LABORATORY OF NUCLEAR MEDICINE AND RADIATION BIOLOGY
900 VETERAN AVENUE
UNIVERSITY OF CALIFORNIA, LOS ANGELES, CALIFORNIA 90024
AND DEPARTMENT OF RADIOLOGICAL SCIENCES
UCLA SCHOOL OF MEDICINE, LOS ANGELES, CALIFORNIA 90024

These studies were supported by Contract #DE-AM33-76-SF00012 from the U.S. Department of Energy and the University of California.

Prepared for the U.S. Department of Energy,
under Contract #DE-AM33-76-SF00012

THE SIGNAL AMPLIFICATION TECHNIQUE (SAT),
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Reducing Image Noise in Computed
Tomography

Michael E. Phelps, Sung-Cheng Huang,
Edward J. Hoffman, David Plummer,
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**THE SIGNAL AMPLIFICATION TECHNIQUE (SAT):
An Approach for Improving Resolution and Reducing Image Noise
in Computed Tomography**

Michael L. Phelps, Song-Cheng Huang, Edward J. Hoffman,
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Abstract: Signal amplification technique

Source: Staff of the Department of Energy contract DE-AM00-76-SF000012 and
contracts E-40400 and E-401-NS15554

Division of Biophysics,
Department of Neurological Sciences & The Laboratory of Nuclear Medicine,
Laboratories of Biomedical and Environmental Sciences,
UCLA School of Medicine, Los Angeles, California

Key words: Positron Computed Tomography
Signal amplification
Spatial resolution
Noise reduction

ABSTRACT

Spatial resolution improvements in computed tomography (CT) have been limited by the large and unique error propagation properties of this technique. The desire to provide maximum image resolution has resulted in the use of reconstruction filter functions designed to produce tomographic images with resolution as close as possible to the intrinsic detector resolution. Thus, many CT systems produce images with excessive noise with the system resolution determined by the detector resolution rather than the reconstruction algorithm. CT is a rigorous mathematical technique which applies an increasing amplification to increasing spatial frequencies in the measured data. This mathematical approach to spatial frequency amplification cannot distinguish between signal and noise and therefore both are amplified equally. We report here a method in which tomographic resolution is improved by using very sharp detectors to selectively amplify the signal and not noise. Thus, this approach is referred to as the signal amplification technique (SAT). SAT can produce dramatic improvements in image resolution without increases in statistical noise or dose because increases in the cutoff frequency of the reconstruction algorithm are not required to improve image resolution. Alternatively, in cases where image counts are low, such as in rapid dynamic or receptor studies, statistical noise can be reduced by lowering the cutoff frequency while still maintaining the best possible image resolution. A possible system design for a positron CT system with SAT is described.

INTRODUCTION

Computed Tomography (CT) is a technique for mathematically reconstructing cross sectional images from projections of an object. It is equivalent to collecting a sufficient number of independent linear equations to mathematically solve for the unknown distribution of tissue attenuation coefficients in x-ray CT, tissue radionuclide concentration in emission CT, acoustical interactions in ultrasound CT and parameters related to magnetic moments in NMR CT. This approach has a fundamental basis in the solution of simultaneous algebraic equations. In the discretely sampled, spatial frequency band limited case of real tomographs, the requirements (i.e. sampling, spatial frequency weighting, noise propagation, etc) for image reconstruction have been well developed under various criteria for optimization and uniqueness (i.e. minimum mean square error, maximum likelihood, maximum entropy, etc.)

Operationally, these approaches have employed a variety of Fourier based (i.e. Radon and Fourier transforms, Fast Fourier Transforms and Convolution) or algebraic techniques to reconstruct the tomographic image from projections at multiple angles around the object. It is this mathematical basis that also separates CT from previous forms of medical tomography that employed scanning approaches from combinations of moving detectors, x-ray beams or objects. The extensive literature and details of these approaches have been reviewed elsewhere (1-5).

Improvements in spatial resolution for all forms of CT have been limited by its unique error propagation properties which amplify the effect of errors and noise, especially those with high frequency components. The primary source of difficulty arises from the relationship between statistical noise in the reconstructed image and spatial resolution. If a constant ratio is maintained between spatial resolution and sampling distance (e.g. at least

2/1), then the noise (variance) is inversely proportional to the third power of the spatial resolution (2, 6-10, appendix). Thus, an improvement in resolution by a factor of 2 requires an increase of a factor of 8 in the total number of image counts if the noise level per resolution element is to remain constant as resolution is improved.

The primary approach to solving this problem has been based on tomograph designs which attempt to collect the largest possible number of events per image. In positron CT (PCT), the designs include new detector materials for higher intrinsic efficiency, small system diameters to increase the solid angle for detection, and multiple plane designs which increase efficiency in comparison to single plane geometries. The strengths and weaknesses of these design considerations are discussed elsewhere (8,9,11,13-32). Recently a technique has been discussed in which the location of a positron decay between two detectors is located approximately by measuring the difference in arrival time of the two annihilation photons at the coincidence detectors. This approach uses the high time resolution of CsF detectors and is referred to as time of flight, TOF (33-36). The TOF information is combined with the conventional reconstruction technique to reduce statistical noise in the image.

In the work reported here, we have studied the effect of reducing the detector resolution or beam width to a value well below the expected final reconstructed image resolution and demonstrate that the proper combination of the intrinsic detector resolution and reconstruction filter function can produce significant improvements in image resolution without increasing image noise. Alternatively, one can maintain good image resolution while significantly reducing image noise. This is accomplished by selectively amplifying the signal and not noise and thus this approach is referred to as the signal amplification technique, SAT (30). These investigations were carried out by

theoretical analysis, computer simulations and experimental measurements.

IMAGE RECONSTRUCTION PROBLEM WITH FINITE DETECTOR RESOLUTION: DEFINITION OF PROBLEMS AND THEORETICAL CONSIDERATIONS

In CT the computation is performed as if the detector width were zero (i.e. scan points are represented by line integrals). The finite detector width is only taken into account in choosing the sampling distance and by limiting the cutoff frequency (limit of high frequency recovery) of the reconstruction algorithm. The criteria for choosing the cutoff frequency in terms of the finite detector resolution (more properly the frequency modulation or modulation transfer function, MTF, of the detector) has not been well characterized in the literature. Normally, the cutoff frequency is chosen high enough to produce a tomographic image with a spatial resolution equal to or slightly less than the detector resolution in emission CT or beam width in x-ray CT.

The result of the finite detector resolution or beam width is a blurring of the final reconstructed image. This blurring is equivalent to a 2 dimensional convolution of the actual cross sectional structure with the detector line spread function (assuming system has a spatially invariant response). Thus, if one started with conventional blurring tomography (figs 1B and 2A), referred to as linear superposition of back projections, LSPB (i.e. simple backprojection), the image would contain two sources of blurring (fig 2A). The first is the $1/r$ blurring inherent to LSPB. This can be removed, within the limits set by the cutoff frequency and therefore, the detector resolution, by convolving the transverse scan profiles with a ramp filter or some approximation of this filter such as a Shepp-Logan (46) type filter before back projection. This approach is referred to as linear superposition of filtered

back projections (LSFBP) and represents a true image reconstruction technique (commonly referred to as convolution in CT). This is shown in figure 1C, D and 2B. The resulting image, however, contains the residual blurring due to the finite detector resolution (12 mm FWHM in fig. 2B) which is the second form of blurring inherent to CT.

