LA-UR -93-855

CONF-9308127--2

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36

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SUBMITTED TO: 13TH INTERNATIONAL SYMPOSIUM ON NONLINEAR ACOUSTICS

Acoustics, Bergen Norway, June 28 - July 2, 1993.

MEASUREMENT OF NONLINEAR ELASTIC RESPONSE IN ROCK BY THE RESONANT BAR METHOD

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ABSTRACT

In this work we are studying the behavior of the fundamental (Young's) mode resonant peak as a function of drive amplitude in rock samples. Our goal from these studies is to obtain nonlinear moduli for many rock types, and to study the nonlinear moduli as a function of water saturation and other changes in physical properties. Measurements were made on seven different room dry rock samples. For one sample measurements were taken at 16 saturation levels between 1 and 98%. All samples display a "softening" nonlinearity, that is, the resonant frequency shifts downward with increasing drive amplitude. In extreme cases, the resonant frequency changes by as much as 25% over a strain interval of 10^{-7} to $\sim 4 \times 10^{-5}$. Measurements indicate that the nonlinear response is extremely sensitive to saturation. Estimates of a combined cubic and quartic nonlinear parameter Γ range from approximately -300 to -10^9 for the rock samples. Measurements on PVC and aluminum show no detectable bending implying that $|\Gamma|$ is unmeasurably small using this method.

1. Introduction

Recent laboratory and field experiments have demonstrated that earth materials have an enormous nonlinear elastic response, comparable to fluid containing gas bubbles[1,2,3,4,5,6,7,8]. The ramifications of this response may ultimately effect many areas of research in geoscience including seismology, where the spectral distortion of seismic waves during propagation must be be considered[9]. Other areas of research include rock mechanics and materials science where the nonlinear response may be used to characterize materials. In addition, characterization of material property change by monitoring nonlinear response may be of value. These changes include variations in water saturation for porous media, change in response to variations in stress, change induced by fatigue damage, etc.

Our purpose here is to explore the nonlinear phenomenon of resonant peak bending in rock samples and to characterize the nonlinear response of different rock samples. We present a nonlinear coefficient derived from resonant peak bending for several rock samples, and the change of nonlinear modulus with changes in water saturation for one

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rock sample. These measurements are compared with the measured Young's modulus E_0 and specific dissipation constant Q.

Nonlinear resonance has been discussed by many authors and treatments can be found in texts on vibration[10] and differential equations[11]. It is not our purpose here to review theory and the resonance equation will be only briefly covered in the following section.

2. Theory

Young's modulus E_0 is obtained from the fundamental resonant frequency measured at low drive voltage in the strain interval of 10^{-7}-10^{-8}. From the bar density ρ and length x the modulus is,

$$E_0 = \rho C_E^2 = \rho \frac{x^2 \omega_0^2}{\pi^2} \tag{1}$$

where ω_0 is the fundamental resonant bar frequency at linear elastic strain, and C_E is the Young's mode velocity at ω_0.

The equation of motion for a discrete system (consisting of a spring-mass-dashpot) is

$$\ddot{u} + \frac{\omega_0}{Q}\dot{u} + \omega_0^2\, u\,(1 + \Gamma u^2) = F \cos(\omega t), \tag{2}$$

where u is mass displacement, $\dot{u}$ is velocity, $\ddot{u}$ is acceleration, Q is specific dissipation (inversely proportional to attenuation), F is the amplitude of the oscillating force and Γ is a measure of the strength of the nonlinearity. Solution of Eq (2) leads to the frequency response of the system,

$$\left(\frac{\Omega}{Q}\right)^2 \zeta_1^2 + \left(1 - \Omega^2 + \frac{3}{4}\eta\,\zeta_1^2\right)^2 \zeta_1^2 = 1, \tag{3}$$

where

$$\Omega = \frac{\omega_m}{\omega_0}, \quad \zeta_1 = \frac{\omega_0^2 u}{F}, \quad \text{and} \quad \eta = \frac{F}{\omega_0^2}\Gamma,$$

and ω_m is the frequency corresponding to the maximum acceleration. Under the simplifying assumptions $Q >> 1$ (small attenuation) and $\eta << Q^{-1}$ (nonlinearity negligible compared to attenuation) one has,

$$\frac{\Delta\omega}{\omega_0} = \ddot{u}_m^2 \left(\frac{3\Gamma}{8\omega^4 Q^2}\right) \tag{4}$$

where $\Delta\omega = (\omega_m - \omega_0)$ and $\ddot{u}_m^2$ is the square of the maximum acceleration. Our experiments are configured to measure $\ddot{u}$, Q, ω, and ω_0. Therefore, we can determine Γ as predicted for a discrete system.

