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ANALYSIS OF SLIFER DATA FROM UNDERGROUND EXPLOSIONS

M. Heusinkveld

January 12, 1979

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ANALYSIS OF SLIFER DATA FROM UNDERGROUND EXPLOSIONS

ABSTRACT

We derive a formula describing the distance-time relationship of a shock wave moving outward from an underground explosion. We compare our calculated results with those computed using the LASL and BOTE formulas and with slifer data obtained from field experiments. We find that for many of the field events, our derived curve provides a better fit than do the LASL or BOTE formulas. We also present methods for the determination of the detonation energy W under three conditions: a) where time and distance are known accurately, b) where there is an unknown offset of time and distance, and c) where there is an unknown offset of both time and distance. These methods are applied with moderate success to a set of (t,r) data supplied by Goldwire.

INTRODUCTION

This report studies the relation of arrival time vs radial distance for the outward-moving shock from an explosion, which has been used to estimate the energy of a nuclear explosion since the first days of nuclear testing. The earliest tests were conducted in the atmosphere, and Taylor¹ extensively analyzed this relation for atmospheric detonations. Because the properties of air are uniform and well-documented, he was able to obtain agreement between his calculated results and the observed shock-arrival times. Shock-arrival-time measurements of underground nuclear tests, for which the transmitting media are less uniform; and less well-known, were first made using switches or other transducers at discrete locations, and Los Alamos Scientific Laboratory (LASL) and Lawrence Livermore Laboratory (LLL) used such measurements to determine nuclear energies. The "slifer" system² was developed later at LLL. This technique continuously measures the position of the shock front by detecting the point at which an electric cable extended toward the detonation center is crushed. Recently, LASL has begun measuring the shock position using time domain reflectometry (TDR).

Slifer systems have supplied the bulk of the time-vs-position data on terrestrial shock waves generated in the underground nuclear testing program at the Nevada Test Site (NTS). A large amount of data has been acquired, and this report analyzes these data.

Although the term "hydrodynamic yield measurements" has also been used, this term implies that the only measurements used were those that did not extend down into the region where nonlinear effects become significant. We therefore use the term "slifer data."

SEMI-EMPIRICAL FORMULAS

LASL AND SLA EMPIRICAL FORMULAS

Workers at Sandia Laboratories in Albuquerque (SLA) observed that the (t,r) relation for an earth medium was approximately linear when plotted in log-log form. For an underground explosion with a yield of 1 kt, they deduced the empirical relationship

$$r = 5.986 t^{0.4396} \quad (1)$$

for alluvium and

$$r = 5.699 t^{0.4097} \quad (2)$$

for granite, where r is the distance (in m) from the center of the detonation and t is the elapsed time (in ms) from t_0 .

Later, using data collected from a larger number of nuclear events than available at SLA and from computer calculations, LASL obtained the relation³

$$\frac{r}{W^{1/3}} = \alpha \left(\frac{t}{W^{1/3}} \right)^\beta, \quad (3)$$

where r is distance (in m), W is energy output (in kt), t is time (in ms), and the preferred values for α and β are 6.29 and 0.475, respectively. This formula was stated to be most accurate for a 1-kt event over the hydrodynamic region extending about 0.16-0.60 scaled ms or about 2.6-4.9 m. For larger events, these limits were increased by a factor of $W^{1/3}$.

BOTE FORMULA

It has been argued that at large distances, the shock velocity should approach the sonic velocity of the medium. Rambo and Kurtz proposed the BOTE relation^{4,5}

$$R_s = R_{s0} \left(\frac{\alpha W}{\rho_0} \right)^{0.2} t^{0.4} + ct, \quad (4)$$

where R_s is the shock radius, R_{s0} is the reference radius for the hydrodynamic regime, α is a constant dependent on the medium, ρ_0 is the density of the medium, and c is the sonic velocity. An alternate form, in which the energy scaling is apparent, is obtained by rearrangement:

$$\frac{R_s}{W^{1/3}} = R_{s0} \left[\left(\frac{\alpha}{\rho_0} \right)^{0.2} \left(\frac{t}{W^{1/3}} \right)^{0.4} \right] + \frac{ct}{W^{1/3}}. \quad (5)$$

This formula is related to the approximate solution of the motion of a shock wave in a gamma-law gas, as given by Chernyi.⁶ Although Eq. (4) can be solved for the sonic velocity at large distances, addition of this velocity has no physical basis at small distances.

THEORETICAL FORMULA

DERIVATION OF THE (t,r) RELATION

Over the pressure range encountered in slifer applications, a linear relation exists between shock velocity U and particle velocity u :⁷

$$U = a + bu. \quad (6)$$

This relation holds with constant values for a and b for materials in the hydrodynamic regime where prevailing stresses greatly exceed the material strength and where there are no phase changes. We have assumed this ideal situation here.

The second Rankine-Hugoniot jump condition relates the shock pressure to these velocities:

$$P = \rho_0 u U, \quad (7)$$

where ρ_0 is initial density of the unshocked medium. It has been assumed that the ambient pressure is negligible. The particle velocity u is eliminated by solving Eq. (6) for u and substituting the result in Eq. (7):

$$P = \frac{\rho_0 U(U-a)}{b} . \quad (8)$$

The Grüneisen gamma relates the driving pressure at the shock front to the energy density:

$$P = \frac{\Gamma e}{V} , \quad (9)$$

where e is energy per unit mass; e/V is thus energy per unit volume. In using this equation, we assume that it includes kinetic effects in addition to the strictly internal-energy effects. The energy density e/V at the shock front is related to the average energy density throughout the sphere of shocked material by a relation of the form

$$e = f \bar{e} , \quad (10)$$

where, in this idealized analysis, f is assumed to be a constant. Equation (10) introduces some (but not all) of the deficiencies of the similarity solution, but permits a solution to be obtained for the shock propagation relations without requiring solutions for the motions within the sphere.

