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CHARGE DIVIDING MECHANISM ON RESISTIVE ELECTRODE
IN POSITION-SENSITIVE DETECTORS*

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October 1978

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CHARGE DIVIDING MECHANISM ON RESISTIVE ELECTRODE IN POSITION-SENSITIVE DETECTORS*

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Abstract

A complete charge-division mechanism, including both the diffusion and the electromagnetic wave propagation on resistive electrodes, is presented. The charge injected into such a transmission line divides between the two ends according to the ratio of resistances and independently of the value of the line resistance, of the propagation mechanism and of the distribution of inductance and capacitance along the line. The shortest charge division time is achieved for $R_L = 2\pi(L/C)^{1/2}$, where R , L , C are resistance, inductance and capacitance per unit length and l is the length of the line.

Introduction

Position determination of the particle along the anode wire by charge division has been used and reported by several authors. The underlying physical processes, however, have been studied and understood only for diffusive approximation of the transmission line. Within this approximation, which neglects completely the inductance of the line, (and within an additional assumption about the line uniformity) it was shown^{1,2} that the charge division method is inherently linear. That is, the ratio of the charges collected at the two ends of the anode is the inverse ratio of the wire resistances between the injection point and the wire ends (see Fig. 1).

$$\frac{q_A}{q_B} = \frac{l-x_0}{x_0} \quad (1)$$

The possibility to use the charge division as an unambiguous second coordinate readout for drift chambers requires that the relation¹ is satisfied for a practical detector with a realistic value of the anode resistance. The value of the anode resistance is limited by the requirement of the drift time measurement. If the anode resistance were too big, the rise time of the signal passing through the anode would increase causing the degradation of the drift time measurement.

For typical values of anode resistance used in drift chambers, the diffusion approximation of the transmission line equation is not an adequate approximation and a complete description is needed.

As shown in Fig. 1, the anode wire with the surrounding electrodes can be considered as a transmission line with distributed R , L , C parameters (resistance, inductance, and capacitance per unit length, respectively, and $G = 0$).

In this paper, the propagation of signals along such a transmission line is studied theoretically. It is shown first that for any anode line with $R \neq 0$ there are two different propagation mechanisms always present,

- (a) electric diffusion
- (b) electromagnetic wave propagation.

The relative importance of each propagation mechanism depends on the line parameters. If the total ohmic resistance $l \cdot R$ of the anode is larger than the critical resistance $R_c \cdot l = 2\pi\sqrt{L/C}$ the electric diffusion mode of propagation dominates. If $l \cdot R < 2\pi\sqrt{L/C}$, the electromagnetic wave propagation is the dominant propagation mechanism. For the critical value of the anode resistance, both mechanisms are equally important.

It is shown that for the critical value of the anode resistance, the deterioration of the signal rise time due to the diffusion properties of the resistive anode is negligible for all practical applications. Moreover, it is shown that the critical value of the anode resistance minimizes the discharge time of the line, that is the time needed for the charge division method to give its asymptotic value which depends only on the position of the incident particle.

The asymptotic ratio of charges flowing out from the two ends of the anode wire is the inverse ratio of the wire resistances between the injection point and the wire ends, that is Eq. (1). This ratio does not depend on L or C of the line even in the general case where both $L(x)$ and $C(x)$ are functions of the position along the anode (including the lumped L, C added to the line). Only the time needed to reach the asymptotic ratio depends on all parameters of the line.

Theoretical conclusions are compared with the measurement.

Theory of the Charge Dividing Mechanism for a Uniform Line

The signal propagation along a resistive transmission line can be solved for a special case of a homogeneous line where all three parameters (resistance R , inductance L , and capacitance C per unit length) are constant along the length of the line.

The differential equation describing electromagnetic behavior of the line is the well known telegrapher's equation.

$$\frac{\partial^2 u(x,t)}{\partial x^2} = CL \frac{\partial^2 u(x,t)}{\partial t^2} + RC \frac{\partial u(x,t)}{\partial t} \quad (2)$$

$u(x,t)$ - voltage
 x - position coordinate of the line
 t - time
 R, L, C - resistance, inductance, and capacitance of the line per unit length.