Figure 2C illustrates the removal of the finite detector (i.e. to the limit for the original data set in this study) while retaining the 1/r blurring of LBP or simple back projection (note high spatial frequency amplification which provides recognition of fine structure). Applying a conventional LBP CT reconstruction to remove 1/r blurring yields the image shown in figure 2D which has had both forms of blurring removed.

It should be pointed out that while figure 2C shows the importance of high frequency amplification by improving the intrinsic detector resolution, the 1/r blurring of LBP results in a superposition of these data on a large background (i.e. the detailed features of the brain occur as small amplitude variations on a large background). If statistical noise were added to these images then the noise level would have to be very low (i.e. image counts very high) to preserve the structural features of the object because the image contrast is very low.

Image Restoration

Bracewell (47,48) recognized the blurring effect in CT due to the finite detector resolution and proposed a technique to improve image resolution by taking the finite detector resolution into account. This approach attempts to restore the true signal content of the object and is therefore referred to as image restoration (48). Bracewell (48) and Verly and Bracewell (49) have developed the mathematical basis for restoration and the form of the integrals

over a finite beam width for x-ray CT. The proposed strip integral kernel allows for extended source/detector geometries, non-uniform x-ray emission and detector sensitivity, and eccentricity of the system response. These authors clearly recognized the difficulties involved in this approach due to limitations involved in spatially variant point response functions, statistical factors, discrete sampling intervals and other imperfections of real systems.

A major difficulty in this approach to improve resolution results from the fact that one is trying to recover the higher spatial frequencies of the measured signal that have low amplitudes in the intrinsic response function of the detector (i.e. a low signal-to-noise, S/N). This condition of course results from the finite detector resolution which attenuates the spatial frequencies of the object with the highest attenuation occurring at the highest spatial frequencies. For example, as seen in figure 3A, very few frequencies have modulation transfer function (MTF) values of 100% and there is a rapid decline in the MTF or amplitude of the higher frequencies that one would like to recover in the final image. Increasing the amplitude of these higher frequencies must be done from a low S/N environment with large weighting factors which amplify the noise as well as the signal. In addition, even if the data are well sampled, any inconsistencies (i.e. errors) in the point response function will also be amplified.

This problem is further compounded in CT since the reconstruction filter function gives, in general, increasing weight to increasing spatial frequencies (fig. 3B) and therefore any errors that occur at high frequencies will receive the highest amplification. Since restoration is applied to the higher frequencies, the reconstruction process will produce a high amplification of any inconsistencies or statistical noise generated from the restoration process. This increasing weighting with increasing frequencies in the CT recon-

struction algorithm is the reason why small errors at high spatial frequencies appear as large errors in the final reconstructed images. In CT design trade offs, it is clearly better that errors or imperfections occur at low spatial frequencies so that they will be suppressed rather than amplified. It is also important to avoid errors that are correlated or reinforced such as an error that sweeps through a fixed arc to yield reinforced circular artifacts (50).

Although the formulation of an approach to take the finite detector resolution (and variations that occur with this response width) into account can provide a better basis for dealing with data taken with actual CT devices, significant improvements in resolution with this approach will be limited for the reasons discussed above.

Signal Reproduction Technique (SAT)

In CT, a reconstruction filter function is typically employed which will produce a point or line spread function response in the reconstructed image that is equal to or slightly worse than the intrinsic response of the detector. Normally resolution is improved by increasing the intrinsic detector resolution and the cutoff frequency of the filter in a corresponding fashion. This typically results in a linear increase in resolution and a power function increase in image noise because of the inverse third power relationship between resolution and image counts discussed earlier. Since CT, and also restoration, can be viewed as a high spatial frequency amplification techniques, one needs a method of amplifying the higher frequencies that avoids the noise amplification of these approaches. This problem has motivated us to examine other approaches to resolution improvement that would possibly circumvent or lessen this problem of noise amplification in CT. An approach described by Phelps et al (30) is to dramatically increase the

intrinsic detector resolution while using a reconstruction filter function with a cutoff frequency well below the highest spatial frequencies in the detector MTF. Thus the image resolution is improved by having greater intrinsic amplitude in the intermediate frequencies (fig 3A). This approach selectively amplifies the signal and not noise in the scan data. This method provides improved image resolution after reconstruction without a concomitant increase in image noise. Alternatively, the increased frequency amplitude allows a reduction in image noise while maintaining the best possible image resolution in cases where it is necessary to use a lower cutoff frequency due to low image counts (i.e. rapid dynamic or receptor studies).

The above can be understood from the following considerations. Assume that sampling is sufficient so as not to pose a limit in the reconstruction, then the image resolution (MTF_i) is related to the detector intrinsic resolution (MTF_d) and the reconstruction filter function as

$$MTF_i(f) = MTF_d(f) \{FRF(f)/f\} \quad (1)$$

where f is spatial frequency and $FRF(f)$ is the frequency response function of the reconstruction filter. Assuming both the detector and image line spread function (LSF) can be represented by Gaussian functions, then

$$MTF_i(f) = \exp \left\{ -\frac{2\pi^2 f^2}{2.36^2} W_i^2 \right\} \quad (2)$$

and

$$MTF_d(f) = \exp \left\{ -\frac{2\pi^2 f^2}{2.36^2} W_d^2 \right\} \quad (3)$$

where W_i and W_d are the full width at half maximum (FWHM) of the image and detector LSF, respectively. Thus, for a given detector resolution, if one desires a particular image resolution, the FRF(f) of the reconstruction filter should have the following form.

$$\text{FRF}(f) = |f| \exp \left(-\frac{2\pi^2 f^2}{2.36^2} (W_i^2 - W_d^2) \right) \quad (4)$$

$$= |f| \exp \left(-\frac{2\pi^2 f^2}{2.36^2} (W_f^2) \right) \quad (5)$$

where

$$W_f^2 = W_i^2 - W_d^2 \quad (6)$$

In other words, if a filter of this form with parameter W_f is used, the resultant image resolution, W_i , will be $(W_d^2 + W_f^2)^{1/2}$.

