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GAMMA RAYS FOR PEDESTRIANS¹

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Abstract

Nuclear gamma radiation does not have many of the properties taken for granted in atomic or molecular radiation and necessary for lasers. The basic science and technology underlying these differences and the proposed methods of overcoming difficulties resulting from them are not properly understood. Considerable illumination in this interdisciplinary problem could be provided by some back-of-the-envelope calculations and simple experimental surveys by small groups of students and postdocs with an elementary knowledge of the nuclear and solid state physics which is evidently not familiar these days to laser physicists.

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Any realistic discussion of gamma ray lasers must take into account not only the unavailability of mirrors and lenses used to control the paths of optical photons but also the very different physics present in nuclear gamma ray transitions and not present in the transitions at longer wave lengths used for conventional lasers. Laser action between two energy levels depends upon the fact that the transition from the upper level to the lower level takes place primarily by the emission of a photon, that the photon emitted in the transition has the right properties to cause the inverse transition in another atom or molecule, and that it is relatively easy to produce a population inversion by pumping with photons. This does not happen in normal nuclear transitions.

The 14 KeV nuclear transition in ^{57}Fe does not normally take place by photon emission. The probability of photon emission is less than 10%. This small number of photons normally emitted do not generally have the proper energy to produce the inverse transition in another ^{57}Fe nucleus. Furthermore, there is no simple way to “pump” a population inversion of these two levels by irradiation with photons; there is only a tiny probability that a photon incident on a sample will produce a nuclear transition. The dominant absorption mechanism for photons is electronic, not nuclear, and the probability that a photon incident upon a sample will produce a nuclear transition rather than being absorbed by other means is generally less than one in a million.

These differences must be understood and taken into account before any gamma ray laser design can be considered. It is necessary to find ways to increase the probability of photon emission, to find ways to get the emitted gamma ray to have the proper energy and to find new efficient ways of achieving a population inversion.

SOME SIMPLE SCALE DIFFERENCES

Some insight into the nature of these differences which indicate where normal intuition from optical transitions breaks down for gamma rays can be seen by examining characteristic scales for the two cases. Let R denote the size of the radiating system; e.g. nucleus, atom or molecule, D denote the distance between neighboring radiators (the lattice constant in a crystal), x denote the amplitude of zero-point and thermal oscillations of the radiator, and k denote the wave number of the radiation. Then for normal optical laser transitions,

$$x \ll R \sim D \ll (1/k). \quad (1a)$$

Thus

$$kx \ll kR \sim kD \ll 1, \quad (1b)$$

For nuclear gamma ray transitions,

$$R \ll x \sim (1/k) \ll D \quad (2a)$$

and

$$kR \ll kx \sim 1 \ll kD \quad (2b)$$

In both cases $kR \ll 1$, indicating that the long wave length approximation and the multipole expansion are valid. But the fact that kx is of order unity for the gamma ray case and kD is large introduces new problems of coherence for lasers. The motion of the radiating nucleus in a lattice introduces a phase modulation that tends to destroy the coherence. The Debye-Waller factor $e^{-k^2\langle x^2 \rangle}$ expresses the reduction in intensity of the coherent radiation due to zero-point and thermal motion. The fact that kD is large means that there are large phase differences between radiation from nearest neighbors, and that considerable effort is needed to control this phase in order to maintain coherence.

These effects can be seen explicitly by examining the matrix element for the amplitude emitted from two neighboring nuclei

$$A = \langle \psi(\vec{x}_1, \vec{x}_2) | e^{i\vec{k} \cdot \vec{x}_1} | \psi(\vec{x}_1, \vec{x}_2) \rangle, \quad (3)$$

where $\vec{x}_1$ and $\vec{x}_2$ are the coordinates of the nuclei relative to their equilibrium positions. Then for gamma rays

$$\vec{k}_\gamma \cdot \vec{D} \gg 1 \quad (4a)$$

and

$$k_\gamma^2 \langle x^2 \rangle \sim 1. \quad (4b)$$

The reduction in the coherent intensity due to thermal and zero-point motion is seen to be given by the Debye-Waller factor $e^{-k^2\langle x^2 \rangle}$.