The first term in Eq. (2) represents electromagnetic wave propagation and the second term represents diffusion. The boundary conditions due to the low impedance preamplifiers at the end of the line,

$$u(0,t) = u(l,t) = 0 \quad (3)$$

where l is the length of the line.

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The Eq. (2) with boundary conditions(3) can be solved by the Fourier series method. Let us assume the solution in the form

$$u(x,t) = \sum_{n=1}^{\infty} X_n(x) \cdot T_n(t) \quad (4)$$

Introducing (4) into (2), we can separate variables x and t and write:

$$\frac{1}{X_n} \frac{\partial^2 X_n(x)}{\partial x^2} = \frac{C}{T_n} \left(L \frac{\partial^2 T_n(t)}{\partial t^2} + R \frac{\partial T_n(t)}{\partial t} \right) = -\lambda_n^2 \quad (5)$$

The first equality of the Eq. (5), together with boundary conditions(3) is a regular Sturm-Liouville problem. It defines the eigenvalue λ_n^2 and its solutions are:

$$X_n(x) = A_n \sin \lambda_n x \quad \lambda_n = \frac{n \cdot \pi}{L} \quad n = 1, 2, \dots \quad (6)$$

where all A_n have to be defined from the initial conditions. The time equations then become,

$$CL \frac{\partial^2 T_n}{\partial t^2} + RC \frac{\partial T_n}{\partial t} + \lambda_n^2 T_n = 0 \quad (7)$$

which for each n is a simple linear second order differential equation with constant coefficients - a well known oscillator equation. Assuming the solution of the form

$$T(t) = e^{\alpha_n t} \quad \text{the auxiliary equation for } \alpha_n \text{ is}$$

$$CL \alpha_n^2 + RC \alpha_n + \lambda_n^2 = 0 \quad (8)$$

with the solution for α_n

$$\alpha_n^{(1,2)} = \frac{-RC \pm \sqrt{(RC)^2 - 4CL\lambda_n^2}}{2CL} = \frac{-RC \pm \sqrt{D_n}}{2CL} \quad (9)$$

The discriminant D_n in Eq. (9) depends on the degree of harmonics n . Solution of the Eq. (7) can be (1) underdamped, (2) overdamped, or (3) critically damped according to the value of D_n which includes the parameters of the line and the degree of the harmonic.

To write down the solutions explicitly, we have to specify the initial conditions. Without losing the generality, we can assume the pulse charge injection at point x_0 along the line. The first initial condition is:

$$u(x, t=0) = Q/C \cdot \delta(x-x_0) \quad (10)$$

The second condition simply states that the current in the line with a nonzero inductance cannot change discontinuously. (See also Appendix.) We can rewrite the second initial condition using the continuity equation

$$(A1) \cdot \frac{\partial i(x,t)}{\partial x} + C \frac{\partial u(x,t)}{\partial t} = 0 \quad \text{as} \quad \frac{\partial u(x,t=0)}{\partial t} = 0$$

After a straightforward but cumbersome calculation, we can write the solution of Eq. (2) as

$$u(x,t) = \frac{2Q}{CL} \cdot \sum_{n=1}^{\infty} \sin \frac{\pi \cdot n}{L} x_0 \cdot \sin \frac{\pi \cdot n}{L} x \cdot T_n(t) \quad (11)$$

(1) For the underdamped harmonics ($D_n < 0$)

$$T_n(t) = e^{-\frac{R}{2L} t} \cdot \left(\cos \omega_n t + \frac{R}{2L\omega_n} \cdot \sin \omega_n t \right)$$

$$\omega_n = \frac{\sqrt{-D_n}}{2LC} = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{n\pi}{L}\right)^2} \cdot \frac{1}{LC} - \left(\frac{R}{2L}\right)^2 \quad (12)$$

(2) For the overdamped harmonics ($D_n > 0$)

$$T_n(t) = A_n^{(1)} \cdot e^{\alpha_n^{(1)} t} + A_n^{(2)} \cdot e^{\alpha_n^{(2)} t}$$

with

$$\alpha_n^{(1,2)} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{n\pi}{L}\right)^2} \cdot \frac{1}{LC} \quad (13)$$

and

$$A_n^{(1)} = -\frac{\alpha_n^{(2)}}{\alpha_n^{(1)} - \alpha_n^{(2)}}; \quad A_n^{(2)} = \frac{\alpha_n^{(1)}}{\alpha_n^{(1)} - \alpha_n^{(2)}}$$