In Eq. image noise (variance) is related to the reconstruction filter by (Appendix)

$$\sigma^2/m^2 = \frac{2D^3}{N} \int_{-\infty}^{\infty} \text{FRF}(f)^2 df \quad (7)$$

where σ^2/m^2 is the fractional variance, D is the object diameter and N is the total number of recorded events. Equation 7 shows that the image noise level is the same if D , N and the filter function are constant. For FRF of the form shown in eq 4,

$$\int_{-\infty}^{\infty} \text{FRF}(f)^2 df = \frac{2.36^3 (\pi)^{1/2}}{8\pi^3 W_f^2} \quad (8)$$

$$= \frac{0.4635}{\pi^2} (W_1^2 - W_0^2)^{-3/2} \quad (9)$$

and thus by eq. 1

$$S_{f,m}^2 \leq \frac{0.4635^2}{\pi^2} (W_1^2 - W_0^2)^{-3/2} \quad (10)$$

Since the term $(W_1^2 - W_0^2)$ is in the denominator of eq. 10, the greater the difference between W_1 and W_0 or more simply the smaller the value of W_0 , the lower the image noise for a given image resolution.

On the basis of the above, the improvement in image resolution by increasing the intrinsic detector resolution without changing the image noise level (without changing the reconstruction filter function) can be seen from the following. If a reference intrinsic detector resolution (W_{RD}) and a reference reconstruction filter W_1 are chosen such that W_1 is given by $(1+u)W_{RD}$, where $u > 0$, then from eq. 9

$$W_1^2 = (W_1^2 - W_0^2) + W_0^2 \quad (11)$$

$$W_1^2 = ((1+u)^2 W_{RD}^2 - W_{RD}^2 + W_0^2)^{1/2} \quad (12)$$

or

$$W_1/W_{RD} = ((1+u)^2 - 1 + (W_0/W_{RD})^2)^{1/2} \quad (13)$$

where u is the fractional loss in image resolution compared to the intrinsic resolution, which is suffered by a CT system. The effect of DFI can be

examined by making comparisons to different reference situations in which the image resolution is worse than a reference detector resolution by (100)%. Thus the values of λ_1 are chosen to be $((1+\alpha)^2 - 1)^{1/2} \lambda_{PD}$.

Equation 10 is plotted in figure 4 for various values of α (i.e. where the reference image resolution is 5, 10, 20, etc % worse than the reference detector resolution). These curves show that the image resolution is significantly improved as the intrinsic detector resolution is improved even though the reconstruction filter function is not changed and therefore image noise is kept constant. It is also apparent that the image resolution improvement is greatest when α is small. A small value of α is the norm in conventional image resolution is usually quoted in the literature (5,6) as ranging from 10% worse than or equal to the intrinsic detector resolution of beam width. Figure 4 and eq. 10 show that there is a point of diminishing returns in image resolution. This is due to the fact that the amplitude of the signal and the noise are approaching 100% at these detector resolutions (5,6) and there isn't room for large increases in amplitude within the dynamic range. In addition, the reconstruction filter function is becoming the determining factor in the image resolution rather than the intrinsic detector resolution, as was initially the case. However, for the case of finest detector resolution, increases in the filter cutoff frequency (i.e. extending the range of frequency amplification, Fig. 5), of course with some noise amplification, will extend the DAI approach.

In situations where the number of counts is low, image noise can be reduced while still maintaining a reasonably good image resolution as shown by eq. 10. For a reference consider a conventional reconstruction with a ramp filter with a cutoff frequency $F = 1/\lambda_1$. In this case the integral of $(\sin x)^2$ in eq. 7 is $2/3\lambda_1^3$ and thus the image noise is given by (appendix)

$$(\sigma^2/m^2)_{\text{ramp}} \approx \left(\frac{253}{1.5N} \right) \left(\frac{1}{N_1^3} \right) \quad (14)$$

The degree of noise suppression with eq 14 as a reference is given by the ratio of eq 16 to eq 14.

$$\begin{aligned} \text{noise suppression (variance)} &= \frac{(0.4635)1.5}{2} (1 - W_d^2/N_1^2)^{-3/2} \\ &= 0.07044(1 - W_d^2/N_1^2)^{-3/2} \end{aligned} \quad (15)$$

When the intrinsic detector resolution is very small compared to the reconstruction resolution ($W_d^2/N_1^2 \rightarrow 0$), then

$$\text{noise reduction} = \text{uniform} \quad (16)$$

Thus, for a fixed number of counts and image resolution, the image noise decreases as the detector resolution is decreased and a limit is reached as the resolution approaches zero ($N_1 \rightarrow \infty$). At this limit the image noise is only about 1/3 of the case with $N_1 = 1$ and when a ramp reconstruction filter is used. A plot of the relationship given by eq. 15 is shown in figure 5 as the solid line. The dashed line shows the relationship if the FWHM values of the filters were set to zero for all frequencies above $1/N_1$ as is the case for the ramp filter used as a reference. The noise reduction occurs as a result of the fact that with a progressively smaller intrinsic detector resolution, one can maintain the same image resolution with a progressively lower cutoff frequency in the reconstruction filter function (Figs 3B). While it is true, as stated earlier, that image noise increases as the 3rd power (eqs 6, 14) of

the cutoff frequency in the filter function (i.e. spatial resolution), it is equally true that noise decreases at a third power rate as the cutoff frequency is lowered. A limit is reached when λ_1 is large compared to λ_0 (eq 10). This occurs when the filter function is the dominant factor, as opposed to the detector resolution, in determining the image resolution. When the filter becomes dominant then the image resolution decreases directly with the decrease in the cutoff frequency even though the intrinsic detector resolution is unvarying time.

MATERIAL AND METHODS

Image Simulations

A computer simulation data base was generated for bar phantoms, brain slices, and cylinders and dog head slices. The bar phantom consisted of a series of bars and spaces of different widths. A base set of simulated transverse slices was generated by sampling (and sample width) and 1.125° angular increments of 1.0° were convolved with Gaussian functions of different full widths at half maximum (FWHM) to produce scan profiles representing different detector resolution. These data were then used to reconstruct images of the bar phantom with different combinations of detector resolutions and reconstruction filter functions. In all cases the filter was either a ramp or a Shepp-Logan (40) type. The frequency response functions of the filters used in this work are shown in figure 3a. The final image resolution was assessed by measuring the peak to valley amplitude of the bars and spaces (5).

Simulations of an actual human brain slice were performed to examine a realistic object with a wide range in the size (i.e. spatial frequencies) of structures. Brain slices stained with Mulligan's stain to visually enhance gray/white matter delineation were digitized with a video digitizer in 1 x 1

mm format with the brain slice in 1 to 1 scale (see fig 9). A threshold technique was used to separate gray and white matter structures and CSF fluid which were given image values of 8, 2, and 1, respectively as described elsewhere (52). These data were employed to generate transverse scan profiles (1 mm sampling and sample width) at 1.5° angular spacing over 180°. These transverse scan profiles were convolved with different Gaussian shaped detector resolutions and statistical noise was added with a pseudorandom number generator for various levels of total image counts. In these final data sets for image construction, the transverse and angular sampling were 2.45 mm and 1.5°. From these data, images were reconstructed using a Shepp-Logan (46) type filter (fig 3b) and a convolution reconstruction algorithm.

Images of uniform cylinders were also reconstructed for different detector resolutions and levels of statistical noise using a simulated data set.

Simulated cylindrical phantoms (17 cm diam) were reconstructed both with and without photon attenuation to examine effect of attenuation on the distribution and magnitude of statistical error.

