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THE MECHANISMS OF FAST NEUTRON PENETRATION

IN THICK LAYERS OF SODIUM

LONG-YANN HUANG

1975

NOTICE

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ABSTRACT

A series of computer experiments have been carried out to elucidate the penetration mechanisms of fast neutrons through thick layers of sodium such as occur in LMFBR designs. As a one-dimensional approximation of the actual situation, the calculations have mainly concentrated on the flux 5 meters from a plane isotropic fission source in an infinite sodium medium. Most of the transport calculations were made with the moments-method code, BMT, using a 496 energy point grid. Previously developed methods for reconstructing the flux from the spatial moments were used, except that a set of biorthogonal polynomials ~~were~~ ^{was} constructed suitable for expansion of the flux in terms of a Gaussian weight function. The moments-method technique lends itself to easy and economical changes of the input cross section data. A large number of such modified cross section sets, built around the ENDF/B-III set, were used in separate calculations designed variously to emphasize or eliminate one or more particular transport processes.

By means of such calculations, it has been shown that as the energy decreases below 190 KeV, the

flux spectrum at 5m is increasingly dominated by an age-diffusion process that is quantitatively close to conventional age theory if the age is suitably chosen. On this basis, it has been possible to explain the observed very soft low energy spectrum ($\sim 1/E^2$ per unit energy around 10 eV). Physically, this type of transport mechanism accords well with the fact the total cross section is nearly constant over most of the lethargy range of about 11 from 190 KeV to a few eV, with almost negligible absorption. The near-Gaussian spatial dependence of this process is in convolution with an approximate exponential behavior that characterizes the high energy transport, and dominate the spectrum above 200 KeV.

It has been possible to discriminate three reasonably distinct processes which form components of the high-energy exponential. In one a major portion of the transport occurs at energies corresponding to deep local minima of the total cross section. The second is characterized by the importance of transport by source neutrons at very high energies, 5 MeV or above. In the third there is a multiple-collision buildup of the slowing down neutrons with no single cross section

feature or energy region predominating. Each of these components is separately capable of leading to a near-exponential deep penetration behavior with roughly the same relaxation length. For a fission source and ENDF/B-III cross sections, at 5 meters each contributes about equally to the total flux in the important 100 KeV to 600 KeV range. However, under changed conditions one or another of the mechanisms may dominate. Thus, with a "softer" source characteristic of the LMFBF core leakage spectrum, penetration via the 297 KeV minimum will predominate. The relative importance of the other two mechanisms depends sensitively on the anisotropy of elastic scattering, the amount of inelastic scattering and the energy distribution of inelastically scattered neutrons. Relatively modest decreases in the ratio of inelastic to elastic scattering or in the fractional energy loss per scattering markedly increase the deeply penetrating flux.

Conclusions from this picture of neutron penetration in sodium are made as to the types of transport calculations that can be successfully made in shield design, and the accuracies needed in future cross section measurements.

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I. Introduction

In most of the present designs of the sodium-cooled fast breeder reactor, the reactor core sits in a big pot of liquid sodium. A schematic diagram of the "pot" system is shown in Figure I-1⁽¹⁾. In this system, a large tank filled with liquid sodium contains all the primary heat-transfer equipment, including the intermediate heat exchangers. This arrangement, consequently, makes the dimensions of the "pot" to be on a heroic scale. For example, the thickness of sodium above the fuel elements in the FTR reactor design is approximately 15 ft and the diameter of the pool is approximately 20 ft⁽²⁾. The sodium coolant which surrounds the reactor core, therefore, constitutes a major portion of the neutron shield. Thus, there is considerable interest in finding out the manner in which fast neutrons, typically from a fission source, penetrate through a distance the order of 5 meters in liquid sodium. The purpose of this study is primarily to investigate these neutron penetration mechanisms.

In this study, the investigation will not provide any specific calculation of the neutron penetration through the sodium in any actual reactor design, nor even

the devising of computational methods to do such calculations more accurately or quicker. Rather the study is designed to elucidate in a simplified geometry what are the sort of life histories characteristic of neutrons that penetrate so far in sodium, how in fact do the cross section properties of sodium influence and affect the penetrating flux. Armed with this comprehension of the penetration mechanisms one can then tell which sodium cross section features need to be measured more accurately and which are good enough for the job of shielding calculation. And more important, one then has the understanding to predict at least qualitatively how the neutrons will behave in circumstances of different geometries, source energies, distances and so on.

To provide answers to these questions, transport calculations were made to investigate the sensitivity of the penetrating flux to the interactions between neutrons and nuclei in the sodium medium. Specifically, the neutron penetration mechanisms in the sodium medium were investigated through calculations of the effects of changes in the input microscopic cross section data on the penetrating flux.

The requirement of making many changes in input cross section data led to the choice of the moments-method for solving the neutron transport equation, because of the feasibility of changing discrete-energy cross section data used in this method. But the use of the moments-method limits the problems treated to be infinite medium calculations, although calculations with the discrete ordinates method have verified that infinite medium conclusions apply also to the finite slab, as will be discussed in Section IV-3. It also presents something of a problem in reconstructing the spatial dependence of the flux from the spatial moments. However, in exchange, very detailed spectrum calculations (496 points) may be made for quite modest computer expenditures, even where the scattering phenomena are complicated functions of energy and angle⁽³⁾. Since the dimensions of the sodium "pot" are approximately 5 meters, a one dimensional calculation which neglects the negligible side leakage was employed. The computer code, BMT⁽⁴⁾, developed at Columbia University, was selected as the calculation tool. The one-dimensional discrete-ordinates code, ANISN⁽⁵⁾, was also used for the finite slab calculations. The methods used for the reconstruction of the spatial dependence of the flux from the spatial moments are

given as Appendices.

Chapter II reviews the moments and discrete-ordinates methods of solving the Boltzmann transport equation. The basic set of the sodium cross sections used, ENDF/B-III MAT 1156⁽⁶⁾, in the transport calculations is discussed in Chapter III. A description of how the calculations were performed, and what assumptions were involved is presented in Chapter IV. Chapter V describes in detail the analyses performed on the calculations and the deductions obtained from them as to the character of the penetration mechanisms that determine the flux spectrum in thick layers of sodium. The reader's attention is directed particularly to the statement of 9 "conclusions" given at the start of Chapter V, which present in capsule form the deductions made as to these penetration mechanisms, and which form the framework for the ensuing discussion. Chapter VI contains a summary and conclusions to be drawn from this study.

Figure I-1: A Schematic Diagram of the "pot" system of a Sodium-cooled Fast Breeder Reactor Design.

II. Theory

II. 1. The Boltzmann Transport Equation:

The time-independent neutron transport equation, which is obtained from a conservation of neutrons in phase space, has the following form^(7,8,9):

$$\vec{\Omega} \cdot \nabla N(\vec{r}, \vec{\Omega}, E) + \Sigma_T(E) N(\vec{r}, \vec{\Omega}, E) = \int dE' \int d\Omega' \Sigma_s(E' \rightarrow E, \vec{\Omega}' \rightarrow \vec{\Omega}) N(\vec{r}, \vec{\Omega}', E') + S(\vec{r}, \vec{\Omega}, E)$$

(II - 1)

where

$\vec{\Omega}$	= unit direction vector,
∇	= gradient operator,
N	= neutron angular flux,
$\vec{r}$	= spatial vector,
E	= energy,
Σ_T	= macroscopic total cross section,
$\Sigma_s(E' \rightarrow E, \vec{\Omega}' \rightarrow \vec{\Omega}) dE d\Omega'$	macroscopic differential scattering cross section for transfers from energy E' to energy E range dE and from direction $\vec{\Omega}'$ to $\vec{\Omega}$ in element of solid angle $d\Omega'$,
S	= external source strength.

The left hand side of the above equation represents the net loss of neutrons in a given unit element of phase space through convection (the first term) and collision (the second term) out of the given phase space, while the right hand side of this equation represents the net gain of neutrons from the external neutron source (the fourth term) and of neutrons scattered into the element of phase space from any other region of phase space (the third term).

Here, equation (II-1) has also already implied that:

1. the medium is homogeneous, i.e. the cross sections are not functions of position, $\vec{r}$;
2. the total cross section is not a function of the individual directions, $\vec{\Omega}$ or $\vec{\Omega}'$, i.e. neither the neutrons nor the target nuclei are polarized.

For a plane source in a plane-symmetric system, equation (II-1) is reduced to

$$\begin{aligned} \mu \cdot \frac{\partial N(x, \mu, E)}{\partial x} + \Sigma_T(E) N(x, \mu, E) \\ = \int dE' \int d\Omega' \Sigma_s(E' \rightarrow E, \omega_L) N(x, \mu', E') + S(x, \mu, E) \end{aligned}$$

(II-2)

Here, x is the direction normal to the plane, μ is the cosine of the angular between the direction of neutron and x -direction, and w_L is the cosine of the scattering angle in the laboratory system, or

$$w_L = \vec{\Omega} \cdot \vec{\Omega}' \quad (\text{II- 3})$$

Now, define

$$F(x, \mu, E) = \int dE' \int d\Omega' \Sigma_s(E' \rightarrow E, w_L) N(x, \mu', E') \quad (\text{II- 4})$$

Then equation (II-2) becomes

$$\mu \frac{\partial N}{\partial x} + \Sigma_T N = F + S \quad (\text{II- 5})$$

Equation (II-5) is the basic equation we will use in the following sections.

II. 2. The Moments Method:

The moments method was developed by Spencer and Fano^(10,11,12) and has been extensively applied to the calculation of gamma ray penetration by Goldstein and Wilkins⁽¹³⁾. This method expands the neutron angular flux into a series of angular and spatial moments. Therefore, for any given moment, the neutron transport equation can be reduced to a simplified equation with only the energy as a variable. And the moments can thus be evaluated numerically with high accuracy even where the scattering phenomena are complicated functions of energy and angle⁽³⁾. A complete derivation of the moments equation is given in References 4, 7 and 13.

To obtain the moments equation, one first expands the neutron angular flux into a Legendre polynomial series as follows.

$$N(x, \mu, E) = \sum_{\ell} \frac{2\ell+1}{4\pi} N_{\ell}(x, E) P_{\ell}(\mu) \quad (\text{II- 6})$$

where

$$N_{\ell}(x, E) = \int d\Omega N(x, \mu, E) P_{\ell}(\mu) \quad (\text{II- 7})$$

and P_l is the Legendre polynomial of the l th order. By operating with $\int d\Omega P_l(\mu)$ on both side of equation (II-5) and using the addition theorem for Legendre polynomial, one obtains

$$\begin{aligned} \frac{l+1}{2l+1} \frac{\partial N_{l+1}(x, E)}{\partial x} + \frac{l}{2l+1} \frac{\partial N_{l-1}(x, E)}{\partial x} + \Sigma_T(E) N_l(x, E) \\ = F_l(x, E) + S_l(x, E) \end{aligned} \quad (\text{II- 8})$$

where

$$\begin{aligned} F_l(x, E) &= \int d\Omega F(x, \mu, E) P_l(\mu) \\ &= \int dE' \Sigma_{s,l}(E' \rightarrow E) N_l(x, E') \end{aligned} \quad (\text{II- 9})$$

$$\Sigma_{s,l}(E' \rightarrow E) = \int d\Omega_L \Sigma_s(E' \rightarrow E, \omega_L) P_l(\omega_L) \quad (\text{II-10})$$

$$S_l(x, E) = \int d\Omega S(x, \mu, E) P_l(\mu) \quad (\text{II-11})$$

and

$$N_l(x, E) = 0 \quad l < 0 \quad (\text{II-12})$$

Now define the spatial moments as

$$N_{n,l}(\bar{E}) = \frac{1}{n!} \int_{-\infty}^{\infty} dx x^n N_l(x, E) \quad (\text{II-13})$$

Then by operating $\frac{1}{n!} \int_{-\infty}^{\infty} dx x^n$ on both sides of equation

(II-8), one obtains the moments equation as follows.

$$\begin{aligned} \Sigma_T(E) N_{n,l}(\bar{E}) - \frac{l+1}{2l+1} N_{n,l+1}(\bar{E}) - \frac{l}{2l+1} N_{n,l-1}(\bar{E}) \\ = \int dE' \Sigma_{S,l}(E' \rightarrow E) N_{n,l}(\bar{E}') + S_{n,l}(E) \end{aligned} \quad (\text{II-14})$$

where

$$S_{n,l}(\bar{E}) = \frac{1}{n!} \int_{-\infty}^{\infty} dx x^n S_l(x, E) \quad (\text{II-15})$$

and

$$N_{n,l}(\bar{E}) = 0 \quad n < 0 \text{ or } l < 0 \quad (\text{II-16})$$

Equation (II-14) represents a set of interlinked equations in one variable, E. For problems involving only down-scattering and absorption, this set of equations can be

solved numerically by starting at the $N_{0,0}$ moment at the highest energy point in the problem, at which the integral term vanishes; all moments of interest are then calculated at that energy point before proceeding to the next lower energy point. The integral term is evaluated from the values of the moments at the higher energy points assuming some type of behavior for the variation of the moments between the energy points.

Note that for a rigorous solution of $N_{n,l}$ a prior knowledge of only $N_{n-1,l-1}$ and $N_{n-1,l+1}$ is required. For a particular energy in the problem, the moments equations are solved in the diagonal sequence as indicated in Figure II-1. For the plane isotropic case, all moments whose sum of indice, $n+l$, is odd or $n < l$ have been shown to vanish⁽⁷⁾. The order of solution is then as shown in Figure II-2. The detailed consideration of numerical calculation is given in Reference 4.

Figure II-1: The Order of Solution of the Moments Equations
for a General Case.

		l					
		0	1	2	3	4	5
n	0	1	2	4	7	11	16
	1	3	5	8	12	17	
	2	6	9	13	18		
	3	10	14	19			
	4	15	20				
	5	21					

Figure II-2: The Order of Solution of the Moments Equations
for a Plane Isotropic Case.

		l					
		0	1	2	3	4	5
n	0	1					
	1		2				
	2	3		4			
	3		5		7		
	4	6		8			
	5		9				
	6	10					

II. 3. The Discrete Ordinates S_N Method:

The essential basis of the discrete- S_N method is that the angular distribution of the neutron flux is evaluated for a number of discrete angle variables (8,14,15,16). By considering enough such discrete directions, it is possible, in principle, to obtain a solution of the transport equation to any desired degree of accuracy. In the solution of practical problems by discrete ordinates techniques, a discrete energy variable is usually introduced, by means of a multigroup approximation, and a discrete space mesh is used for the spatial coordinates. Consequently, all the independent variables of the time-independent neutron transport equation, namely, space $\vec{r}$, direction $\vec{\Omega}$, and energy E are treated as discrete.

II. 3. 1. The Multigroup Equations:

The first step in the development of multigroup equations is to divide the energy range in the problem into a finite number of intervals, say G , with arbitrary width as shown in Figure II-3, where E_g is interval

boundary, $g = 0, 1, 2, \dots, G$ and $E_{\min} = E_G E_{G-1} \dots E_1 E_0 = E_{\max}$. The next step is to integrate the energy dependent equation (II-5) over the group energy from E_g to E_{g-1} . The equation for the g th group takes the following form:

$$\mu \frac{\partial N^g(x, \mu)}{\partial x} + \Sigma_T^g N^g(x, \mu) = F^g(x, \mu) + S^g(x, \mu) \quad (\text{II-17})$$

where

$$N^g(x, \mu) = \int_{E_g}^{E_{g-1}} dE N(x, \mu, E) \quad (\text{II-18})$$

$$\Sigma_T^g = \frac{1}{N^g(x, \mu)} \int_{E_g}^{E_{g-1}} dE \Sigma_T(E) N(x, E, \mu) \quad (\text{II-19})$$

$$F^g(x, \mu) = \int_{E_g}^{E_{g-1}} dE F(x, \mu, E) \quad (\text{II-20})$$

and

$$S^g(x, \mu) = \int_{E_g}^{E_{g-1}} dE S(x, \mu, E) \quad (\text{II-21})$$

If $F(x, \mu, E)$ is expanded into a Legendre polynomial series as follows:

$$F(x, \mu, E) = \sum_{\ell} \frac{2\ell+1}{4\pi} F_{\ell}(x, E) P_{\ell}(\mu) \quad (\text{II-22})$$

where $F_l(x, E)$ is defined as in equation (II-9), then

$$\begin{aligned} F^g(x, \mu) &= \int_{E_g}^{E_{g+1}} dE \sum_l \frac{2l+1}{4\pi} F_l(x, E) P_l(\mu) \\ &= \sum_l \frac{2l+1}{4\pi} \int dE' N_l(x, E') \int_{E_g}^{E_{g+1}} dE \Sigma_{s,l}(\vec{E}' \rightarrow \vec{E}) \end{aligned} \quad (\text{II-23})$$

If the integral over E' in equation (II-23) is expressed as the sum of integrals over all energy groups. i.e.

$$\int dE' = \sum_{g'} \int_{E_{g'}}^{E_{g'+1}} dE', \quad (\text{II-24})$$

Equation (II-23) becomes

$$\begin{aligned} F^g(x, \mu) &= \sum_l \frac{2l+1}{4\pi} \sum_{g'} \int_{E_{g'}}^{E_{g'+1}} dE' N_l(x, E') \int_{E_g}^{E_{g+1}} dE \Sigma_{s,l}(\vec{E}' \rightarrow \vec{E}) \\ &= \sum_l \frac{2l+1}{4\pi} \sum_{g'} \Sigma_{s,l}^{g' \rightarrow g} N_l^{g'}(x) \end{aligned} \quad (\text{II-25})$$

where

$$N_l^g(x) = \int_{E_g}^{E_{g+1}} N_l(x, E) dE \quad (\text{II-26})$$

$$\Sigma_{s,l}^{g' \rightarrow g} = \frac{\int_{E_{g'}}^{E_{g'+1}} dE' N_l(x, E') \int_{E_g}^{E_{g+1}} dE \Sigma_{s,l}(\vec{E}' \rightarrow \vec{E})}{\int_{E_g}^{E_{g+1}} dE' N_l(x, E')} \quad (\text{II-27})$$

The transport equation (II-17) becomes

$$\begin{aligned} \mu \frac{\partial N^g(x, \mu)}{\partial x} + \sum_r^g N^g(x, \mu) = S^g(x, \mu) \\ + \sum_l \frac{2l+1}{4\pi} \sum_{g'} \bar{\Sigma}_{s,l}^{g' \rightarrow g} N_l^{g'}(x) \end{aligned} \quad (\text{II-28})$$

The multigroup equations (equation (II-28)) are coupled to one another through the scattering term. This set of equations derived is rigorous through the definitions of group constants (equations (II-19) and (II-27)). The approximations are first introduced in the evaluation of these constants. The total and transfer cross sections must be weighted by the angular flux and its Legendre components, respectively, according to their definitions. However, neither the angular flux nor its Legendre components are known before solving the transport equation. Various weighting functions have to be selected to approximate them.

II. 3. 2. The Discrete S_N Equations:

Consider equation (II-28) for a set of discrete directions $\{\mu_m\}$, $m = 1, 2, \dots, M$; if the integral over angle is evaluated by a numerical quadrature, i.e.

$$N_{\ell}(x, E) = 2\pi \int_{-1}^{+1} d\mu N(x, \mu, E) P_{\ell}(\mu) \\ \simeq \sum_m 2\pi \omega_m N(x, \mu_m, E) P_{\ell}(\mu_m)$$

(II-29)

where the ω_m 's are the quadrature weights, and

$$\sum_m \omega_m = 1 \quad (II-30)$$

Or

$$N_{\ell}^g(x) \simeq \sum_m 2\pi \omega_m N^g(x, \mu_m) P_{\ell}(\mu_m) \quad (II-31)$$

Then equation (II-28) is approximated by

$$\mu_m \frac{\partial N^g(x, \mu_m)}{\partial x} + \sum_r^g N^g(x, \mu_m) = S^g(x, \mu_m) \\ + \sum_{\ell} \frac{2\ell+1}{2} \sum_{g'} \sum_m \omega_m \Sigma_{s,\ell}^{g' \rightarrow g} N^{g'}(x, \mu_m) P_{\ell}(\mu_m) \quad (II-32)$$

This set of M coupled differential equations can be solved readily by finite difference techniques once the boundary conditions and the characters of the problem are specified. Consider a set of spatial intervals as shown in Figure II-4, where T is the thickness of the

slab. The derivative term in equation (II-32) is then approximated by finite difference, such as

$$\left. \frac{\partial N^g(x, \mu_m)}{\partial x} \right|_{\text{Interval } I} \approx \frac{N^g(x_i, \mu_m) - N^g(x_{i-1}, \mu_m)}{x_i - x_{i-1}} \quad (\text{II-33})$$

Then equation (II-32) is approximated by

$$\begin{aligned} \mu_m \frac{N^g(x_i, \mu_m) - N^g(x_{i-1}, \mu_m)}{x_i - x_{i-1}} + \sum_T \frac{1}{2} N^{g,2}(\mu_m) \\ = \sum_l \frac{2l+1}{2} \sum_{g'} \sum_m \omega_m \sum_{s,l} \frac{g' \rightarrow g}{\Sigma_{s,l}} N^{g',I}(\mu_m) P_l(\mu_m) \\ + S^{g,I}(\mu_m) \end{aligned} \quad (\text{II-34})$$

where $N^{g,I}(\mu_m)$ is the g -th group neutron flux evaluated in the mid-point of interval I and in direction μ_m . Now, there are three unknowns, namely, $N^g(x_i, \mu_m)$, $N^g(x_{i-1}, \mu_m)$ and $N^{g,I}(\mu_m)$, in equation (II-34). One of them is, in fact, known, either from boundary condition or from calculations in the adjoining intervals. For example, for 0 (the incoming direction), the commonly used "vacuum" boundary condition means that $N^g(x_I, \mu_m) = 0$ for any g or m . One other equation is required for use with equation (II-34) to allow calculation of the unknown flux. Usually, a relationship between those three unknowns is used, i.e.

$$N^{g,i}_{(\mu_m)} = f(N^g(x_i, \mu_m), N^g(x_{i-1}, \mu_m)) \quad (\text{II-35})$$

For example, the simple symmetric and arithmetic mean relation as follows:

$$N^{g,i}_{(\mu_m)} = \frac{N^g(x_i, \mu_m) + N^g(x_{i-1}, \mu_m)}{2} \quad (\text{II-36})$$

is frequently used. Thus, one can solve equation (II-34). Ordinarily, iterations are performed since the scattering term in equation (II-34) depends on the flux of the particular energy group of interest through the in-group scattering term.

Figure II-3: Division of the Energy Range.

Figure II-4: Division of the Spatial Range.