(3) For the critically damped harmonics ($D_n = 0$)

$$T_n(t) = e^{-\frac{R}{2L} t} \cdot \left(1 + \frac{R}{2L} t \right) \quad (14)$$

Let us study the decay time of the line, that is, the time needed to discharge the line. We shall see that the fundamental harmonic ($n=1$) has the slowest decay rate and, therefore, it defines the discharge time of the line. The line parameters R, L, C define the main harmonic to be either underdamped, overdamped, or critically damped. According to the character of the solution of the main harmonic equation we can define an RCL line to be:

- (1) underdamped line for $\sqrt{L/C} < 2\pi \sqrt{R/C}$
- (2) overdamped line for $\sqrt{L/C} > 2\pi \sqrt{R/C}$ or
- (3) critically damped line for $\sqrt{L/C} = 2\pi \sqrt{R/C}$

As it is well known and can be seen from the explicit solution¹¹ critical damping gives the most rapid decay ($\sim e^{-R_C/2L} t$). Higher harmonics ($n = 2, 3, \dots$) of a critically damped line have periodic solutions with the same exponential damping factor as the fundamental harmonics. Initial condition of an impulse charge injection at a certain position along the transmission line gives a white spectrum of the coefficient of the Fourier harmonics. We can see that the critically damped line gives the discharge time roughly corresponding to the decay time of the fundamental harmonic independent of the initial excitation of the line.

For the underdamped line, all harmonics (including the fundamental harmonic) are decaying as $e^{-R/2L}$ (Eq. 12). The line resistance R is smaller than the critical resistance R_C and so the decay time of an underdamped line is longer than the decay time for the critically damped line.

For the overdamped line at least the first harmonic has an aperiodical solution. Its slowly decaying exponential limits the discharge rate of the line to be lower than the discharge rate of the critically damped line. It is interesting to note that the propagation in a line with nonzero inductance cannot be purely diffusive. No matter how large the resistance R of the line is, the solutions of Eq. (11) are underdamped above a certain value of n . These solutions are needed to limit the speed of the signal propagation compared to the pure diffusion. Obviously, the periodic harmonics of the overdamped line are decaying as $e^{-R/2L}$, where $R_L > R_C$, therefore, much faster than the fundamental harmonic which dominates the behavior of the line for $t > L/R$.

We have just shown that the critically damped line gives the shortest decay time. Let us gain some insight into physical processes defining and limiting its rate of the decay.

For a critically damped line, the diffusion and the electromagnetic wave propagation are equally important. We can rewrite the time constant of the common exponential factor in the solution for the critically damped line (11) as

$$\tau_c = \frac{2L}{R_{crit}} = \frac{2L \cdot l}{2\pi\sqrt{L/C}} = \frac{l}{\pi v} = \frac{\tau_0}{\pi} \quad (15)$$

where $v = 1/\sqrt{LC}$ is the speed of the electromagnetic wave propagation along the line (in vacuum) and $\tau_0 = l/v$ is the time for the signal to travel the single length of the line. The decay time constant of the critically damped line is about one third of the time needed for light to travel the length of the line.

It is interesting to compare this time constant to the time constant obtained for a critically damped line from a well known diffusive approximation of an RCL line. For such an RC line, the time constant is

$$\tau_D = \frac{RC l^2}{\pi^2} \quad (\text{Ref. 2}),$$

which for $lR = 2\pi\sqrt{L/C}$, gives

$$\tau_D = \frac{2l}{\pi v} = \frac{2\tau_0}{\pi} = 2\tau_c \quad (16)$$

We see that for the diffusive approximation, the decay time constant is twice as long as for the realistic case. In the diffusive approximation, the charge from the central region of the line has to diffuse all the way to the end of the line. If the realistic case is considered, the charge from the central region is "pushed away" by the electromagnetic wave propagation mechanism resulting in a shorter discharge time constant.