An important point regarding Eq (2) is that if Γ is positive the resonance curve will bend upward in frequency (hardening nonlinearity), and and if Γ is negative the resonance curve will bend downward in frequency (softening nonlinearity). Eq (2) is also hysteretic in its behavior in that it depends on which direction the driving frequency is swept, meaning that the amplitude is not uniquely determined by the applied forcing function. This behavior will be illustrated in the results section.

The discrete system assumption implies that stress σ, strain ε, and displacement u are homogeneous in the sample as a function of time. In reality, this is not the case. Stress and strain are maximum at the center of the bar (in absolute value) and minimum at the bar ends. Displacement is maximum at the bar ends and minimum in the bar center. Work on the solution to the elastic resonance equation of motion is continuing.

3. Experimental Procedure

The experimental configuration for obtaining frequency versus acceleration measurements from a sample is described as follows. Two function generators serve as a voltage to frequency converter. A ramp voltage function output by one function generator is fed to a second function generator to create a frequency sweep interval. The interval is chosen to encompass frequencies well above and well below the fundamental resonant mode of the sample. The signal is amplified and acoustically excited by an electromagnetic (coil/magnet) source oriented parallel to the axis of the sample. The magnet is affixed to the sample. The signal is detected by use of a calibrated accelerometer, is pre-amplified, and is then fed to a graphics tablet where the signal is time averaged to obtain frequency versus maximum acceleration. The signal is also fed to an oscilloscope for monitoring the time series signal. Measurements are made of both upward and downward frequency sweeps over the chosen interval. Typically, 5-10 experiments are conducted at successively increasing drive voltages over the same frequency interval in order to monitor resonant peak shift. A single sweep is typically 5-20 minutes in duration.

Measurements of seven different rock samples were made. Comparative studies were conducted using the relatively elastically linear materials aluminum and PVC. For one rock sample, Meule sandstone, measurements were taken at 16 different water saturation levels between 1-98%. For the saturation measerements, the sample was saturated after evacuation under vacuum and measurements were made as the rock dried under room conditions. Densities were estimated from the dry weight and the measured porosity. Sample lengths ranged from 0.39-1.0 m and diameters ranged from 2.5 to 5 cm.

4. Results and Discussion

Figure 1 shows a sample sequence of resonance curves for nine different excitation levels of Fontainebleau sandstone. The solid lines represent downward frequency sweeps and the dashed lines represent upward sweeps. The direction of bending shows that the behavior is softening. This is the case for all measurements taken. The hysteretic behavior mentioned above is clearly seen as intensity is increased.

In extreme cases (dry Meule sandstone), the resonant frequency changes by as much as 25% over a strain interval of 10^{-7} to ~4×10^{-5}. Experiments conducted in aluminum and polyvinylchloride PVC showed no detectable nonlinear behavior. PVC has a similar Q to many rocks, and because Γ is inversely proportional to Q^2 [Eq. (4)], is a better comparison than aluminum. The resonant curves for PVC are shown in Figure 2.

The values of E_0, Q and Γ for seven different rock types are shown in Table 1. All samples were room dried for the experiments with the exception of Fontainebleau sandstone which was oven dried under vacuum. Table 2 shows these values as a function of saturation in Meule sandstone.