Within the sphere defined by the shock front, the average energy per unit volume is

$$\frac{\bar{e}}{V} = \frac{W}{\frac{4}{3}\pi r^3} , \quad (11)$$

where W is the energy of the explosive device and r is the distance to the explosion center. The driving pressure, derived from Eqs. (9)-(11), is

$$P = \frac{3\Gamma f W}{4\pi r^3} . \quad (12)$$

Combination of Eqs. (8) and (12) to eliminate the pressure term gives the relation

$$\frac{\rho_0 U(U-a)}{b} = \frac{3\Gamma f W}{4\pi r^3} .$$

Rearrangement gives

$$U^2 - aU = \frac{3b\Gamma f W}{4\pi r^3 \rho_0} . \quad (13)$$

For convenience, we substitute

$$g^3 = \frac{3b\Gamma f W}{\pi \rho_0 a^2} \quad (14)$$

in Eq. (13), where g is a reference distance. This gives a relation for the shock velocity in quadratic form:

$$U^2 - aU - \frac{a^2 g^3}{4r^3} = 0 . \quad (15)$$

One solution is

$$U = \frac{a}{2} \left[1 + \left(1 + \frac{g^3}{r^3} \right)^{1/2} \right], \quad (16)$$

and only the positive root of the solution has physical significance.

The shock arrival time is given by the relation

$$t = \int_0^r \frac{dr}{U} \quad (17)$$

$$= \frac{2}{a} \int_0^r \frac{dr}{1 + \left(1 + \frac{g^3}{r^3} \right)^{1/2}}. \quad (18)$$

Multiplying both the numerator and denominator of the integrand by $(1 + g^3/r^3)^{1/2} - 1$ gives

$$t = \frac{2}{a} \int_0^r \frac{r^3}{g^3} \left[\left(1 + \frac{g^3}{r^3} \right)^{1/2} - 1 \right] dr. \quad (19)$$

Equation (19) can be expressed in terms of a dimensionless time τ and a dimensionless distance x by substituting

$$x = \frac{r}{g}, \quad (20)$$

$$\tau = \frac{at}{g}. \quad (21)$$

In these dimensionless units, Eq. (19) becomes

$$\tau = 2 \int_0^x x^3 \left[\left(1 + \frac{1}{x^3} \right)^{1/2} - 1 \right] dx. \quad (22)$$

We found no simple solution for this integral.

In the dimensionless variable form, the shock velocity as a function of position is derived from Eq. (16):

$$\phi = \frac{1}{2} \left[1 + \left(1 + \frac{1}{x^3} \right)^{1/2} \right]. \quad (23)$$

The dimensionless velocity ϕ is defined by the relation

$$\phi = \frac{x}{\tau} = \frac{U}{a}. \quad (24)$$

BEHAVIOR AT EARLY TIME

Although an analytic solution is not available, the asymptotic behavior at early time for strong shocks can be determined. In this case, the term $1/x^3$ is large, and the expression $[(1/x^3 + 1)^{1/2}]$ can be expanded in series form with $1/x^3$ as the leading term:

$$\begin{aligned}
\tau &= 2 \int_0^x x^3 \left[\left(\frac{1}{x^3} \right)^{1/2} + \frac{1}{2} \left(\frac{1}{x^3} \right)^{-1/2} - \frac{1}{8} \left(\frac{1}{x^3} \right)^{-3/2} + \dots - 1 \right] dx \\
&= 2 \int_0^x \left[x^{3/2} - x^3 + \frac{1}{2} x^{9/2} - \frac{1}{8} x^{15/2} + \dots \right] dx \\
&= \frac{4}{5} x^{5/2} - \frac{1}{2} x^4 + \frac{2}{11} x^{11/2} - \frac{1}{34} x^{17/2} + \dots
\end{aligned} \tag{25}$$

The first term on the right-hand side leads to the similarity solution:

$$\tau = \frac{4}{5} x^{5/2}, \tag{26}$$

$$t = \frac{4}{5} \frac{r^{5/2}}{a g^{3/2}}, \tag{27}$$

$$r = \left(\frac{5}{4} \right)^{2/5} a^{2/5} g^{3/5} t^{2/5} \tag{28}$$

$$= \left(\frac{5}{4} \right)^{2/5} \left(\frac{3b\Gamma W}{\pi \rho_0} \right)^{1/5} t^{2/5}. \tag{29}$$

As reported by Zel'dovich and Raizer,⁸ the corresponding Chernyi solution for a strong shock in a homogeneous atmosphere is

$$R = \left(\frac{5}{4} \right)^{2/5} \left(\frac{3(\gamma-1)(\gamma+1)^2}{\pi(3\gamma-1)} \right)^{1/5} \left(\frac{W}{\rho_0} \right)^{1/5} t^{2/5}. \tag{30}$$

These solutions are identical except that the solid-medium case [Eq. (29)] contains the factor $b\Gamma$ whereas the Chernyi solution for the gaseous medium [Eq. (30)] contains the factor $(\gamma-1)(\gamma+1)^2/(3\gamma-1)$. In the gaseous case reported by Zel'dovich and Raizer, the shock velocity-particle velocity relation is $U = u(\gamma+1)/2$, whereas in the solid case [Eq. (29)], it is $U = bu$ (when $a \ll bu$); b consequently corresponds to $(\gamma+1)/2$. For strong shocks, the solid state parameter Γ is roughly equivalent to the parameter $\gamma-1$ for gases; the quantity f in Eq. (29) therefore corresponds to the quantity $2(\gamma+1)/(3\gamma-1)$ in the Chernyi solution.

The term a is not present in the early shock phenomena because we have implicitly assumed that $a \ll U$ in the approximation. The next term of the series expansion does contain a , and a is used to establish the length and time scales for the problem. If the term a were not present, the problem would contain no length scale, and the only dimensional quantities would be the total energy and the density. A similarity solution would be valid for such a problem, assuming that the Γ term is constant.

BEHAVIOR AT LATE TIME

The (t,r) behavior at late time, when the time-of-arrival curve asymptotically approaches the sonic velocity, is found by series expansion of Eq. (22):

$$\begin{aligned}\tau &= 2 \int_0^x x^3 \left[\left(1 + \frac{1}{x^3} \right)^{1/2} - 1 \right] dx \\ &= K + 2 \int_{x_0}^x x^3 \left[1 + \frac{1}{2x^3} - \frac{1}{8x^6} + \frac{1}{16x^9} + \dots - 1 \right] dx,\end{aligned}$$

where

$$K = 2 \int_0^{x_0} x^3 \left[\left(1 + \frac{1}{x^3} \right)^{1/2} - 1 \right] dx.$$

The quantity x_0 must be large enough ($x_0^3 \gg 1$) so that the series expansion of the remaining integrand converges quickly over the integral from x_0 to x . Integration of Eq. (22) gives the relation:

$$\tau = (x - x_0) + \frac{1}{8} \left(\frac{1}{x^2} - \frac{1}{x_0^2} \right) - \frac{1}{40} \left(\frac{1}{x^5} - \frac{1}{x_0^5} \right) + \dots + K. \quad (31)$$

Conversion to dimensionless units gives

$$at = (r - r_0) + \frac{g}{8} \left(\frac{1}{r^2} - \frac{1}{r_0^2} \right) - \frac{g^6}{40} \left(\frac{1}{r^5} - \frac{1}{r_0^5} \right) + \dots + K. \quad (32)$$

Equation (32) describes the velocity a at large distances.