We also note that $\vec{k} \cdot \vec{D}$ satisfies Bragg condition for constructive interference between the two terms in Eq. (5) only at Bragg angle. Coherence is washed out when the intensity is averaged over angles and reduces the enhancement of superradiance.

Additional scale differences are simply seen in the energy domain; namely that the energy of optical radiation is much less than atomic ionization energies, while the energy of nuclear gamma rays is much greater

$$\hbar\omega_{\text{opt}} \ll E_I \ll \hbar\omega_\gamma. \quad (5a)$$

This allows energy to be lost by ionization processes absent in conventional optical lasers. The kinetic energy of the recoil of an atom because of the momentum carried by the emitted photon is much less than the natural line width of the radiative transition in the optical case. In the gamma ray case the recoil energy is very much larger than the natural line width and is of the same order as the thermal energy or the Debye temperature of the crystal.

$$\Gamma \ll \frac{(\hbar k_\gamma)^2}{2M} \sim K\theta_D. \quad (5b)$$

For the 14 KeV line in ^{57}Fe

$$\frac{(\hbar k_\gamma)^2}{2M} = \frac{(1.4)^2 \times 10^8}{2 \times 57 \times 10^9} \sim \frac{1}{570} \text{ e.v.} \gg \Gamma. \quad (6a)$$

For a 50 KeV line in a nucleus with mass ~ 50

$$\frac{(\hbar k_\gamma)^2}{2M} \approx \sim KT. \quad (6b)$$

SOME IMPORTANT CONSEQUENCES OF THE SCALE DIFFERENCES

A. Frequency shift effects

1. The electromagnetic radiation emitted in a transition between two energy levels of an isolated atom or molecule has the right frequency to induce the inverse transition between the same two levels in another atom or molecule. The electromagnetic radiation emitted in a transition between two energy levels of an isolated nucleus does not have the right frequency to induce the inverse transition between the same two levels in another nucleus. The recoil kinetic energy, negligible for atomic and molecular transitions, is crucial for nuclear transitions.

2. The ratio of the natural line width to the energy of a nuclear gamma ray can be many orders of magnitude smaller than for the atomic or molecular case. Thus appreciable line shifts which destroy laser action can be produced in nuclear transitions by tiny effects which are completely negligible in atomic or molecular transitions. Some examples are Doppler shifts produced by tiny velocities, or small differences in external fields acting on different atoms. Nuclear transitions have even been affected by the gravitational red shift of a photon passing between two nuclei.

B. Phase Shift Effects

1. The wave length of electromagnetic radiation emitted in atomic and molecular transitions is long in comparison with the distance between nearest neighbors in normal matter. The wave length of electromagnetic radiation emitted in nuclear transitions is short in comparison with the distance between nearest neighbors in normal matter. There is therefore an appreciable phase shift in the propagation of nuclear gamma radiation between neighboring nuclei, which can affect phase coherence.
2. The wave length of electromagnetic radiation emitted in atomic and molecular transitions is long in comparison with the amplitude of thermal or zero point motion in normal matter. The wave length of electromagnetic radiation emitted in nuclear transitions is of the same order as the amplitude of thermal or zero point motion in normal matter. Thus this motion can destroy phase coherence.

C. Loss of Energy by Radiationless Transitions

1. The dominant mechanism for the transition between an upper and a lower energy level of an isolated atom or molecule is the emission of a photon. In most transitions of interest for lasers, the dominant mechanism for the transition between an upper and a lower energy level of an isolated nucleus is the ejection of an atomic electron (internal conversion) *not* photon emission.
2. The dominant absorption mechanism in matter for photons emitted by electrons in atomic or molecular transitions is absorption by bound electrons which then make other atomic or molecular transitions; hopefully the desired transition for a laser. The dominant absorption mechanism in matter for photons emitted by nucleons in

nuclear transitions is still absorption by electrons which recoil into the continuum and cannot induce other nuclear transitions like the desired transition for a graser.