Discussion of the Results for the Uniform Line

Equation (11) gives the complete solution of the problem, that is, the analytical expression for the voltage $u(x, t)$ across the line as a function of position coordinate x , time t and injection point x_0 . From this we can directly derive the time dependence of the currents flowing out of the ends of the line. The current rise time is the most important factor for the timing information used for the measurement of the drift coordinate. The decay rate of the current defines the discharge time of the line, that is, the time needed for the charge ratio to reach the asymptotic value. Its length defines the pile-up time and limits the rate capability of the charge division method.

The explicit relations for the current flowing out of the ends of the line are cumbersome and provide little insight into the physical phenomena. We shall, rather, present here the results of the calculation done by computer to show the signal propagation along a two meter long line used as a model, that is the voltage across the line $u(x, t)$ as a function of x with t as a parameter. Also, the currents at the ends of the line as seen by preamplifiers and charges collected by the preamplifiers as a function of time are shown.

Results are presented for three damping conditions; underdamped, (Figs. 2-4), overdamped, (Figs. 5, 6), and critically damped, (Figs. 7, 8). The total resistance of the overdamped line is twice the value of the critical resistance, the underdamped resistance is one half of the resistance for the critically damped line. We can see the relative importance of the diffusive and electromagnetic wave propagation for each case. Typical anode wire parameters are used ($c \approx 10 \text{ pF/m}$, $Z_0 \approx 330 \Omega$).

For the underdamped line, where the electromagnetic wave propagation mechanism dominates, there are large reflections at the shorted line ends (Figs. 2, 3). The remaining charge on the line changes the sign and it takes several reflections to discharge the line (Fig. 4).

The overdamped line attenuates the wave propagation very fast and after a few nanoseconds, only diffusive propagation is present. Then, the voltage distribution along the line is dominated by the fundamental harmonic. This harmonic is symmetrical with respect to the center of the line independent of the injection point (Fig. 5). The line discharge is relatively slow (Fig. 6).

In the critically damped line both the electromagnetic wave propagation and the diffusion are equally important. There is a reflected wave of the opposite polarity which tends to cancel the direct diffusive charge on the line (Fig. 7), decreasing the total discharge time. We can see that in 20 nanoseconds (≈ 10 times the τ_c for a 2 m long line under consideration) the remaining charge on the line is less than 0.1%.

The rise time of the current pulse at the line end is about 1 ns, and about 40% of the total charge is delivered to the preamplifier within this time (Fig. 8).

On Figs. 2-8, we have considered an example of a 2'm long anode wire with a capacitance per unit length $C = 10$ pF. We shall now derive the scaling relations for voltage, current and charge with the line parameters.

For proportional and drift chambers, the anode capacitance per unit length depends only logarithmically on the anode wire diameter and on the distance between the anode and the surrounding electrodes. The value of $C = 10$ pF/m taken in the above example is a typical value for most chambers. Given the C-value, then with $\epsilon_r = 1$, $\epsilon_0 = 1$, $Z_0 = \sqrt{L/C} \approx 330 \Omega$. Therefore, the critical resistance Z_{R_c} is typically $2 K\Omega$.

The length of the wire is a very important parameter. Decay time constant is proportional to the wire length as should be expected for the electromagnetic wave propagation mechanism (Eq. 15). The critically damped line defines the anode resistance in such a way that also the diffusive time constant is proportional to the wire length (Eq. 16). (This is the reason why Eq. (15) is so simple.)

Similarly, it can be shown that all time constants (and thus the rise time degradation) are proportional to the line length. Figures 2-8 can, therefore, be used for any length $\ell_1 = K \cdot \ell$ of the line with the following transformations:

$$\begin{aligned} t &\rightarrow t_1 = K \cdot t \\ x &\rightarrow x_1 = K \cdot x \\ u(x, t) &\rightarrow u_1(x_1, t_1) = \frac{u}{K}(x, t) \\ i(x, t) &\rightarrow i_1(x_1, t_1) = \frac{i(x, t)}{K} \\ Q(t) &\rightarrow Q(t_1) = Q(t) \end{aligned} \quad (17)$$

where subscript 1 denotes the new scaled variables.