III. Sodium Cross Section

III. 1. Introduction:

The transport of neutrons in a bulk medium fundamentally depends upon the ways in which neutrons interact with nuclei of the medium. The measure of those interactions is termed cross sections. Therefore, it is necessary and important to examine the cross sections used in the transport calculations.

Natural sodium is monoisotopic, that is, 100 % sodium 23. So only a pure sodium-23 medium was considered. The new version of the ENDF/B III cross section set of sodium, Material Number 1156⁽⁶⁾, was chosen as the basic set of sodium cross section for the neutron transport calculations. The variations of cross sections with neutron energy, as stated in the ENDF/B file, are shown in Figures III-1 to III-3⁽²⁰⁾.

SODIUM-23

ENDF/B MAT NO. 1156

Figure III-1

$^{11}\text{Na}^{23}$

SODIUM-23

ENDF/B MAT NO. 1156

Figure III-2

Figure III-3

III. 2. General Behavior of Sodium Cross Sections:

Sodium exhibits almost every type of neutron cross-section peculiarity, which is possible for a non-fissionable nucleus, namely, (n,n) , (n,n') , (n,γ) , (n,d) , (n,p) , and $(n,2n)$, in some energy range or another. Above 5 MeV the total cross section exhibits only minor fluctuations about an average level of about 1.6 barns. At the other end of the energy range, it is substantially constant at about 3.2 barns below 200 KeV except at two wide scattering resonances and for a few very narrow absorption resonances of negligible significance. (The resonance data and corresponding results of resonance integral and non-escape probability calculations⁽²¹⁾ are shown in Tables III-1 to III-3.) In the range from 200 KeV up to 5 MeV, the total cross section has large fluctuations from between values of 1.2 barns and 8.3 barns, with an average value of about 4.5 barns, which is higher than that in any other region. There are a number of valleys in this region, particularly at 297 KeV, 521 KeV, 1.885 MeV and 3.075 MeV. Table III-4 shows the location of dips and the values of the cross sections at the minima. It is noted that only the one at 297 KeV is the real minimum of the total cross section over the

entire energy range. However, it is very narrow.

Radiative capture in sodium is very small except at some absorption resonances and at lowest energies. The absorption cross section is less than 1 millibarn in the energy range above 10 KeV, and varies according to the $1/v$ law below 1 KeV with value rising to about 0.1 barns at 1 eV.

Now let us examine the cross section behavior in some detail, starting at the upper energy and proceeding downward, just as the neutron does. Above about 10 MeV the angular distribution of elastic scattering is strongly peaked in the forward direction as would be expected⁽¹⁹⁾. The average cosine of the scattering angle, $\bar{\mu}$, as shown in Figure III-3, is greater than 0.65. In this region a neutron suffering an elastic scattering loses, on the average, less than 3 % of its energy. The average increase in lethargy per elastic collision, $\bar{\xi}$, as a function of neutron energy is shown in Figure III-4. A neutron with energy of 15 MeV, for example, loses only about 336 KeV after an elastic collision. But it is not likely to make many elastic collisions. The elastic cross section is considerably

less than one half of the total cross section. Non-elastic reactions dominate in this range. About 15 % of the non-elastic cross section is true absorption chiefly via the (n,p) and (n,α) reactions, and, of course, the (n,γ) reaction although this is negligible in magnitude. Most of the non-elastic reactions, however, produce one or more secondary neutrons at considerably reduced neutron energy via (n,n') and $(n,2n)$ reactions.

The energy range spanned by the secondary neutrons is very wide with the highest energies resulting, of course, from inelastic scattering to the lowest possible excited levels of the target nuclei. There are eighteen (18) discrete levels with the minimum threshold energy of 470 KeV. Table III-5 shows the Q values and relevant data for these levels. It is noted that all inelastic reactions to discrete levels are considered in the evaluation to drop to zero above about 10 MeV, as shown in Table III-5. This arise because above about 9 MeV of excitation the levels are so closely spaced that they are, in effect, considered as a continuum and therefore give rise to a continuous spectrum of seconsary neutrons. This continuous spectrum takes on the form of a Maxwellian evaporation spectrum⁽¹⁹⁾ with a nuclear

temperature⁽¹⁹⁾ of the range from 1.65 MeV to 2.15 MeV. At the highest incident energies another neutron production reaction, (n,2n) reaction, is present. However, this reaction is relatively less important for a fission source than the inelastic scattering in view of its higher threshold energy (12.95 MeV) and its lower cross section, as shown in Figure III-2. The secondary neutrons from the (n,2n) reaction are twice as numerous as those from inelastic reactions, and have also a continuous spectrum on the form of a Maxwellian distribution with a nuclear temperature in the range between 0.1 MeV to 0.41 MeV. Thus, for an incident neutron with energy of 15 MeV, the spectrum of secondary neutrons stretches all the way from about 14.7 MeV to low KeV region, with maximum in 0.41 MeV to 2.15 MeV range. Further, the spectrum is far from smooth with many gaps in the higher energy portion.

As one goes down in incident energy, the (n,2n) reaction drops out at about 13 MeV and the continuum (n,n') reaction drops out at about 9 MeV. However, the total non-elastic cross section stays about the same, with all of the difference going into cross sections of the relatively low-lying discrete levels. The spectrum of secondary neutrons, therefore, has a lot of spikes

and probably hardens as the incident energy decreases from 9 MeV to 3 MeV.

Below about 7 MeV, the (n,α) and (n,p) cross sections begin to decrease rapidly although the total inelastic cross section remains, on the average, nearly constant at about 0.9 barns, as shown in Figure III-2. These two reactions, (n,α) and (n,p) , drop out completely at 4.04 MeV and 3.77 MeV, respectively. However, the total inelastic cross section in this energy range still stays about the same. Since the general level of the total cross section increases as the energy decreases, the difference, namely the elastic cross section also increases. Below 2 MeV the total cross section begins to show increased structures. The fluctuation of the total cross section increases as the energy decreases but ceases at about 200 KeV. Below 2 MeV, inelastic scattering is reduced to that arising from just one excited level at 470 KeV, which reduces neutron energy not much more than it is by elastic scattering. The chance of inelastic scattering is also down, on the average to about 20 % per collision. Thus, at lower energies elastic scattering is the dominant slowing-down mechanism. Energy loss per elastic collision is also considerably more effective,

as shown in Figure III-4. The reason, of course, is that the angular distribution of elastic scattering, as shown in Figure III-3, is much less anisotropic than it is at higher energies. The average cosine of the scattering angle, $\bar{\mu}$, as shown in Figure III-3, drops to about 0.35 on the average. The average energy loss therefore increases to about 5.8 % per collision. A 2.0 MeV neutron therefore loses about 100 KeV after an elastic collision, on the average.

The threshold for inelastic scattering is 470 KeV. Below that the only slowing-down mechanism is by elastic scattering. Thus, the neutrons can survive many collisions since absorption is still very small. By 200 KeV it becomes possible to resolve individual resonances. The angular distribution for elastic scattering is completely isotropic in the center of mass system, and the average loss of energy after elastic collision is about 8.5 %.

Below 1 KeV, the total cross section is nearly constant, and so is the elastic cross section. The absorption increases according to the $1/v$ law as energy decreases, as expected⁽¹⁹⁾.

Table III-1: Parameters of Sodium ENDF/B
Cross Section Evaluation.

Isotope	Sodium - 23
Fractional Abundance	1.0
Lower Energy Limit (eV)	600.0
Upper Energy Limit (eV)	1.50×10^5
Nuclear Spin	3/2
Spin Scattering Length (A^+)	0.53
Spin Scattering Length (A^-)	0.0
Number of ℓ States	2

Table III-2: Sodium Resonance Widths

INDEX	ℓ	ENERGY (eV)	J	RESONANCE WIDTHS (eV)		
				TOTAL	NEUTRON	RADIATION
1	0	2.850+3	1	410.47	410.0	0.47
2	0	1.147+5	2	13.72	11.0	2.72
3	1	7.530+3	1	1.512	0.012	1.50
4	1	3.540+4	3	1.62	0.86	0.76
5	1	5.300+4	2	1201.48	1200.0	1.48
6	1	1.295+5	3	1.874	0.374	1.50
7	1	1.391+5	2	4.83	3.33	1.50

Table III-3: Resonance Integrals and Non-escape Probability for Sodium Resonances.

INDEX	RESONANCE ENERGY (eV)	RESONANCE INTEGRAL*(b)	NON-ESCAPE PROBABILITY*(%)
1	2.850 E+03	1.175 -2 [#]	3.904 -0
2 ^{\$}	1.147 E+05	2.822 -4	9.372 -2
3 ^{\$}	7.530 E+03	9.324 -4	3.098 -1
4 ^{\$}	3.540 E+04	6.882 -4	2.287 -1
5	5.300 E+04	4.539 -4	1.508 -1
6 ^{\$}	1.295 E+05	5.639 -5	4.755 -2
7 ^{\$}	1.391 E+05	1.556 -4	1.312 -1

* At temperature of 1300° K.

[#] 1.175 -2 reads as 1.175×10^{-2} .

\$ Resonances which were ignored in moments calculations.

** Total non-escape probability = 4.866 %.

*** Absorption which was not considered in moments calculations = 0.811 %.

Table III-4: Sodium Cross Section Dips.

INDEX	ENERGY	TOTAL CROSS	ELASTIC CROSS	REMARK
<u> </u>	<u>(MeV)</u>	<u>SECTION (barns)</u>	<u>SECTION (barns)</u>	<u> </u>
1	3.075	1.5817	0.9767	
2	1.885	1.72	1.3257	
3	0.521	1.96	1.9479	Widest
4	0.297	1.20	1.19933	Deepest

Table III-5: Discrete Inelastic Levels for Sodium

INDEX	Q VALUE (MeV)	MAXIMUM CROSS SECTION (b)	ENERGY AT WHICH MAX. OCCURS, MeV	ENERGY * BOUNDARY, MeV
1	0.439	0.776	1.955	10.5
2	2.078	0.2275	3.925	10.0
3	2.393	0.09259	3.891	10.0
4	2.64	0.06817	3.925	10.05
5	2.705	0.09027	4.115	10.05
6	2.983	0.2747	3.277	10.5
7	3.68	0.054	6.20	10.0
8	3.88	0.1175	6.20	10.5
9	4.43	0.0489	5.38	10.0
10	4.77	0.0745	6.675	9.6
11	5.38	0.04375	8.65	12.0
12	5.53	0.02972	8.17	11.5
13	5.76	0.09	9.055	12.0
14	5.955	0.08	7.865	11.0
15	6.0785	0.08	7.865	11.0
16	6.27	0.07	8.65	11.0
17	7.11	0.06	8.65	12.0
18	7.79	0.06	9.65	12.5

* Energy boundary above which discrete inelastic scattering does not occur.

XI BAR FOR THE ELASTIC SCATTERING IN THE SODIUM MEDIUM

Figure III-4.

III. 3. Cross Sections for Moments Calculations:

For moments calculations, the ENDF/B cross sections were first processed with PROCENDF code⁽²²⁾ into a set of cross sections suitable as input to moments code, BMT⁽⁴⁾. There are about 2,100 point data for total and elastic cross sections, respectively, in the ENDF/B file. Because of the limitation of 700 data points in the BMT code, these two sets of cross sections were preselected before fitting into BMT code. Since the deep penetration of neutrons is sensitive to the cross section used⁽¹⁸⁾, the selection was based on the physical characteristics of cross section. However, in view of the necessity of quite lots of points to represent a resonance or a cross section minimum⁽¹⁷⁾, some fine and narrow resonances were smoothed out to reduce the number of data points. Only wide resonances at energies of 2.85 KeV and 53.0 KeV were taken into consideration. The other five (5) narrow resonances were ignored. The effect of this approximation results only in the neglect of absorption less than 1 %, as shown in Table III-3. In energy range of about 2 MeV to 3 MeV, some minor fluctuations were also smoothed out to reduce the number of data points. The total and elastic cross sections

used for moments calculations are shown in Figures III-5 and III-6. To test the effect of this smoothing of cross sections on the penetrating flux, several calculations (cases in Category L, Appendix E,) with finer cross section data, taking those narrow resonances and/or minor fluctuations into consideration, were also performed. They showed that the removal of these resonances and the smoothing of the particular fluctuations have insignificant effect on the penetrating flux, qualitatively and quantitatively.

The charged-particle reactions, (n,p) and (n,α) reactions, were treated as absorption, since we were not interested in the secondary charged particles. The neutron-production reaction, $(n,2n)$ reaction, was also treated as absorption due to the limitation of BMT code. The effect of neglecting this reaction for a fission source is very small since its threshold energy is so high and its cross section is so low. In fact, the number of source neutrons above the threshold is only 9.2×10^{-5} . It is surely an overestimate to assume that one half of them make $(n,2n)$ collisions. These 9.2×10^{-5} neutrons come out of the reaction with energies that can be approximated by a fission source truncated below 0.542

MeV (case 3-, cf. Appendix E). On this basis, and assuming the (n,2n) secondary neutrons are born about 25 cm from the source, it can be estimated that the (n,2n) secondary neutrons contribute 3.8×10^{-4} % to the 5m flux at 100 KeV.

The other cross sections remained the same as they were in the ENDF/B-III file.

Figure III-5

Figure III-6

III. 4. Cross Sections for ANISN Calculations:

For ANISN⁽⁵⁾ calculations, the ENDF/B cross sections were processed with AMPX code⁽²³⁾ into a set of group averaged cross sections having a 78 group structure with a P-3 approximation for the elastic scattering angular distribution. The 78 group set, whose structure is shown in Table III-6, was tailored to the sodium cross section characteristics. The weighting function used in averaging the reaction cross sections was $1/E\Sigma_T(E)$, where $\Sigma_T(E)$ was the total cross section at energy E. The group averaged reaction cross sections are shown in Table III-7. However, it is clear from this table that this set of group structure did not represent the sodium cross section structures very well, particularly in the energy ranges of cross section minima and resonances. For example, the averaged cross section in the group contained the 297 KeV minimum (Group 42) was 2.31 barns, while the point cross section at the bottom of this minimum was 1.20 barns. On the other hand, the group averaged cross section in the group contained the 53.0 KeV resonance (Group 48) was 4.71 barns, while the point cross section at the top of this resonance was 37.1 barns. Thus, results calculated with this set of group structure

have a large error in these energy regions. However, as will be discussed in Chapter V, this has major effects only in the regions of minima and resonances themselves, but has minor effects on regions at other energies. Thus, the group structure does not affect the integrated penetrating flux qualitatively.

All ANISN calculations performed were with S-16 angular quadrature and a distributed fission source at the origin of the slabs. The direction cosines and their corresponding weights used are shown in Table III-8. Among the ANISN calculations, only two of them will be reported in this study. One is the case with a 5m sodium slab, and the other is the one with a 6m sodium slab. Both cases were with a comparable spatial-interval division. The largest spatial interval in these divisions was 4 cm.

Table III-6: MULTIGROUP STRUCTURE

GROUP	ENERGY RANGE	LETHARGY RANGE	GROUP	ENERGY RANGE	LETHARGY RANGE
1	1.2040E 07 - 1.4191E 07	-0.350 - -0.250	40	3.0192E 05 - 3.0942E 05	3.475 - 3.500
2	1.1331E 07 - 1.2840E 07	-0.250 - -0.125	41	2.9452E 05 - 3.0197E 05	3.525 - 3.550
3	1.0719E 07 - 1.1331E 07	-0.125 - -0.075	42	2.8725E 05 - 2.9452E 05	3.550 - 3.575
4	9.5123E 06 - 1.0719E 07	-0.075 - -0.050	43	2.7324E 05 - 2.8725E 05	3.600 - 3.650
5	8.1873E 06 - 9.5123E 06	0.050 - 0.200	44	1.9255E 05 - 2.7324E 05	3.600 - 3.950
6	7.0469E 06 - 8.1873E 06	0.200 - 0.350	45	1.5764E 05 - 1.9255E 05	3.950 - 4.150
7	6.3763E 06 - 7.0469E 06	0.350 - 0.450	46	1.1109E 05 - 1.5764E 05	4.150 - 4.500
8	5.2205E 06 - 6.3763E 06	0.450 - 0.650	47	5.9462E 04 - 1.1109E 05	4.500 - 5.125
9	4.9659E 05 - 5.2205E 06	0.650 - 0.700	48	4.0068E 04 - 5.9462E 04	5.125 - 5.500
10	4.8432E 06 - 4.9659E 06	0.700 - 0.725	49	2.6058E 04 - 4.0068E 04	5.500 - 5.950
11	4.6070E 06 - 4.8432E 06	0.725 - 0.775	50	2.4799E 04 - 2.6058E 04	5.950 - 6.000
12	4.4933E 06 - 4.6070E 06	0.775 - 0.800	51	2.3579E 04 - 2.4799E 04	6.000 - 6.050
13	4.2741E 06 - 4.4933E 06	0.800 - 0.850	52	1.9305E 04 - 2.3579E 04	6.050 - 6.250
14	4.0657E 06 - 4.2741E 06	0.850 - 0.900	53	9.1188E 03 - 1.9305E 04	6.250 - 7.000
15	3.8674E 06 - 4.0657E 06	0.900 - 0.950	54	5.5308E 03 - 9.1188E 03	7.000 - 7.500
16	3.1664E 06 - 3.8674E 06	0.950 - 1.150	55	4.0973E 03 - 5.5308E 03	7.500 - 7.800
17	3.0082E 06 - 3.1664E 06	1.150 - 1.175	56	3.0354E 03 - 4.0973E 03	7.800 - 8.100
18	3.0119E 06 - 3.0082E 06	1.175 - 1.200	57	2.8635E 03 - 3.0354E 03	8.100 - 8.158
19	2.5924E 06 - 3.0119E 06	1.200 - 1.350	58	2.7455E 03 - 2.8635E 03	8.158 - 8.200
20	2.4560E 06 - 2.5924E 06	1.350 - 1.400	59	2.6126E 03 - 2.7455E 03	8.200 - 8.250
21	2.2313E 06 - 2.4560E 06	1.400 - 1.500	60	2.4852E 03 - 2.6126E 03	8.250 - 8.300
22	1.9691E 06 - 2.2313E 06	1.500 - 1.625	61	1.5073E 03 - 2.4852E 03	8.300 - 8.800
23	1.9205E 06 - 1.9691E 06	1.625 - 1.650	62	9.6112E 02 - 1.5073E 03	8.800 - 9.250
24	1.8731E 06 - 1.9205E 06	1.650 - 1.675	63	4.5400E 02 - 9.6112E 02	9.250 - 10.000
25	1.8268E 06 - 1.8731E 06	1.675 - 1.700	64	2.7536E 02 - 4.5400E 02	10.000 - 10.500
26	1.7371E 06 - 1.8268E 06	1.700 - 1.750	65	1.6702E 02 - 2.7536E 02	10.500 - 11.000
27	1.6530E 06 - 1.7371E 06	1.750 - 1.800	66	1.0130E 02 - 1.6702E 02	11.000 - 11.500
28	1.3534E 06 - 1.6530E 06	1.800 - 2.000	67	6.1442E 01 - 1.0130E 02	11.500 - 12.000
29	1.1080E 06 - 1.3534E 06	2.000 - 2.200	68	3.7267E 01 - 6.1442E 01	12.000 - 12.500
30	9.0718E 05 - 1.1080E 06	2.200 - 2.400	69	2.2603E 01 - 3.7267E 01	12.500 - 13.000
31	7.4274E 05 - 9.0718E 05	2.400 - 2.600	70	1.3710E 01 - 2.2603E 01	13.000 - 13.500
32	6.0810E 05 - 7.4274E 05	2.600 - 2.800	71	8.3153E 00 - 1.3710E 01	13.500 - 14.000
33	5.3665E 05 - 6.0810E 05	2.800 - 2.925	72	5.0435E 00 - 8.3153E 00	14.000 - 14.500
34	5.2340E 05 - 5.3665E 05	2.925 - 2.950	73	3.0590E 00 - 5.0435E 00	14.500 - 15.000
35	5.1047E 05 - 5.2340E 05	2.950 - 2.975	74	1.8534E 00 - 3.0590E 00	15.000 - 15.500
36	4.9787E 05 - 5.1047E 05	2.975 - 3.000	75	1.1254E 00 - 1.8534E 00	15.500 - 16.000
37	4.7359E 05 - 4.9787E 05	3.000 - 3.050	76	8.3368E 01 - 1.1254E 00	16.000 - 16.300
38	3.5084E 05 - 4.7359E 05	3.050 - 3.350	77	6.2506E 01 - 8.3368E 01	16.300 - 16.588
39	3.0962E 05 - 3.5084E 05	3.350 - 3.475	78	4.1399E 01 - 6.2506E 01	16.588 - 17.000