Figure 3 shows saturation versus E_0 and Γ normalized to their respective 98% saturation values. The coefficient Γ appears to be far more sensitive to changes in

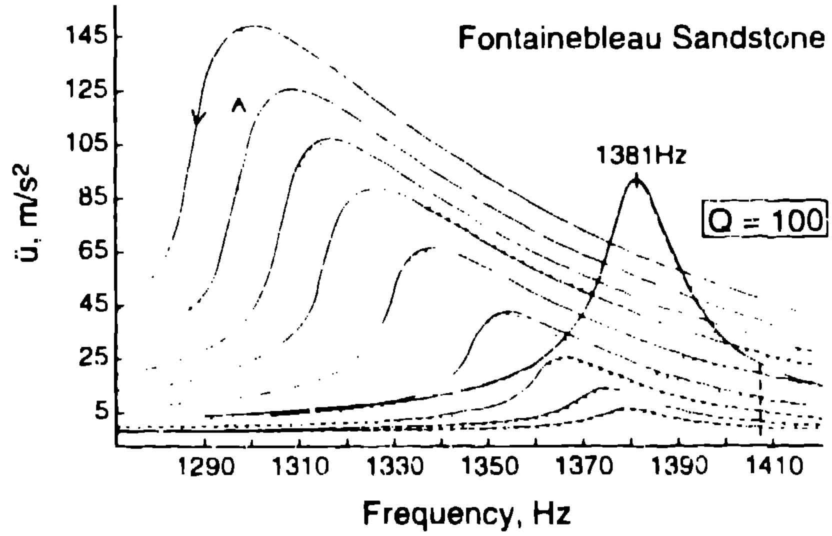

Figure 1. Acceleration ü versus frequency for nine excitation levels in Fontainebleau sandstone. The large peak at 1381 Hz is at an increased acceleration scale to show the character of the linear behavior. The maximum acceleration corresponds to a strain of $3.7\text{x}10^{-5}$. Solid line = downward sweep, dashed line = upward sweep.

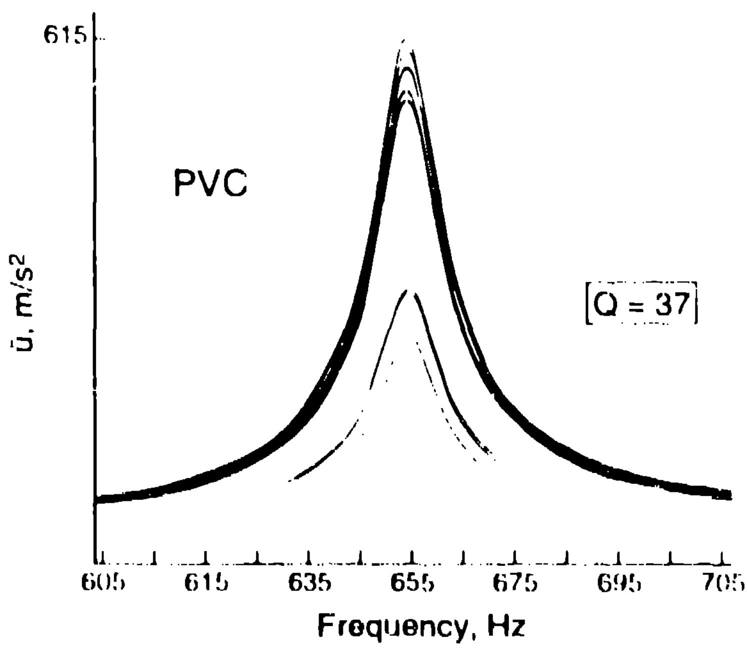

Figure 2 Acceleration ü versus frequency for eight excitation levels in PVC

saturation than E_0. The values of Γ are only approximate. This is because the fit implied by Eq. (4) was not always good. In addition, for the saturation study the experiment was only conducted once and therefore the precision is not known.

In rock, we have found that Γ is negative, not merely from resonance results but also from pulse mode[9] and static pressure experiments. The implication is that the time average modulus is weaker at elevated strains. Finally, we have not quantified the relationship between Γ and nonlinear coefficients determined in other ways e.g., β and δ determined for the propagating wave case[9]. This work is continuing.

Table 1. Dynamic Young's modulus, Q, and nonlinear modulus Γ obtained from resonance experiments. Density assumed to be 2.2 g/cm^3.