Scaling relations (17) can be seen directly from the Eqs. (11-14). If $R/R_c \propto 1/\ell$ in (12), (13), and (14), Eq. (11) can be written as

$$\mu(\xi, \vartheta) = \frac{20}{C\ell} \cdot \sum_{n=1}^{\infty} X_n^{(1)}(\xi) \cdot T_n^{(1)}(\vartheta) \quad (18)$$

This is a universal function of the normalized variables $\xi = x/\ell$ and $\vartheta = t/\ell$. The only dependence on the length of the wire is in the coefficient of this equation.

Charge Dividing Mechanism for the General Case

In the Appendix, the proof of relation (1) (linearity of the charge division) is given for the general case where all line parameters $R(x)$, $L(x)$, $C(x)$ are the function of the position along the line. (We can arrive at the same conclusion for the special case of the uniform line directly from Eq. (11)). It states that the asymptotic ratio of the charges flowing out from the ends depend only on the ratio of the wire resistances and is independent of the capacitance and the inductance of the line.

Experimental Results

The linearity of the charge division method was tested several times and was reported in detail in Refs. 2 and 3, where also a more complete list of references can be found.

To test the rise time degradation of the current signal along the resistive line is much more difficult. The maximum degradation of the signal rise time occurs

for the injection point at the center of the line. In this worst case the rise time degradation for a critically damped line is only about 20% of the time needed for the electromagnetic wave to propagate along the full length of the line, i.e., for a 2 m long resistive line about 1 ns (Fig. 8). To detect such a small contribution to the overall rise time at the anode wire end, the injected signal and the preamplifier have to have a comparable rise time.

We used the common base preamplifier described in Ref. 3 where $22 K\Omega$ collector resistor of the first transistor was replaced by a $1 K\Omega$ resistor. (The collector current was increased to 2 mA.) The pulse response of the preamplifier is shown in Fig. 9(a).

Figures 9(b) and (c) show signal waveforms at the output of the preamplifier, taken with a storage oscilloscope. Figure 9(b) is a 5.9 keV X-ray at $x = 0$ seen at the near end. The rise time of about 5 ns is consistent with the diffusion of the electron cloud during its drift to the anode. This diffusion time was obtained with a gas mixture of 60% Ar and 40% C_2H_4 .

Figure 9(c) shows the same X-ray source at $x = 1$ m, that is, at the middle of the anode wire. The amplitude is $\frac{1}{2}$ of the previous signal waveform, but the rise time is the same. We can conclude that the rise time degradation of the signal along the 2 m long resistive wire is smaller than 2 ns.

Conclusions

The intrinsic linearity of the charge division method is particularly important for a large detector system with many wires, where a single two-point position calibration at the ends of the wire can be used. The measured particle position is independent of the reactive parameters L and C of the line. This is important in large systems where differences in L and C from wire to wire can be expected. Finally, the position calibration is also independent of the variations in the wire resistance with time and/or temperature.

From the studies of the response time of the line, we conclude that the resistance for wires of one to two meters in length should be between $R_c \ell$ and $2 R_c \ell$, that is, between 2 and 4 kohms. This is a compromise between the requirement for low noise (high wire resistance) and for minimum response time (critical damping). With these values of wire length and resistance, the discharge time of the line is shorter than the current pulse of the proportional chamber, which is determined by the drift velocity of positive ions. The degradation of the signal current rise time due to the propagation along the anode is small, since the signal rise time for a single cluster of primary ionization is limited to 5 to 10 nanoseconds by the diffusion during the electron drift to the anode. Therefore, the response time of such a line does not limit the rate and the timing accuracy in drift chamber applications.

These results prove the compatibility of the charge division method with the drift time measurement. This is important for construction of large drift chambers with unambiguous readout and of certain types of time projection chambers.

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APPENDIX

The anode wire and the surrounding electrodes (Fig. 1) represent a transmission line with distributed R, L, C parameters (resistance, inductance and capacitance per unit length, respectively), and conductivity $G = 0$.

As it is well known, the current and voltage as functions of position x along the wire and time t have to satisfy two telegrapher's equations.

$$\frac{\partial u(x,t)}{\partial x} + L(x) \frac{\partial i(x,t)}{\partial t} + R(x)i(x,t) = 0 \quad (A1)$$

$$\frac{\partial i(x,t)}{\partial x} + C(x) \frac{\partial u(x,t)}{\partial t} + (G u(x,t)) = 0$$

Boundary conditions for the anode wire terminated by low impedance amplifiers (shorted) are,

$$u(0,t) = u(l,t) = 0 \text{ for } t \geq 0 \quad (A2)$$

where l is the length of the anode.

