT has the added advantage of producing a time-domain output that is theoretically linear and, therefore, easier to use in a detection algorithm. Furthermore, its computational requirements are closer to those of the FFT than to those of the FCT, which are much greater.

8. GENERAL CONCLUSIONS

We have presented several methods of detecting and tracking the frequency of a chirping signal. All perform well in low levels of background noise and chirp rates. The simplest, the IIR two pole enhancer/tracker is appropriate where computational power is limited and where the chirping parameters can be estimated to some degree. The matched chirp transform, MCT, is able to detect the signal in higher noise backgrounds provided an estimate can be made of the chirp parameters and sufficient computational capability is available. We have tried to present sufficient information on each method for an assessment of the potential value of each in the user's application.

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University of New Mexico
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