Simulated reconstructions of a dog heart were also performed by digitizing an actual cross sectional slice through the midplane of the heart. Data format and reconstructions were the same as discussed above for the brain slice except no noise was added to the images. The contrast values were 1 for left ventricle, 0.5 for right ventricles and 0.5 for a small 3 x 12 mm ischemic defect.

Experimental Line Spread Functions

Line spread functions (LSF) were measured for different size and shape bismuth germanate (BGO), cesium fluoride (CsF) and NaI(Tl) detectors using the UCLA tomographic simulator (53). The line source was 1 mm diam, 10 cm long and contained ^{64}Cu . All measurements were performed with detector pairs

separated by 60 cm with the line source midway between the two detectors. The finite size of the line source was removed by deconvolution since it was a significant portion of the LSF for the smaller detectors studied. LSFs were simultaneously recorded with energy thresholds at 100 keV and for photopeak only.

Tomographic Studies

Tomographic studies were performed with the NeuroECAT (EG&G/ORTEC, Oak Ridge, Tenn.) (31,32). This system has an intrinsic detector resolution of 7.4 mm (high resolution mode) and 9.8 mm (low resolution mode) with a transverse and angular sampling of 2.6 mm and 1.8/5°, respectively. Image reconstruction is performed using a convolution based algorithm with a Shepp-Logan type filter with selectable cutoff frequencies. The axial resolution is 13.4 mm.

A 20 year old normal volunteer was injected intravenously with 10 mCi of [18 F]-fluoro-2-deoxy-glucose (FDG) which was prepared by the method described elsewhere (33). Approximately 40 min were allowed for near steady state conditions before scanning was initiated (35,36). Photon attenuation was corrected using a geometric technique with an elliptical boundary around the head (17). A study was also performed in the same manner with FDG administered intravenously to a 20 kg dog to image the myocardium. Measured values of attenuation with an external ring source of 68 Ga positron activity (17) were used to compensate for photon attenuation.

RESULTS

Resolution Improvement with SAT at Constant Image Noise

Figure 6 illustrates the SAT approach with the simulated reconstruction of a bar phantom. The lower right hand image is a conventional reconstruction

with a 12 mm detector resolution and a filter sufficient to produce an image resolution of 12.4 mm. As expected, the 5.5 mm bars are unresolved. The other images in figure 6 demonstrate the image resolution improvement as the detector resolution is improved from 12 to 9, 6, and 3.5 mm without changing the CT filter. Since the same reconstruction filter is used, the image noise will be the same in each image (see Fig 8B). The effect of SAT was also examined with the bar phantom simulation by using different filter functions to alter the ratio of the reference image resolution to reference detector resolution (i.e. different values of α in eq 13). The values of $\alpha = 3, 20, 33$ and 40% are shown in figure 4 and compare well to the theoretical predictions. Figure 5 is an example with $\alpha = 3\%$.

Figures 7A and 8 and 8A illustrate the resolution improvement with reconstructed images of brain slices with different intrinsic resolutions and number of image counts without changing the filter functions. These studies illustrate the effect of the signal amplification technique with the actual spatial frequency distribution of structures in the human brain. Filter F in figure 5b was used for all of the images of figures 7 and 8. While figures 7A and 7b with 20 and 1 million counts clearly show the resolution improvement with SAT, figure 8A with 250K counts shows more clearly how the SAT is able to raise the high spatial frequency structures of the brain above the noise level in an image. With the 12 mm detector resolution, the spatial frequency amplitude of the structures (signals) are suppressed to the level of the noise. However, as SAT is increased one reaches the point at which, even with this few number of image counts, the S/N ratio is so high that the image noise is barely perceived. The actual image resolution in these studies (figs 7 and 8) are estimated to be 12.4, 9.6, 6.8, 5.2 and 4.1mm with the intrinsic detector resolutions of 12, 9, 6, 4, and 2 mm respectively.

Figure 8b, is the reconstruction of uniform cylinders under the same condition (i.e. number of image counts, sampling, filter function and changes in detector and image resolution) as in figure 8A. This study illustrates the constant noise level in the image reconstructions of figures 7 and 8A. The actual noise level (% standard deviation) for the different uniform cylinders was ± 3.09 , ± 3.8 and ± 27.5 for 20, 1 and 0.250 million counts respectively, which is in agreement with theoretical predictions published previously (8-11). These values are independent of the intrinsic detector and image resolution. The only noticeable change in the images of figure 8B is the sharpening of the outer edge (i.e. high frequency component of the cylinder) with the increased image resolution from SAT.

All of the simulation studies in this work have neglected the effect of photon attenuation which effectively redistributes the recorded counts and contributes noise because there is a higher density of observed counts at the edge of the object relative to the higher attenuating central region. The magnitude of this effect was examined by reconstructing the uniform phantom with the same total number of image counts as in figures 7 and 8 but redistributing them in proportion to the attenuation of 511 keV photons. This produced an average increase in the noise in a center 5 cm diameter region of 17% over the value in the image with no attenuation and no change in a 1 cm thick ring 5 mm from the outer edge. Thus, neglecting photon attenuation in this work produced a minor perturbation on the results presented here.

To compare the analytical and simulated results to actual tomographic results, figure 9 shows a simulated and measured brain section at the level of basal ganglia in which each image contains about 5 million counts, intrinsic detector resolution of about 8 mm and image resolution of 10 mm. There is a good correspondence in image quality between the simulated and actual tomogra-

pic image even though the latter contains summing in the axial direction due to the resolution of 13.5 mm in this direction.

Noise Suppression at Constant Image Resolution

Noise suppression is illustrated in figure 10 in which the image resolution is maintained at 12.4 mm but lower cutoff frequency filters (figure 3B) are used as the detector resolution is reduced from 12 to 9, 6, and 4 mm. With only 100K counts in the image, the reconstruction with the 12 mm detector resolution is dominated by statistical noise. However, the noise suppression capabilities of SMT progressively reduce the statistical noise to the point that the image becomes limited by the finite detector resolution. The analytical relationship for noise suppression is given by eqs 10 and 15, the latter being shown graphically in figure 9. Depending upon the number of counts and detector resolution, the cutoff frequency of the filter can be altered to provide an optimal compromise between statistical noise and spatial resolution. For example, once a limit is set for an actual intrinsic detector resolution and number of image counts that will be recorded in a particular study, then the filter cutoff frequency can be raised to the point at which improvements through high frequency signal amplification are limited by statistical noise (fig 11) as long as the sampling requirements are met.

Intrinsic Detector Resolution

It is frequently stated that the FWHM resolution of a positron coincidence system is about 40 to 45% of the diameter of the detector. This is however, rarely true. This fraction is dependent on the width, depth, shape and type of detector material, energy discrimination and location of the source between the two detectors (the best resolution is at the center point

between the two detectors). The following sequences show the relationships for the value of this fraction: BGO>CsF>NaI; rectangle>square>circle; deep detector>shallow detector; low energy threshold>high energy threshold; narrow detector>wide detector.