GROUP	TOTAL	ELASTIC	INELASTIC	(N,2N)	ABSORPTION	(N,GAMMA)	(N,P)	(N,ALPHA)
1	1.62722E 00	7.57621E -01	6.61483E -01	9.79931E -03	1.90321E -01	2.04102E -04	5.08287E -02	1.47280E -01
2	1.60466E 00	6.64205E -01	7.21369E -01	0.0	2.1912E -01	1.86451E -04	6.10234E -02	1.57902E -01
3	1.55084E 00	6.00615E -01	7.63374E -01	0.0	2.04855E -01	1.78906E -04	6.99943E -02	1.34681E -01
4	1.50573E 00	5.90227E -01	8.02534E -01	0.0	1.84967E -01	1.76420E -04	8.07135E -02	1.04071E -01
5	1.57415E 00	5.93518E -01	8.66172E -01	0.0	1.14466E -01	1.72649E -04	5.33890E -02	6.09041E -02
6	1.66023E 00	6.86291E -01	9.02337E -01	0.0	7.16950E -02	1.69113E -04	4.79901E -02	2.34458E -02
7	1.69927E 00	7.55541E -01	9.07151E -01	0.0	3.65719E -02	1.66800E -04	3.22068E -02	4.20430E -03
8	1.85503E 00	9.27356E -01	9.05700E -01	0.0	2.19665E -02	1.64275E -04	2.07623E -02	1.40001E -03
9	1.84947E 00	9.60495E -01	8.85471E -01	0.0	3.50385E -03	1.62093E -04	2.80666E -03	5.35093E -04
10	1.85231E 00	9.85492E -01	8.64347E -01	0.0	2.46751E -03	1.61455E -04	1.86499E -03	4.41071E -04
11	1.93952E 00	1.08693E 00	8.51165E -01	0.0	1.52578E -03	1.60825E -04	1.01524E -03	3.49717E -04
12	1.83538E 00	9.94327E -01	8.40036E -01	0.0	1.01433E -03	1.60121E -04	5.95247E -04	2.58916E -04
13	2.03343E 00	1.18093E 00	8.51753E -01	0.0	7.38371E -04	1.60003E -04	4.00680E -04	1.77693E -04
14	2.01501E 00	1.13379E 00	8.80703E -01	0.0	4.29451E -04	1.60003E -04	2.04407E -04	6.50411E -05
15	2.03424E 00	1.13058E 00	9.02027E -01	0.0	2.30894E -04	1.60486E -04	6.95461E -05	8.62033E -07
16	2.24426E 00	1.41368E 00	8.30411E -01	0.0	1.60243E -04	1.65704E -04	2.53899E -06	0.0
17	1.90215E 00	1.24921E 00	6.52772E -01	0.0	1.70693E -04	1.70693E -04	0.0	0.0
18	1.79503E 00	1.19829E 00	5.96577E -01	0.0	1.71746E -04	1.71746E -04	0.0	0.0
19	2.43144E 00	1.73033E 00	6.50927E -01	0.0	1.75722E -04	1.75722E -04	0.0	0.0
20	2.81185E 00	2.15425E 00	6.57424E -01	0.0	1.80380E -04	1.80380E -04	0.0	0.0
21	2.91866E 00	2.22294E 00	6.95539E -01	0.0	1.84079E -04	1.84079E -04	0.0	0.0
22	3.04108E 00	2.42827E 00	6.12620E -01	0.0	1.89510E -04	1.89510E -04	0.0	0.0
23	2.68349E 00	2.03611E 00	6.47187E -01	0.0	1.93290E -04	1.93290E -04	0.0	0.0
24	1.85051E 00	1.3948E 00	4.53293E -01	0.0	1.94532E -04	1.94532E -04	0.0	0.0
25	2.14576E 00	1.73024E 00	4.15324E -01	0.0	1.95719E -04	1.95719E -04	0.0	0.0
26	2.78427E 00	2.15534E 00	6.28728E -01	0.0	1.97700E -04	1.97700E -04	0.0	0.0
27	2.56693E 00	1.97689E 00	5.89839E -01	0.0	2.00474E -04	2.00474E -04	0.0	0.0
28	2.56037E 00	2.16258E 00	3.97585E -01	0.0	2.07036E -04	2.07036E -04	0.0	0.0
29	3.21219E 00	2.59795E 00	6.14123E -01	0.0	2.18005E -04	2.18005E -04	0.0	0.0
30	3.77648E 00	3.27362E 00	5.02625E -01	0.0	2.30235E -04	2.30235E -04	0.0	0.0
31	4.64875E 00	4.30401E 00	3.44460E -01	0.0	2.65806E -04	2.65806E -04	0.0	0.0
32	5.59664E 00	5.33109E 00	2.65208E -01	0.0	3.43457E -04	3.43457E -04	0.0	0.0
33	4.04978E 00	3.92943E 00	1.20015E -01	0.0	3.36827E -04	3.36827E -04	0.0	0.0
34	4.04139E 00	3.42785E 00	1.32395E -02	0.0	3.03498E -04	3.08498E -04	0.0	0.0
35	1.99159E 00	1.98014E 00	1.11542E -02	0.0	2.95783E -04	2.95783E -04	0.0	0.0
36	2.17927E 00	2.16283E 00	1.61599E -02	0.0	2.83014E -04	2.83014E -04	0.0	0.0
37	2.40454E 00	2.38459E 00	1.96911E -02	0.0	2.65208E -04	2.65208E -04	0.0	0.0
38	3.50931E 00	3.58899E 00	3.12747E -05	0.0	2.84071E -04	2.84071E -04	0.0	0.0
39	3.12850E 00	3.12746E 00	0.0	0.0	5.40707E -04	5.40707E -04	0.0	0.0
40	3.72496E 00	3.72428E 00	0.0	0.0	6.84457E -04	6.84457E -04	0.0	0.0
41	2.73846E 00	2.73776E 00	0.0	0.0	6.90258E -04	6.90258E -04	0.0	0.0
42	2.30842E 00	2.30772E 00	0.0	0.0	6.95771E -04	6.95771E -04	0.0	0.0
43	3.05458E 00	3.05429E 00	0.0	0.0	6.90529E -04	6.90529E -04	0.0	0.0
44	4.37156E 00	4.37082E 00	0.0	0.0	7.33774E -04	7.33774E -04	0.0	0.0
45	3.31782E 00	3.31708E 00	0.0	0.0	7.39669E -04	7.39669E -04	0.0	0.0
46	3.42539E 00	3.42474E 00	0.0	0.0	6.50748E -04	6.50748E -04	0.0	0.0
47	3.60547E 00	3.60543E 00	0.0	0.0	2.47068E -04	2.47068E -04	0.0	0.0
48	4.70675E 00	4.70521E 00	0.0	0.0	1.53380E -03	1.53380E -03	0.0	0.0
49	3.81234E 00	3.81125E 00	0.0	0.0	1.09699E -03	1.09699E -03	0.0	0.0
50	3.91937E 00	3.91914E 00	0.0	0.0	2.24858E -04	2.24858E -04	0.0	0.0

GROUP	TOTAL	ELASTIC	INELASTIC	(N,2N)	ABSORPTION	(N,GAMMA)	(K,P)	(N,ALPHA)
51	3.94776E 00	3.94753E 00	0.00	0.00	2.25199E-04	2.25199E-04	0.00	0.00
52	4.02867E 00	4.02844E 00	0.00	0.00	2.28933E-04	2.28933E-04	0.00	0.00
53	4.55553E 00	4.55525E 00	0.00	0.00	2.81981E-04	2.81981E-04	0.00	0.00
54	6.55997E 00	6.55871E 00	0.00	0.00	9.81613E-04	9.81613E-04	0.00	0.00
55	1.23031E 01	1.23031E 01	0.00	0.00	2.02076E-06	2.02076E-06	0.00	0.00
56	4.25832E 01	4.25832E 01	0.00	0.00	1.66418E-06	1.66418E-06	0.00	0.00
57	3.00626E 02	3.00626E 02	0.00	0.00	1.44501E-06	1.44501E-06	0.00	0.00
58	3.42759E 02	3.42759E 02	0.00	0.00	1.40120E-06	1.40120E-06	0.00	0.00
59	2.02165E 02	2.02165E 02	0.00	0.00	1.36350E-06	1.36350E-06	0.00	0.00
60	1.01990E 02	1.01990E 02	0.00	0.00	1.32575E-06	1.32575E-06	0.00	0.00
61	1.12530E 01	1.12530E 01	0.00	0.00	1.10066E-06	1.10066E-06	0.00	0.00
62	4.21955E 00	4.21449E 00	0.00	0.00	5.15577E-03	5.15577E-03	0.00	0.00
63	3.36310E 00	3.35517E 00	0.00	0.00	7.93106E-03	7.93106E-03	0.00	0.00
64	3.21635E 00	3.20036E 00	0.00	0.00	7.99203E-03	7.99203E-03	0.00	0.00
65	3.17654E 00	3.16816E 00	0.00	0.00	8.37957E-03	8.37957E-03	0.00	0.00
66	3.15751E 00	3.14079E 00	0.00	0.00	9.21631E-03	9.21631E-03	0.00	0.00
67	3.15133E 00	3.14079E 00	0.00	0.00	1.05468E-02	1.05468E-02	0.00	0.00
68	3.15167E 00	3.13918E 00	0.00	0.00	1.24909E-02	1.24909E-02	0.00	0.00
69	3.15250E 00	3.13662E 00	0.00	0.00	1.58831E-02	1.58831E-02	0.00	0.00
70	3.15306E 00	3.13351E 00	0.00	0.00	2.03480E-02	2.03480E-02	0.00	0.00
71	3.15670E 00	3.13063E 00	0.00	0.00	2.60717E-02	2.60717E-02	0.00	0.00
72	3.17471E 00	3.14125E 00	0.00	0.00	3.34591E-02	3.34591E-02	0.00	0.00
73	3.19721E 00	3.15425E 00	0.00	0.00	4.29624E-02	4.29624E-02	0.00	0.00
74	3.22246E 00	3.16730E 00	0.00	0.00	5.51650E-02	5.51650E-02	0.00	0.00
75	3.25124E 00	3.18041E 00	0.00	0.00	7.08326E-02	7.08326E-02	0.00	0.00
76	3.27019E 00	3.19181E 00	0.00	0.00	8.63774E-02	8.63774E-02	0.00	0.00
77	3.30878E 00	3.20874E 00	0.00	0.00	1.00039E-01	1.00039E-01	0.00	0.00
78	3.34566E 00	3.22639E 00	0.00	0.00	1.19271E-01	1.19271E-01	0.00	0.00

Table III-8: Angular Quadrature for ANISN Calculations.

<u>INDEX</u>	<u>COSINE (μ)</u>	<u>WEIGHT</u>	<u>Wt. X COSINE</u>
1	9.89401 -1	1.35762 -2	1.34323 -2
2	9.44574 -1	3.11267 -2	2.94015 -2
3	8.65613 -1	4.75792 -2	4.11860 -2
4	7.55404 -1	6.23145 -2	4.70726 -2
5	6.17876 -1	7.47979 -2	4.62158 -2
6	4.58017 -1	8.45782 -2	3.87382 -2
7	2.81603 -1	9.13017 -2	2.57108 -2
8	9.50124 -2	9.47253 -2	9.00007 -3
9	-9.50124 -2	9.47253 -2	-9.00007 -3
10	-2.81603 -1	9.13017 -2	-2.57108 -2
11	-4.58107 -1	8.45782 -2	-3.87382 -2
12	-6.17876 -1	7.47979 -2	-4.62158 -2
13	-7.55404 -1	6.23145 -2	-4.70726 -2
14	-8.65631 -1	4.75792 -2	-4.11860 -2
15	-9.44574 -1	3.11267 -2	-2.94015 -2
16	-9.89401 -1	1.35762 -2	-1.34323 -2
17	-1.00000	0.0	0.0

IV. Description of Transport Calculations

IV. 1. Methods and Assumptions:

Most of the transport calculations in this work were made with the moments-method code BMT⁽⁴⁾, typically with a 496 point energy grid covering the range from 15 MeV to 1.02 eV. More than 100 cases of moments calculations were performed, with each case assigned an index number. The index will be used to refer to a particular transport calculation in the following discussions. Appendix E gives a complete list of the cases and a brief description of special characteristics pertaining to each case. Only those features different from the standard case (case 0), are listed.

The standard problem treated was neutron transport in an infinite, homogeneous pure sodium-23 medium with a plane isotropic fission neutron source. The energy dependence of the fission source was approximated by Watt's fission spectrum, where the fraction of neutrons emitted with energies in the range from E to $E + dE$ was given by⁽³²⁾

$$S(E) dE = 0.48394 \exp(-E) \sinh(\sqrt{2E}) dE \quad (\text{IV-1})$$

for $0 \leq E \leq 15$ MeV. The spectrum with the above parameters closely approximates the experimental U^{235} fission spectrum. The average energy of the fission neutrons found by integrating over the spectrum, viz.

$$\bar{E} = \int_0^{\infty} E \cdot S(E) dE \quad (IV-2)$$

is 1.98 MeV. On the other hand, the most probable neutron energy, that is, the energy corresponding to the peak of the $S(E)$ curve is only about 0.72 MeV.

The atomic density of the sodium medium was assumed to be 0.025×10^{24} atoms/cm³ (equivalent to 0.954 grams/cm³.) This density was about 1 % higher than the average density of the sodium samples used in the ORNL benchmark experiment (i.e. 0.945 grams/cm³ (2)). The 1 % difference means anywhere up to 20 % difference in flux at 5m. However, this quantitative difference in flux does not change the pictures of the deep penetration qualitatively.

The basic cross section set used was that of ENDF/B-III, MAT 1156⁽⁶⁾, around which more than hundred odd variant sets were constructed by altering one or

more cross section features. The general behavior of sodium cross sections is discussed in Chapter III.

A P-8 approximation for the elastic scattering angular distribution in the center of mass system was assumed for the input to the BMT code. The code automatically calculated the transfer cross sections on the basis of a P-5 approximation to the elastic scattering kernel in the laboratory system. On the basis of previous calculations with other materials ⁽¹⁸⁾ and the studies for sodium ^(40,41), these approximations are expected to affect the flux by no more than 1%.

Since ENDF/B-III assumes that the inelastic scattering in sodium is isotropic in the laboratory system, it was simplest to use the infinite mass approximation in calculations with the BMT code. However, this approximation is not good enough for levels with small Q values where the energy loss in the laboratory system due to nuclear recoil can be comparable to Q. It is also a poor approximation for any Q value immediately near the threshold where the effective nuclear mass (see Reference 4) is much smaller than the actual mass. Therefore, the infinite mass approximation is, at least,

not good for the lowest level in sodium, whose Q value is only 439 KeV, and especially near threshold. To test the effect of this approximation, a set calculations for the finite mass approximation assuming isotropy in the center of mass system was also performed.

The 496 point energy grid used in the moments calculations was not of fixed lethargy spacing; rather the spacing from point to point was arbitrary, depending mainly on the variation of sodium cross section with neutron energy. Basically, there were 3 to 4 energy points selected for each collision interval. More closely-spaced points were selected in energy ranges where the neutron cross sections fluctuate relatively rapidly with energy, particularly in ranges of cross section valleys and resonances, since the deep-penetration flux is very sensitive to the energy grid chosen in the vicinity of cross section minima⁽¹⁷⁾. Table IV-1 shows the distribution of energy points through the whole energy range. As discussed in the last Chapter, sodium cross sections are relatively smooth in the highest and lowest portions of energy range of interest. Therefore, fewer energy points were taken in these ranges, as shown in intervals 1 and 7 in Table IV-1. For the range from 5 MeV down to 0.2

MeV, (a lethargy interval of 3.22) representing approximately 20 % of the whole lethargy of interest, there were 233 points, or about 47 % of the total number of energy points.

The results obtained directly from moments calculations are, of course, only the spatial moments of the fluxes and not the fluxes themselves. Therefore, reconstructions of the spatial flux distribution from moments had to be performed. Reconstructed fluxes at the distance of 5 meters from the source, which was of particular interest as discussed in Chapter I, were studied throughout this work.

Table IV-1: Distribution of Energy Grid

<u>INTERVAL</u>	<u>ENERGY RANGE</u> <u>(MeV)</u>	<u>LETHARGY</u> <u>RANGE</u>	<u>NO. OF ENERGY</u> <u>POINTS</u>
1	15.0 - 5.0	1.10	30
2	5.0 - 3.0	0.51	54
3	3.0 - 1.0	1.10	90
4	1.0 - 0.5	0.69	48
5	0.5 - 0.3	0.51	18
6	0.3 - 0.2	0.41	23
7	0.2 - 1.02-6	12.19	233
Total	15.0 - 1.02-6	16.51	496

IV. 2. Reconstruction of Spatial Distribution from the Spatial Moments:

In addition to the restriction to infinite homogeneous media, the moments-method for solving the neutron transport equation also has the drawback that it provides only the spatial moments of the neutron flux and not the neutron flux itself. However, a variety of procedures, both old⁽⁷⁾ and new⁽²⁴⁾, are available for reconstructing the spatial distribution of the flux from moments. The ranges of the applicability of different schemes for reconstruction are strongly dependent upon the physical characteristic of the flux to be reconstructed. Therefore, the selection of methods for use should have physical justification.

A number of different schemes^(7,12,25,26) were tried to reconstruct the spatial distribution from the set of moments obtained from the standard run (case 0). Two of these methods were finally chosen for practical use because of better internal consistency and because they accorded with the physical nature of the penetration processes. One of the two methods was the biorthogonal polynomial method^(7,12,25) to a Gaussian weight function,

and the other was the simple exponential fitting method. Suitable biorthogonal polynomials to the Gaussian weight do not seem to have been reported in the literature previously, and they were therefore constructed by what is in principle a Gram-Schmidt procedure^(11,12), as detailed in Appendix A. Standard fitting procedures to an exponential are reviewed in Appendix B. The former method was applied to the energy range below 0.05 MeV and the latter to the energy range above 0.05 MeV.

As will be shown below, in Section V-2, the conditions for the age theory approximation are fairly well satisfied below 50 KeV, and the spatial distribution of the flux is to a first crude approximation given by a Gaussian. The product of a Gaussian and a polynomial in distance should therefore describe the spatial distribution quite closely. The most elegant and straight forward way of finding the polynomial which fits the calculated even spatial moments is to use a biorthogonal polynomial expansion with a Gaussian weight. The rapid convergence of this expansion in the reconstructed flux proves this choice to be correct.

On the other hand, the fluxes at higher energies

are expected to be asymptotically exponential^(8,9,19).

In this case, the moment ratio function, defined as

$$R_{2n}(E) = \frac{N_{2n+2,0}(E)}{N_{2n,0}(E)} \quad (IV-3)$$

where n is integer, can be shown⁽⁴³⁾ to be related to the square of the average relaxation length of the flux, which is the average distance over which the flux decays a factor of e . (A brief derivation is shown in Appendix B.) Thus, the quantity $R_{2n}(E)$ viewed as a function of index n gives a fair representation of the local relaxation length of the flux as a function of distance. The information contained in such a curve ($R_{2n}(E)$ vs. n), called the "signature" curve of the flux at energy E ⁽⁴³⁾, gives a clue of whether the flux approaches its asymptotic value, and the ratios of moments of higher order give the approximate value of the local relaxation length. Figure IV-1 shows some examples of the signature curves as taken from the calculation of the standard case in sodium (case 0). It is clear from this figure that the fluxes at these energies (5.0, 2.13 and 0.1 MeV) very rapidly approach an asymptotic exponential with a

relaxation length of about 28 cm. For these signature curves, the rapid approach to a constant value implies that the relaxation length of the deeply penetrating flux does not change significantly in the region of space contributing to the higher moments⁽⁴³⁾. It confirms the asymptotic simple exponential character of the deeply penetrating flux. Thus, from this information we can successfully reconstruct the local flux (i.e. in the neighborhood of 5m) by the simple exponential fitting method. Appendix B gives more complete details.

The reconstructed energy spectrum of the neutron scalar flux at 5m for case 0 is shown in Figure IV-2. The high energy portion of this curve was obtained by reconstructing the spatial moments with the simple exponential fitting method, while the lower energy portion was reconstructed by the biorthogonal polynomial method to a Gaussian weight function. In the overlapping region, around 50 KeV, of the two reconstructing schemes, the difference in the reconstructed flux is within 3 %. In Figure IV-3, a corresponding spectrum of the 5m flux obtained by a ANISN calculation for a 6m sodium slab⁽⁴⁰⁾, in which case the edge effect on the flux at 5m is negligible, is also shown. In this case, the calculation

was made with a 78-group cross sections, processed by the AMPX code⁽²³⁾, as discussed in Section III-4. The difference between these two spectra is within 10 % except in regions of cross section minima and resonances. In these regions, the difference between these two spectra is substantially large, since the 78-group structure does not represent the cross section characters in these regions very well. The group average cross sections in regions of the minima are overestimated. As a result, the 5m flux at the energies of cross section minima is calculated by ANISN to be lower than that obtained by the moments calculation by a factor more than 10, especially at the 297 KeV minimum, since the deep penetrating flux is very sensitive to the cross section of the minima^(17,18). Similarly, the 5m flux in the 53 KeV resonance region obtained by ANISN calculation is higher than that obtained by the moments method by a factor more than 10. In regions immediately below the cross section minima, the difference of these two spectra is also a little higher than that at other energies due to the incorrect group cross section at the minima, which reduces the buildup of the penetrating flux at energies just below the minima. Overall, however, the agreement of the reconstructed flux to that calculated

by ANISN is very good.

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$$R_n = \sqrt{N_{\pi+250} / N_{\pi+0}} \quad (cm)$$

figure IV-1

SODIUM (NR 1156) RECONSTRUCTED FLUX (CASE 0)

Figure IV-2

Figure IV-3

IV. 3. Validity of the Application of the Infinite Medium Conclusions to the Finite Slab:

One of the restrictions of using the moments method for solving the neutron transport equation is that it can only perform infinite medium calculations. Thus, the conclusions gathered from the moments calculations, as will be discussed in the next Chapter, primarily apply only to an infinite medium case. However, if the leakage effect due to the presence of the boundary of the medium considered in the spectrum of the flux in a finite slab is local, i.e. the leakage of neutrons from the boundary does not affect the neutron penetration processes in the distances far from the boundary of the medium, the infinite-medium theory of neutron penetration could be extended and applied to the finite slab. To investigate the validity of the application of the infinite-medium conclusions to the finite slabs in sodium case, an examination of the difference of the spatial distribution of the penetrating flux between an infinite medium case and a finite slab case is necessary.

Two Anisn calculations were made. One was for a 5m slab, the other was for a 6m slab. Both cases used

the same cross section set and other parameters. At the 5m distance in the 6m slab, the leakage effects are negligibly small, and the flux at this point can be taken as representative of the infinite medium case. The flux at 5m in the two slabs was then compared to see the effect of the boundary leakage on the spectrum of the 5m flux. Table IV-2 shows this comparison at some energy groups. This comparison indicates that the leakage effect at high energy group are very small, however, the effect accumulates to the lower energy groups. At group 9 (from 4.97 MeV to 5.22 MeV), for example, the difference of the 5m flux in these two cases is less than 10 %, while at group 76 (from 0.834 eV to 1.13 eV), the difference is a factor of more than 2.5.

But comparison of the spatial distribution of these two calculations show that the edge effect is a local phenomenon. Table IV-3 to Table IV-6 show this comparison at some energy groups. It is clear from these tables that the fluxes in the two slabs do not differ substantially for all points more than 50 cm from the boundary. Thus, the neutron penetration mechanisms are not affected by the boundary except at distances very close to the boundary. And therefore, the infinite

medium theory of the neutron penetration in sodium can be applied to thick finite slabs. Naturally, one would expect that when the slabs are thin (1m or less), and the leakage becomes large, that there would be qualitative differences in the mechanisms that determine the spectra in the infinite and finite media respectively. Some evidence for the importance of the leakage phenomena in thin slabs has recently been found⁽⁴¹⁾.

Table IV-2: Comparison of the 5m fluxes in a 5m and a 6m slabs.

GROUP	ENERGY RANGE (eV)	FLUX (6m SLAB)	FLUX (5m SLAB)
1	1.28+7-1.42+7*	1.12 -11	1.08 -11
9	4.97+6-5.22+6	2.78 -10	2.41 -10
15	3.87+6-4.07+6	3.80 -10	3.06 -10
30	9.07+5-1.11+6	1.08 -08	6.40 -09
36	4.98+5-5.10+5	4.60 -09	3.02 -09
46	1.11+5-1.58+5	1.87 -07	9.24 -08
62	9.61+2-1.51+3	1.74 -06	7.07 -07
66	1.01+2-1.67+2	2.47 -05	1.18 -05
76	8.34-1-1.13-0	1.72 -04	6.72 -05

* 1.42+7 reads $1.42 \times 10^{+07}$.