A numerical rather than analytic integration of Eq. (22) gives $\tau = 9.355$ when $x = 10$, and this value is used for K when Eq. (31) is solved for $x_0 = 10$.

When $x_0 = 10$, all terms except $(x - x_0)$ on the right-hand side of Eq. (31) are very small and may be neglected; the equation may be written

$$\begin{aligned}\tau &= x - 10 + 9.355 \\ x &= \tau + 0.645.\end{aligned} \quad (33)$$

The value 0.645 is the y-intercept of the asymptotic line as indicated in the (τ, x) plot (Fig. 1). In Fig. 2, the same relation is shown for the (t, r) plot for wet tuff (discussed later). The y-intercept is at $r = 3.92$, and the slope equals 3.00. [In this report, the independent variable plotted along the abscissa is listed first in all descriptions, e.g., in a (t, r) plot, t (time) is the independent variable. In the LASL work, radius was regarded as the independent variable and time as the dependent variable; using this convention, they thus described their slier data as r - t data.]

In principle, this asymptotic line on the (t, r) plot (Fig. 2) defines values for both a and g in dimensional units such as metres and milliseconds. Stated explicitly,

$$r_1 = 0.645 \text{ g}, \quad (34)$$

where r_1 is the intercept. The slope of the line gives the value for a , which in this case is 3.00 m/ms. The value for g is calculated to be 6.08 m. In this report, we have used the consistent set of units based on metres, milliseconds, megagrams, gigapascals, and gigajoules. One kiloton TNT equivalent equals 4184 GJ, and 1 kbar equals 0.1 GPa.

If values for p_0 and b are known from laboratory measurements on samples of the medium, and if Γ and f are determined empirically from earlier field tests, then in principle the energy W can be determined by this intercept method [Eqs. (25)-(34)]. Better methods of determining W , involving all the data, are presented later in this report.

FIG. 1. Linear (τ, x) plot from derived formula showing y-intercept.

FIG. 2. Linear (t, r) plot from derived formula showing y-intercept.

DIMENSIONLESS SHOCK PARAMETERS

The results of the numerical integration of Eq. (22) are shown in Figs. 1 and 3 in linear and logarithmic form, respectively. The slope of the curve in Fig. 3 changes from 0.4 to 1.0 with increasing time, indicating the transition from the shock regime to the low-pressure regime. The velocity a is not the sonic velocity, but rather the y-intercept of the curve of particle velocity vs shock velocity for the medium.

The relation for dimensionless shock velocity is $\phi = U/a$, as given in Eq. (24). The particle velocity and the pressure can also be converted to dimensionless form. Because $u = (U-a)/b$, the conversion unit chosen for u is the factor b/a :

$$\psi = \frac{b}{a} u, \quad (35)$$

where ψ is the dimensionless particle velocity. This definition of ψ permits simplification of subsequent formulas. In dimensionless units, the particle velocity is related to the shock velocity by the equation

$$\psi = \phi - 1. \quad (36)$$

From Eqs. (23) and (36), ψ is related to dimensionless distance by the relation

$$\psi = \frac{1}{2} \left[\left(1 + \frac{1}{x^3} \right)^{1/2} - 1 \right]. \quad (37)$$

The shock pressure is converted to dimensionless pressure using the relation $P = uU\rho_0$. Using Eqs. (24) and (35), the conversion factor chosen for pressure is

$$p = \frac{b}{a^2 \rho_0} P \quad (38)$$

and the equation for dimensionless pressure p becomes

$$p = \psi \phi. \quad (39)$$

FIG. 3. Logarithmic (τ, x) plot from derived formula.

This pressure p is thus related to dimensionless distance by the product of the expressions for ϕ and ψ , given in Eq. (23) and (37), respectively:

$$p = \frac{1}{4x^3} \quad (40)$$

Figures 4 and 5 compare P , ϕ , and ψ as functions of dimensionless distance. The pressure follows a radial attenuation of x^{-3} over the entire range, whereas ϕ and ψ are attenuated at a rate of $x^{-3/2}$ at the near distances. The dimensionless velocities ϕ and ψ separate at dimensionless distances of about 0.1 (Fig. 4) and exhibit markedly differing slopes at larger dimensionless distances. This indicates that the presence of the a term at small radial distances appreciably affects the slopes of the (t, r) curves. These quantities are plotted as functions of dimensionless time in Figs. 6 and 7.

For fully saturated tuff, we found that 3m/ms is an approximate value for a , and that 1.11 is the corresponding value for the slope b of the shock velocity-particle velocity line (see below). If we assume a density ρ_0 of 2.2 mg/m³, $\Gamma = 1$, $f = 1$, and $W = 1$ kt, then the characteristic length g is 6.1 m.

At a dimensionless shock velocity of $\phi = 1.5$, the dimensionless particle velocity is $\psi = \phi - 1 = 0.5$. Because $U = a\phi$ and $u = a\psi/b$, the actual shock velocity and particle velocity are 4.5 m/ms and 1.35 m/ms, respectively.

The dimensionless shock pressure, $p = \phi\psi$, is numerically equal to 0.75 units when the velocities have these values. The actual corresponding shock pressure P is 100 kbar [Eq. (38)]. The scaled distance at this time is 0.693 [Eq. (40)], and the dimensionless time is 0.24 units (read from Fig. 3). The corresponding real distances and times are 4.23 m and 0.49 ms, respectively, since $r = gx$ and $t = gr/a$.

As stated earlier, the hydrodynamic region as defined by LASL extends to about 0.60 ms. Figure 4 shows that beyond the point $x = 0.693$, the value of the a term very strongly influences the shock velocity curve, as indicated by the increased curvature of the (x, ϕ) line. At earlier times, the slope of the velocity curve on the logarithmic plot has already been perturbed, although a straight line might still approximate this curve over a limited range.