Measured resolutions for selected detector sizes and materials are shown in table 1. For these sizes and detector materials, the percentage of the detector width represented by the FWHM of a line spread function (LSF) ranges from about 44 to 68% with the largest % corresponding to the smallest detector width. Thus a limit to intrinsic detector resolution with discrete detectors is reached more rapidly than would be expected if these percentages were constant for all size detectors.

PCT SYSTEM DESIGN FOR SAT

The SAT approach generally emphasizes the use of smaller detectors in PCT systems, however for large gains by this technique, the detector should be as small as feasible or practical. An underlying assumption in the data presented here is that the system detection efficiency has remained constant while the detector size is decreased. Although it is not a strictly necessary condition, a large decrease in efficiency could negate the gains from SAT.

At the present time, BGO with its high atomic number, high density and non hygroscopic nature is the most appropriate detector for applying SAT. For example, a 4 mm wide BGO detector has a coincidence efficiency of about 61% for a 100 keV threshold and 51% for photopeak (28). An equivalent NaI(Tl) or CsF detector have corresponding values of about 23 and 7.3% respectively (28). BGO crystals need only be separated by the thickness of the light reflecting material on its surface, while NaI(Tl) and CsF require hermetically sealed encasement. Thus the packing fraction of BGO can easily be as high as

95% with a corresponding geometric efficiency of 90%. The best reported packing distance for rectangular NaI(Tl) detectors is 2.3 mm by Derenzo et al (14), giving a packing fraction of 84% and a geometric efficiency of 40%.

Lower photofraction and lower overall stopping power of NaI(Tl) and CsF also increases the cross talk between adjacent detectors (i.e. low stopping power and photofraction produce significant penetration and scatter). This can be decreased with interdetector shielding (12,19,27,28,57) but this further reduces the packing fraction. The magnitude of cross talk due to interdetector scatter (i.e. neglecting penetration which is also higher for NaI(Tl) and CsF) was found by Derenzo et al (14) to be about 30% with 8 mm wide NaI(Tl) detectors with no interdetector shielding. This produces a 30% decrease in system efficiency if these events are rejected, as done by Derenzo et al (14), or a worsening of spatial resolution if they are accepted. With 4 mm wide NaI or CsF crystals this loss would even be higher. In preliminary measurements in our laboratory, crystal to crystal scattering was measured with 4.5 mm BGO detectors and was found to be only 6% with a 100 keV threshold. This value is in good agreement with the theoretical prediction of Derenzo (26).

An apparent disadvantage of BGO relative to NaI(Tl) and CsF is its poorer coincidence time resolution. However, the combination of decay time and light yield for CsF (5nsec, 5% relative to NaI(Tl)), NaI(Tl) (230 nsec, 100%) and BGO (300 nsec, 8% relative to NaI(Tl)), the requirement to provide a long enough coincidence resolving time to obtain uniform sensitivity over the full system field of view (2 to 3 nsec for neuro system and 4 to 6 nsec for whole body system) and the system time jitter have produced tomographs with system time resolutions with NaI(Tl) detectors of 10 to 20 nsec (14,15,18,23,57-64), with CsF of 6 nsec (brain system; (29)) and BGO with 16 to 25 nsec

(19,20,26,31,32,65). The better coincidence time resolution of CsF and NaI yields a lower random coincidence rate for a given singles rate. However, the critical issue is the statistical quality of the image data for a given amount of activity. In an environment in which randoms are measured and subtracted from the data, their primary effect is to lower statistical accuracy (i.e. consider an extreme example in which BGO time resolution is 4 times CsF: 200 true events with 20% randoms of a BGO system will have a statistical accuracy of $(200 + 2 \times 0.2 \times 200)^{1/2}/200 = 8.4\%$, while 100 true events with 5% randoms in a CsF device will have $(100 + 2 \times 0.05 \times 100)^{1/2}/100 = 10.5\%$ statistical accuracy). Thus because of the higher intrinsic and geometric efficiency and the high 511 keV to low energy scatter efficiency with BGO detectors, the coincidence timing advantage of CsF and NaI(Tl) can be offset (27,30). This is particularly true with the small detectors required to take full advantage of the SAT approach. In this case, the higher efficiency will more than offset even the timing resolution of CsF while yielding an efficiency that would be 3 to 4 times CsF and NaI (27,30). For example, Derenzo et al (20) found an increase in efficiency of 2.3 in their tomograph when the 6 mm wide NaI detectors were replaced with 9.5 mm wide BGO detectors (larger BGO were used because packing distance required for NaI detector cans was not necessary with BGO). This difference would be larger for the smaller detectors used with SAT.

The actual intrinsic spatial resolution in positron computed tomography (PCT) systems is determined by a number of factors. Positron range normally contributes only a small, although significant with some radionuclides, degradation (Table 2; 16,66-71) for the common radionuclides used in PCT (i.e. ^{11}C , ^{13}N , ^{15}O , ^{18}F , ^{82}Rb , and ^{68}Ga). The error due to the fact that a small percentage of β^+ annihilations occur with the emission of the two 511 keV photons

at an angle slightly different from 180° , produces a relatively small contribution to resolution of about 1.3 to 2.0 mm with detector separations of 60 to 100 cm (69-72). Although, these factors have not been of significance in existing PET systems, they become more important in designs employing the SAT approach with very fine intrinsic detector resolution. However, the effect of these factors are lessened since the final intrinsic spatial resolution will be the convolution of all of these factors. For example, if the FWHM of the detector, $\pm$ range and angulation were 3, 1 and 1 mm, respectively, and were all assumed to be Gaussian, (good assumption for the detector and angulation error but moderate assumption for the positron range in which the distribution tends to be more sharply peaked at the center and broader at the base (10,66,71)), then the overall FWHM resolution would be about 3.32 mm.

A possible system design using the SAT approach would be a circular ring (maximal geometric efficiency) with a close packed set of 4.5 mm wide BGO detectors. This system could remain stationary and still provide a linear sampling resolution of about 2.45 mm (i.e. allowing for 0.4 mm interdetector spacing and using the coincidences between adjacent opposing detectors to yield data sampling twice that of the interdetector spacing: 20) and angular sampling of about 0.6° . This system would provide high system detection efficiency and the advantages of SAT with a 3 to 4 mm intrinsic detector resolution (Table 1). The stationarity of the system would provide an equivalent capability in a static or dynamic mode and would remove artifacts associated with cardiac and respiratory motion that can occur with moving detector systems. The combinations of the high detection efficiency and SAT would provide dramatic increases in spatial resolution and noise suppression for dynamic studies. The durability and stability of BGO is also considered a realistic advantage of this detector material.

The above system has the geometric design of the Donner tomograph developed by Derenzo, Budinger and coworkers (20) with the addition of the small 800 detectors to allow improved sampling and the signal amplification technique.