Table IV-3: Comparison of the Spatial Distribution of
the Flux in a 5m Sodium Slab to that in a
6m Sodium Slab.(Group 15: 3.87 - 4.07 MeV)

DITANCE (cm)	FLUX IN A 6m SLAB (n/cm ² /sec/group)	FLUX IN A 5m SLAB (n/cm ² /sec/group)
1.5	1.4690 E -01	1.4690 E -01
50.0	8.5520 E -03	8.5520 E -03
102.0	1.0288 E -03	1.0288 E -03
150.0	1.5850 E -04	1.5850 E -04
202.0	2.1692 E -05	2.1691 E -05
250.0	3.5376 E -06	3.5376 E -06
302.0	5.0559 E -07	5.0559 E -07
350.0	8.5206 E -08	8.5205 E -08
402.0	1.2563 E -08	1.2562 E -08
450.0	2.1727 E -09	2.1676 E -09
462.0	1.4035 E -09	1.3950 E -09
474.0	9.0728 E -10	8.9248 E -10
482.0	6.7855 E -10	6.5655 E -10
486.0	5.8687 E -10	5.5597 E -10
490.0	5.0761 E -10	4.8333 E -10
498.0	3.7985 E -10	3.0628 E -10

Table IV-4: Comparison of the Spatial Distribution of the Flux in a 5m Sodium Slab to that in a 6m Sodium Slab. (Group 30: 0.907-1.11MeV)

<u>DISTANCE</u> <u>(cm)</u>	<u>FLUX IN A 6m SLAB</u> <u>(n/cm²/sec/group)</u>	<u>FLUX IN A 5m SLAB</u> <u>(n/cm²/sec/group)</u>
1.5	1.3518 E -00	1.3518 E -00
50.0	1.9362 E -01	1.9362 E -01
102.0	2.9569 E -02	2.9569 E -02
150.0	5.0226 E -03	5.0226 E -03
202.0	7.1845 E -04	7.1844 E -04
250.0	1.1812 E -04	1.1812 E -04
302.0	1.6658 E -05	1.6658 E -05
350.0	2.7361 E -06	2.7361 E -06
402.0	3.8865 E -07	3.8860 E -07
450.0	6.4592 E -08	6.4224 E -08
462.0	4.1292 E -08	4.0662 E -08
474.0	2.6410 E -08	2.5316 E -08
482.0	1.9611 E -08	1.8006 E -08
486.0	1.6900 E -08	1.4940 E -08
490.0	1.4565 E -08	1.2478 E -08
498.0	1.0821 E -08	6.3974 E -09

Table IV-5: Comparison of the Spatial Distribution of
the Flux in a 5m Sodium Slab to that in a
6m Sodium Slab. (Group 46: 111 - 158 KeV)

<u>DISTANCE</u> <u>(cm)</u>	<u>FLUX IN A 6m SLAB</u> <u>(n/cm²/sec/group)</u>	<u>FLUX IN A 5m SLAB</u> <u>(n/cm²/sec/group)</u>
1.5	2.5214 E -00	2.5214 E -00
50.0	1.1634 E -00	1.1634 E -00
102.0	2.8957 E -01	2.8957 E -01
150.0	6.3080 E -02	6.3080 E -02
202.0	1.0578 E -02	1.0578 E -02
250.0	1.8972 E -03	1.8972 E -03
302.0	2.8231 E -04	2.8231 E -04
350.0	4.7543 E -05	4.7543 E -05
402.0	6.8156 E -06	6.8141 E -06
450.0	1.1296 E -06	1.1180 E -06
462.0	7.2067 E -07	7.0172 E -07
474.0	4.5983 E -07	4.2893 E -07
482.0	3.4083 E -07	2.9798 E -07
486.0	2.9343 E -07	2.4288 E -07
490.0	2.5263 E -07	1.9862 E -07
498.0	1.8727 E -07	9.2381 E -08

Table IV-6: Comparison of the Spatial Distribution of
the Flux in a 5m Sodium Slab to that in a
6m Sodium Slab. (Group 76: 0.834 - 1.13 eV)

<u>DISTANCE</u> <u>(cm)</u>	<u>FLUX IN A 6m SLAB</u> <u>(n/cm²/sec/group)</u>	<u>FLUX IN A 5m SLAB</u> <u>(n/cm²/sec/group)</u>
1.5	4.4807 E -01	4.4807 E -01
50.0	4.0584 E -01	4.0584 E -01
102.0	2.9818 E -01	2.9818 E -01
150.0	1.8840 E -01	1.8840 E -01
202.0	9.6458 E -02	9.6458 E -02
250.0	4.5104 E -02	4.5104 E -02
302.0	1.7281 E -02	1.7281 E -02
350.0	6.3825 E -03	6.3812 E -03
402.0	1.9534 E -03	1.9475 E -03
450.0	6.0213 E -04	5.7726 E -04
462.0	4.4268 E -04	4.0875 E -04
474.0	3.2445 E -04	2.7664 E -04
482.0	2.6310 E -04	2.0294 E -04
486.0	2.3675 E -04	1.6916 E -04
490.0	2.1293 E -04	1.4073 E -04
498.0	1.7198 E -04	6.7162 E -05

V. Results of Transport Calculations

V. 1. Summary of Analysis of Transport Calculations:

In the course of this study, more than 100 separate moments calculations were performed. Each case was specially designed to elucidate and support one or more points in our theory of penetration mechanisms. In this study, not all parts of the spectrum are of equal interest. We have concentrated on two energy regions of particular importance. One is the part below 1 KeV which contains most of the integrated flux and contributes almost all response of low energy detectors, e.g. unmodified Bonner ball detectors⁽²⁾. The other is the region from about 100 KeV to 600 KeV, which is the most important region for subsequent penetration into iron in LMFBR designs⁽³⁹⁾ and for neutron biological dose⁽³⁹⁾. In describing the transport mechanisms responsible for these and other portions of the spectrum the term "dominate" or "dominance" will be frequently used, but in a very specific technical meaning. A particular cross section phenomenon or energy range is said to "dominate" the flux in some part of the spectrum, when the magnitude of the flux and its spatial behavior is almost completely determined by the cross sections in the given range or

the specific phenomenon, to the exclusion of almost all other cross section behaviors.

The conclusions or theorems gathered from these moments calculations may be summarized as follows:

CONCLUSION 1: Below 1 KeV the spectrum of the 5m flux is in rough, but not exact, agreement with the predictions of age theory.

CONCLUSION 2: At all energies below about 200 KeV the age-like behavior is in convolution with an approximately exponential component with an asymptotic relaxation length of about 28 cm.

CONCLUSION 3: The approximate exponential behavior, characterized by a relaxation length of about 28 cm, is a high energy penetration phenomenon, present up to highest energies in the flux spectrum arising from a full fission source.

CONCLUSION 4: The cross section minima in the sodium cross section provide one source of this high

energy exponential behavior, but there are others of nearly equal importance from high energies. However, the 297 KeV minimum, which is the deepest, is on verge of being the dominant feature for the flux in 50 to 250 KeV range.

The next three conclusions relate to the region above 5 MeV where the low level of the average total cross section (relative to other energy regions) and the marked forwardly peaked anisotropy of elastic scattering provides conditions for easy penetrations. These conclusions summarize the studies undertaken to assess the relative importance of this energy region to the detected flux below about 600 KeV.

CONCLUSION 5: The source region above about 4 MeV contributes a disproportionate share of the 5m flux in 100 to 600 KeV region. This behavior is independent of the presence of cross section minima.

CONCLUSION 6: Source neutrons born above about 5 MeV penetrate more easily than total source, but do not dominate the 5m flux below 600 KeV. At increasingly larger distances this may however become the dominant source energy range.

CONCLUSION 7: The above energy range can be made to be dominant at 5m but only by large changes in cross sections.

CONCLUSION 8: Even without any source neutrons above 5 MeV, flux still builds-up so as to have an asymptotic relaxation length of about 28 cm.

CONCLUSION 9: The high energy transport mechanism, outside of the local effects of the cross section minima, is a multiple-scattering buildup of scattered neutrons, distinct on the one hand from age-theory transport and on the other hand from transport dominated by a particular cross section phenomenon such as a deep minimum. It possesses the following characteristics:

a. Neutrons contributing to the 5m flux have undergone many scatterings, the order of 30 to 40 collisions for the 100 to 200 KeV region.

b. It differs from age theory in that spatial behavior is not Gaussian but closer to exponential.

c. The transport process is spread out over the entire energy range. No one energy region contributes the bulk of the spatial transport for flux in the 100 to 600 KeV range.

d. Inelastic scattering, at all energies where significant, decreases the penetrating flux, because it increases the rate of slowing down and decreases the forward transport, relative to elastic scattering.

e. Anisotropy of elastic scattering, at all energies, increases the penetrating flux, because it increases forward transport and decreases rate of slowing down, relative to isotropic elastic scattering, but it is *not* solely a transport cross section correction.

The supporting evidences for the above conclusions are presented and discussed in detail in the following sections.

V. 2. CONCLUSION 1: Below 1 KeV the Spectrum of the 5m Flux is in Rough, but not Exact, Agreement with the Predictions of Age Theory.

As shown in Figure IV-2, the energy spectrum of the 5m flux is very soft in the low energy range. It can be estimated from this plot that in the region below 1 KeV the spectrum of the 5m flux per unit lethargy behaves roughly like $1/E$. This is a very unusual behavior for neutron deep penetration. For most materials encountered in shielding, for example, iron, water, carbon, concrete etc. the low energy end of the spectrum rapidly reaches an equilibrium form. Further, this equilibrium spectrum is usually similar to the infinite medium spectrum, i.e. for constant cross section, constant in flux per unit lethargy. For example, Figure V-1 shows the flux spectrum per unit lethargy for a point isotropic fission source at the center of a 5m-radius sphere of carbon⁽²⁹⁾, where the cross section at low energies is also constant. Here the flux spectrum per unit lethargy is nearly constant, -- over a range in energy of a factor of 10 the spectrum changes by a factor of about 1.39. By contrast, the flux

spectrum per unit lethargy in the sodium case shown in Figure IV-2, changes a factor of 6.15 over the same range in energy (from 100 eV to 1 KeV)!

A glance at the total cross section for sodium (Figure II-5) shows that except for interruptions by two strong scattering resonances at 2.85 KeV and 53.0 KeV the total cross section is nearly flat from 190 KeV down to 1 eV. Even omitting the intervals of the resonances, where the cross sections are so high that no appreciable transport occurs, there is a lethargy range of about 10 over which the total cross section is constant, the scattering is isotropic in the center of mass system, and the absorption is small except at the lowest energies. Since the average lethargy increase per collision in sodium is only 0.0853, under these circumstances, the assumptions of age theory^(8,26,44) are nearly as well satisfied in this region as one could expect to find in nature.

It is natural, therefore, to examine, as the lowest approximation, what Fermi age theory predicts for the low energy spectrum. On this theory, the slowing-down

density, which is proportional to the flux per lethargy under the condition of constant cross section, is given for a plane isotropic monoenergetic source by

$$q(x,u) = \xi \sum_I N(x,u) = \frac{S}{\sqrt{4\pi\tau}} e^{-\frac{x^2}{4\tau}} \quad (V-1)$$

where $q(x,u)$ = the slowing-down density.

$N(x,u)$ = the scalar flux.

S = the external source strength.

τ = the Fermi age, given by:

$$\tau(u) = \int_0^u \frac{du'}{3 \xi \Sigma_T^2 (1 - \bar{\mu}_0)} \quad (V-2)$$

with $\bar{\mu}_0$ = the average cosine of the scattering angle.

ξ = the average lethargy increase per collision.

Here, equation V-2 is based on the assumption of no absorption. In equation V-1, the lethargy dependence of the spectrum is contained in τ , principally in the exponential. Since τ increases with lethargy, then at a fixed distance, x , the exponent decreases, and the flux can be expected to increase with lethargy, in agreement with observations. Quantitatively, if $N(x,u)$ were to behave like

$$N \propto \frac{1}{E^n} \quad (V-3)$$

then n would be given by

$$n = - \frac{d \ln N}{d \ln E} = - \frac{d \ln N}{d u} \quad (V-4)$$

If we apply this formula to the age theory prediction for N , we obtain a formula for the effective " n " as a function of the spectrum energy

$$n = \left(\frac{x^2}{4\tau} - \frac{1}{2} \right) \frac{1}{\tau} \frac{d\tau}{du} \quad (V-5)$$

Now, we need to evaluate the Fermi age for a fission source in sodium. If Fermi age theory were correct the ratio of the second spatial moment to the zeroth spatial moment should be exactly the Fermi age. In any case, in the region of constant cross section, τ should be a linear function of lethargy, so that $d\tau/du$ should be constant. Figure V-2 shows the ratios of the second spatial moment to the zeroth moment as a function of energy as calculated by the moments method for the sodium medium. It is clear that except for the regions of the two resonances τ is indeed a linear function of u at lower energies, with a

slope about $650 \text{ cm}^2/\text{lethargy}$, which agrees with elementary predictions by equation V-2. Using the data of Figure V-2, equation V-5 predicts that the flux spectrum per unit lethargy behaves like $1/E^{0.8}$ roughly around 1.6 eV and falls off more steeply at higher energies. Thus the age theory predicts a spectrum that rises even more rapidly with decreasing energy than is observed, i.e. a very soft spectrum.

In Figure V-3, the calculated spatial distribution of the flux at 2.05 eV is plotted relative to the Gaussian predicted by age theory using the age taken from Figure V-2. If the Gaussian age theory prediction were correct the plot would be a straight horizontal line. The first impression is that the age theory prediction is remarkable good --- out to 5m the deviation from the Gaussian never amounts to more than a factor of 2.6, although the flux itself falls by 4 decades in that distance. The second impression is that the deviations from age theory are clearly significant. Since the abscissa is the square of the distance, if there had simply been an error in choosing the age the plot would have been a straight line. Obviously there are non-negligible departures from the Gaussian even at this low energy.

The quantitative differences might possibly be explained by the effects of absorption, the nature of the effective source, and, of course, by breakdowns in age theory, which will be discussed in the next section. But clearly the nature of the low energy spectrum in sodium at deep penetrations qualitatively agrees with what age theory predicts.

Figure V-1

Figure V-2

DISTANCE, FROM THE SOURCE, Z (m)

Figure V-3: FLUX AT 2.05 eV

FLUX * EXP (Z*Z/4/tau) * 1.0E-05

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KEUFFEL & ESSER CO.

V.3. CONCLUSION 2: At All Energies Below about 200 KeV
the Age-like Behavior is in Convolution
with an Approximately Exponential
Component with an Asymptotic Relaxation
Length of about 28 cm.

The age theory thus can be successfully applied to sodium case to predict the behaviors of the energy spectrum of the penetrating flux in the low energy end qualitatively, as discussed in the last section. A question then naturally arises: Why do the other materials behave the way they do? Why do they have a spectrum shape nearly constant in lethargy and nearly independent of distance? The situation in these materials is actually well known and has been extensively studied^(17,30,31). Where the spectrum exhibits an equilibrium low energy shape, the cross section at high energies characteristically has one or more deep minima going to well below the average level of the cross section. These minima most often arise from destructive interference between a resonance and the potential scattering. Sometimes the minimum is a broad dip arising in the course of the long range behavior of the cross section. In any case, the region of the cross section minimum provides a favorable route of easy penetration through large distances, and the

rest of the slowing-down process occurs in the immediate vicinity of the detector. The apparent spectrum at the detection point is then roughly the same as would be seen in the vicinity of the original source, i.e. the infinite medium spectrum. The variation with distance, however, is not given by the random-walk of the Gaussian, but by the unscattered flight down the minimum, i.e. roughly an exponential.

One can give a semi-quantitative form to this picture by considering the spectrum as determined by the convolution of two transport kernels; one an exponential representing travel at the energy of the minimum, the other an age-theory Gaussian describing the rest of the slowing down and transport process:

$$N(x, u) \sim \frac{A}{\sqrt{4\pi z}} \int_{-\infty}^{\infty} e^{-\frac{|x'|}{\lambda}} e^{-\frac{(x-x')^2}{4z}} dx' \quad (V-6)$$

The relaxation length λ at the minimum should be nearly the same as the mean-free-path, $1/\Sigma_T$, if the minimum is so narrow that any scattering throws the neutron out of the minimum. If λ is sufficiently large that, for positive x ,

$$x \lambda \gg 2z \quad (V-7)$$

then the contribution in Equation (V-6) for x' negative is so small that the absolute sign on x' can be neglected and the integral may be approximated by

$$N(x,u) \rightarrow A \exp(\tau/\lambda^2) \exp(-x/\lambda) \quad (V-8)$$

$$x > 0$$

Thus, under these conditions, the flux falls off exponentially with the relaxation length λ , and the dependence on u occurs only through τ/λ^2 factor, so that

$$\frac{d \ln N}{d u} = \frac{1}{\lambda^2} \frac{d \tau}{d u} \quad (V-9)$$

which is usually quite small. At the other extreme, if τ is so large that

$$x \lambda \ll 2 \tau \quad (V-10)$$

then the effect of the added path at the minimum can be easily shown to be negligible. Equation (V-6) can be rewritten as

$$N(x,u) \sim \frac{A}{\sqrt{4\pi\tau}} \int_{-\infty}^{\infty} e^{-\frac{|x-x'|}{\lambda}} e^{-\frac{x'^2}{4\tau}} dx' \quad (V-11)$$

Under the assumptions of equation (V-10) the variable x' in the Gaussian can be replaced by x and the integral can be approximated by

$$N(x, u) \rightarrow \frac{2Az}{\sqrt{4\pi z}} e^{-\frac{x^2}{4z}} \quad (V-12)$$

Thus the model qualitatively covers both forms of behaviors that of the low energy spectrum in sodium on the one hand and in iron, carbon etc. on the other hand.

The question arises: Is the low energy spectrum of sodium also characterized by a convolution of kernels or is it an example of a "pure" age-diffusion? To answer this question, we have to examine the penetrating flux in the sodium case in more detailed manner and at various energies.

First let us examine an idealized sodium with constant cross section, isotropic elastic scattering and no absorption, and for a monoenergetic source, which is the situation providing the closest approximation to the age theory that could be found in a material of the same mass as sodium (Case CX2). In this case, there should be no exponential penetration mechanism present, since the cross section is constant. Thus if the age theory were true, the spatial moments of the scalar flux, which would be a pure Gaussian, would be given by

$$N_{2n,0} = A \cdot \frac{z^n}{n!} \quad (V-13)$$

Therefore, the plot of $(N_{2n,0}/N_{0,0})/(z^n/n!)$ against the index n will lie along a horizontal line with a value of unity, if the spatial dependence of the flux is purely Gaussian. Curve in Figure V-4 shows such a plot for the idealized sodium at a lethargy of 10.2 from the source energy (case CX2). It can be seen that the deviations from the Gaussian are substantial. When the cross sections are scaled up to be comparable to the flat region in real sodium, it can be shown that the true flux in this case at 5m is about $2\frac{1}{2}$ times that predicted by age theory. Therefore, even in the best of the cases, age theory can not be used quantitatively for deep penetration in the shield. Nonetheless, this curve gives some idea of the deviation expected from the Gaussian spatial dependence in sodium for a "pure" age-diffusion mechanism. Curve B in Figure V-4 is a similar plot for actual sodium at an energy of 1.02 eV, which corresponds to about the same stretch of constant cross section as in the previous idealized case. It is clear that the deviations are larger, perhaps signalling the existence of a non-age diffusion penetration which is just try to show itself.

One may also argue that the deviations from the idealized sodium case are due to the effects of absorption at low energies and the fact the source here is not monoenergetic but is a fission source. To test the effect of the absorption at low energies on the deviations from the idealized sodium case, the standard problem was rerun with nothing changed but with the absorption below 10 KeV set to zero (case 0NC). The ratios of $(N_{2n,0}/N_{0,0})/(\tau^n/n!)$ at 1.02 eV of these two cases (Cases 0 and 0NC) are shown in Table V-1 for comparison. It is clear from this table that the absorption at low energies has little effect on the ratio $(N_{2n,0}/N_{0,0})/(\tau^n/n!)$. Thus, the absorption at low energies is not the factor making the deviations of the spatial distribution of the penetrating flux in sodium from the Gaussian function larger than that in the idealized sodium. Next, consider a set of monoenergetic sources. Then the slowing down density from this set of sources is given by

$$q(x,u) = \sum_I N(x,u) = \sum_i \frac{S_i}{\sqrt{4\pi Z_i(u)}} e^{-\frac{x^2}{4Z_i}} \quad (V-14)$$

where S_i is the source strength of the i th monoenergetic source and $Z_i(u)$ is the age at lethargy u due to the i th source. The spatial moments for the same flux are given by

$$N_{2n,0}(u) = \sum_i A_i \cdot S_i \cdot \frac{z_i^n(u)}{n!} \quad (V-15)$$

where A_i is the normalization factor for the i th source. Now, if one breaks the energy range of the fission source into a set of energy intervals, and consider approximately the partial fission source in each energy interval as a monoenergetic source, then from the calculations with this set of partial fission sources (cases in Category A, Appendix E), one has

$$z_i(u) = \frac{N_{2,0}^i(u)}{N_{0,0}^i(u)} \quad (V-16)$$

Thus, one can calculate, according to equation (V-15), approximately the spatial moments from a fission source predicted by age theory. The ratios of the normalized moments $(N_{2n,0}/N_{0,0})$ calculated in case 0 to that calculated according to equation (V-15) are shown in Curve C in Figure V-5. Curves A and B in this figure are the same as in Figure V-4, since $(N_{2n,0}/N_{0,0})/(N_{2n,0}^{age}/N_{0,0}^{age})$ is equivalent to $(N_{2n,0}/N_{0,0})/(z^n/n!)$ in these two cases. It is clear that, at least, in the distance of interest, the deviations from the idealized sodium are still very large. This shows the existence of some other penetration mechanisms.