Injection of charge into the line defines the initial conditions,

$$\begin{aligned} u(x,0) &= V(x) & 0 \leq x \leq l \\ i(x,0) &= I(x) \end{aligned} \quad (A3)$$

We are interested in the total (asymptotic) charge flowing out from each end

$$\begin{aligned} -Q_a &= Q(0) = \int_0^\infty i(0,t)dt \\ Q_b &= Q(l) = \int_0^\infty i(l,t)dt \end{aligned}$$

This suggests the integration of the Eqs. (A1) with respect to t from 0 to ∞ . (Special case of the Laplace transform solution for $s = 0$.) Introducing the notation

$$\begin{aligned} U(x) &= \int_0^\infty u(x,t)dt \\ Q(x) &= \int_0^\infty i(x,t)dt \end{aligned} \quad (A4)$$

and using the initial conditions (A3), we obtain by integrating (A1),

$$\begin{aligned} \frac{\partial U(x)}{\partial x} + R(x) Q(x) &= L(x) I(x) \\ \frac{\partial Q(x)}{\partial x} &= C(x) V(x) \end{aligned} \quad (A5)$$

The second equation of (A5) can be integrated again with respect to x

$$Q(x) = \int_0^x C(y) V(y) dy + Q(0) \quad (A6)$$

Substituting the expression (A6) into the first equation (A5), after integration, we obtain

$$U(x) = -Q(0) \int_0^x R(z) dz - \int_0^x dz R(z) \int_0^z C(y) \cdot V(y) dy + \int_0^x L(z) I(z) dz \quad (A7)$$

We used here the integrated boundary conditions (A2)

$$U(0) = \int_0^\infty u(0,t)dt = 0$$

Knowing that $U(l) = 0$ as a consequence of the same boundary condition, we can evaluate expression (A7) for $x = l$ and calculate $Q(0)$ from it.

$$Q(0) = \frac{1}{\int_0^l R(z) dz} \cdot \left[\int_0^l L(z) I(z) dz - \int_0^l dz R(z) \int_0^z C(y) V(y) dy \right] \quad (A8)$$

Substituting $Q(0)$ into (A6), we can evaluate $Q(x)$ for $x = l$,

$$\begin{aligned} Q(l) &= \int_0^l C(y) V(y) dy + \frac{1}{\int_0^l R(z) dz} \cdot \\ &\quad \left[\int_0^l L(z) I(z) dz - \int_0^l dz R(z) \int_0^z C(y) V(y) dy \right] \end{aligned}$$

or after a short manipulation we can write:

$$Q(l) = \frac{1}{\int_0^l R(z) dz} \cdot \left[\int_0^l L(z) I(z) dz + \int_0^l dz R(z) \int_z^l C(y) \cdot V(y) dy \right] \quad (A9)$$

The expressions (A8) and (A9) give the total asymptotic charge collected from the ends of the anode wire as functions of the transmission line parameters $R(x)$, $L(x)$, $C(x)$ and the initial condition $V(x)$ and $I(x)$. To apply the above equations for our case, we have to find the initial conditions corresponding to the impulse charge injection into the transmission line.

Assuming that the line was completely discharged at time $t = 0$, the injected charge causes the potential of the anode to move so the first initial condition (A3) can be written as

$$u(x,0) = V(x) = \frac{q(x)}{C(x)} \quad (A10)$$

where $q(x)$ is the linear density of the injected charge. The initial condition (A10) is correct also in the

limiting case of the point injection where $q(x) = Q \cdot \delta(x-x_0)$. If the line has a zero inductance, the corresponding differential equation is of the first order with respect to time and only one initial condition would be required. (Current at time $t = 0$ is defined by the differential equation and the first initial condition.) If the line has a nonzero inductance $L(x)$, the injected charge does not excite the magnetic energy at $t = 0^+$ therefore

$$\frac{1}{2} I(x)^2 \cdot L(x) = 0$$

and

$$I(x) = 0 \quad (A11)$$

In any case, the product $I(x) \cdot L(x)$ in Eqs. (A8), (A9) is equal to zero which, together with the initial condition (A10), allows us to write

$$Q(0) = \frac{-1}{\int_0^l R(z) dz} \left[\int_0^l dz R(z) \int_0^z q(y) dy \right] \quad (A12)$$

$$Q(l) = \frac{1}{\int_0^l R(z) dz} \left[\int_0^l dz R(z) \int_z^l q(y) dy \right] \quad (A13)$$