DERIVED (t, r) RELATION FOR A POROUS MEDIUM

Perret and Bass⁹ observed experimentally that, in some porous media, the pressure decays with respect to distance at a rate greater than $1/r^2$. This may result either from conversion of energy to an unavailable form or from degradation of the shape of the pressure wave. In this case, Eq. (12) may be replaced by the relation

FIG. 4. Logarithmic plot of dimensionless pressure, shock velocity, and particle velocity as functions of dimensionless distance.

FIG. 5. Linear plot of dimensionless pressure, shock velocity, and particle velocity as functions of dimensionless distance.

FIG. 6. Logarithmic plot of dimensionless pressure, shock velocity, and particle velocity as functions of dimensionless time.

FIG. 7. Linear plot of dimensionless pressure, shock velocity, and particle velocity as functions of dimensionless time.

$$P = \frac{3\Gamma\Gamma W}{4\pi r^3} \left(\frac{\lambda}{r} \right)^\epsilon, \quad (41)$$

where ϵ represents the additional pressure decrement factor and λ is a characteristic length for energy dissipation. The shock velocity is then determined by combining Eqs. (8) and (41) to eliminate the pressure terms:

$$\frac{\rho_0 U(U-a)}{b} = \frac{3\Gamma\Gamma W}{4\pi r^3} \left(\frac{\lambda}{r} \right)^\epsilon; \quad (42)$$

$$U = \frac{a}{2} \left\{ 1 + \left[1 + \frac{g^3}{r^3} \left(\frac{g}{r} \right)^\epsilon \left(\frac{\lambda}{g} \right)^\epsilon \right]^{1/2} \right\}. \quad (43)$$

As before, the term g is defined as

$$g = \left(\frac{3b\Gamma r W}{\pi \rho_0 a^2} \right)^{1/3}.$$

For convenience, we set $g = \lambda$. Because ϵ is a small number, we would expect the quantity $(\lambda/g)^\epsilon$ to have a value near unity. Equation (43) can then be rewritten as

$$U = \frac{a}{2} \left\{ 1 + \left[1 + \left(\frac{g}{r} \right)^n \right]^{1/2} \right\}, \quad (44)$$

where $n = 3 + \epsilon$. The dimensionless form of this equation is

$$\phi = \frac{1}{2} \left\{ 1 + \left[1 + \left(\frac{1}{x} \right)^n \right]^{1/2} \right\}, \quad (45)$$

where the conversion scales for distance and velocity are the same as before, given in Eqs. (20) and (24).

The time for the stress wave to reach a given radial distance is specified by the integral

$$t = \frac{2}{a} \int_0^r \frac{dr}{1 + \left[1 + \left(\frac{g}{r} \right)^n \right]^{1/2}}. \quad (46)$$

The corresponding equation for dimensionless time, using Eqs. (20) and (21), is

$$\tau = 2 \int_0^x x^n \left\{ \left[1 + \left(\frac{1}{x} \right)^n \right]^{1/2} - 1 \right\} dx. \quad (47)$$

Because $u = (U - a)/b$, the particle velocity is given by the expression

$$u = \frac{a}{2b} \left\{ \left[1 + \left(\frac{g}{r} \right)^n \right]^{1/2} - 1 \right\}, \quad (48)$$

and the dimensionless particle velocity has the form

$$\psi = \frac{1}{2} \left\{ \left[1 + \left(\frac{1}{x} \right)^n \right]^{1/2} - 1 \right\}. \quad (49)$$

The pressure is obtained from Eqs. (7), (44), and (48):

$$P = \frac{a^2 \rho_0}{4b} \left(\frac{g}{r} \right)^n. \quad (50)$$

Using Eq. (38) or (39), the dimensionless form is

$$p = \frac{1}{4x^n}. \quad (51)$$

The behavior at early time is found in the same manner as before. A series expansion of the integrand of Eq. (47) followed by integration gives the result

$$\tau = \frac{4x^{(n+2)/2}}{n+2} - \frac{2x^{n+1}}{n+1} + \frac{2x^{(3n+2)/2}}{3n+2} - \frac{x^{(5n+2)/2}}{2(5n+2)} + \dots \quad (52)$$

The first term of the expansion gives the asymptotic behavior at early time. The (t, r) solution for the case in which only the first term is used is

$$r = \left(\frac{n+2}{4} \right)^{2/(n+2)} (ag^{n/2})^{2/(n+2)} t^{2/(n+2)}. \quad (53)$$

The corresponding LASL formula can be stated in the form

$$r = \alpha W^{(1-\beta)/3} t^\beta. \quad (54)$$

From Eqs. (53) and (54), we see that n can be related to β :

$$\beta = \frac{2}{n+2}, \quad (55)$$

$$n = \frac{2(1-\beta)}{\beta}. \quad (56)$$

The value $\beta = 0.475$, as used in the LASL formula, leads to the value $n = 2.21$. The value $\beta = 0.4$ gives $n = 3$, which is the value assumed in Eq. (12) in the earlier derivation.

Equation (47), which gives the relation between τ and x , has been integrated numerically with the CURVE program¹⁰ for the values $n = 2.5, 3, 3.25$, and 3.5 . Figure 8 shows the results in logarithmic form. A corresponding set of curves for shock velocity ϕ vs x [Eq. (45)] is given in Fig. 9. The parameter x can be eliminated between Eqs. (47) and (49) to give the (τ, ϕ) relation. Figure 10 gives the (τ, ϕ) curves for the same values of n listed above.

Because the energy dissipation length λ was arbitrarily set equal to g , these curves coincide at $x = 1$. For lack of a more basic derivation, the missing amplitude coefficient $(\lambda/g)^4$ in Eq. (43) must be determined by comparison of arrival-time curves [Eq. (46)] with experimental (t, r) curves.

FIG. 8. Logarithmic plot of dimensionless distance vs dimensionless time for $n = 2.5, 3, 3.25$, and 3.5 .

FIG. 9. Logarithmic plot of dimensionless shock velocity vs dimensionless distance for $n = 2.5, 3, 3.25$, and 3.5 .

FIG. 10. Logarithmic plot of dimensionless shock velocity vs dimensionless time for $n = 2.5, 3, 3.25$, and 3.5 .

COMPARISON AMONG THE (t,r) FORMULAS

COMPARISON OF LASL AND DERIVED (t,r) FORMULAS

Because the LASL formula [Eq. (3)] gives good results in determinations of energy output from underground nuclear detonations, a comparison between the results derived here and the LASL formula is useful.