In order to show the existence of exponential penetration mechanism in the sodium case more explicitly, we examine the ratios of successive spatial even moments in another way. According to equation (V-12), if the spatial dependence of the scalar flux were pure Gaussian, the ratio of successive even moments could be written

$$\frac{N_{2n-2,0}(u)}{N_{2n,0}(u)} = \frac{n}{\tau} \quad (V-17)$$

Suppose the spatial dependence of the flux were purely exponential, i.e.

$$N_0(x,u) \sim A \cdot \exp(-|x|/\lambda) \quad (V-18)$$

with resultant even moments

$$N_{2n,0}(u) \sim 2A \lambda^{2n+1} \quad (V-19)$$

The corresponding ratio of moments would then be

$$\frac{N_{2n-2,0}(u)}{N_{2n,0}(u)} = \frac{1}{\lambda^2} \quad (V-20)$$

Thus, if we plot the ratio $N_{2n-2,0}/N_{2n,0}$ against the index

n, the ratio will lie along a straight line passing through the origin if the flux is a pure Gaussian, while the ratio will be constant if the flux is exponential. Figure V-6 shows such a plot at energies of 1.02 eV, 100 eV, and 100 KeV in the sodium case (case 0). The reason of showing the ratios of the moments at 100 KeV is that the neutron age, Z , at this energy is very small, and so another penetration mode, if it exists, has a better chance of not being swamped by the Gaussian. It is clear from this figure that there are substantial deviations from the pure Gaussian distribution for the spatial dependence of the fluxes at these energies. Further, the curve for energy at 100 KeV, clearly demonstrates that the penetration at large distance is almost entirely exponential, with a relaxation length, λ , that can be determined to be about 28 cm. On the other hand, the spatial distribution of the flux at 1.02 eV is more like a Gaussian than like an exponential. And the situation at energy of 100 eV is in between.

Another method to show the existence of the exponential penetration mechanism stems from the observation that if there were some way of reducing the age, τ , in sodium, then there would be a better chance of the additional exponential mechanism showing up. Clearly, τ can be reduced if the cross section, say in the flat region, were considerably increased. (See equation (V-2).)

Therefore, a moments calculation (Case OLEXI) was made in which the total cross section was increased to 8.07 barns below 244 KeV except where the two resonances naturally gave higher cross sections. In this case, the absorption cross section was kept the same as in the standard case (Case 0). Over most of the flat region, the effect corresponds to increasing the cross section by a factor of about 2.55 and decreasing the slope of the age, $d\tau/du$, by a factor of 2.55^2 or 6.5. (See equation (V-2).) The curve OLEXI in Figure V - 7 shows the age as given by the ratio of $N_{2,0}/N_{0,0}$ for this "pseudo-sodium"; the decreased values are clear. Figure V - 8 shows the 5m flux for this case. It is clear from this figure that the low energy end of the spectrum is now nearly flat, looking much like that for carbon. (cf. Figure IV-2) Further, the curve of ratios of $N_{2n-2,0}/N_{2n,0}$ against the index n at 1.02 eV, as shown in Figure V - 9, is nearly constant for

high order n. Clearly, it demonstrates the existence of the exponential penetration mechanism with an asymptotic relaxation length, λ , that can be determined from this curve to be about 28 cm.

At 1.02 eV and 5 meters, with the standard cross section set (Case 0) and $\lambda = 28.3$ cm, the dimensionless ratio involved in the limiting form of the convolution integral has the value

$$\frac{x\lambda}{2z} \approx \frac{14.150}{14.400} \approx 1.$$

Clearly, the exponential mechanism is not dominant. For the case OLEXI, the same ratio is

$$\frac{x\lambda}{2z} \approx \frac{14.150}{4.600} \approx 3;$$

the condition for the exponential behavior is more satisfied. Further, equation (V-9) predicts an energy dependence of

$$\frac{d \ln N}{d u} = \frac{1}{\lambda^2} \frac{dz}{d u} \approx 0.125$$

for case OLEXI (where absorption is not important.); the moments calculation actually shows a value of 0.110. The agreement is quite satisfactory, considering the crudity of the model. Therefore, the age-like behavior is in convolution with an exponential component with an

asymptotic relaxation length of about 28 cm. This conclusion can be extended to all energies below about 200 KeV, as evidenced by the signature curves shown in Figure V-10.

Table V-1.: Ratios $(N_{2n,0}/N_{0,0})/(\tau^n/n!)$ at 1.02 eV for
Cases 0 and ONC.

<u>n</u>	$(N_{2n,0}/N_{0,0}) / (\tau^n/n!)$	
	<u>Case 0</u>	<u>Case ONC</u>
0	1.0000	1.0000
1	1.0000	1.0000
2	1.0401	1.0397
3	1.1247	1.1233
4	1.2642	1.2609
5	1.4769	1.4704
6	1.7935	1.7813
7	2.2647	2.2425
8	2.9750	2.9349
9	4.0679	3.9955
10	5.7947	5.6620

Figure V-4

A. MOCK-UP SODIUM AT $U = 10.2$

B. REAL SODIUM AT 1.02 eV.

Figure V-5

A. MOCK-UP SODIUM AT $u = 10.2$.

B. REAL SODIUM AT 1.02 eV .

C. REAL SODIUM AT 1.02 eV ,
CONSIDERING ENERGY
DEPENDENCE OF SOURCE.

$$\frac{(N_{2n,0} / N_{0,0})}{(N_{2n,0}^{AGE} / N_{0,0}^{AGE})}$$

Figure V-6

COMPARISON OF AGES FOR CASES ØLEXI AND Ø

Figure V-7

Figure V-8

Figure V-9

Figure V-10

V. 4. CONCLUSION 3: The Asymptotic Exponential Behavior, Characterized by a Relaxation Length of about 28 cm, is a High Energy Penetration Phenomenon, Present up to Highest Energies in the Flux Spectrum arising from a Full Fission Source.

As shown in the last section, there exists in sodium some kind of penetration mechanisms manifested at energies below about 200 KeV by an exponential component with a relaxation length of about 28 cm. Does this phenomenon appear only in this particular energy range? To answer this question, we have to examine the signature curves, as given by the square roots of ratios of succeeding even spatial moments, at other energy points. This examination reveals that the signature curves at energies higher than about 200 KeV fall or rise to a plateau of about 28 cm or longer. In Figure V-11, the signature curves of several energy points between 230 KeV and 7.0 MeV are shown. It is clear from this figure that the spatial distribution at these energies is approximately an exponential with an asymptotic relaxation length of about 28 cm except at the highest energies,

e.g. 7.0 MeV, where the relaxation length rises to about 30 cm. (see Conclusion 6 below.) The approach of the spatial dependence of the flux to an exponential is shown more explicitly for the same energies in Figure V-12. The flux is plotted relative to an exponential with a relaxation length of 28 cm. Over most of this region, the difference from this simple exponential are less than a factor of 2, even though the flux itself falls by about 7 decades in this distance. The most prominent deviations are at 7.0 MeV, where a relaxation length of 30 cm seems more appropriate. Therefore, the asymptotic relaxation length of about 28 cm to 30 cm is present up to highest energies.

An examination of the calculations for partial fission sources (cases in Category A, Appendix E) shows that the 28 cm asymptotic relaxation length does not manifest itself until there are some source neutrons born above the 297 KeV minimum. Attention is therefore naturally turned to the role of the cross section minima in establishing the deep penetration exponential behavior.

Figure V-11

Figure V-12

A. FLUX AT 7.000 MeV (X 1000)

B. FLUX AT 4.515 MeV (X 100)

C. FLUX AT 2.130 MeV (X 10)

D. FLUX AT 0.230 MeV (X 0.1)

FLUX * EXP (Z / 28.0)

V. 5. CONCLUSION 4: The Cross Section Minima in the Sodium Cross Section Provide one Source of this High Energy Exponential Behavior, but there are Others of nearly Equal Importance from high Energies. However, the 297 KeV minimum, which is the deepest, is on verge of being the dominant feature for the flux in 50 to 250 KeV range.

As discussed in Section V-3, any neutron that lands in one of the cross section minima will travel a much greater distance before collision than a neutron at an energy corresponding to the average or peak cross section. The variation of flux with distance of this kind of penetration is given roughly by an exponential. It is also observed in the last section that cross section minima can provide one source of high energy exponential behavior. The question remains whether this process dominates the flux in 50 to 250 KeV range. In some materials, we know this mechanism does dominate the spectrum, e.g. deep penetration in iron⁽¹⁷⁾.

In sodium there are a number of valleys exhibited in the total cross section, particularly at 297 KeV,

521 KeV, 1.885 MeV and 3.075 MeV (cf. Table III-4), with the one at 297 KeV the deepest and the one at 521 KeV the widest. There are narrow spikes in the flux spectrum at these energies as shown in Figure IV-2. These "spikes" show that the flux at the energy of the minima arises from neutrons that penetrate a good fraction of the distance while in the minima themselves. To test the importance of these minima for the rest of the spectrum, a separate flux calculation (Case OS) was made with these four minima "filled up". By this, it is meant that the elastic cross section in the vicinity of the minima was increased so that the total cross section was brought up to the level of the neighboring energy regions, and no indication of a minimum at any of these energies remained in the total cross section curve. The ratio of the standard 5m flux to that with the smoothed cross section set is shown in Figure V-13. The results are disappointing. "Filling up" the four cross section dips has a large effect in the region of the minima themselves, but the effects do not persist outside of the minima. At 265 KeV, the elimination of the minima has reduced the flux only by a factor of 2. By 1.02 eV, the reduction is only 17 % but that is understandable, for the age-like diffusion is taking over. It should be noted that in materials where

the cross section minima really dominate the transport process, filling up the minima reduces flux by several orders of magnitude^(17,30,31). Therefore, in sodium, cross section minima can provide one source of high energy exponential behavior, but penetration via the minima does not dominate the transport process.

Examination of the asymptotic relaxation lengths in 50 to 250 KeV range shows that smoothing of cross section minima has reduced it by only a few percent. Clearly, with a full fission source there are other penetration mechanisms of nearly equal importance. That these additional mechanisms arise only in the regions of the spectrum above about 1 MeV can be demonstrated, for example, by truncating the fission source at 868 KeV (Case 3) and again investigating the effects of filling up the minima at 297 KeV and 521 KeV (Case 3S). The dramatic results are shown in Figure V-14. Not only does the presence of the minima increase the 5m flux at their energies by more than a factor of 100,000, but the effects persist to lower energies, and still amount to a factor of almost 1,300 at 100 KeV. Clearly, for source neutrons below about 1 MeV transport in the cross section minima does dominate the penetrating flux in the 50 to 250 KeV

range. But note that at a few eV range in both the full-fission source case and the partial-fission source case, the minima have a negligible influence, because transport to such low energies occurs by age-like diffusion in energy regions well below the locations of the minima.

A similar calculation with a truncated fission source, this time below 907 KeV, was performed in Reference 41. This calculation was made with the ANISN code⁽⁵⁾ for an infinite sodium slab 5m thick. The forward flux at 5m, taken from Reference 41, is shown in Figure V-15. It is clear from the huge steps in the spectrum that the source of the flux in 50 to 250 KeV range at 5m is primarily from those neutrons penetrating through the cross section minima at 297 KeV and 521 KeV. The relatively flat plateau down into the KeV region also shows that the minima are the dominant cross section feature for flux in 50 to 250 KeV range in this case.

But, if we truncated the fission source at higher energies (Cases 4 to 7), the importance of the minima progressively decreases as the upper cutoff energy is increased. This is shown in Table V-2 which lists the decrease in the 5m flux at 100 KeV upon "filling up" the

minima for various partial fission source (cases 4S to 7S). Even inclusion of fission neutrons up to 1.6 MeV cuts the effect of the minima on the 5m flux at 100 KeV by about a factor of 100. Some other penetration mechanism exists for these high energy source neutrons that is of approximately equal importance with transport via the minima.

However, the 297 KeV minimum, which is the deepest, is on verge of becoming important as seen in case 3S. To inquire further into the importance of this minimum, a number of separate flux calculations (cases ODDA to ODDF) for a full fission source were performed with higher or lower total cross section at bottom of the 297 KeV minimum. In these cases, the total cross sections in the neighborhood of the 297 KeV minimum were increased or decreased in such manner that the maximum change occurred at the bottom of the minimum, i.e. at 297 KeV, and the change in cross section tapered off to zero at 287 KeV, where the original cross section is at the average level in the neighborhood of the minimum. The formulism used for this change is detailed in Appendix C. The effects on the 5m flux at 100 KeV are summarized in Table V-3. These results are also shown in a more

sophisticated form in Figure V-16. In this figure, the abscissa is the logarithm of the ratio of the new total cross section at the bottom of the 297 KeV minimum, σ , to that of case 0, $\sigma_0 = 1.2$ barns, while the ordinate is the logarithm of the ratio of the 5m flux at 100 KeV for each new calculation, R , to that for case 0, R_0 . The slope of this curve at the origin may be defined as the sensitivity of the 5m flux at 100 KeV to the total cross section at the bottom of the 297 KeV minimum^(36,40,41). Clearly, its effect is nonlinear. Filling up by itself (case ODDF) reduces the 5m flux at 100 KeV by only about 20 %. But deepening the minimum by about 40 % in the lowest cross section (case ODDA) increases the same flux by more than a factor of 11. A positive error in the ENDF/B-III cross section at the 297 KeV minimum is not practicably significant. But even a relative small negative error in this cross section can lead to large uncertainty in the deeply penetrating flux between 50 and 300 KeV.

Table V-2: Ratios of the 5m Flux at 100 KeV

CASES	HIGHEST ENERGY OF FISSION SOURCE, MeV	RATIO OF THE 5m FLUXES
3 / 3S	0.868	1306.5
4 / 4S	1.600	14.7
5 / 5S	2.860	3.5
6 / 6S	3.925	3.3
7 / 7S	5.076	2.4
0 / 0S	15.000	2.0

Table V-3: Effects of the Cross Section at the
297 KeV Minimum on the 5m flux at
100 KeV.

CASE	CROSS SECTION AT <u>297 KeV (barns)</u>	RATIO OF THE FLUX TO <u>THAT OF CASE 0</u>
0	1.20	1.000
ODDA	0.75	11.31
ODDB	0.90	2.247
ODDC	1.10	1.135
ODDD	1.25	0.964
ODDE	1.50	0.889
ODDF*	2.71	0.817

* Case ODDF was with the minimum at 297 KeV "filled up".

THE STANDARD CASE VS. THE CASE WITH SMOOTHED CROSS SECTION SET (0/0S)

Figure V-13

CASE 2 VS. CASE 0S AND CASE 3 VS. CASE 3S

Figure V-14

Figure V-15.

Figure V-16

SENSITIVITY OF THE 5m FLUX AT 100 KeV TO THE
TOTAL CROSS SECTION AT THE 297 KeV MINIMUM

V. 6. CONCLUSION 5: The Source Region above about 4 MeV
Contributes a Disproportionate Share
of the 5m Flux in 100 to 600 KeV
Region. This Behavior is Independent
of the Presence of Cross Section Minima.

An examination of the total cross section of sodium, (Figure III-1. or Figure III-5) shows that above about 4 MeV the average cross section curve falls monotonically, and above 5-6 MeV is below the level of all except the deepest minima. Further the angular distribution for elastic scattering in the same energy region becomes progressively more peaked in the forward direction. Reflecting this behavior of the angular distribution, Figure III-4 shows that the average lethargy change per elastic collision falls steeply with increasing incident energy above about 4 MeV. One would expect as a result of these cross section properties that source neutrons born above 4-5 MeV would penetrate more easily than neutrons at almost any other energy. It might not be surprising therefore if this region of the source were to dominate the transport of neutrons reaching 5m at energies above the "age-dominated" portion of the spectrum. To test this possibility, the contribution of various portions of the source spectrum to the 5m flux was calculated.

A plane isotropic fission neutron source at the

origin of the sodium medium was assumed in the moments calculations. The energy dependence of the fission source was approximated by Watt's fission spectrum, as described in Section IV-1. Several moments calculations (cases in Category A, Appendix E) were made with a partial fission source to study the relative contribution to the flux from source neutrons with energies in different portions of the whole energy range. The results derived from these calculations are summarized in Table V-4. The calculations show that high energy source neutrons contribute disproportionately to the penetrating flux. Thus only above 3 MeV does a source energy interval ΔE_0 contribute a large fraction of the 100 KeV flux at 5m than the fraction it forms of the source. And the 4.3% of the total source 5 MeV contributes almost 33% of the flux at 100 KeV measure at 5m.

Furthermore, this behavior is independent of the presence of cross section minima as deduced from another set of calculations with a partial fission source and smoothed cross set (Cases 1PS to 9S). The relative contributions derived from these calculations are summarized in Table V-5. Source neutrons born above about 5 MeV contribute about 42% of the 5m flux at 100 KeV. The decreased contribution of the higher energy portions of the source when the minima are present indicates that the minima give source neutrons below 5 MeV a better chance of penetration in competition with the source neutrons above 5 MeV. In any case, it is clear from the evidence of the relative source contribution that the energy region above 5 MeV is favored for ease of deep penetration.

Table V- 4: Relative contribution of source neutrons to the
5m flux in the sodium medium.

Cases Involved	Source Energy (MeV)	Percent of source	Percent of the 5m flux at		
			<u>265 KeV</u>	<u>100 KeV</u>	<u>20.4 KeV</u>
1	0.000 - 0.040	0.4	0.00	0.00	0.00
1 & 2	0.040 - 0.350	8.1	0.05	0.05	0.06
2 & 3-	0.350 - 0.542	6.3	0.19	0.20	0.23
3-& 3	0.542 - 0.868	11.5	0.44	0.47	0.42
3 & 3+	0.868 - 1.250	13.4	0.78	0.75	0.97
3+& 4	1.250 - 1.600	10.2	0.95	1.23	1.38
4 & 5	1.600 - 2.860	28.0	11.1	12.9	16.2
5 & 6	2.860 - 3.925	11.7	27.1	28.4	29.9
6 & 7	3.925 - 5.076	6.1	23.0	22.6	21.8
8 & 9	5.076 - 7.000	3.1	22.7	21.3	18.9
9	7.000 -15.000	1.2	13.8	12.1	10.1

Table V-5 : Relative contribution of source neutrons to the 5m
flux in sodium; Smoothed cross section set.

Cases Involved	Source Energy (MeV)	Percent of Source	Percent of the 5m flux at		
			<u>265 KeV</u>	<u>100 KeV</u>	<u>20.4 KeV</u>
1PS & 0S	0.000 - 0.040	0.4	0.000	0.000	0.000
1PS & 2PS	0.040 - 0.350	8.1	0.000	0.001	0.007
2PS & 3PS	0.350 - 0.868	17.8	0.002	0.019	0.126
3PS & 4PS	0.868 - 1.600	23.6	0.15	0.357	0.984
4PS & 5PS	1.600 - 2.860	28.0	6.10	8.84	14.11
5PS & 6PS	2.860 - 3.925	11.7	20.57	23.44	25.85
6PS & 7PS	3.925 - 5.076	6.1	25.81	25.30	23.90
7PS & 9S	5.076 - 7.000	3.1	27.17	24.74	21.22
9S	7.000 -15.000	1.2	20.20	17.30	13.80

V. 7. CONCLUSION 6: Source Neutrons Born above about 5 MeV

Penetrate more easily than Total Source,
but do not Dominate the 5m Flux below 600
KeV. At increasingly larger Distances this
may become the Dominant Source Energy Range.

It was concluded in the previous section that neutrons born above about 4 MeV contribute more to the 100-600 KeV flux at 5m than their proportionate share of the source. Here it will be shown that of these the source neutrons above 5 MeV penetrate more easily than the rest of the source and thus could possibly become the dominant contributor. The asymptotic relaxation lengths in this energy range verify this conclusion. It has already been noted in Section V-4 that the flux at 7 MeV has a slightly higher asymptotic relaxation length. Table V-6 shows the asymptotic relaxation lengths at some other energies, as given by the square roots of the ratios of the 18th and 16th spatial moments, to which the flux in the neighborhood of 5m contributes. It is clear from this table that neutrons born above 5 MeV have longer relaxation length than at any other energy except at the 297 KeV minimum. The penetration ability of neutrons above 5 MeV is then better than the others.

In cases 8 and 9, with a partial fission source above 5.076 MeV and 7 MeV, respectively, the relaxation lengths in 100 to 600 KeV range are longer than those of case 0, as shown in Table V-7. For example, at 100 KeV, the relaxation length is 28.28

cm for total source, but goes up to 30.04 cm and 30.92 cm for partial sources above 5.076 MeV and 7 MeV, respectively. For the partial source between 5.076 and 7 MeV the relaxation length of the 100 KeV flux at 5m is about 29.5 cm, still substantially higher than the total source. But for the partial source between 3.925 and 5.076 MeV the similar relaxation length is 28.64 cm, indistinguishable from the total source. And for the next lower partial source, 2.86 to 3.925 MeV, the same relaxation length has fallen to 27.46 cm.

so source neutrons born above about 5 MeV definitely penetrate more easily than total source, and they thus form a physically distinctive portion of the source spectrum. But they do not dominate the 5m flux below 600 KeV, since the increase of the relaxation lengths in case 8 is only a few percent. When the minima are "filled up", the relative importance of these high energy source neutrons improves a little bit. In Table V-8, the relaxation lengths of case 0S, 8S and 9s are shown. Compared the numbers in Table V-7 and V-8, we can see that the relaxation length at 100 KeV increases 8.0% in case 8S compared to case 0S, while it increases only 6.2% in case 8 relative to case 0. But these are still only modest changes, and even in the absence of the cross section minima the high energy source neutrons still do not dominate the 5m flux below 600 KeV. On the basis of these observations one would expect, however, that with sufficiently thick shields, 7m or more, this portion of the source would overwhelm all other source neutrons in contributing to the flux in the 100 KeV-1 MeV range and would dominate the transport process.

Table V-6: The Mean-Free-Path and Asymptotic Relaxation Length in the Neighborhood of 5m in Sodium.

ENERGY (MeV)	MEAN-FREE-PATH <u>$1/\Sigma_T$ (cm)</u>	ASYMPTOTIC RELAXATION <u>LENGTH * (cm)</u>
1.430E+01	24.39	30.87
1.300E+01	25.00	31.42
1.185E+01	24.86	30.81
1.005E+01	25.60	29.97
8.170E+00	25.81	29.98
7.000E+00	23.88	29.27
6.200E+00	22.35	28.88
5.380E+00	20.83	28.40
3.000E+00	17.94	27.95
1.000E+00	13.10	27.78
2.970E-01	33.33	30.91
2.650E-01	11.76	28.27
1.000E-01	11.38	28.28
2.040E-02	9.85	28.36

* Asymptotic relaxation length = R_{16} (see equation (IV-3))

Table V-7: The Asymptotic Relaxation Lengths in the
Neighborhood of 5m in Sodium of Cases 0,
8, and 9.

ENERGY (KeV)	ASYMPTOTIC RELAXATION LENGTH (cm)		
	<u>CASE 0</u>	<u>CASE 8</u>	<u>CASE 9</u>
601.0	27.78	29.43	30.40
502.5	27.79	29.54	30.50
405.0	27.79	29.56	30.51
299.0	27.79	29.60	30.55
265.0	28.27	29.87	30.75
190.0	28.28	29.91	30.80
100.0	28.28	30.04	30.92

Table V-8: The Asymptotic Relaxation Lengths in the
Neighborhood of 5m in Sodium of Cases 0S,
8S, and 9S.