CONCLUSION

Computed tomography is a method which employs a mathematical technique that can be used to reconstruct images up to the limits set by the detector resolution or beam width. A limiting feature of computed tomography is its large error propagation and noise amplification and the fact that at any given spatial frequency it amplifies both signal and noise equally. The SAT approach proposed here has the specific advantage of amplifying signal and not noise. The desire to provide the highest resolution possible in the tomographic image has resulted in attempts to produce image resolution as close as possible to the detector resolution or beam width. Because of this many CT systems produce images with excessive statistical noise. Understanding of the principles of SAT and the dominant role of the intrinsic detector resolution or beam width, can lead to designs of tomographs of higher capability and a better understanding of the limitations of present designs.

The resolution improvement of SAT without significant noise amplification not only improves delineation of small structures (figs 6-8,12,13) and lessens partial volume effects (73), but it also raises the signal above the noise level associated with a given number of image counts (fig 8). This reduces the need to increase radiation dose or counting time to reduce the noise level (i.e. SAT increases signal content rather than the opposite approach of reducing noise to yield an acceptable S/N ratio for perception of structures). Low spatial resolution suppresses the amplitude of the signal toward the levels of the noise and to achieve a good S/N, higher doses or longer

imaging times are required to suppress noise. When statistical noise is clearly the dominant factor in a particular application, such as in rapid dynamic or receptor studies, SAT can be used to suppress image noise while maintaining good resolution (figs 8, 10).

The high resolution of a SAT system also provides an inherent discrimination against random and scatter coincidences in PCT. These sources of noise are of low spatial frequency and thus as the higher frequency signals are amplified these components of image noise becomes less significant. For example, the amplitude of the structures that are few mm to a few cm are amplified relative to the random and scatter coincidence background which has a spatial frequency of tens of cm.

SAT is of limited value in single photon CT because the efficiency of most systems decreases as the square of the collimator hole size. Thus, since the collimator is the dominant factor in resolution even in the scintillation camera the efficiency will be decreasing and therefore noise will be increasing as one improves the detector resolution to employ SAT. The principles of SAT do however provide a criteria for better optimizing single photon CT.

The principle of SAT does increase the viability of the multiwire gas proportional counters (MWPC) which have high intrinsic spatial resolution for PCT (74,75). This could offset their low efficiency and relatively poor coincidence time resolution to a significant degree. Improvements in efficiency and coincidence time resolution have been made recently by Jeavons et al (76) which improves these possibilities but MWPC are still less favorable than BGO detectors. In x-ray CT such systems as the GE scanners have demonstrated high image resolution and lower dose due to the signal amplification inherent to the thin detector channels of their gas detectors. Limits occur in this system, as in many other newer x-ray CT scanners, because of the

open/closed ratio of the detectors due to the channel plates or septa. One would prefer very thin detector channels or beam widths with very small or no interposing septa to decrease the system efficiency or increase patient dose. A possible approach in x-ray CT would be with image intensifier tubes specially designed or shielded to limit scatter (i.e. maintain high unscattered to scatter radiation ratio at each detector channel) and to have low inherent noise characteristics.

The high timing resolution of CsF for time of flight (TOF) measurements does offer the potential of noise suppression in PCT as proposed by Alessandri et al (30), Mottant et al (34) and Ter Pogossian et al (35). With the small size of the opening, one for measurement with CsF offers little improvement but with large openings such as the silicon tube, the noise suppression could be significant. For reduced cone beam and with large object size, the amount of noise suppression, however, has been estimated to be about a factor of 2 by Alessandri et al (30), more than 100 by Singer et al (36) and about 4 by Ter Pogossian et al (35) as compared to conventional PCT. These values are preliminary estimates and those for real systems must await the developments of the technology.

Many sources of noise occur in CT in general or in PCT in particular (Table). Many of these noise components are generated or amplified in ways that are particular to CT. Because CT inherently amplifies high frequencies it can take small errors at high spatial resolution and dramatically increase their magnitude in the final image. On the other hand, CT can be forgiving of low frequency errors and will for the same reasons, not enhance them. Because data from a series of measurements around a cross section are employed, correlated errors at high spatial frequencies are reinforced to produce disruptive artifacts such as streaks or circles even though they are a very small errors.

at each point in the original scan data. In the convolution algorithm, each point in the measured scan profile is convolved with a filter function in which, of all the other scan points, the adjacent points receive the highest weight. Thus any uncertainties or errors between nearest neighbor points should be minimized since they will receive the largest weight in the reconstruction process. For example, a system geometry in which nearest neighbor points were collected at distant times when object or activity motion could occur would be particularly sensitive to errors. Redundant sampling such as employed in the polygon PET systems (11), or over scanning in x-ray CT systems provides a great deal of stability and protection from artifacts and noise in the data. However, the full capability of CT requires system design and construction that minimizes the many sources of noise beyond simple counting statistics. A full extension of the general principles of traditional imaging or reconstruction to CT should be done with caution due to the particular characteristics of error propagation and the signal and noise amplification of CT. The design examples here for the DKT approach are not unique but rather illustrate the general property of this technique. Optimization must be performed for each particular environment for which CT is employed.

approximate estimation of statistical noise in Emission Computed Tomography (ECT).

The noise variance (σ^2) in CT images reconstructed with a filtered back projection algorithm has an upper bound given by (6).

$$\sigma^2 \leq \frac{2}{N_0} \sum_{i=1}^{N_0} V_i(\Delta x) \int_{-F}^F (FRF)^2 df \quad (6)$$

where (Δx) is the Nyquist sampling distance, N_0 is the total number of angular views, V_i 's are the largest noise variances of the data values in each angular projection. $F = 1/(2(\Delta x))$ is the Nyquist frequency corresponding to the sampling distance Δx , and FRF is the frequency response of the reconstruction filter.

Since the detector measurements have to be corrected for attenuation before the image is reconstructed, this operation will enhance noise. Hence, the noise propagation associated with the correction factor has to be considered. Let $f(x,y)$ be the object distribution, $h(t,\theta)$ be the attenuation property along a line (t,θ) which has a perpendicular distance t from the center and an orientation θ in the x - y coordinates, i.e.,

$$h(t,\theta) = \int_{-\infty}^{\infty} f(x,y) ds \quad (7)$$

where ds is the line element perpendicular to the coordinates (t,θ) . Now, let $n(t,\theta)$ be the noise associated with $h(t,\theta)$, which corresponds to $N(t,\theta)$. Then, the mean value of

$$h(t,\theta) + n(t,\theta) = h(t,\theta) + N(t,\theta) \quad (8)$$

is the true value of $h(t,\theta)$. The standard deviation of each measurement $n(t,\theta)$ associated with the filtered back projection algorithm, it can be estimated by

$$\sigma(t,\theta) = \frac{1}{A(t,\theta)} \sqrt{\sum_{i=1}^{N_0} V_i(\Delta x) \int_{-F}^F (FRF)^2 df} \quad (9)$$

assuming Poisson distributed noise,

$$\sigma^2(h(t,\theta)) = \frac{\bar{n}(t,\theta)}{A^2(t,\theta)(\Delta t)^2 c^2} \quad (10)$$