NEW PROBLEMS IN THE NUCLEAR DOMAIN RELEVANT TO GAMMA RAY LASERS

We now discuss the problems arising from these differences. The large differences in wave length, frequency, energy and momentum between gamma radiation and optical radiation have important physical implications both in the energy-momentum domain and in the space-time domain. Gamma rays have large momenta, producing recoil shifts larger than the natural line width, as noted in Eq. (5b). They have energies larger than atomic ionization energies as noted in Eq. (5a). Thus gamma ray emission in nuclear transitions competes with radiationless transitions in which the energy is carried off by an atomic electron. The overwhelmingly dominant absorption mechanism in matter for nuclear gamma rays is electronic, rather than nuclear, and does not produce further nuclear excitation. Gamma ray wave lengths are so much shorter than interatomic distances that phase differences between radiation from nearest neighbors can completely destroy coherence effects as noted in Eqs. (2-4). These effects all deserve serious attention. They lead to difficulties which may prove to be insurmountable.

A. New Problems with the Lasing Transition

Two important effects present in gamma ray transitions are the recoil shift and internal conversion. The recoil shift changes the energy of the gamma ray emitted in a hopefully lasing transition by such a large amount that it makes the gamma ray useless for the excitation of another nucleus. The shifted gamma ray is thus lost to further laser action. Internal conversion takes most of the energy of the lasing transition out of the radiation field and converts it into incoherent electron emission also useless for further laser action. Some ways must be found to eliminate large energy losses due to the emission of recoil-shifted gamma rays and conversion electrons in order to make the construction of a gamma ray laser feasible.

In addition the phenomenon of superradiance is very different for nuclear gamma rays, whose wave lengths are short compared to interatomic spacings, in contrast to the case of optical radiation. Most of the coherence effect is lost in the nuclear case.

1. The Recoil Shift

Any atom or nucleus which emits a photon must recoil with a momentum equal and opposite to the photon momentum. The energy of a photon emitted in the transition between two levels is then less than the energy difference between the two levels by the kinetic energy of the recoil. For optical transitions, the recoil shift is much less than the natural line width and can be ignored. For gamma rays the recoil shift is orders of magnitude larger than the natural width as noted in Eqs.(6).

The only known method for overcoming this difficulty is by using the Mössbauer effect, in which the atom making the transition is bound in a crystal and the recoil momentum is taken up by the whole crystal with negligible energy loss. However, the probability of such a Mössbauer transition depends upon the energy of the gamma ray, the ambient temperature and the Debye temperature of the crystal in a function of these variables known as the Debye-Waller factor. The requirement that the Debye-Waller factor must be near unity to avoid wasting energy with the emission of recoil-shifted photons places serious constraints on the values of the above-mentioned variables.

2. Internal Conversion

Unlike atomic and molecular transitions, where the photon energy is smaller than ionization energies, the energy of a nuclear transition is considerably greater than the ionization energy of the atom as noted in Eq. (5a). It is therefore possible for the transition between two nuclear levels to take place with the energy emitted by ejecting an atomic electron, rather than in the form of radiation. In common transitions, like the Mössbauer transition in ^{57}Fe , the probability of this electron ejection process called internal conversion is more than an order of magnitude greater than the probability of photon emission. The only methods proposed for eliminating this loss due to internal conversion are by the use of coherent effects like the Bormann effect¹ or superradiance² which can enhance the relative probability of photon emission. Unlike the Mössbauer effect, which is well understood theoretically and confirmed experimentally, there has been no convincing theoretical prediction confirmed by experimental verification of the possibility of overcoming the loss due to internal conversion.