ENERGY (KeV)	ASYMPTOTIC RELAXATION LENGTH (cm)		
	<u>CASE 0S</u>	<u>CASE 8S</u>	<u>CASE 9S</u>
601.0	27.38	29.15	30.20
502.5	27.35	29.17	30.22
405.0	27.33	29.20	30.25
299.0	27.31	29.23	30.28
265.0	27.29	29.27	30.31
190.0	27.28	29.30	30.34
100.0	27.25	29.42	30.44

V.8. CONCLUSION 7: The Source Range above 5 MeV can be made to be Dominant at 5m but only by Large Changes in Cross Sections.

As concluded in the last section, source neutrons born above about 5 MeV penetrate more easily than total source. To investigate the nature of the penetration mechanism of these neutrons, the characteristics of the calculated spectrum of the flux at above 5 MeV were examined more carefully.

As shown in Table V-6, the asymptotic relaxation lengths at energies above 5 MeV are about 30 cm or more, which are at least 20% larger than the mean-free-paths given by $1/\Sigma_T$. It should be remembered that in the case of a narrow minimum the asymptotic relaxation length is never more than $1/\Sigma_T$. There are two familiar situations where the asymptotic relaxation length can be larger than $1/\Sigma_T$. One is the age-type transport when $x \ll \sqrt{2\tau}$. But the age-like process implies a large number of collisions, and the high energy range is too restricted (and the effective absorption is too high) to permit a description in terms of the age theory. The other circumstance is in the monoenergetic scattering case in which there is an

asymptotic relaxation length always larger than $1/\Sigma_T$ (8, 9, 33, 34, 35). It seems far fetched to describe the high energy portion of the spectrum in terms of monoenergetic scattering. But suppose we set up a "top" group composed, say, of all neutrons above 5 MeV. There is no scattering into this group because there is no higher group. The behavior of the group flux can be considered to correspond to a monoenergetic problem with a scattering cross section determined by the group self-scattering, and an absorption corresponding to the scattering out of the group. Most of this out-scattering will arise from inelastic collisions. In this region, a large part of the inelastic scattering comes from excitation of levels of 2.08 MeV or higher and in most cases that would be enough to throw the neutron out of the group. The energy loss from the elastic scattering is so small, at least at these energies, that it mostly contributes to the in-group scattering. Thus, we can hope to apply the results of monoenergetic theory to the "group" flux composed of all neutrons above 5 MeV.

However, in the usual monoenergetic scattering case, the asymptotic relaxation length is appreciably larger than $1/\Sigma_T$ only if absorption is small. It is usually stated that with 10 % absorption, the asymptotic

relaxation length is 90 % larger than $1/\Sigma_T$, but with 50 % absorption, it is only about 4.4 % larger^(35,36). At the energies of interest, inelastic cross section is more than 50 % of the total cross section and the process is effectively all absorption so far as the "top" group is concerned. The asymptotic relaxation length should still be almost the same as $1/\Sigma_T$. But the facts recited above hold only if the elastic scattering is isotropic in the center of mass system. If the angular distribution of the elastic scattering is highly anisotropic as in our case, the situation is very different. With realistic anisotropies, even high absorption does not have as much effect in reducing the asymptotic relaxation length as in the case of isotropic scattering⁽³⁷⁾. For an anisotropy of elastic scattering reasonably similar to that expected for sodium in this energy region, even with a 50 % absorption, the asymptotic relaxation length would be about 70 % longer than $1/\Sigma_T$ ⁽³⁷⁾.

To verify this conclusion, some moments calculations were performed. First the problem with a partial fission source above 5.076 MeV (case 8) was rerun with nothing changed except that the elastic scattering in the source region was taken as isotropic in the center

of mass system (case 8A) *. The asymptotic relaxation length in the source region fell to just about $1/\Sigma_T$, and the high energy flux was correspondingly decimated. However, the 5m flux at 100 KeV was not greatly affected --- only reduced by a factor of about 2, as shown in Figure V-17. Thus, the high energy penetration is not dominant enough to take a neutron all the way to the vicinity of the detector. Instead, a high energy neutron that reaches the detector in the fraction of an MeV range travels only a small initial part of its path at high energies, the rest is done by slowing down and transport below 5 MeV.

Another moments calculation, in the opposite direction, had more dramatic effect. The inelastic scattering at energies above about 5 MeV was set equal to zero and the elastic cross section increased to keep the total cross section constant (case 0I8E). As a result, the effective absorption for the "top" group is drastically reduced. The asymptotic relaxation length of the calculated

* In all situations where calculations were made for "isotropic" elastic scattering, isotropy in the center of mass system is meant.

high energy flux jumps up to greater than 50 cm, clearly much greater than that of any other mechanism, as shown in Table V-9. The 5m flux at high energies is drastically increased by a factor of about 10,000, as shown in Table V-10. However, the magnitude of this flux increase became more moderate below 3 MeV. This is because of the inelastic scattering is still present in this energy region, which as we shall see decreases the penetrating flux at any given energy. Nonetheless, the 5m flux at 100 KeV also increased by a factor of 33. A similar calculation was performed in which the inelastic cross section everywhere was replaced by elastic cross section (case OIE), with results also shown in Tables V-9 and V-10, and in Figure V-18. Here, the 5m flux at 100 KeV increased a factor of 340, in line with the above argument.

Another calculation was made in which the inelastic scattering was replaced by elastic scattering, but the angular distribution of the replaced elastic scattering was kept isotropic in the laboratory system (case OIEAA). The calculated 5m flux at 100 KeV increased by only a factor of 33, compared to 340 in case OIE. The anisotropy of elastic scattering replacing the inelastic scattering thus plays an important role.

However, the effect is not simply that using the transport cross section in place of the total cross section. To verify this conclusion, case 01E was rerun with a P-1 approximation for the angular distribution of elastic scattering in the center of mass system (case 01EA1), i.e. with the linear anisotropy that gives rise to the customary transport cross section approximation. The ratios of the 5m flux for case 01E to that for case 01EA1 are summarized in Table V-11. We see that the 5m flux above 5 MeV calculated in case 01EA1 is less by more than a factor of 6, while even the flux the flux below 5 MeV is reduced by a factor of about 2. Thus, the effect of the anisotropy can not be accounted for by the use of a simple transport cross section correction.

While the complete conversion of inelastic to elastic scattering is, of course, unrealistic, it suggests that the flux should be sensitive to a small decrease in the inelastic cross section with corresponding increase in the elastic cross section. Indeed, a 10 % decrease in the inelastic cross section everywhere, with corresponding increase in the elastic cross section, which is well within the uncertainties of the cross section measurement (38), increases the 5m flux at 100 KeV by more than 50 %

(case 01D). In other direction, a 10 % increase in the inelastic cross section everywhere with corresponding decrease in the elastic cross section decreases the same flux by more than 30 % (case 01A). Thus, in actual sodium, the high energy penetration mechanism helps make the flux particularly sensitive to uncertainties in the inelastic cross section. However, at least for the cross section in the ENDF/B-III set, it is not the dominant transport mechanism, and it can be made to be dominant only by large changes in cross sections, particularly the inelastic cross section.

Table V-9: Comparison of the Asymptotic Relaxation Lengths.

ENERGY (MeV)	SQUARE ROOT OF RATIO OF THE 20th and 18th SPATIAL MOMENTS (cm)		
	<u>CASE 0</u>	<u>CASE 01E</u>	<u>CASE 018E</u>
7.00 E-00	29.32	50.13	50.13
5.00 E-00	28.30	51.29	51.21
3.00 E-00	28.05	51.83	51.39
2.13 E-00	27.98	51.96	51.34
1.00 E-00	27.86	52.11	51.40
4.25 E-01	27.83	52.16	51.44
1.00 E-01	28.33	52.27	51.61
2.04 E-02	28.33	52.27	51.61
5.26 E-04	28.49	52.31	51.69
1.05 E-05	30.35	52.32	51.44
1.02 E-06	31.98	52.26	50.64

Table V-10 : Comparison of the 5m Fluxes.

ENERGY (MeV)	Ratio of the 5m Fluxes	
	<u>OIE / O</u>	<u>OI8E / O</u>
7.00 E-00	8.08 E+03	8.08 E+03
5.00 E-00	1.14 E+04	4.88 E+03
3.00 E-00	8.56 E+03	1.62 E+02
2.13 E-00	5.69 E+03	2.08 E+02
1.00 E-00	2.87 E+03	1.13 E+02
4.25 E-01	1.08 E+03	7.62 E+01
1.00 E-01	3.41 E+02	3.28 E+01
2.04 E-02	1.79 E+02	1.78 E+01
5.26 E-04	7.78 E+01	9.16 E-00
1.05 E-05	1.02 E+01	2.11 E -00
1.02 E-06	4.92 E-00	1.40 E-00

Table V-11: Comparison of the 5m Fluxes for
Cases OIE and OIEA1.

<u>ENERGY (MeV)</u>	<u>FLUX (OIE) / FLUX (OIEA1)</u>
1.300E+01	177.0
1.120E+01	30.6
9.055E+00	22.7
7.000E+00	17.1
5.380E+00	6.57
3.000E+00	2.71
2.130E+00	2.17
1.000E+00	1.73
6.010E-01	1.70
2.650E-01	1.60
1.000E-01	1.52
2.040E-02	1.44
5.260E-04	1.27

Figure V-17

Figure V-18

V. 9. CONCLUSION 8: Even Without any Source Neutrons above 5 MeV, the Flux still Builds-up so as to have an Asymptotic Relaxation Length of about 28 cm.

As discussed in Section V-6, source neutrons born above about 5 MeV contribute a disproportionate share of the penetrating flux. Although the transport mechanisms leading to this state of affairs are interesting, these source neutrons are not the overwhelming contributor to the flux in the 100 KeV range. They do not even provide the majority. Source neutrons below 5 MeV contribute about 67% of the 5m flux at 100 KeV as shown in Table V-4. Even eliminating the effect of the cross section minima, source neutron below 5 MeV still contribute about 58% of the same flux as shown in Table V-5. Examination of the asymptotic relaxation lengths in the neighborhood of 5m for the case with a partial fission source truncated at 5.076 MeV (case 7) shows that flux still builds up to have an asymptotic relaxation length of about 28 cm even without source neutrons above 5 MeV. In Table V-12, the 5m relaxation length for a complete fission source compared to those for a partial fission source truncated at 5.076 MeV are tabulated. Even by 3.925 MeV, well above the highest energy minimum, the partial source relaxation

length is more than 90 % of the full source, and is up to 97 % figure by 100 KeV.

Table V-12: Relaxation Lengths in the Neighborhood of 5m
in Sodium Medium of Cases 0 and 7.

<u>ENERGY (MeV)</u>	<u>$\bar{\lambda}_7$ (cm)</u>	<u>$\bar{\lambda}_0$ (cm)</u>	<u>$\bar{\lambda}_7 / \bar{\lambda}_0$</u>
5.000	24.86	28.16	0.8828
3.925	25.57	28.08	0.9106
3.000	26.18	27.95	0.9367
2.130	26.25	27.87	0.9419
1.005	26.38	27.78	0.9496
0.870	26.39	27.78	0.9500
0.601	26.39	27.78	0.9500
0.542	26.39	27.78	0.9500
0.480	26.47	27.79	0.9525
0.265	27.51	28.27	0.9731
0.100	27.48	28.28	0.9717

V. 10. CONCLUSION 9: The High Energy Transport Mechanism, outside of the Local Effects of the Cross Section Minima, is a Multiple Scattering Buildup of Scattered Neutrons, Distinct on the one hand from Age-theory Transport and on the other hand from Transport Dominated by a particular Cross Section Phenomenon such as a deep Minimum.

When the minima in the cross sections are filled up, and all source neutrons above 5 MeV excluded, we are thus left with a flux in the hundreds-KeV range which is still a substantial fraction of the original flux. Further, the rate of penetration, the relaxation length, is almost at the same level at a depth of 5m as the complete problem. The value of this relaxation length, about 27.5 cm as shown in Table V-12, is about twice the largest mean-free-path length as determined by the smoothed total cross section anywhere in the energy range below 5 MeV. The nature of this remaining penetration process is difficult to characterize succinctly. It shares many of the properties of age diffusion, but equally strongly differs from age-like penetration in other

properties, most notably in having a nearly exponential spatial behavior (cf. Figures V-11 and V-12) rather than the Gaussian. It seems to resemble most closely the process of the buildup of scattered photons in gamma ray penetration, where the buildup factors, too, are fairly close to exponential. With due care to separate extraneous effects, some of the distinctive properties of this buildup mechanism can be derived from calculations made with the full source or with the correct cross sections.

As with age-like penetration, this transport process is spread out over the entire energy range. No one energy region dominates the penetration flux in the 100 to 600 KeV region. For example, the case with a partial fission source above 7.0 MeV (case 9) was rerun with nothing changed except all total and partial cross sections above 7.0 MeV were increased by 20 % (case 9XA). The 5m flux above 7.0 MeV was correspondingly reduced by a factor more than 30. However, the 5m flux in the 100 to 600 KeV region was not greatly affected. In this region, the 5m flux was reduced by only a factor less than 2, as shown in Figure V-19. Clearly, changes in a limited range of the cross sections have major effect on the penetrating flux in that range, but have much less

effect on fluxes at considerably lower energies. Obviously, only a small part of the transport occurs in the restricted energy range. Similar phenomena happened in the calculations with the partial fission source above 3.925 MeV (series 6P). making the elastic scattering isotropic in the center of mass system in the source region (case 6PA) reduced the 5m flux at 100 KeV by a factor of only 2, much less effect than in the source range. But making the elastic scattering isotropic in the center of mass system everywhere (case 0A6P) reduced the same flux by a factor of 14. Even greater reductions occur at energies above the 297 KeV minimum. (cf. also the comparison between cases 8 and 8A above, Section V-8, page V-62.)

Another property of the "buildup" penetration shared with age-diffusion is that it involves many collisions. We have developed a technique, described in Appendix D, of using moments-method calculations (or any other deterministic transport computational technique) to estimate approximately the average number of collisions suffered by the neutrons before reaching the detector. Briefly, it involves repeating the calculation with an additional absorption "tag" which is a small fixed percentage of the total collision cross section every-

where. The number of collisions so calculated in sodium are shown in Table V-13 both at the 5m position and for the infinite medium spectrum. It is clear from this table that neutrons contributing to the 5m flux for the 100 to 200 KeV range have undergone many collisions, the order of at least 30 to 40. It is somewhat surprising that even with this substantial number of collisions the penetration remains nearly exponential. After all, a two dimensional random walk with 30 to 40 steps has an end point which, according to the law of large numbers, is distributed fairly closely to the Gaussian of age theory. A possible clue to this unexpected behavior may lie in the figures for the number of collisions for the infinite medium spectrum which, of course, is physically determined solely by the average lethargy increase per collision. In the energy region between 100 and 265 KeV, the number of collisions to reach the 5m detector is nearly twice as large as the average number of collisions for the infinite medium spectrum. For the very low energy end of the spectrum, where age theory almost holds, the absolute difference in these quantities is reduced, and the fractional difference is quite small. It seems likely that the larger number of collisions required to penetrate to 5m at, say, 100 KeV arises from an excess of small-

angle elastic scatterings. The random walk is no longer random but constrained by having small angular changes of directions; in the jargon of the probability theorist, it is a "stiff" chain of steps. Such a constrained random walk approaches the Gaussian distribution only after an exceedingly large number of steps. After all, in a completely "straight ahead" sequence of collisions, the distribution always remains exponential.

As in the case of the buildup in gamma ray penetration, the "buildup" penetration in sodium is affected by two factors: one is the penetration power of neutrons and the other is the rate of slowing-down. Anything which retards the rate of slowing-down or increases the penetration power increases the buildup and therefore the deep-penetrating flux. Conversely, any mechanism which increases the slowing-down rate or decreases the penetration power of neutrons will decrease the buildup and therefore the penetrating flux. However, the effects of these two factors are not completely independent. The interplay of these two factors is shown in the role of inelastic scattering. We have seen how sensitive the penetrating flux is to the interchange of inelastic scattering to elastic scattering in Section

V-8. The effects of the replacement of inelastic cross section by elastic cross section come into two parts: first, the angular distribution of scattered neutrons becomes strongly anisotropic instead of isotropic in the original case, which increases the penetration power of scattered neutrons; secondly, the rate of slowing-down is decreased due to the less energy loss after scattering. It seems that the latter effect is the most effective one among the two. This can be shown in case OIEAA. In this case, inelastic scattering everywhere was replaced by elastic scattering, but the angular distribution of the elastic scattering which replaced inelastic scattering remained isotropic, the same as the original inelastic scattering. The effect of this replacement is thus only on the neutron slowing-down rate. The 5m flux at 100 KeV accordingly increased by a factor of 34. Should the angular distribution of both the original elastic scattering and the one replacing inelastic scattering be the same of the original elastic scattering, i.e. very strongly forward peaked (case OIE), the same flux would increase by a factor of 340. That means that the 5m flux at 100 KeV increased by only a factor of about 10 due to the increase of neutron penetration power, when the inelastic scattering was replaced by the elastic scattering. Clearly,

the effect of inelastic scattering on the penetrating flux is more prominent on increasing the slowing-down rate than on decreasing the penetration power. Indeed, even a quite minor change which affects only the slowing-down rate of inelastic scattering can produce considerable change in the penetrating flux. One of such is to consider the effect of nuclear recoil in the inelastic scattering. (All previous calculations used an infinite mass approximation for the inelastic scattering. No nuclear recoil was considered.) Except very near the threshold of the inelastic level, where the cross section is very small, the recoil effect only slightly increases the energy loss upon inelastic scattering from the lowest level at 470 KeV, and is negligible for the higher levels. Nevertheless, putting in nuclear recoil for all levels (case OIFM), lowers the 5m flux at 100 KeV by 34 % and by 50 % at around 750 KeV where the effect is at maximum. Even only putting in nuclear recoil for the lowest level only (case OIFN1) reduces the 5m flux at 100 KeV by 27 % and by 45 % at around 750 KeV where the effect is also at maximum, as shown in Figure V-20. Another example is to consider the effect of changing the Q values of the inelastic levels. Decreasing all Q values by 100 KeV (case OIQD1), which is a very small fraction of energy increase after

each inelastic scattering except for neutrons originally with energies near the threshold, the 5m flux at 100 KeV increases by more than 35 %. Even much more drastic effects appear when decreasing all Q values by 250 KeV (case 0IQD) or by 400 KeV (case 0IQD2), as shown in Figure V-21. Thus, the penetrating flux is inordinately sensitive to details of the energy loss in inelastic scattering.

On the other hand, change of anisotropy of elastic scattering affects not only the neutron penetration power but also the slowing-down rate. Making elastic scattering isotropic not only decreases forward transport (i.e. decreases penetration power), but also increases the energy loss per collision (i.e. increases the slowing-down rate). To verify this conclusion, a separate calculation in which the angular distribution of elastic scattering was isotropic in the center of mass system everywhere (case 0A) was made. The ratios of the standard 5m flux to that for isotropic case are shown in Figure V-22. The 5m flux at 100 KeV reduced by a factor of about 10. Even greater reductions occurred at energies higher than the 297 KeV minimum. However, the effect of anisotropy is spread out over the entire energy range.

No one energy region dominates. To verify this conclusion,

the calculations with partial fission sources were rerun with nothing changed but the angular distribution of elastic scattering was isotropic in the source region (cases 4PA, 5PA, 6PA, 8A and 9A). The effects of these changes on the 5m flux at 100 KeV are summarized in Table V-14. The effect of making elastic scattering isotropic on the 5m flux increases as the range of isotropic scattering increases, as would be expected. Further, the effects of anisotropy of elastic scattering on the penetrating flux persist to very low energies. It is clear by comparing the first two lines in Table V-14 that the anisotropy in the energy range below 1.60 MeV, (where the original angular distribution is nearly isotropic below about 700 KeV,) still has a substantial effect which is even greater than any other energy range. However, it is not solely a transport-cross-section correction. If it were, the ratio of the standard flux to that with only transport-cross-section calculation (case 0A1), at all energies, would be a constant number. In actual case, as shown in Figure V-23, the ratios at higher energies are much greater than that at lower energies. So it is not only a transport-cross-section correction. But for flux below 100 KeV, the linear transport-cross-section approximation is good enough

when no source neutrons above MeV region are present.

In summary of this section, we conclude that the high energy "buildup" penetration possesses the following characteristics:

- a. Neutrons contributing to the 5m flux have undergone many scatterings, the order of 30 to 40 collisions for the 100 to 200 KeV region.
- b. It differs from age diffusion in that spatial behavior is not Gaussian but closer to exponential.
- c. The transport process is spread out over the entire energy range. No one energy region contributes the bulk of the spatial transport for flux in the 100 to 600 KeV range.
- d. Inelastic scattering, at all energies where significant, decreases the penetrating flux, because it increases the rate of slowing-down and decreases the forward transport, relative to elastic scattering.
- e. Anisotropy of elastic scattering, at all energies, increases the penetrating flux, because it increases forward transport and decreases rate of slowing-down, relative to isotropic elastic scattering. But it is not solely a transport-cross-section correction.

Table V-13: Estimation of Number of Collisions
in Sodium (case OCP3).

<u>ENERGY (MeV)</u>	<u>AT 5m POSITION</u>	<u>INFINITE MEDIUM</u>
2.65 E-01	34.9	16.5
1.00 E-01	42.4	23.2
2.04 E-02	60.5	41.4
5.26 E-04	102.6	84.6
1.05 E-05	144.5	131.0
1.03 E-06	169.9	158.8

Table V-14: Ratios of the 5m Fluxes at 100 KeV.

CASES COMPARED	LOWEST SOURCE ENERGY (MeV)	RATIO
0 / 0A	0.0	9.43
4P / 4PA	1.60	3.76
5P / 5PA	2.86	2.02
6P / 6PA	3.925	1.95
8 / 8A	5.076	1.71
9 / 9A	7.00	1.60

Figure V-19

Figure V-20

CASE 0100 VS. CASE 0, CASE 01001 VS. CASE 0 AND CASE 01002 VS. CASE 0

Figure V-21

CASE 0 VS. CASE 0A. ELASTIC SCATTERING IS ISOTROPIC IN THE C.M. SYSTEM.