Asymptotic charges $Q_a = -Q(0)$ and $Q_b = Q(l)$ in expressions (A12) and (A13) do not contain $L(x)$ and $C(x)$ so the charge ratio depends only on the resistive properties of the anode.

For the usual case of the uniform anode resistance $R(l) = R = \text{constant}$, one integration in expressions (A12) and (A13) can be performed and for the charge ratio we obtain,

$$\frac{Q_a}{Q_b} = \frac{\int_0^l q(y) (l-y) dy}{\int_0^l q(y) \cdot y dy} \quad (A14)$$

Relation (A14) is the generalized charge division formula. It also proves that the charge division determines the centroid of the injected (or induced) charge. For the special case of the point injection $q(x) = Q\delta(x-x_0)$, it reduces to the relation (1).

References

1. S. Kalbitzer and W. Melzer, Nucl. Instr. & Meth. 56 (1967) 301.
2. J. L. Alberi and V. Radeka, IEEE Trans. Nucl. Sci., NS-23 (Feb. 1976) 251-258.
3. V. Radeka and P. Rehak, IEEE Trans. Nucl. Sci., NS-25 (1978) 46-52.

Figure 1

Resistive anode of a proportional detector with surrounding electrodes represents a transmission line with distributed R, L, C parameter. The charge Q is injected at position x_0 at the time $t = 0$. The notation for currents and voltages is shown.

Fig. 2. Voltage along the 2m long anode wire at different times after an infinitely sharp charge was injected at the wire center for the under-damped line with the total resistance $4R$ equal $\frac{1}{2}$ of the critical resistance.

Fig. 3. Voltage along the 2m long anode wire at different times after an infinitely sharp charge was injected 68 cm (1/3 of the wire length) from the left end of the wire. The line is underdamped with the resistance equal $\frac{1}{2}$ of the critical resistance.

Fig. 4. (P18) Charge at the end of the same underdamped line as a function of time in ns for an impulse of charge injected at $X = 68$ cm, solid - at the near end, dashed - at the far end.

(P19) Charge collected at both ends (solid) and charge remaining on the line (dashed).

(P20) Charge ratio $QL/(QL + QR)$ as a function of time (solid).

Fig. 5. Voltage along the 2m long anode wire at different times after an impulse of charge was injected 68 cm from the left end of the wire. The line is overdamped with the resistance equal twice the critical resistance.

Fig. 6. (P84) Charge at the end of the same overdamped line as a function of time in ns for an impulse of charge injected at $X = 68$ cm, solid - at the near end, dashed - at the far end.

(P85) Charge collected at the both ends (solid) and charge remaining on the line (dashed).

(P86) Charge ratio $QL/(QL + QR)$ as a function of time in ns (solid).

Fig. 7. Voltage along the 2m long anode wire at different times after an impulse of charge was injected 68 cm from the left end of the wire. The line is critically damped, that is, the wire resistance equals the critical resistance.

Fig. 8. (P51) Charge at the end of the same critically damped line as a function of time for an impulse of charge injected at $X = 68$ cm, solid - at the near end, dashed - at the far end.

(P52) Charge collected at the both ends (solid) and charge remaining on the line (dashed).

(P53) Charge ratio $QL/(QL + QR)$ as a function of time in ns (solid).

(P54) Current at two ends of the line as a function of time in ns, solid - at the near end, dashed - at the far end, dotted - the sum (right scale)

(a)

(b)

(c)

Figure 9

Signal waveforms at the output of the current pre-amplifier:

- a) after an impulse excitation with a pulser with 20 ps risetime.
- b) 5.9 KeV X-ray source at $x_0 = 0$.
- c) the same X-ray source at $x_0 = 1$ m (middle of the line).