The LASL formula may be rewritten as

$$r = \alpha W^{(1-\beta)/3} t^\beta. \quad (57)$$

The shock velocity is given by the relation

$$\begin{aligned} U &= \frac{dr}{dt} \\ &= \beta \alpha W^{(1-\beta)/3} t^{\beta-1}. \end{aligned} \quad (58)$$

The shock velocity as a function of radial distance is found from Eqs. (57) and (58):

$$U = \alpha^{1/\beta} \beta W^{(1-\beta)/3} r^{-(1-\beta)/\beta}. \quad (59)$$

The (t,r) relation, the (t,u) relation, and the (r,U) relation can all be represented on a logarithmic plot as straight lines with slopes of β , $-(1-\beta)$, and $-(1-\beta)/\beta$, respectively.

No characteristic units are specified in the LASL (t,r) formula; we therefore use the same α and g reference values as in the LLL derivation. There is, however, an inconsistency here; if there were no reference

values in the formula, a similarity solution should apply. This would lead to the value $\beta = 0.4$ whereas the LASL value is $\beta = 0.475$. (In other words, α is not a dimensionless constant.) By setting $x = r/g$ and $\tau = at/g$, where $g = 6.1$ and $a = 3$, and by using the value $\alpha = 6.29$ and $\beta = 0.475$, the dimensionless form of the LASL formula given by Eq. (57) becomes

$$x = 1.445 W^{0.175} \tau^{0.475} \quad (60)$$

Using this formula, a curve for an energy W of 1 kt and the curve from Eq. (22) are plotted in Fig. 11. The two curves agree quite well over the range of 0.06-1.0 in dimensionless time; this corresponds to a range of 0.12-2.03 ms. The LASL region of best slope on this logarithmic plot extends from 0.16 to 0.60 ms or from $\tau = 0.32$ to $\tau = 1.22$.

RELATION OF LASL AND DERIVED FORMULAS TO EXPERIMENTAL PRESSURE AND PARTICLE-VELOCITY DATA

The validity of the experimental value of β in the LASL formula can be checked. Perret and Bass⁹ of SLA compiled the results of underground shock measurements for many events at NTS, and obtained empirical relations for several of the shock parameters. They found that in the hydrodynamic region, the peak pressure follows the relation

$$p = 7.710(R/W^{1/3})^{-2.96 \pm 0.19} \quad (61)$$

where p is pressure (in kbar), R is radial distance (in m), and W is explosively released energy (in kt). This formula is valid for radial distance from about 2.5 to 8 scaled m with corresponding pressures of 500 to 20 kbar. The exponent -2.96 closely matches the value -3 required in a similarity solution, and the latter value is assumed in the derivation given here.

Measurements have been made with velocity detectors, which move along with the medium and therefore measure particle velocities. From these data, Perret and Bass obtained the equation

$$u = 24,300(R/W^{1/3})^{-1.87 \pm 0.05} \quad (62)$$

FIG. 11. Comparison of $(\log \tau, \log x)$ curves from LASL formula and from derived formula.

where u is particle velocity in m/s. Because $p = \rho_0 u U$, the shock velocity can be found from Eqs. (61) and (62). Assuming a density ρ_0 of 2 Mg/m³, the resulting shock velocity is

$$U = 15,860(R/W^{1/3})^{-1.09} \quad (63)$$

When the values $\beta = 0.475$ and $\alpha = 6.29$ are inserted into the LASL formula [Eq. (59)], the following numerical equation for shock velocity is obtained:

$$U = 21,940(R/W^{1/3})^{-1.105} \quad (64)$$

The exponents of Eqs. (63) and (64) agree well, and the coefficients agree approximately. The exponent of -1.09 from the field data would actually lead to a value for β of 0.478. One should remember that Eqs. (61) and (62) represent straight-line fits to the data of Perret and Bass only over the region where Perret and Bass consider the motions to be hydrodynamic.

The values calculated from the derived formulas for pressure and for particle velocity can also be checked against the SLA measurements. The dimensional form of Eq. (37) is

$$u = \frac{a}{2b} \left\{ \left[1 + \left(\frac{g}{r} \right)^3 \right]^{1/2} - 1 \right\} \quad (65)$$

Similarly, the dimensionless formula for pressure [Eq. (40)] is converted to dimensional form using Eq. (38):

$$P = \frac{\rho_0 a^2}{4b} \left(\frac{g}{r} \right)^3 \quad (66)$$

The values $a = 3$, $b = 1.11$, and $g = 6.1$ are again used, and Fig. 12 plots the resulting curves and the experimental data of Perret and Bass. The good agreement of the theoretical pressure curve with the experimental data extends to a scaled distance of 8 m/kt^{1/3}. In this derivation, we assumed that the pressure would follow a $1/r^3$ decay as indicated in Eq. (12). The experimental data showed a deviation at distances greater than 8 m/kt^{1/3}.

The particle velocity curves are in similar good agreement to this same distance; at greater distance the derived-particle-velocity curve falls below the experimental points (Fig. 12). This deviation indicates that the similarity solution no longer applies beyond this distance. Because $g = 6.1$ in this comparison, the pressure and particle-velocity curves of Figs. 4-7 become less reliable at scaled distances greater than $x = 1.3$. The corresponding dimensionless time is $\tau = 0.7$ (from Fig. 1).

COMPARISON OF DERIVED AND BOTE (t,r) FORMULAS

Kurtz⁵ demonstrated that the BOTE model gives results in good agreement with data from field experiments. Consequently, it is desirable to determine whether or not the BOTE form gives results that are comparable to those from the derived formulas. The BOTE form as presented by Kurtz is

$$R_B = \left(\frac{5}{2} \right)^{2/5} \left[\frac{3}{4\pi} \frac{(\gamma-1)(\gamma+1)^2}{(3\gamma-1)} \right]^{1/5} \left(\frac{W}{\rho_0} \right)^{1/5} t^{2/5} + ct \quad (67)$$

$$= \left(\frac{5}{4} \right)^{2/5} g_B^{3/5} c^{2/5} t^{2/5} + ct, \quad (68)$$

FIG. 12 Pressure and particle velocity data of Perret and Bass and results from derived formula.

where

$$g_B^3 = \frac{3(\gamma-1)(\gamma+1)^2 W}{\pi(3\gamma-1)c^2 \rho_0} \quad (69)$$

Equation (69) is similar in form to Eq. (28) and can also be converted to dimensionless form with the substitutions

$$x_B = \frac{R_B}{g_B}, \quad (70)$$

$$\tau_B = \frac{ct}{g_B}. \quad (71)$$

With these substitutions, the dimensionless form of the BOTE equation is

$$x_B = \left(\frac{5}{4}\right)^{2/5} \tau_B^{2/5} + \tau_B. \quad (72)$$

The results of the numerical integration of Eq. (22) are given in Figs. 13 and 14 in linear and logarithmic form, respectively. The plots of the dimensionless BOTE equation (72) are also given with these results. The results are similar, but over the range covered, the BOTE curves lie considerably above the numerical integration curves.