Figure V-22

CASE 0 VS. CASE 0A1. P 1 APPROXIMATION FOR THE ELASTIC SCATTERING

Figure V-23

VI. Summary and Conclusions

The preceding sections have built up a picture of the neutron penetration mechanisms which determine the flux at distances of about 5 meters from a fission source. It has been shown that at low energies (100 KeV or lower) the spectrum is primarily determined by something close to age-diffusion in which only the low energy cross sections are significant. At higher energies the flux, it has been demonstrated, is increasingly controlled by a balance of three mechanisms primarily operating at upper-KeV and MeV ranges:

1. transport in cross section minima,
2. transport primarily at energies above 5 MeV,
3. a multi-collision transport more or less uniformly distributed in space-and-energy, leading to an effective exponential asymptotic behavior. For lack of a better terminology, this process is described as "buildup" or "drift".

The first mechanism depends only on the total cross section in the region of the minima. But the other two are also sensitive to the anisotropy of the scattering process and (particularly in the last) to the details of

the slowing down process in inelastic collisions. Each of the mechanisms could produce, were the other two absent, roughly equivalent deep-penetration relaxation lengths. For the given cross section set and source distribution, no one mechanism dominates.

The shield designer, confronted with the problem of treating neutron penetration through distances typical of LMFBR situations, is likely to seek answers to three kinds of questions:

1. What degree of computational sophistication is needed to calculate the flux spectrum in sodium at deep penetrations?
2. What sort of cross sectional uncertainties should be required of the sets of cross sections provided by the evaluator and measurers?
3. How does the situation change for other penetrations, source distributions or geometries?

The picture of the penetration process developed here can provide at least qualitative answers to most of these questions and can guide intelligently in developing the quantitative answers. Detailed examples would be very dependent on specific design situations, and lengthy

discussion would be inappropriate here, but certain points are clear and obvious on the basis of the above description of transport mechanisms:

1. For design-type calculations it might well be possible to use relatively simple approximations to the transport equation for particular parts of the flux spectrum. Thus, at the low energy end we have seen that from only the zeroth and second spatial moment the flux out to 5m can be predicted to about a factor of two. A carefully designed "age" type theory using correct second moment could be expected to predict the spectrum up to about 100 KeV quite well.

2. Clearly, as concerns the low energy part of the spectrum the most important piece of data is the total cross section in the "flat" region below 100 KeV

--- everything else is secondary. In the higher energy region, we have seen that the most sensitive pieces of data are the total cross section everywhere, but especially in the minima, both the narrow ones 3 MeV and lower, and the broad one above 5 MeV, and the inelastic cross sections. Because of the non-linear effects of changes in the cross section, the uncertainties are of more consequence in one direction than another. Thus, a lower limit on the total cross section in the region of the minima is more significant than an upper limit. In the angular distribution of elastic scattering, only the first few legendre polynomial coefficients are important. Coefficients above $\ell = 3$ are unimportant, and even the $\ell = 3$ term is not likely to be significant except in high energy calculations. Overall, the picture of the penetration mechanisms in sodium will not be changed significantly, when the cross sections change within the limit of the estimated uncertainties of the sodium cross section data in the ENDF/B-III file⁽³⁸⁾ (cf. Cases OERR+ and OERR-).

3. The investigations reported here should be valid where ever the penetration is so deep that overall leakage from the system is small. The relative importance

of the various mechanisms discussed should to that extent be independent of geometry. What happens for slabs so thin that leakage is significant at all energies can not be determined from the picture developed here. Other studies⁽⁴¹⁾ have suggested that with thin ($\leq 1\text{m}$) bare slabs, leakage effects dominate the situation and the flux is relatively insensitive to cross section features.

On the other hand, the effects of changing source distribution are quite clear and may be drastic. The near balance between various penetration mechanisms in the KeV-MeV region can be profoundly disturbed if the source distribution is significantly shifted from the fission spectrum. A softened source, such as might be typical of the LMFBF core, would be extremely sensitive to the low energy minima, particularly at 297 and 521 KeV, and little else in the cross section picture might matter. On the other side, with a so-called "thermo nuclear" source the higher energy penetration around 7 MeV probably would dominate and set the pace for the relaxation length of the entire spectrum down to about 1 KeV or lower. Other cross section phenomena would have only local effect on the spectrum shape.

It is hoped that the insight into the penetration mechanisms of fast neutrons in thick layers of sodium obtained here will help in the design further transport calculations and will influence future cross section measurements.

VII. References

1. F. L. Culler, Jr. and W. O. Harms, Phys. Today, 25, No. 5, 28 (1972)
2. R. E. Maerker, F. J. Muckenthaler, and C. E. Clifford, "The ORNL Benchmark Experiment for Neutron Transport in Thick Sodium," ORNL-TM-3960, Oak Ridge, Tennessee (1972)
3. C. R. Weisbin, L. J. Lidofsky, and H. Goldstein, Trans. Am. Nucl. Soc., 12, 402 (1969)
4. C. R. Weisbin, "BMT: A New Moment Solution of the Neutron Transport Equation," Ph. D. Thesis, Columbia University, New York (1969)
5. W. W. Engle, Jr., "A User Manual for ANISN, a One-dimensional Discrete Ordinates Transport Code with Anisotropic Scattering," K-1693, ORNL, Oak Ridge, Tennessee (1967)
6. ENDF/B-III Cross Section, Na 1156, National Cross Section Center, Brookhaven National Laboratory (1972)
7. H. Goldstein, Fundamental Aspects of Reactor Shielding, Addison-Wesley Publishing Company, Massachusetts (1959)
8. B. Davison, Neutron Transport Theory, Oxford at the Clarendon Press, London (1957)
9. G. I. Bell and S. Glasstone, Nuclear Reactor Theory,

- Van Nostrand Reinhold Company, New York (1970)
10. L. V. Spencer and U. Fano, *Phy. Rev.*, 81, 464 (1951)
 11. L. V. Spencer and U. Fano, *J. Research Nat. Bur. of Standards*, 46, 446 (1951)
 12. U. Fano, L. V. Spencer and M. J. Berger, "Penetration and Diffusion of X-rays," *Handbuch der Physik*, Vol. XXXVIII/2. (1959)
 13. H. Goldstein and J. E. Wilkins, Jr., "Calculation of the Penetration of Gamma Rays," NYO-3075 (1951)
 14. H. Greenspan, C. N. Kelber and D. Okrent, Computing Methods in Reactor Physics, Gordon and Breach, New York (1969)
 15. K. D. Lathrop, *Reactor Technology*, 15, 107 (1972)
 16. W. W. Engle, M. A. Boling and B. W. Colston, "DTF-II, A One-dimensional, Multigroup Neutron Transport Program," NAA-SR-10951, Atomic International (1966)
 17. W. E. Preeg, "Flux Sensitivity to Cross Section Data for Iron and Oxygen," Ph. D. Thesis, Columbia University (1970)
 18. O. Ozer, "Deep Penetration of Neutrons in Attenuating Media," Ph. D. Thesis, Columbia University (1968)
 19. J. R. Larmarsh, Introduction to Nuclear Theory, Addison-Wesley, New York (1966)
 20. D. E. Cullen and P. J. Hlavac, "ENDF/B Cross Sections,"

BNL-17100 (ENDF-200), Brookhaven National Laboratory.
(1972)

21. L. Dresner, Resonance Absorption in Nuclear Reactors, Pergamon Press, New York (1960)
22. PROCENDF, Columbia University, New York
23. N. M. Greene and et. al., "AMPX: A Modular Code System for Generating Coupled Multigroup Neutron-Gamma Libraries from ENDF/B," ORNL-TM- 3706 (AMPX-I), to be Published (1974)
24. P. Soran and H. Goldstein, Trans. Am. Nucl. Soc., 13, 405 (1970)
25. P. D. Soran, "Reconstruction of Neutron Spatial Distribution from Spatial Moments," Ph. D. Thesis, Columbia University (1971)
26. J. Certainse, "A Solution of the Neutron Transport Equation, Part III: Reconstruction of a Function from its Moments," NYO-6270 (1956)
27. R. E. Marshark, Rev. of Mod. Phys., 19, 185 (1947)
28. R. C. Muller, Nucl. Sci. Eng., 6, 160 (1959)
29. J. Ching, Oak Ridge National Laboratory, Private Communication (1972)
30. H. Goldstein and et. al., "The Mechanisms of Fast Neutron Transport in Iron," Columbia University (1971)

31. J. Ching, E. Oblo and H. Goldstein, "Application of a Discrete-energy, Discrete Ordinates Technique to the Study of Neutron Transport in Iron," ORNL-TM-4235, ORNL, Oak Ridge, Tennessee (1973)
32. L. J. Templin, "Reactor Physics Constants," ANL-5800, Argonne National Laboratory (1963)
33. K. M. Case and P. F. Zweifel, Linear Transport Theory, Addison-Wesley, Massachusetts (1967)
34. K. M. Case, Ann. Phys., 9, 1 (1960)
35. H. Goldstein, "Notes on Transport Theory," Columbia University, New York (1970)
36. H. Goldstein, Columbia University, Private Communication (1971)
37. E. Oblo, K. Kin, and H. Goldstein, Nucl. Sci. Eng., 54, 72 (1974)
38. N. C. Paik and T. A. Pitterle, "Evaluation of Sodium-23 Neutron Data for the ENDF/B Version III File," To be Published (1971)
39. F. R. Mynatt, "Fast Reactor Analytical Shielding Progress Report for May and June 1973," ORNL-TM-4401, p. 50, ORNL, Oak Ridge, Tennessee (1973)
40. H. Goldstein and et. al., Trans. Am. Nucl. Soc., 18, 364 (1974)
41. H. Goldstein and et. al., "Sensitivity Study in

Sodium," Columbia University, New York, To be Published.

- 43. H. Goldstein, "Properties of Neutron Transport Directly Derivable from the Spatial Moments of the Flux," Columbia University, New York (1970)
- 44. G. Placzek, "The Spatial Distribution of Neutrons Slowed Down by Elastic Collision," MDDC-2 (1946)

Appendix A: Reconstruction of Spatial Distribution
from Moments: the Biorthogonal
Polynomials Method

One of the most extensively applied techniques for the reconstruction of the spatial dependence of the flux from the available moments is the so-called the polynomial expansion method^(7,12). In this method, two sets of orthonormal polynomials, which are dependent upon the weighting function chosen, are employed. Consider a single variable function $f(x)$, whose first several moments are known are defined as

$$f_n = \int_X dx x^n f(x) \quad (A-1)$$

where X is the domain of the function $f(x)$. To obtain a polynomial representation of $f(x)$, we need a preselected weighting function $W(x)$ which is dependent upon the nature of $f(x)$ and two sets of linear independent polynomials $U_n(x)$ and $U_n^+(x)$ such that

$$\int_X dx W(x) U_m(x) U_n^+(x) = \delta_{mn} \quad (A-2)$$

Then, in terms of $W(x)$ and its associated polynomials

the desired expansion is

$$f(x) = W(x) \sum_0^N A_n U_n(x) \quad (A-3)$$

with

$$A_n = \int_X dx U_n^+(x) f(x) \quad (A-4)$$

where f_N is the highest moment known. Mathematically, the A_n 's are uniquely given by the first $n+1$ moments of $f(x)$. Thus a knowledge of the f_n 's is sufficient for determining the expansion.

For a given $W(x)$, $U_n(x)$ is assumed to contain all powers of x up to x^n to retain its generality. In regard to the $U_n^+(x)$, two cases are usually encountered: (1) Successive moments are available to be used, in which case $U_n^+(x) = U_n(x)$ and the two sets of polynomials are said to be self-adjoint. (2) Due to the symmetry properties of $f(x)$, only even (or odd) moments exist, in which case U_n^+ 's must similarly be even (or odd) so that the two polynomial sets form a biorthogonal system.

Since the polynomials have the properties of

orthonormality and are finite and linear, the corresponding sets of a given weighting function can be constructed practically. We only consider an even biorthogonal system with domain of $(0, \infty)$ here. Let

$$U_m(x) = \sum_{i=0}^m a_{mi} x^i \quad (A-5)$$

$$U_n^+(x) = \sum_{j=0}^n b_{nj} x^{2j} \quad (A-6)$$

with

$$\int_0^{\infty} dx W(x) U_m(x) U_n^+(x) = \delta_{mn} \quad (A-7)$$

where both m and n are less than or equal to N . It is noted here that there are not enough conditions to get unique solutions for a_{mi} 's and b_{nj} 's. We arbitrarily set

$$b_{n0} = 0 \quad \text{for any } n \quad (A-8)$$

Then the normalization equation (A-7) becomes

$$\sum_{i=0}^m \sum_{j=0}^n a_{mi} b_{nj} H_{ij} = \delta_{mn} \quad (A-9)$$

where

$$H_{ij} = \int_0^{\infty} w(x) x^{i+2j} dx \quad (A-10)$$

Or in a matrix form, equation (II-9) becomes

$$\underline{H} \cdot \underline{B} = - \underline{h} \quad (A-11)$$

$$\underline{H}' \cdot \underline{A} = \underline{g} \quad (A-12)$$

where

$$\underline{H} = \begin{pmatrix} H_{01} & H_{02} & \dots & H_{0n} \\ H_{11} & H_{12} & \dots & H_{1n} \\ \dots & \dots & \dots & \dots \\ H_{n-1,1} & \dots & \dots & H_{n-1,n} \end{pmatrix} \quad (A-13)$$

$$\underline{B}^T = (b_{n1} \ b_{n2} \ b_{n3} \ \dots \ b_{nn}) \quad (A-14)$$

$$\underline{h}^T = (H_{00} \ H_{10} \ \dots \ H_{n-1,0}) \quad (A-15)$$

$$\underline{A}^T = (a_{n0} \ a_{n1} \ \dots \ a_{nn}) \quad (A-16)$$

$$\underline{H}' = \begin{pmatrix} \underline{h}^T & H_{n0} \\ \underline{H}^T & \underline{g}' \end{pmatrix} \quad (A-17)$$

$$\underline{g}'^T = (H_{n1} \ H_{n2} \ \dots \ H_{nn}) \quad (A-18)$$

$$\underline{g}^T = (0 \ 0 \ \dots \ \frac{1}{b_{nn}}) \quad (A-19)$$

where T means transpose. The solution of the two matrix equations (equations (A-11) and (A-12)) can be easily obtained in the order of b_{nj} 's and a_{mi} 's (n from 0 to N).

For the case of choosing an exponential function as the weighting function, the biorthogonal system has been investigated in detail in Reference 12. Here we are interested in the case of using a Gaussian function as a weighting function. The following are the first few polynomials U_n , U_n^+ of the biorthogonal system associated with the weighting function $\exp(-x^2/2)$ obtained by solving equations (A-11) and (A-12).

$$U_0^+(x) = 1$$

$$U_1^+(x) = 1 - x^2$$

$$U_2^+(x) = 1 - \frac{5}{2} x^2 + \frac{1}{2} x^4$$

$$U_3^+(x) = 1 - \frac{9}{2} x^2 + 2 x^4 - \frac{1}{6} x^6$$

$$U_4^+(x) = 1 - 7 x^2 + \frac{41}{8} x^4 - \frac{11}{12} x^6 + \frac{1}{24} x^8$$

$$U_5^+(x) = 1 - 10 x^2 + \frac{85}{8} x^4 - \frac{73}{24} x^6 + \frac{7}{24} x^8 - \frac{1}{120} x^{10}$$

$$U_0(x) = \sqrt{\frac{2}{\pi}}$$

$$U_1(x) = \sqrt{\frac{2}{\pi}} - x$$

$$U_2(x) = \sqrt{\frac{2}{\pi}} - 2x + \sqrt{\frac{2}{\pi}} x^2$$

$$U_3(x) = \sqrt{\frac{2}{\pi}} - 3x + 3\sqrt{\frac{2}{\pi}} x^2 - \frac{1}{2} x^3$$

$$U_4(x) = \sqrt{\frac{2}{\pi}} - 4x + 6\sqrt{\frac{2}{\pi}} x^2 - 2x^3 + \frac{1}{3}\sqrt{\frac{2}{\pi}} x^4$$

$$U_5(x) = \sqrt{\frac{2}{\pi}} - 5x + 10\sqrt{\frac{2}{\pi}} x^2 - 5x^3 + \frac{5}{3}\sqrt{\frac{2}{\pi}} x^4 - \frac{1}{8} x^5$$

We also found a general form for $U_n(x)$, i.e.

$$U_n(x) = \sum_{i=0}^n (-1)^i \binom{n}{i} D_i x^i \quad (A-20)$$

here

$$D_i = \frac{\left(\frac{i}{2}\right)! 2^{i/2}}{i!} \sqrt{\frac{z}{\pi}} \quad i \text{ even} \quad (\text{A-21})$$

$$D_i = \frac{1}{2^{(i-1)/2} \left(\frac{i-1}{2}\right)!} \quad i \text{ odd} \quad (\text{A-22})$$

Appendix B: Reconstruction of Spatial Distribution from Moments: the Simple Exponential Fitting Method

The range of applicability of the biorthogonal polynomial method for reconstruction of spatial distribution from moments is limited by the weight-function chosen. In the problem treated, the choice of weight-function was a Gaussian function. But for all physical problems, the spatial distribution is expected to be exponential at distance not too close to the source. Therefore, it is expected that the biorthogonal polynomial method would no longer obtain good results at higher energies. So another method, the simple exponential-fitting method, has been developed which is applicable very well at higher energies.

It is shown by Goldstein⁽⁴³⁾ that the ratios of the higher moments give local relaxation lengths approximately on an assumption of a nearly-exponential flux behavior. Let us define the logarithmic derivative, $\lambda(x,E)$, of the scalar flux $N_0(x,E)$ as

$$\frac{1}{\lambda(x,E)} = - \frac{1}{N_0(x,E)} \frac{d}{dx} N_0(x,E) \quad (B-1)$$

If the flux were represented by a single exponential, λ would, of course, be constant in x . Now, from the definition of moments

$$N_{2n,0}(\bar{E}) = \frac{1}{(2n)!} \int_{-\infty}^{\infty} x^{2n} N_0(x, \bar{E}) dx \quad (\text{B- 2})$$

Through integration by parts, we get

$$N_{2n,0}(\bar{E}) = - \frac{1}{(2n+1)!} \int_{-\infty}^{\infty} x^{2n+1} \frac{d}{dx} N_0(x, \bar{E}) dx \quad (\text{B- 3})$$

or

$$N_{2n,0}(\bar{E}) = \frac{1}{(2n+1)!} \int_{-\infty}^{\infty} x^{2n+1} \frac{N_0(x, \bar{E})}{\lambda(x, \bar{E})} dx \quad (\text{B- 4})$$

If we define a spatial average of $\lambda(x, \bar{E})$ as

$$\frac{1}{\bar{\lambda}_{2n+1}(\bar{E})} = \frac{\int_{-\infty}^{\infty} x^{2n+1} \frac{N_0(x, \bar{E})}{\lambda(x, \bar{E})} dx}{\int_{-\infty}^{\infty} x^{2n+1} N_0(x, \bar{E}) dx} \quad (\text{B- 5})$$

Then

$$N_{2n,0}(\bar{E}) = \frac{1}{\bar{\lambda}_{2n+1}(\bar{E})} N_{2n+1,0}(\bar{E}) \quad (\text{B- 6})$$

Therefore, in terms of this quantity, $\bar{\lambda}$, which has the

dimensions of length and plays the role of a suitably averaged relaxation length, the ratio of successive even moments is given by

$$R_{2n}(\bar{E}) = \frac{N_{2n+2,0}(\bar{E})}{N_{2n,0}(\bar{E})} = \bar{\lambda}_{2n+2}(\bar{E}) \bar{\lambda}_{2n+1}(\bar{E}) \quad (\text{B-7})$$

So if the signature curves of moments, given as the curve of $\sqrt{R_{2n}}$ vs. n , at some energies approach equilibrium values rapidly, it is clear that the fluxes at those energies are asymptotically exponential. In other words,

$$R_{2n}(\bar{E}) = (\bar{\lambda}_{asp})^2 \quad (\text{B-8})$$

or

$$N_0(x, \bar{E}) = A(\bar{E}) \exp(-|x|/\bar{\lambda}_{asp}) \quad (x) \gg 0 \quad (\text{B-9})$$

where $\bar{\lambda}_{asp}$ is the asymptotic relaxation length, and $A(\bar{E})$ may be determined from the definition of moments. That is

$$A(\bar{E}) = \frac{N_{2n,0}(\bar{E})}{2(\bar{\lambda}_{asp})^{2n+1}} \quad (\text{B-10})$$

In order to select the proper moment to determine the coefficient $A(E)$, we should look at the integrand of equation (B-2) defining the moments. It can be shown that there is no more than one maximum for the higher value of n with a roughly exponential flux at large distance⁽⁴³⁾. The location of the maximum is determined by

$$\frac{d}{dx} [x^{2n} N_0(x, E)] = 0 = 2n x^{2n-1} N_0(x, E) + x^{2n} \frac{d}{dx} N_0(x, E)$$

or

$$x_{2n, \max} = 2n \lambda(x, E) \quad (\text{II-11})$$

If the integrand in the moments definition is narrowly peaked about this maximum, then any given moment could be closely associated with the flux at the location of this maximum. Based on this assumption, we can find the suitable moment which can be used to evaluate the coefficient $A(E)$ at a particular location x by

$$2n(x) \sim \frac{x}{\bar{\lambda}_{asp}} \quad (\text{B-12})$$

In this way, the order of moment, $2n$, is usually not an integer. The two moments with orders close to the value given by equation (B-12) are both used to evaluate the coefficient $A(E)$. It is found that the difference between

the fluxes obtained from those two moments is within 5 %.
Then the average value of those two fluxes is taken.

Thus, from the signature curve of moments, we may find the local relaxation length, and the suitable moments which may be used to represent the flux at a distance of interest. The local flux may, of course, be evaluated.

Appendix C: Formulus for Changing Cross Sections in the Region of the 297 KeV Minimum

In order to investigate the importance of the 297 KeV minimum to the neutron deep penetration, a number of moments calculations in whiach the total cross sections in the neighborhood of this minimum was increased or decreased were performed. In these cases, the total cross sections in the region of the 297 KeV minimum were increased or decreased in such a manner that the maximum change occurred at the bottom of the minimum and the change tapered off to zero at both ends of the minimum interval. The lower end of this minimum interval was chosen at 287 KeV, where the cross section reached the value of the average cross section outside the minimum. The upper end was chosen at 299 KeV, which was the first one data point just above the bottom of the minimum in the ENDF/B-III file. The total cross section at the upper end is much higher than that at the lower end. However, we have selected a number of solution points in the upper side of the minimum to insure the convergence of the flux calculation at the bottom of the minimum.