3. The Bormann Effect

The Bormann effect¹ involves the propagation of electromagnetic waves through a

crystal in a mode which greatly reduces absorption. The crystal lattice acts somewhat like a wave guide, and there is a mode in which the field has a minimum or a node at the positions of the atoms in the lattice. Although this effect reduces the absorption of the radiation by the atoms and increases the transmission, it would generally also reduce the stimulated emission of radiation by nuclei. The absence of a strong radiation field at the nucleus thus will not help laser operation. It has been suggested that one might be able to eat one's cake and have it, too, by having a Borrmann mode in which the electric field vanishes at the nucleus but the magnetic field does not. If the internal conversion is dominated by electric multipoles and the radiative transition by magnetic multipoles, there is a possibility of having a stimulated emission of magnetic multipole radiation while the internal conversion is suppressed. This places additional constraints on the choice of a suitable nucleus for the transition, since most nuclear transitions are dominated by a single multipole which carries the minimum change of angular momentum, and this multipole is the same for radiation and internal conversion. Considerable basic research will be needed to establish whether the Borrmann effect exists and has the desired properties in nearly perfect crystals relevant to gamma ray lasers. There will then remain the open question of how the effect is destroyed in a practical crystal with defects and radiation damage.

4. Superradiance

Dicke has shown how the rate of radiative transitions can be enhanced by collective effects.² There are two effects: (1) coherence in single-photon transitions involving different atoms, (2) multiphoton coherence effects.

We first consider the case where only a single atom in a crystal is excited, and coherent effects occur because any one of N atoms may be excited and the many-body wave function is a coherent superposition of states in which a different atom is excited. The transition probability is then enhanced by a factor N and the lifetime of the excited level reduced by a factor N from the lifetime of a single atom. However, this depends upon a coherent excitation and a coherent radiation in which all contributions from different atoms are in the same phase. This is possible as long as the wave length of the radiation is large compared to the size of the radiator.

For nuclear gamma transitions this effect no longer occurs, because the interatomic distances are no longer negligible in comparison with the wave length of the radiation as shown in Eqs.(3-4). The relative phases of the contributions of radiation from different

nuclei depend upon the wave length of the radiation, the distance between atoms and the angle of the radiation, in a manner similar to the case of X-ray scattering by crystals. Although it is possible to adjust the phases of the radiation to interfere constructively in certain directions in a crystal; e.g. Bragg directions, this enhancement occurs only in a very small solid angle around the Bragg direction, and is mainly at the expense of radiation in other directions. The result of the coherence is primarily to change the angular distribution of the radiation, but not its overall intensity, thus giving little or no change in the lifetime of the excited state.

A simple derivation of this result has recently been given.³ We summarize this derivation here.

Consider a system of N atoms or nuclei, in which one is in an excited state and all the rest are in the ground state. The wave function for the superradiant state has an equal probability for any one of the N atoms or nuclei to be in the excited state and is a coherent superposition of such singly excited systems:

$$|S\rangle = (1/\sqrt{N}) \sum_i e^{i\phi_i} |g, e_i\rangle, \quad (7)$$

where $|g, e_i\rangle$ denotes the state in which the i th atom is in its excited state and all the remaining atoms are in their ground state and ϕ_i is an arbitrary phase factor chosen to maximize coherence effects. Let J denote the operator describing the radiative transition to the ground state of the entire system, denoted by $|G\rangle$, with the emission of a photon of wave vector $\vec{k}$. Then the transition matrix element is given by

$$\langle G|M|S\rangle = (1/\sqrt{N}) \sum_i \langle G|J|g, e_i\rangle e^{i\phi_i} e^{-i\vec{k}\cdot\vec{r}_i}. \quad (8)$$

The intensity of the radiation is then

$$|A|^2 = \sum_i \sum_j |a|^2 e^{i(\phi_i - \phi_j)} e^{-i\vec{k}\cdot(\vec{r}_i - \vec{r}_j)}, \quad (9a)$$

where

$$a = (1/\sqrt{N}) \langle G|J|g, e_i\rangle. \quad (9b)$$

Note that the choice

$$\phi_i = \vec{k} \cdot \vec{r}_i + 2n\pi \quad (10)$$

gives an amplitude proportional to N and an intensity proportional to N^2 .