The first term on the right-hand side of Eq. (72) describes the limiting behavior at early time, as given by Eq. (26), and the second term on the right describes the limiting form at late time, as given by the first term of Eq. (32). Simply adding these two terms, as in the BOTE equation, fails to give the results of the numerical integration. We found, by trial and error, that if numerical coefficients are used for the two right-hand terms in Eq. (72), a better fit can be obtained. Again using trial and error, a moderately good fit over a limited range was obtained with the relation

$$x = 0.974 \left(\frac{5}{4}\right)^{2/5} \tau_B^{2/5} + 0.654 \tau_B. \quad (73)$$

Figures 15 and 16 compare the plot of this equation with the results of the numerical integration of Eq. (22). These curves agree moderately well over part of the range, but this fact has no particular significance. The LASL formula, with the two adjustable parameters α and β , gives a straight line on the logarithmic plot.

FIG. 13. Comparison of BOTE (τ, x) results with derived results (linear plot).

FIG. 14. Comparison of BOTE (τ, x) results with derived results (logarithmic plot).

FIG. 15. Comparison of (τ, x) results from adjusted BOTE formula with derived results (linear plot).

FIG. 16. Comparison of (τ, x) results from adjusted BOTE formula with derived results (logarithmic plot).

with slope and vertical position adjusted to fit the experimental data. In the BOTE equation, another adjustable parameter is included that, in effect, adds curvature to this line to give a better fit. Curvature added using BOTE is not the same as that obtained by us from the derived Eq. (19), and the two curves agree only over a limited range.

APPLICATION OF FORMULAS TO SLIFER DATA

COMPARISON OF SLIFER DATA WITH RESULTS FROM LASL (t, r) FORMULA

Because of the scaling method used here, where time and distance are both scaled as $1/W^{1/3}$, the pressures and energy densities are identical in the scaled and unscaled systems. Consequently, the same equation of state of the medium applies to both systems unless there are time-dependent effects. In principle, all the scaled (t, r) curves should coincide for the same medium.

Scaled (t, r) curves for some of the representative events are shown in Figs. 16-20. Figure 17 shows the scaled (t, r) results for Fondutta, Estuary, Cheshire, and Fontina, all conducted in saturated tuff at Pahute Mesa at NTS. Also shown on Fig. 17 is the reference line from the LASL formula given in Eq. (3) for a 1-kt yield. These experimental curves fit the LASL line fairly closely over a limited range, but all the experimental curves show an upward curvature not allowed for in the LASL formula.

Figure 18 gives the scaled (t, r) curves for the events Reblochon, Chiberta, and Marsilly, all conducted in saturated tuff at Yucca Flat and located near each other. These curves agree with each other quite well, indicating that the observed curvature is a characteristic of the shock mechanics in this particular medium. The curve for Marsilly is somewhat lower in this logarithmic plot, indicating either that the energy output was lower than the value used in scaling this event or that the seismic coupling was lower for some unknown reason.

Figure 19 gives scaled (t, r) results for some events conducted in unsaturated alluvium or tuff at Yucca Flat, including Chevre, Anacostia, and Cashmere. These curves show a slope consistently less steep than the slope of the LASL curve. Figure 20 shows results for some additional events in unsaturated alluvium or unsaturated tuff at Yucca Flat, including Cathay, and Chaenactis. These curves show the irregularities frequently encountered in slifer work that make it practically impossible to obtain reliable energy estimates from the data. These irregularities may result from perturbations attributable to emplacement hardware, medium inhomogeneity, or electronic malfunction.

FIG. 17. Comparison of scaled (t,r) curves for Fondutta, Estuary, Cheshire, and Fontina with LASL (t,r) curve.

FIG. 18. Comparison of scaled (t,r) curves for Reblonch, Chiberta, and Marsilly with LASL (t,r) curve.

FIG. 19. Comparison of scaled (t,r) curves for Chevre, Anacostia, and Cashmere with LASL (t,r) curve.

FIG. 20. Comparison of scaled (t,r) curves for Can, Cathay, and Chaenactis with LASL (t,r) curve.

The data presented here were selected as representative events from the large number of events that have been studied at NTS. In Fig. 21, scaled (t,r) curves from events in several different media and the LASL (t,r) curve are plotted together. Although the curves for the individual events diverge from the LASL line, the envelope of these individual curves fits the LASL line quite well. It is this circumstance that has enabled LASL to make good energy determinations in the past using the LASL formula.

FIG. 21. Comparison of scaled (t,r) curves for events in several media with LASL (t,r) curve.

COMPARISON OF SLIFER DATA WITH RESULTS FROM DERIVED (t,r) FORMULA

The slifer (t,r) curves of Figs. 17-21 show that the slifer results depend on the medium. The degree of saturation probably had a strong effect on the results. Figure 22 shows the shock-velocity vs particle-velocity curves in silicon sand that contained different amounts of water.¹¹ The slopes b of the U_s - u_p curves appear similar in all three cases, but the a values (the y-intercepts) are about 1.56, 2.0, and 2.6 for the cases of 20%, 50%, and 100% water saturation, respectively.

Figure 23 shows data for saturated tuff and for dry tuff,¹² and Fig. 24 gives data for basalt.¹¹ From these data and from the silicon sand results, values for the quantities a , b , and ρ_0 were selected; these values specify g (Table I). Using Eqs. (19) and (20), the values for a and g were used to convert the dimensionless (t,r) results from Eq. (22) to the dimensional form.

The experimental (t,r) results from Fondutta, Estuary, Cheshire, and Fontina and the derived (t,r) curve using the wet tuff parameters are plotted in Figs. 25 and 26. There is quite good agreement between the experimental data and the derived curves for the wet tuff. A similar comparison is made for Reblochon, Chiberta, and Marsilly in wet tuff at Yucca Flat. The agreement with the derived curves as shown in Figs. 27 and 28 is very good for these events. In the corresponding curve of Fig. 18, the curvature of the experimental line could only be approximated by the straight line from the LASL formula.