Substituting eq. 5A into eq. 5A, the variance of $K(t, \theta)$ becomes

$$\sigma^2(K(t, \theta)) = \frac{K(t, \theta)}{A(t, \theta)(\Delta\tau)} \quad (6A)$$

or

$$\sigma^2(K(t, \theta)) = \frac{K(t, \theta)^2}{n(t, \theta)} \quad (7A)$$

In eq. 7A, V_1 is the largest value of $\sigma^2(K(t, \theta))$ among all measurements in the run angular modulation. Applying eq. 7A, V_1 can be represented by

$$V_1 = \max_t \left(\frac{K(t, \theta)^2}{n(t, \theta)} \right) \quad (8A)$$

For a uniform distribution

$$V_1 = \frac{1}{n_c} \sum_{i=1}^N V_i \int_{-F}^F (FRF)^2 df \quad (9A)$$

For a non-uniform distribution is a uniform cylinder, the above formula can be reduced. If the cross-section of a cylinder be denoted D with a uniform distribution. The largest value of V_1 can be shown to correspond to the lines that pass through the center of the

$$V_1 = (\pi^2 D^2)/n_c \text{ for all } i \quad (10A)$$

where n_c is the mean value of the corresponding detector measurements (with proton attenuation). Thus, eq (9A) becomes

$$\sigma^2 \leq \frac{\pi^2 D^2 (\Delta x)}{N_0 n_c} \int_{-F}^F (FRF)^2 df \quad (11A)$$

For object diameters smaller than about 30 on and with an attenuation coefficient near 0.1 cm^{-1} , the detector measurements do not vary much across the transverse scan profile of the object. In other words, the total number of photons (N) detected for one image can be approximated by

$$N = N_0 \frac{D}{(\Delta x)} n_c \quad (12A)$$

where $D/(\Delta x)$ is the number of transverse scan measurements per angular projection within the diameter of the object. Utilizing the relationship of eq 12A in eq 11A, one obtains

$$\sigma^2/m^2 \leq \frac{\pi D^3}{N} \int_{-F}^F (FRF)^2 df \quad (13A)$$

If the resolution is used for reconstruction, then

$$\int_{-F}^F (FRF)^2 df = 1/(12(\Delta x)^3) \quad (14A)$$

$$\sigma^2/m^2 \leq \frac{\pi D^3}{12N(\Delta x)^3}$$

Thus, the percentage noise level in a uniform cylindrical object can be estimated by a function of the object diameter, resolution element size and the total number of detected photons. Eq. 14A is similar in form to that reported by Buzinger et al. (2), and more closely resembles the reported equation for nonuniform distributions (77).

For the case where $\Delta x = (2\Delta x)$, then

$$\text{then } \sigma^2/m^2 \leq \frac{\pi D^3}{1.5NN_0^3} \quad (15A)$$

FIGURE CAPTIONS

Fig. 1 Illustration of linear superposition of back projections (LSBP) or simple back projection and the CT technique of linear superposition of filtered back projections (LSFBP). A) Example of data collection format for transaxial tomography consisting of transverse scans at discrete angles around cross section of interest. For simplicity, the cross section contains all zeros except for the object in the center. b) LSBP in which each scan point is projected back across the image (i.e. back projection) at the position it was recorded. This process is repeated for each angle and data are added linearly on top of each other (i.e. linear superposition). This approach produces positive data in the regions that should be zero with their magnitude decreasing as a function the radius, r , away from the object in the center (i.e. $1/r$ blurring). The principle of this approach was developed by SODAGE in 1921 for x-ray section tomography and subsequently refined by others (37). This same approach was initially applied by Kuhl, Edwards and coworkers (38,39) to single photon section tomography and to positron section tomography by Rankowitz et al (40) and Robertson et al (41). c) The blurring effect of LSBP can be removed by mathematically solving the tomographic problem, or equivalently, adding a mathematically derived filter to LSBP to correct for $1/r$ blurring (i.e. convolving the scan data with the inverse of the blurring function). This approach is referred to as linear superposition of filtered back projection (LSFBP) and represents true computed tomography. Image in C illus-

strates partial removal of $1/r$ blurring with an insufficient number of angular projections as contrasted to D with sufficient angular sampling. A ramp reconstruction filter function is shown to illustrate the increasing weight (w) given to increasing spatial frequencies (v). The first PCT systems developed that employed LSFBP were by Phelps, Hoffman, Ter Pogossian and coworkers (12,42,43) and Brownell et al (13) and for single photon tomography by Kuhl, Edwards, Ricci and coworkers (44), Budinger et al (1) and Keyes et al (45).

Fig. 2 Illustration of the two forms of blurring in the tomographic data: $1/r$ and finite detector resolution. A) Reconstruction of a brain slice with both types of blurring, simple back projection ($1/r$ blurring) and a 12 mm detector resolution. B) Removal of $1/r$ blurring by reconstruction with a convolution CT algorithm to the limit set by a 12 mm detector resolution. Thus, resultant image is blurred due to the finite detector resolution. C) Reconstruction of the brain slice by simple back projection (i.e. with $1/r$ blurring) but removal of blurring due to finite detector resolution (i.e. signal amplification). Note the fine structure is better perceived but image contains a blurring background since CT was not employed to remove $1/r$ blurring. D) Removal of both forms of blurring. Images were noise free (see text for effect of adding noise).

Fig. 3 A) Modulation transfer functions (i.e. spatial frequency response) of detectors with FWHM resolutions of 12, 9, 6, 4 and 2 mm. Vertical line shows position of a conventional cutoff frequency for a ramp reconstruction filter that would be used to produce a tomographic image with a resolution slightly worse than the detector of FWHM = 12

mm. Note amplification of frequencies between 0 and this cutoff frequency as detector resolution is improved. B) Frequency response functions of different reconstruction filters. Filters A, B, C, E, F and G are Shepp/Logan type filters (i.e. their FRFs are described by $k\{\sin(f/k)\}$) and D is the ramp. Filter F is linear approximation of the ramp filter D with less weighting at the intermediate frequencies, extension to higher frequencies and no sharp cutoff frequency that can produce overshoots at structure boundaries. Filter F has slightly higher noise amplification than D but provides some weighting at the higher frequencies for the higher frequency responses of high resolution detectors. Filters F, E, C, B and A were used with progressively higher detector resolutions to suppress noise while maintaining a constant image resolution. C) Image modulation transfer functions from reconstructions with 12, 9, 6, 4 and 2 mm detector resolutions and the same reconstruction filter function (filter F).

Fig. 4. Plots of eq 13 illustrating image resolution (W_i) improvement by improving detector resolution (W_d) without changing reconstruction filter. Examples are shown relative to different reference conditions of α (i.e. references are to filter functions which produce an initial image resolution that is 3, 5, 10, 20, etc % worse than the initial or reference detector resolution). Values of α are shown in % at right of fig. Values on axes are normalized to be applicable for any particular detector and image resolution.