However, this coherence occurs only in the direction of a particular wave vector $\mathbf{k}$. If we assume that the individual sources radiate isotropically, then the total radiation integrated over all angles is given by

$$\int |A|^2 d\Omega = 4\pi \sum_i \sum_j |a|^2 e^{i(\phi_i - \phi_j)} \sin(k|r_i - r_j|)/k|r_i - r_j|. \quad (11)$$

This expression is seen to be proportional to N , not to N^2 , if $k|r_i - r_j| \gg 1$. The factor $k|r_i - r_j|$ in the denominator immediately suppresses all contributions from pairs separated by large distances and insures that there is no N dependence greater than linear for large N . Furthermore, the appearance of the sine rather than the cosine makes it impossible to achieve constructive interference for a set of sources equally spaced on a line. If the distance between nearest neighbors along this line is d and the overall phase difference between the radiation from nearest neighbors is ϕ , then the total radiation from this line for a large crystal is

$$\int |A|^2 d\Omega = 2\pi N_l |a|^2 \left\{ 1 + 2 \sum_n \sin(n\phi)/nkd \right\} \quad (12)$$

where N_l is the number of atoms in this line. The sum is clearly of order $1/kd$, giving a contribution of order N_l if kd is of order unity. Note that if $\sin \phi = 1$, then $\sin(2\phi) = 0$, $\sin(3\phi) = -1$ and the sum will be less than $1/kd$. If $\sin \phi$ is small and positive, the sum approaches $\pi/2kd$.

A dependence on N^2 is obtained only if $k|r_i - r_j| \ll 1$ for all pairs in the sample.

The case where many atoms are simultaneously excited is more complicated, and the above simple argument does not hold. However, the variation of the phase from different radiators as a function of the angle must be considered. The basic physics can be seen by examining simple properties of multiphoton states. Consider the case where three atoms are excited and decay to the ground state by the emission of three photons. The final state of the three photons has the form

$$|\psi_{123}\rangle = a_A^\dagger a_B^\dagger a_C^\dagger |0\rangle, \quad (13)$$

where $a_A^\dagger$, $a_B^\dagger$ and $a_C^\dagger$ denote the creation operators for the three photons emitted by the three atoms. If the three photons are in completely different quantum states, the wave

function ψ_{ABC} is normalized to unity.

$$\langle \psi_{ABC} | \psi_{ABC} \rangle = 1. \quad (14a)$$

If, however, all three photons are in the same quantum state, the normalization of the state is $3! = 6$.

$$\langle \psi_{AAA} | \psi_{AAA} \rangle = 6. \quad (14b)$$

The transition matrix element for the three-atom transition will contain a triple product of photon creation operators having the same form and coefficient regardless of whether the photons are the same or different. Thus the difference between the normalization factors (14a) and (14b) will appear in the final transition probability and give an enhancement factor of $n!$ for the case where n atoms are simultaneously de-excited with the emission of all photons into the same quantum state.

Let us now consider the case where the photon emitted by each atom is expressed as a linear combination of plane wave states with a phase depending upon the position of the radiating atom,

$$a_\mu^\dagger = \frac{1}{\sqrt{S}} \sum_{i=1}^S a_{\vec{k}_i}^\dagger e^{-i\vec{k}_i \cdot \vec{r}_\mu}, \quad (15)$$

where $\mu = A, B$ or C , $\vec{r}_A$, $\vec{r}_B$ and $\vec{r}_C$ denote the positions of the three atoms, and we choose a discrete set of S values of the wave number $\vec{k}$ for convenience. For this case the norm of the state ψ_{ABC} is

$$\begin{aligned} \langle 0 | a_C a_B a_A a_A^\dagger a_B^\dagger a_C^\dagger | 0 \rangle = & 1 + \frac{1}{S^2} \left\{ \left| \sum_{i=1}^S e^{i\vec{k}_i \cdot (\vec{r}_A - \vec{r}_B)} \right|^2 + \left| \sum_{i=1}^S e^{i\vec{k}_i \cdot (\vec{r}_B - \vec{r}_C)} \right|^2 + \left| \sum_{i=1}^S e^{i\vec{k}_i \cdot (\vec{r}_C - \vec{r}_A)} \right|^2 \right\} \\ & + \frac{1}{S^3} \left\{ \sum_{i=1}^S e^{i\vec{k}_i \cdot (\vec{r}_A - \vec{r}_B)} \sum_{j=1}^S e^{i\vec{k}_j \cdot (\vec{r}_B - \vec{r}_C)} \sum_{k=1}^S e^{i\vec{k}_k \cdot (\vec{r}_C - \vec{r}_A)} + \text{c.c.} \right\}. \end{aligned} \quad (16)$$

Note that for the case where the distance between the atoms is small compared with the wave length of the radiation all the exponential factors become unity and Eq. (16) reduces to the form (14b) with the enhancement factor due to the coherence between the photons. For the case where the distance is large compared with the wave length, the sum of the exponential factors become small and Eq. (16) reduces to the form (14a) with no enhancement factor due to coherence.