The (t,r) results for Cannikin, which was detonated in a basalt medium, and the derived curves for basalt and for wet tuff are shown in Fig. 29. The agreement is better for the wet tuff curve, indicating that the constants of the medium were more like those selected for wet tuff than those for basalt.

Figure 19 shows that the slifer results for the media that are not saturated with water differ considerably from the results for the saturated media. The slopes of the (t,r) curves for these events range from 0.35 to 0.38, which are considerably less than the slope of the LASL line (0.475), the slope of the similarity solution (0.40), and the slope of the portion of the derived curve for early times.

At early time, when the pressure is high, the shock-velocity and particle-velocity curves should have the same exponent, as indicated by Eqs. (16) and (37). Because pressure is directly proportional to the product of these two velocities, the exponent of the pressure curve in this region is double that of either velocity. When $\beta = 0.375$, the pressure-distance decrement becomes $n = 3.33$ [Eq. (56)].

The velocities recorded by the slifers in the unsaturated tuff also decrease to very low values. Figure 30 gives the (r,v) curves for several events in unsaturated media for which slifer data were available to large scaled distances. These velocities decreased to approximately 0.4 m/ms.

It is unclear whether the value for term a should actually be as low as 0.4 m/ms or whether neglected factors are producing unexpected effects. At such low velocities, the inelastic waves lag behind the elastic waves, and the waveforms of the inelastic waves at the large distances and low pressures associated with these low velocities are very broad. The (log t, log r) curves are still moderately linear to these distances, however, so

FIG. 22. Shock velocity as function of particle velocity for silicon sand with saturation of 20, 50, and 100 percent.

FIG. 23. Shock velocity as function of particle velocity for Pahute Mesa tuff.

TABLE 1. Properties of shock-transmitting media.

Medium	ρ_0	a	b	g	n
Basalt	2.5	2.4	1.63	7.677	3.0
Wet tuff	2.2	3.0	1.11	6.074	3.0
Dry tuff	1.6	0.4	1.3	27.274	3.4

FIG. 24. Shock velocity as function of particle velocity for Vacaville basalt.

we assumed that the derived formula could still be applied. From the data in Fig. 30, a value of 0.4 was selected for term a , b was set at 1.3 as a characteristic slope (Figs. 22 and 23), and n was set at 3.4, as required by the slopes of the curves for dry tuff (Fig. 19). These values are listed in Table 1. The curve derived using these values, that calculated using the LASL (t,r) formula, and the derived (t,r) curve for saturated tuff are compared in Fig. 31. In Figs. 32 and 33, the experimental data from several events in unsaturated tuff or alluvium are matched to this derived curve; the fit is fair, but not as good as the fits obtained for the events in saturated tuff.

FIG. 25. Comparison of scaled (t,r) curves for Fondutta, Estuary, Cheshire, and Fontina with derived (t,r) curve for wet tuff (logarithmic plot).

FIG. 26. Comparison of scaled (t,r) curves for Fondutta, Estuary, Cheshire, and Fontina with derived (t,r) curve for wet tuff (linear plot).

FIG. 27. Comparison of scaled (t,r) curves for Reb-lochon, Chiberta, and Marsilly with derived (t,r) curve for wet tuff (logarithmic plot).

FIG. 28. Comparison of scaled (t,r) curves for Chiberta with derived (t,r) curve for wet tuff (linear plot).

FIG. 29. Comparison of scaled (t,r) curve for Cannikin with derived curves for wet tuff and for basalt (logarithmic plot).

FIG. 30. Scaled (r,v) curves for Gouda, Chevre, Portola, and Gruyere in unsaturated tuff or alluvium.

FIG. 31. Comparison of the derived (t,r) curve for dry tuff, the derived curve for wet tuff, and the LASL curve.

FIG. 32. Comparison of the scaled (t,r) curve for Gruyere and the derived (t,r) curve for dry tuff.

FIG. 33. Comparison of the scaled (t,r) curves for Gouda and for Portola and the derived (t,r) curve for dry tuff.

METHODS OF ENERGY DETERMINATION USING THE LASL (t,r) FORMULA

Slifer data have been used primarily to determine the energy of underground explosions, and in the work described thus far, curves have been fitted visually rather than numerically. The advantage of a graphical presentation is that deviations and trends are more readily apparent. More precise work requires numerical curve-fitting, however, and the following sections present such procedures.

One complication encountered when using these techniques arises if the position of the effective center of the explosion is unknown, e.g., when the geometry of the energy source is highly asymmetric. In this case, the measured values of radial distance are not absolute, but may contain a fixed but unknown distance error R_0 . An additional complication results when there is also a timing uncertainty t_0 which arises if no zero time signal is available to give an absolute timing reference.

It is possible to determine to some extent what these distance and time corrections actually are from analysis of the slifer data. The R_0 and t_0 corrections may be included in the LASL (t,r) formula, [Eq. (3)] to give the modified form:

$$\frac{r + R_0}{W^{1/3}} = \alpha \left(\frac{t + t_0}{W^{1/3}} \right)^\beta. \quad (74)$$

This equation may be rearranged to the form

$$W = \alpha^{-3/(1-\beta)} (r + R_0)^{3/(1-\beta)} (t + t_0)^{-3/(1-\beta)}. \quad (75)$$

When the corrections R_0 and t_0 are known, these values can be inserted in Eq. (75) and the energy W determined statistically from the table of experimental (t,r) values. Each (t_i, r_i) pair leads to a particular W_i value, and the best estimate of energy W is made by minimizing the sum of the squares of the deviations:

$$S = \frac{1}{n} \sum_{i=1}^n (W - W_i)^2 . \quad (76)$$

The derivative of S , with respect to the energy W , is set equal to zero:

$$\frac{\partial S}{\partial W} = \frac{2}{n} \sum_{i=1}^n (W - W_i) = 0 .$$

This equation gives the best value for W :

$$W = \frac{1}{n} \sum_{i=1}^n W_i . \quad (77)$$

It may be preferable to minimize the sum of the squares of the deviations of the logarithms rather than of the linear values of W , since the errors may be multiplicative rather than additive. To accomplish this, the W values in Eq. (76) are replaced by the logarithms of the W values, leading to the following result:

$$\log W = \frac{1}{n} \sum_{i=1}^n \log W_i . \quad (78)$$

This analysis assumes equal weighting for each point. Non-uniform weighting can be achieved either by applying a specific weight value to each point or by controlling the density of these points along the (t, r) curve.