Fig. 5. Plot of eq 15 showing image noise (variance) reduction with SAT as intrinsic detector resolution (W_d) is decreased relative to a fixed image resolution (W_i). The 100% reference corresponds to the use of

a ramp filter with a cutoff frequency at $1/W_1$. The solid line is for the case when the filter's FRF has the form of eq 5. The dashed line corresponds to the case where the filter's FRF has the form of eq. 5 at frequencies below $1/W_1$ but is equal to zero at frequencies above $1/W_1$ (i.e. like the ramp filter used as a reference).

Fig. 6 Reconstructed images of a simulated bar phantom with 5.5 mm bars and spacings to show image resolution improvement from improving detector resolution without changing the frequency response of the reconstruction filter. Images are noise free but noise level would be constant since filter is unchanged. Intrinsic detector resolutions in mm are shown in upper left hand corner of images (example of $u = 50$ in Fig 5).

Fig. 7 Simulated reconstruction of a brain slice to show image resolution improvement as detector resolution (values in mm are shown on images without changing the filter function) is improved. Filter F in Figure 5 was used for all reconstructions. A) Gray scale images with 20 million counts. B) Color coded version of images shown in A. Note that although fixed values have been assigned to all gray (red) and white (blue) matter structures in the original data set, the partial volume effects of the different resolutions produce different apparent distributions of the values in these structures. C) Gray scale images containing 1 million counts.

Fig. 8 Same as Figure 7 except brain slice images (A) and reconstructed cylinders (B) contain only 250 thousand counts. Note in (A) how resolution improvement with SAT not only provides better delineation of structures but also that the structures (i.e. signals) are progressively and selectively amplified above image noise which is held

at a constant level because the reconstruction filter is unchanged. The constancy of the noise level is shown in (B) with simulated reconstructions of uniform cylinders under identical conditions to (A).

Fig. 9 Comparison of original digitized brain slice, trinary coded brain slice, simulated reconstruction of trinary coded brain slice and actual image of a human subject with the NeuroECAT. The simulated and NeuroECAT images are both with about 8 mm detector resolution, 5 million counts, 10 mm image resolution, 2.78 mm transverse and 1.5° angular sampling. These latter images illustrate the comparison of the simulated studies of this work with an actual tomographic study.

Fig. 10 Simulated reconstructions showing noise suppression from improving detector resolution (shown on images in mm) concomitant with decreasing the cutoff frequency of reconstruction filter (from C to C, B, and A in fig 3B) and maintaining the image resolution at 12.4 mm. Images contain 100 thousand counts.

Fig. 11 Simulated reconstructions of a brain slice with a detector resolution of 4 mm. Images A, B, C, D and E were reconstructed with filters A, B, C, F and G in fig 3B to illustrate the change in image resolution and quality from the condition where the determining factor is the filter function (images A, B and C), to near optimum (image D) and the case where detector resolution becomes dominant (image E) and image noise becomes apparent with only a small increase in resolution. Images all contain 1 million counts.

Fig. 12 Simulated cross sections of a dog heart to show the effect of different image resolutions (shown in mm on images). Relative con-

crasts were 1 and 0.5 in left (LV) and right (RV) ventricle and 0.5 in a small subendocardial defect (3 x 12 mm) in the posterior portion (arrow) of the LV. At 5 mm image resolution the defect is clearly delineated and seen to be limited to the endocardium. At 10 mm image resolution this appears as a less well defined gradient (better seen on numerical data) but at 15 to 20 mm image resolution, it becomes ambiguous as to whether there is transmural defect or wall thinning, and the quantitative description of the degree of ischemia is lower. An actual study at 10 mm image resolution in the dog at the level of the posterior papillary muscle is shown for comparison to simulated results. Anterior is to the left in all images.

Fig. 10 Simulated reconstructions of the brain slice (without noise) to illustrate the delineation of cerebral structures and quantitative measurements as image resolution is progressively improved from 50 to 5 mm. Although all gray matter structures have the same simulated metabolic rate the various resolutions produce different apparent metabolic rates due to the different degrees of partial volume effects. Figure taken from ref 20.

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TABLE 1 Intrinsic Spatial Resolution* For Different Size
BGO, NaI and CsF Detectors

Detector Material	Width (mm)	Height (mm)	Depth (mm)	FWHM (100 keV) (mm)	$\frac{\text{FWHM}}{\text{Width}}^\dagger$ (100 keV) (%)	FWHM (Photop) (mm)	$\frac{\text{FWHM}}{\text{Width}}^\dagger$ (Photop) (%)	FWHM (100 keV) (mm)	$\frac{\text{FWHM}}{\text{Width}}^\dagger$ (100 keV) (%)	FWHM (Photop) (mm)	$\frac{\text{FWHM}}{\text{Width}}^\dagger$ (Photop) (%)
BGO	4.5	30	30	3.08	68.4	2.94	65.3	5.09	113	4.91	109
	11	27	29	6.64	60.4	6.44	58.6	11.5	104	10.5	95.0
	12	14	25	7.15	59.6	6.95	57.2	12.5	104	11.5	95.5
	17	28	30	9.76	57.4	9.46	55.7	17.5	103	15.7	92.4
	19	30	30	10.97	57.7	10.26	54.0	18.8	99	17.8	93.7
NaI(Tl)	8	30	50	4.82	60.3	3.96	49.5	8.84	111	7.33	92.1
	25.4	38.1	76.2	14.5	57.0	13.5	53.0	23.5	92.5	22.5	88.6
	38.1 diam. cycl.		76.2	17.5	45.9	16.7	43.8	30.5	80.1	29.5	77.4
CsF	25.4 diam. cycl.		25.4	11.9	46.7	11.5	45.2	20.5	80.7	19.0	74.7

* Spatial resolutions are from line spread functions with finite size of source removed. FWHM and FWIM refer to full width at half and tenth maximum of line spread function, respectively. The 100 keV and photop (photopeak) refer to energy thresholds.

† FWHM/width refers to the FWHM resolution divided by the detector width.

Table 2: Effect of Postiron Range on Image Resolution

Nuclide	Max beta energy (MeV)	75% Width [†] (mm)	Reference
¹⁸ F	0.633	0.6	Hoffman et al (38)
¹¹ C	0.959	0.9	"
		1.03	Derenzo (41)
¹³ N	1.197	1.3	(*)
¹⁵ O	1.738	2.4	(*)
⁶⁸ Ga	1.898	2.6	Hoffman et al (38)
		2.7	Derenzo (41)
⁶⁷ Ge	3.140	5.0	Hoffman et al (38)
		6.9	Derenzo (41)

area which contains 75% of the area under the LSF. Because the LSF of the postiron range is not Gaussian, the 75% width was chosen since about 75% of the area under a Gaussian curve is at the FWHM.

Value was estimated from interpolation of values given for other radionuclides in table.

Figure 1

Figure 2

Figure 3A

Figure 3B

Figure 22

Figure 4

Figure 5

Figure 7A

Figure 7B

12

9

6

4

2

Figure 7C

12

9

6

4

2

Figure 8A

Figure 8B

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13