The summations in the relation (16) can be evaluated for the case of an isotropic angular distribution where it reduces to the same form as in Eq. (11),

$$\frac{1}{S} \sum_{i=1}^S e^{i\vec{k}_i \cdot (\vec{r}_A - \vec{r}_B)} = \frac{\sin(k|\vec{r}_A - \vec{r}_B|)}{k|\vec{r}_A - \vec{r}_B|}. \quad (17a)$$

For this case

$$\begin{aligned} \langle 0 | a_C a_B a_A a_A^\dagger a_B^\dagger a_C^\dagger | 0 \rangle = & 1 + \frac{\sin^2(k|\vec{r}_A - \vec{r}_B|)}{k^2|\vec{r}_A - \vec{r}_B|^2} + \frac{\sin^2(k|\vec{r}_B - \vec{r}_C|)}{k^2|\vec{r}_B - \vec{r}_C|^2} + \frac{\sin^2(k|\vec{r}_C - \vec{r}_A|)}{k^2|\vec{r}_C - \vec{r}_A|^2} \\ & + 2 \frac{\sin(k|\vec{r}_A - \vec{r}_B|) \sin(k|\vec{r}_B - \vec{r}_C|) \sin(k|\vec{r}_C - \vec{r}_A|)}{k^3|\vec{r}_A - \vec{r}_B| \cdot |\vec{r}_B - \vec{r}_C| \cdot |\vec{r}_C - \vec{r}_A|}. \end{aligned} \quad (17b)$$

The enhancement factor is seen to contain two types of terms (1) positive definite terms depending only upon the distance between one pair of atoms; (2) terms depending upon all three interatomic distances with phases that are oscillating functions of these distances. We again see that eq. (17b) reduces to the two forms (14b) and (14a) for the limiting cases.

This result can be generalized to treat the case of N atoms emitting N photons by defining

$$|\psi_N\rangle = \prod_{\mu=1}^N a_\mu^\dagger \quad (18a)$$

$$|\psi_{N-1}\rangle = \prod_{\mu=1}^{N-1} a_\mu^\dagger \quad (18b)$$

$$|\psi_{N-2}^\rho\rangle = \prod_{\substack{\mu=1 \\ \mu \neq \rho}}^{N-1} a_\mu^\dagger. \quad (18c)$$

where $a_\mu^\dagger$ is defined by Eq. (15).

We now note that

$$\begin{aligned} \langle \psi_N | \psi_N \rangle = & \langle \psi_{N-1} | \psi_{N-1} \rangle + \frac{1}{S^2} \sum_{\rho=1}^{N-1} \left| \sum_{i=1}^S e^{i\vec{k}_i \cdot (\vec{r}_N - \vec{r}_\rho)} \right|^2 \langle \psi_{N-2}^\rho | \psi_{N-2}^\rho \rangle \\ & + \frac{1}{S^2} \sum_{\substack{\rho=1 \\ \rho \neq \sigma}}^{N-1} \sum_{\sigma=1}^{N-1} \sum_{i=1}^S e^{i\vec{k}_i \cdot (\vec{r}_N - \vec{r}_\rho)} \sum_{j=1}^S e^{i\vec{k}_j \cdot (\vec{r}_N - \vec{r}_\sigma)} \langle \psi_{N-2}^\rho | \psi_{N-2}^\sigma \rangle. \end{aligned} \quad (19a)$$