Rock Sample	Saturation	E_0 (GPa)	Q	$-\Gamma$ (m^{-2})
ASI Marble M116	room dry	38.7	1300	2.39×10^9
Carrara Marble M516	room dry	30.3	400	2.12×10^8
Estaillades Limestone S790	room dry	17.6	800	4.80×10^4
Fontainebleau Sandstone 13	oven dry	2.5	100	1.13×10^7
Lavoux Limestone S371	room dry	18.2	~1000	8.44×10^5
Meule Sandstone U107	room dry	8.5	45	1.52×10^3
St. Pantaleon Limestone S591	room dry	17.1	625	6.16×10^6

Table 2. Dynamic Young's modulus, Q, and nonlinear modulus Γ as a function of water saturation in Meule sandstone obtained from resonance experiments. Measured densities used

Rock Sample	Saturation %	E_0 (GPa)	Q	$-\Gamma$ (m^{-2})
Meule Sandstone U107	98	3.39	4.6	4.74×10^4
Meule Sandstone U107	95	3.01	7.7	3.32×10^3
Meule Sandstone U107	88	3.04	8.9	1.18×10^3
Meule Sandstone U107	83	3.05	9.2	4.38×10^4
Meule Sandstone U107	80	3.01	9.3	1.03×10^4
Meule Sandstone U107	69	3.00	9.2	1.05×10^4
Meule Sandstone U107	62	3.04	9.6	3.16×10^5
Meule Sandstone U107	53	3.05	9.6	3.10×10^5
Meule Sandstone U107	40	2.77	10.0	7.84×10^5
Meule Sandstone U107	30	3.01	10.0	1.22×10^6
Meule Sandstone U107	27	3.07	10.0	2.13×10^4
Meule Sandstone U107	16	3.69	10.2	4.21×10^3
Meule Sandstone U107	6	5.56	17.3	4.36×10^5
Meule Sandstone U107	4	7.60	38.0	1.28×10^3
Meule Sandstone U107	3	7.64	36.3	3.70×10^3
Meule Sandstone U107	1	8.43	43.6	3.22×10^3

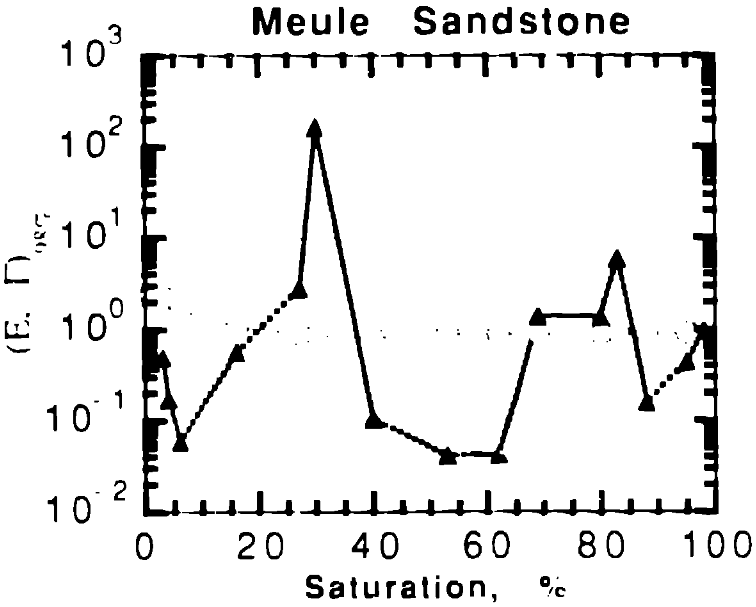

Figure 3. EQ and Γ normalized to their values at 98% saturation.

5. Acknowledgements

Work supported by U. S. Department of Energy Office of Basic Energy Science, Engineering and Geoscience under grant number W 7405-ENG-36, and the Institut Français Du Petrole. We thank Michel Masson for experimental assistance and Katherine McCall, Robert Guyer, Thomas Shankland, Albert Migliori, Igor Beresnev, and Brian Bonner for helpful discussions. G. D. Meegan originally suggested conducting the experiments. We also thank K. McCall for review of the manuscript.

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