In the lower side of the minimum, the total cross sections were changed according to the following expression:

$$\sigma_T^{NEW}(E) = \sigma_T^{OLD}(E) - a \cdot \frac{\sigma_T' - \sigma_T^{OLD}(E)}{[\sigma_T^{OLD}(E)]^2} \quad (C-1)$$

where

$$\sigma_T' = \sigma_T^{OLD}(287 \text{ KeV}) \quad (C-2)$$

$$a \cdot \frac{\sigma_T' - \sigma_T^0}{(\sigma_T^0)^2} = \sigma_T^0 - \sigma_T^{NEW}(297 \text{ KeV}) \quad (C-3)$$

$$\sigma_T^0 = \sigma_T^{OLD}(297 \text{ KeV}) \quad (C-4)$$

or

$$\sigma_T^{NEW}(E) = \sigma_T^{OLD}(E) + (\sigma_T^{NEW}(297 \text{ KeV}) - \sigma_T^0) \left[\frac{\sigma_T' - \sigma_T^{OLD}(E)}{\sigma_T' - \sigma_T^0} \right] \left[\frac{\sigma_T^0}{\sigma_T^{OLD}(E)} \right]^2 \quad (C-5)$$

From the above equation, we have

$$\sigma_T^{NEW}(287 \text{ KeV}) = \sigma_T^{OLD}(287 \text{ KeV}) \quad (C-6)$$

and the maximum change occurs at 297 KeV. On the other hand, in the upper side of the minimum, the cross sections at solution points were interpolated from the two values

of the upper end cross section and the new total cross section at the bottom of the 297 KeV minimum. In all region, the elastic cross sections were set to equal to the difference of the new total cross section and the original absorption cross section.

APPENDIX D: Method for Estimating Number of Neutron Collisions

Consider a neutron slowing down in an infinite medium consisting of a single element with no absorption, and constant cross section. If the initial lethargy of the neutron is taken to be zero, its lethargy u_n after n collisions will be

$$u_n = \sum_{i=1}^n \Delta u_i \quad (D-1)$$

where Δu_i is the increase in lethargy of the neutron at its i th collision. The average value of u_n after n collisions is therefore

$$\overline{u_n} = \sum_{i=1}^n \overline{\Delta u_i} \quad (D-2)$$

where $\overline{\Delta u_i}$ is the average increase in lethargy at the i th collision. If the scattering is isotropic in the center of mass system, $\overline{\Delta u_i} = \xi$ for all collisions, where ξ is the average increase in lethargy per collision, it follows that

$$\overline{u_n} = n \xi \quad (D-3)$$

Equation (D-3) can be used in an obvious way to compute the number of collisions required to increase the average lethargy to the value, $\overline{u_n}$, that is,

$$\eta = \frac{\bar{u}_n}{3} \quad (D-4)$$

Therefore, if the initial energy of the neutron is E_0 and the detector energy corresponding to $\bar{u}_n$ is E , then the average number of collisions $\bar{n}$ is given by

$$\bar{n} = \frac{1}{3} \ln \frac{E_0}{E} \quad (D-5)$$

However, equation (D-5) holds only if all of the following conditions are true.

- a. The neutron source is monoenergetic.
- b. The medium is of no absorption and constant cross section.
- c. Elastic scattering is the only type of collision and its angular distribution is isotropic in the center of mass system.

If any one of the above conditions is not satisfied, e.g. a fission source in the real sodium medium, there is no easy way to find out the average collision numbers to reach a particular detector, except by Monte Carlo method. We have developed a simple approximation to solve this problem.

Consider an integrated quantity, I , which is defined as follows:

$$I = \int dp w(p) N(p) \quad (D-6)$$

where p represents the phase space, $w(p)$ is some kind of weighting function or response function, and $N(p)$ is the neutron flux. If we consider

$$N(p) = \sum_n N(p, n) \quad (D-7)$$

where $N(p, n)$ is the flux of neutrons undergone exact n collisions, then

$$I = \sum_n \int dp w(p) N(p, n) = \sum_n I_n \quad (D-8)$$

where

$$I_n = \int dp w(p) N(p, n) \quad (D-9)$$

Now, consider the same problem with nothing changed but applying a small tag, say, % absorption; uniformly to all collision processes, i.e. elastic and inelastic scattering. By the same token, the flux in this case, $N'(p)$ is given by

$$N'(p) = \sum_n N'(p, n) \quad (D-10)$$

Since the tag is applied uniformly to all collision processes, then

$$N'(p, n) = N(p, n) (1-\epsilon)^n \quad (D-11)$$

And

$$I' = \int dp w(p) N'(p) = \sum_n (1-\epsilon)^n I_n \quad (D-12)$$

If n is not very small, we can approximate

$$\sum_{n_1}^{n_2} I_n \longrightarrow \int_{n_1}^{n_2} J(n) dn \quad (D-13)$$

where it is assumed that n_1 and n_2 are not closely spacing.

Then

$$I \sim \int J(n) dn \quad (D-14)$$

$$I' \sim \int (1-\epsilon)^n J(n) dn \quad (D-15)$$

If we normalize $J(n)$ as follows

$$K(n) = \frac{J(n)}{I} \quad (D-16)$$

then

$$I' = I \int (1-\epsilon)^n K(n) dn \quad (D-17)$$

Here, it is noted that $K(n)$ is a normalized distribution function of collision number. If we simply define

$$(1-\epsilon)^{\bar{n}} = \int (1-\epsilon)^n K(n) dn \quad (D-18)$$

where $\bar{n}$ is the average number of collisions. Then

$$\bar{n} = \ln \left(\frac{I'}{I} \right) / \ln(1-\epsilon) \quad (D-19)$$

On the other hand, if we assume for large n that $K(n)$ approaches a Gaussian function, i.e.

$$K(n) \sim M \exp \left[- \frac{(n-\bar{n})^2}{2\sigma^2} \right] \quad (D-20)$$

where σ is the standard deviation, and has the following

form for medium with no absorption, isotropic scattering, and constant cross section.

$$\sigma = \frac{\sqrt{\bar{n}}}{3} \left(1 - \frac{(1-\beta)^2}{\alpha} \right)^{1/2} \quad (D-21)$$

where

$$\alpha = \left(\frac{A-1}{A+1} \right)^2, \quad A = \text{mass number} \quad (D-22)$$

and M is the normalization factor. Then

$$\begin{aligned} \frac{I'}{I} &= M \int (1-\epsilon)^n e^{-\frac{(n-\bar{n})^2}{2\sigma^2}} dn \\ &= f(\bar{n}, \sigma, n) \end{aligned} \quad (D-23)$$

From equation (D-23), we can solve for $\bar{n}$ and σ if two I' for two different tags are given. If we approximate

$$(1-\epsilon)^n \sim e^{-n\epsilon} \quad (D-24)$$

in equation (D-23), we can get the following analytic forms for $\bar{n}$ and σ .

$$\sigma^2 = 2 \left(\frac{\bar{n}_1 - \bar{n}_2}{\epsilon_2 - \epsilon_1} \right) \quad (D-25)$$

and

$$\bar{n} = \bar{n}_1 + \frac{\epsilon_1^2}{2} \epsilon_1 = \bar{n}_2 + \frac{\epsilon_2^2}{2} \epsilon_2 \quad (D-26)$$

where $\bar{n}_1$ and $\bar{n}_2$ are the average collision numbers calculated from given tags ϵ_1 and ϵ_2 , respectively.

The above method is applied to a medium with the same mass number as sodium but with no absorption, isotropic scattering and constant cross section. The calculated results are summarized in Table D-1 and D-2. In these tables, the theoretical values calculated from equations (D-5) and (D-21) are also listed for comparison. It is clear from these tables that the agreement of the results from the model to the theoretical values are very good. The values of $\bar{n}$ and ϵ calculated are nearly ϵ independent.

We also applied this model to calculate the number of collisions in real sodium medium. It seems that the result is ϵ dependent. This is because the ratio of inelastic cross section to elastic cross section is too small at some energies to get a good flux convergence for a very small tag. So we have to choose a proper value of ϵ , not too large to use

the approximation in equation (D-24), and not too small to get good flux convergence.

Table D-1: Estimation of number of collisions in mock-up
sodium medium

LETHARGY	$\bar{n}_1$	$\bar{n}_2$	$\bar{n}_{th}$
4.60	54.80	54.80	54.47
5.30	63.03	63.03	62.71
6.90	81.96	81.96	81.66
7.60	90.19	90.20	89.89
9.20	109.13	109.13	108.84
9.89	117.36	117.37	117.08
11.49	136.30	136.30	136.03
12.19	144.53	144.53	144.26
13.79	163.47	163.47	163.21
14.17	168.00	168.00	167.74

* $\bar{n}_1$ calculated with $G_1 = 1\%$ and $G_2 = 0.1\%$.

$\bar{n}_2$ calculated with $G_1 = 2\%$ and $G_2 = 0.1\%$.

$\bar{n}_{th}$ calculated from equation (D-5).

Table D-2: Estimation of the standard deviation in mock-up
sodium medium

LEThERGY	σ_1	σ_2	σ_{th}
4.60	4.39	4.52	4.38
5.30	4.71	4.84	4.70
6.90	5.30	5.49	5.37
7.60	5.55	5.76	5.63
9.20	6.13	6.34	6.20
9.89	6.37	6.58	6.43
11.49	6.88	7.10	6.93
12.19	7.08	7.30	7.14
13.79	7.54	7.78	7.59
14.17	7.63	7.88	7.69

* σ_1 calculated with $G_1 = 1\%$ and $G_2 = 0.1\%$

σ_2 calculated with $G_1 = 2\%$ and $G_2 = 0.1\%$

σ_{th} calculated from equation (D-21)

Appendix E: List of BMT Runs

More than 100 cases of BMT calculations have been made in the course of this study. Each calculation has been assigned an index number to be used to refer to a particular case in the discussions. The following is a complete list of cases and a brief description of special characteristics pertaining to each case. Those cases are classified into thirteen categories according to their major feature different from the standard calculation, case 0. Only those features different from case 0 are listed. The standard calculation is described in Section IV-1.

A. Calculations with partial fission sources:

0	The standard case.
1	With a partial fission source at energies ≤ 0.040 MeV.
2	With a partial fission source at energies ≤ 0.350 MeV.
3-	With a partial fission source at energies ≤ 0.542 MeV.
3-P	With a partial fission source at energies > 0.542 MeV.

- 3 With a partial fission source at energies
 ≤ 0.868 MeV.
- 3+ With a partial fission source at energies
 ≤ 1.250 MeV.
- 3+P With a partial fission source at energies
 > 1.250 MeV.
- 4 With a partial fission source at energies
 ≤ 1.600 MeV.
- 5 With a partial fission source at energies
 ≤ 2.860 MeV.
- 6 With a partial fission source at energies
 ≤ 3.925 MeV.
- 6P With a partial fission source at energies
 > 3.925 MeV.
- 7 With a partial fission source at energies
 ≤ 5.076 MeV.
- 8 (7P) With a partial fission source at energies
 > 5.076 MeV.
- 9 With a partial fission source at energies
 ≥ 7.000 MeV.

B. Calculations with the Smoothed Cross Section Set:

In the following cases except case 6PSS, the

standard cross section set was modified by filling the cross section dips up to the level of the average cross section in the vicinity of the cross section dips in the following energy ranges:

0.287 MeV - 0.299 MeV,

0.457 MeV - 0.542 MeV,

1.835 MeV - 1.930 MeV,

3.000 MeV - 3.154 MeV.

0S	With full fission source.
1PS	With a partial fission source at energies > 0.040 MeV.
2PS	With a partial fission source at energies > 0.350 MeV.
3S	With a partial fission source at energies $\leq$ 0.868 MeV.
3PS	With a partial fission source at energies > 0.868 MeV.
4PS	With a partial fission source at energies > 1.600 MeV.
5PS	With a partial fission source at energies > 2.860 MeV.
5P6S	With a partial fission source at energies $\leq$ 3.925 MeV but > 2.86 MeV.

- 6S With a partial fission source at energies ≤ 3.925 MeV.
- 6PS With a partial fission source at energies > 3.925 MeV.
- 7PS (8S) With a partial fission source at energies > 5.076 MeV.
- 9S With a partial fission source at energies ≥ 7.000 MeV.
- 6PSS The standard cross section set was modified by filling up the cross section fluctuations in the following energy ranges:
- 4.115 MeV - 4.200 MeV,
4.428 MeV - 4.565 MeV,
4.900 MeV - 5.076 MeV.
- And the source was a partial fission source at energies > 3.925 MeV.

C. Calculations with Modified Cross Section Set in the Region of the 297 KeV Minimum:

In the following calculations, the standard cross section set in the vicinity of the 297 KeV minimum was modified as described in Appendix C.

ODDA	New total cross section at 297 KeV = 0.75 b.
ODDB	New total cross section at 297 KeV = 0.90 b.
ODDC	New total cross section at 297 KeV = 1.10 b.
ODDD	New total cross section at 297 KeV = 1.25 b.
ODDE	New total cross section at 297 KeV = 1.50 b.
ODDF	New total cross section at 297 KeV = 2.71 b.
	(= total cross section at 287 KeV.)

D. Calculations with Modified Anisotropy of Elastic Scattering:

In the following cases except cases 0A1 and 6A1, the angular distribution of elastic scattering was made to be isotropic in the center of mass system in one or another energy range.

0A	With isotropic everywhere and full fission source.
0AP	With isotropic scattering below 1.10 MeV and full fission source.
3-A(0A3-)	With the same source as case 3- and isotropic scattering in the source region.
3+A(0A3+)	With the same source as case 3+ and isotropic scattering in the source region.

6A (0A6) With the same source as in case 6 and isotropic scattering in the source region.

4PA With the same source as in case 4PS and isotropic scattering in the source region.

5PA With the same source as in case 5PS and isotropic scattering in the source region.

6PA With the same source as in case 6P and isotropic scattering in the source region.

8A (7PA) With the same source as in case 8 and isotropic scattering in the source region.

9A With the same source as in case 9 and isotropic scattering in the source region.

0A6P With the same source as in case 6P and isotropic scattering everywhere.

3-A0 With full fission source and isotropic scattering at energies ≤ 0.542 MeV.

3+A0 With full fission source and isotropic scattering at energies ≤ 1.250 MeV.

6SA With the same assumptions as in case 6S except isotropic scattering in the source region.

0A1 With P-1 approximation for the elastic scattering in the center of mass system.

6A1 With the same source as in case 6 and P-1 approximation for the elastic scattering in the center of mass system.

E. Calculations without Elastic Scattering:

In the following cases, the elastic scattering everywhere was treated as absorption.

0EC With full fission source.

6PEC With the same source as in case 6P.

9EC With the same source as in case 9.

F. Calculations with Modified Inelastic Cross Sections:

0' With different inelastic cut-off energies.

0IC Treated inelastic scattering as absorption.

0IC1 Treated inelastic scattering except the first discrete level as absorption.

0IE Treated inelastic scattering as elastic scattering.

7IE Same as case 0IE except with a partial fission source the same as in case 7.

0IE1 Treated the first-discrete inelastic scattering as elastic scattering.

8IE1 Same as case 0IE1 except with a partial fission source the same as in case 8.

0I8E Made total inelastic cross section at energies > 5.076 MeV to elastic cross section.

8I8E Same as case 0I8E except with a partial fission source the same as in case 8.

0IEAA Made total inelastic scattering to isotropic elastic scattering.

0IEA Same as case 0IE except made all scattering isotropic in the center of mass system.

0IEA1 Same as case 0IE except with P-1 approximation for the elastic scattering in the center of mass system.

7IEA1 Same as case 0IEA1 except with a partial fission source the same as in case 7.

0IND With all inelastic cross sections decreased by 5 % proportionally and with the absorption cross section increased accordingly.

0IA With all inelastic cross sections increased by 10 % proportionally and with elastic cross sections decreased accordingly.

0ID With all inelastic cross sections decreased by 10 % proportionally and with elastic cross sections increased accordingly.

8ID Same as case 0ID except with a partial fission source the same as in case 8.

0ERR- With inelastic cross section decreased by 10 % below 2 MeV, by 15 % above 2 MeV, by 30 % above 8 MeV and with elastic cross sections increased accordingly.

0ERR+ With inelastic cross sections increased by 10 % below 2 MeV, by 15 % above 2 MeV, by 30 % above 8 MeV and with elastic cross sections decreased accordingly.

G. Calculations with Various Treatments of the Inelastic Scattering:

0IFM Using finite mass approximation for all discrete inelastic levels.

0IFM1 Using finite mass approximation for the first discrete inelastic level only.

2+FM1 Same as case 0IFM1 except with a partial fission source at energies > 0.6095 MeV.

0IFM' Same as case 0IFM except putting in 5 more energy points in the upper energy side of the 297 KeV minimum.

0IFM'S Same as case 0IFM' except with the smoothed cross section set as described in Category B.

H. Calculations with Modified Reaction Q Values of Discrete Inelastic Scattering:

0IEQ	With $Q = 0.0$ for all levels.
0IQD	With all Q values decreased by 250 KeV.
0IQD-FM1	Same as case 0IQD except with finite mass approximation for the first level.
2+IQD-FM1	Same as case 0IQD-FM1 except with a partial fission source the same as in case 2+FM1.
0IQD-S	Same as case 0IQD except with the smoothed cross section set described in Category B.
0IQDA	Same as case 0AP except with all Q values decreased by 250 KeV.
0A-IQD	Same as case 0A except with all Q values decreased by 250 KeV.
6IQD	Same as case 0IQD except with a partial fission source the same as in case 6.
0A6-IQD	Same as case 0A6 except with all Q values decreased by 250 KeV.
0IQD1	With all Q values decreased by 100 KeV.
0IQD2	With all Q values decreased by 400 KeV.

I. Calculations with Increased Cross Sections at High Energies:

In the following cases except case 9XA, all cross sections at energies above 0.19 MeV were increased by a factor of 3 proportionally.

0HEXI	With full fission source.
5P6HEXI	With a partial fission source the same as in case 5P6S.
6PHEXI	With the same source as in case 6P.
9HEXI	With the same source as in case 9.
9XA	With the same source as in case 9 and with all cross sections in the source region increased by 20 % proportionally.

J. Calculations with Increased Cross Sections at Low Energies:

In the following calculations, all total cross sections at energies below 244 KeV was set to be 8.07 barns except at the two scattering resonances where the original cross sections were higher than 8.07 barns. Absorption was kept the same as the standard cross section set.

OLEXI With full fission source.

OLEXIFM1 Same as case OIFM1 except with the same cross section set as in case OLEXI.

5P6LEXI Same as case OLEXI except with a partial fission source the same as case 5P6S.

5P6LEXI-S Same as case 5P6LEXI except with the smoothed cross section set in the energy ranges as described in Category B.

5P6LEXI-S1 Same as case OLEXI except with the cross section dip at 3.075 MeV filled up.

5P6LEXI-S2 Same as case 5P6LEXI except with the cross section dips at 297 KeV, 521 KeV and 1.885 MeV filled up.

5P6LEXI-DD Same as case 5P6LEXI except with cross sections in the vicinity of cross section dips modified as described in Category C with the total cross sections at the bottom of the dips were 75 % of the original values.

K. Calculations with Modified Absorption Cross Sections:

ONC With no absorption below 10 KeV.

0C1 Added 1 % absorption everywhere with

$$\sigma_s^{NEW}(E) = \sigma_s^{OLD}(E) - 0.01 \cdot \sigma_T(E) \quad 0.0 \leq E \leq 15.0 \text{ MeV}$$

0C2 Same as case 0C1 except only added 0.1 % absorption.

6PC1 Same as case 0C1 except with a partial fission source the same as case 6P.

6PC2 Same as case 0C2 except with a partial fission source the same as case 6P.

2PSC2 Same as case 2PS except added 0.1 % absorption like case 0C1.

4PSC2 Same as case 4PS except added 0.1 % absorption like case 0C1.

5PSC2 Same as case 5PS except added 0.1 % absorption like case 0C1.

6PSC2 Same as case 6PS except added 0.1 % absorption like case 0C1.

7PSC2 Same as case 7PS except added 0.1 % absorption like case 0C1.

0AC2 Same as case 0A except added 0.1 % absorption like case 0C1.

0IE1C2 Same as case 0IE1 except added 0.1 % absorption like case 0C1.

8IE1C2 Same as case 8IE1 except added 0.1 % absorption like 0C1.

OCP1 Same as case OCl but with

$$\frac{\sigma_s^{OLD}}{\sigma_{s'}^{OLD}} = \frac{\sigma_s^{NEW}}{\sigma_{s'}^{NEW}}$$

$$\sigma_s^{NEW} + \sum_i \sigma_{s'}^{i, NEW} = \sigma_s^{OLD} + \sum_i \sigma_{s'}^{i, OLD} - 0.01 \cdot \sigma_T$$

OCP2 Same as case OCP1 but with 0.1 %
absorption added.

OCP3 Same as case OCP1 but with 0.5 %
absorption added.

6PCP1 Same as case 6P except with 1.0 %
absorption added like case OCP1.

6PCP2 Same as case 6P except with 0.1 %
absorption added like case OCP1.

8IE1CP2 Same as case 8IE1 but with 0.1 % absorption
added like case OCP1.

L. Calculations with Finer Cross Section Set or/and
Finer Energy Grid:

OX With cross sections obtained from direct
interpolation from the PROCENDF output.

OX4P With the same source as in case 4PS and
added 90 energy points in energy range
between 1.60 MeV and 3.00 MeV and using
PROCENDF cross section set in that region.

- OX5P With partial fission source above 2.86 MeV and added 40 energy points in energy range between 3.0 MeV and 4.115 MeV and using the PROCENDF cross section set in that energy range.
- OX6P With the same source as in case 6P and added 40 energy points in energy range between 4.115 MeV and 5.380 MeV and using PROCENDF cross section set in that energy range.
- OX8 With the same source as in case 8 and added 40 energy points in energy range between 5.38 MeV and 7.0 MeV and using PROCENDF cross section set in that energy range.
- OX9 With the same source as in case 9 and added 25 energy points in energy range between 7.0 MeV and 10.95 MeV.

M. Calculations with Mock-up Sodium Cross Sections:

- CX1 With constant cross section, non-absorption, isotropic scattering sodium medium with a monoenergetic source. (lethargy range from 0.0 to 5.5.)

CX2 Same as case CX1 except lethargy range from 0.0 to 14.0.

CX2C1 Same as case CX2 except with 1.0 % absorption.

CX2C2 Same as case CX2 except with 0.1 % absorption.

CX3 With constant cross section, 50 % absorption, anisotropic scattering sodium medium with a constant source from $u = 0.0$ to $u = 2.0$, and with the anisotropy of the elastic scattering the same as the real sodium at 7.0 MeV.

CX3A Same as case CX3 except with isotropic scattering.

CX4 Same as case CX3A except with 50 % inelastic scattering above 0.5 MeV ($Q = 439$ KeV).

CX4-IQD Same as case CX4 except $Q = 189$ KeV.