If we assume Eq. (17a)

$$\begin{aligned} \langle \psi_N | \psi_N \rangle = & \langle \psi_{N-1} | \psi_{N-1} \rangle + \sum_{\rho=1}^{N-1} \frac{\sin^2(k|\vec{r}_N - \vec{r}_\rho|)}{k^2|\vec{r}_N - \vec{r}_\rho|^2} \langle \psi_{N-2}^\rho | \psi_{N-2}^\rho \rangle \\ & + \sum_{\rho=1}^{N-1} \sum_{\sigma=1}^{N-1} \frac{\sin(k|\vec{r}_N - \vec{r}_\rho|) \sin(k|\vec{r}_N - \vec{r}_\sigma|)}{k^2|\vec{r}_N - \vec{r}_\rho| \cdot |\vec{r}_N - \vec{r}_\sigma|} \langle \psi_{N-2}^\rho | \psi_{N-2}^\sigma \rangle. \end{aligned} \quad (19b)$$

The enhancement factor is again seen to contain two types of terms: (1) positive definite terms depending only upon the distance between one pair of atoms; (2) terms depending upon at least three interatomic distances with phases that are oscillating functions of these distances. We again see that for the case where the distance between the atoms is small compared with the wave length of the radiation all the exponential factors become unity and Eq. (19a) reduces to the form (14b) with the enhancement factor due to the coherence between the photons.

$$\langle \psi_N | \psi_N \rangle = \langle \psi_{N-1} | \psi_{N-1} \rangle + (N-1)^2 \langle \psi_{N-2} | \psi_{N-2} \rangle. \quad (20a)$$

We now note that the positive definite sum is of order $N^{1/3}$. We can then write

$$\langle \psi_N | \psi_N \rangle = \langle \psi_{N-1} | \psi_{N-1} \rangle + O(N^{1/3}) \langle \psi_{N-2} | \psi_{N-2} \rangle + \dots \quad (20b)$$

Comparing this result with Eq. (20a) we see that in this case there is an enhancement factor even if the distance between the atoms is large compared with the wave length of the radiation. However, it is only of order $N^{1/3}$ compared to the N^2 for the case where the distance between the atoms is small compared with the wave length of the radiation.

B. New Problems with the Pumping Transition

In addition to the lasing transition itself, there is also the pumping transition necessary to achieve a high population of the excited state in the lasing transition. Here again there are important differences in the nuclear case. The absorption of gamma rays by matter is very different from the absorption of optical photons, and most of the energy of the pumping transition can be expected to be wasted. Furthermore, because there will be no simple relation between the interatomic distances and the wave lengths of the pumping radiation and the lasing radiation, any collective excitation of nuclei in the sample will not have the desirable coherence properties and phases for coherent emission of laser radiation.

1. Energy considerations.

The pumping transition must have a higher energy than the lasing transition in order to reach a level higher than the lasing level, unless there is an isomeric storage level in the vicinity of the lasing level. There must then be a radiation cascade from the pumping level to the lasing level followed by the lasing transition. Because nuclear cross sections are much lower than photoelectric and Compton cross sections, the probability of absorption of pumping photons by the desired nuclear transition will be very small. Factors of 10^{-8} can be expected in typical cases. This implies an enormous waste of energy in the pumping process, and a necessity to dispose of this energy without excessively heating the sample. The Debye-Waller factor is very sensitive to temperature, and unnecessary heating can destroy the efficiency of the Mössbauer effect. There is also the energy of the cascade radiation from the pumping level to the lasing level, which can heat the sample. In addition to heating there is also the possibility of radiation damage by all the undesired photons which can lead to crystal defects and harm coherence effects.

One might envisage an isomeric storage level very close in energy to the lasing level which would require only a small energy for the transition to the lasing level. However, no isomers having this property are known and it is extremely improbable that they should exist.

2. Phase considerations

Since the wave length of the pumping radiation is not commensurate with that of the lasing transition, Bragg conditions for coherent excitation and radiation of these two transitions are incompatible. Thus there is no possibility of a collective excitation of the lasing level in the sample by means of the pumping radiation to give the phase coherence necessary for constructive interference between the radiation from different nuclei in superradiant or Borrmann directions.

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