When the R_0 correction is unknown, this method cannot be used because a value of W_i can no longer be obtained from each (t_i, r_i) pair. Goldwire¹³ presented a solution for the case in which both W and R_0 are unknown. He expressed the (t, r) and W relations of Eq. (74) as follows:

$$R_i = aW^{(1-\beta)}(t_i + t_0)^\beta - R_0 . \quad (79)$$

He rewrote this equation in the form

$$R(t) = cx_1(t) - dx_2(t) ,$$

where

$$c = aW^{(1-\beta)/3} ,$$

$$d = -R_0 ,$$

$$x_1(t) = (t_1 + t_0)^\beta ,$$

$$x_2(t) = 1 .$$

He formed the normal equations, and obtained solutions for $aW^{(1-\beta)/3}$ and for R_0 . By this procedure, he minimized the variance in R_0 and in $W^{(1-\beta)/3}$, which differs from the case where R_0 is already known and the variance in W is minimized.

Methods used in the past when W , R_0 , and t_0 are all unknown have involved minimizing $\Sigma(W-W_i)^2$, essentially by iteratively varying R_0 and t_0 . In some cases these iterative procedures do not converge to reasonable results.

METHODS OF ENERGY DETERMINATION WITH THE DERIVED (t,r) FORMULA

KNOWN R_0 AND t_0 , UNKNOWN W

For convenience, the derived curve for an energy of 1 kt is considered. For some greater value of energy of W kt, the resulting $(\log t, \log r)$ curve is displaced a distance of $\log(W^{1/3})$ both upward and to the right. This corresponds to multiplying the t and r values by $W^{1/3}$, as required by the scaling method used here. Conversely, the energy of an underground explosion greater than 1 kt can be found by determining the distance that the experimental $(\log t, \log r)$ curve must be translated downward and to the left in order to obtain the best fit with the derived curve.

Computation of this diagonal motion can be achieved by establishing two new variables: t/r and tr . The plot of the logarithms of these variables will be of the form $[(\log t - \log r), (\log t + \log r)]$. Application of a scaling factor $W^{1/3}$ to both the distance and the time values of each experimental point then moves it a distance of $2 \log W^{1/3}$ in the vertical direction on the $[\log(t/r), \log tr]$ plot without motion in the horizontal direction.

To determine the best value for $\log W$, the variance in the values of $\log W_i$ is minimized. If the vertical distance between the remapped experimental point i and the remapped reference line is indicated by c_i , then, because $c_i = 2 \log W_i^{1/3} = (2/3) \log W_i$, finding the best value for $\log W$ is equivalent to finding the best value for the distance c . The expressions for the deviation and for the average value of the squares of the deviations are

$$c_i = \log(t_i r_i) - \log(T_i R_i) - c, \quad (80)$$

$$S = \frac{1}{n} \sum_{i=1}^n c_i^2. \quad (81)$$

The capital letters $(T_i R_i)$ refer to the remapped reference curve and the lower-case letters $(t_i r_i)$ refer to the remapped experimental data. The variance is minimized by setting the derivative equal to zero:

$$\frac{\partial S}{\partial c} = 0.$$

This gives the result

$$c = \frac{1}{n} \sum_{i=1}^n [\log(t_i r_i) - \log(T_i R_i)]. \quad (82)$$

Because $c = 2 \log W^{1/3}$, the scaling factor $W^{1/3}$ is

$$W^{1/3} = \text{antilog}(c/2) . \quad (83)$$

and the energy input is

$$W = \text{antilog}(3c/2) . \quad (84)$$

The variance in the value for c is calculated from Eq. (81), and the probable error in W is found as a multiplying factor. These statistical methods must be applied with caution, because the accuracy is usually limited by systematic deviations rather than random ones. As a result, the accuracy is generally not improved by introducing additional points.

KNOWN t_0 , UNKNOWN W AND R_0

When W and R_0 are unknown, a different procedure must be used to determine these two quantities. Goldwire was able to express a yield function and a time function as multiplying quantities in the LASL formulation, but this is not possible with our derived curve.

A method to solve for W in this case consists of differentiating the (t,r) data to obtain the shock velocity as a function of time. This eliminates the distance variable, thus leaving the energy W as the only unknown. Because the velocity remains constant with respect to this scaling, the scale factor affects only the time variable in the (t,v) relation. This scale factor is found by translating the $(\log t, \log v)$ curve horizontally until it gives the best fit to the reference $(\log T, \log V)$ curve.

Because finding the velocity involves differentiation, which accentuates irregularities, smoothing procedures should be used to prevent excessive noise in the velocity curve.

The $(\text{Distance}, \text{Velocity})$ reference curve is obtained from Eq. (16), where the values of a and g are chosen to fit the medium. The $(\text{Distance}, \text{Time})$ curve is obtained from the numerical integration of Eq. (18). The distance variable is then eliminated numerically from these (R,V) and (R,T) sets of data, giving the desired (T,V) table.

The lateral translation distance that gives the best fit between the experimental $(\log t, \log v)$ curve and the reference $(\log T, \log V)$ curve equals $\log W^{1/3}$. The deviation σ_i for an individual point can be expressed as

$$\sigma_i = \log t_i - \log T_i - \log W^{1/3} . \quad (85)$$

The variance is given by the relation

$$S = \frac{1}{n} \sum_{i=1}^n (\log t_i - \log T_i - \log W^{1/3})^2 . \quad (86)$$

This variance is minimized by differentiating Eq. (86) with respect to $\log W^{1/3}$ and setting the result to zero. The resulting expression for $\log W^{1/3}$ is then

$$\log W^{1/3} = \frac{1}{n} \sum_{i=1}^n (\log t_i - \log T_i) . \quad (87)$$

Once W has been calculated, only R_0 is unknown. The value for R_0 can be determined either by fitting the (t,r) curve to the reference (T,R) curve, or by fitting the (r,v) curve to the reference (R,V) curve. The second method is presented here.

Because R_0 is an additive quantity, this fit of the (r,v) curve to the reference curve should be done in linear form. The (R,V) curve was constructed for an energy of 1 kt; for this energy of W kt, the quantity R should be changed to $RW^{1/3}$. The velocity V is independent of W , so that the new reference curve is $(RW^{1/3}, V)$. The fitting procedure then consists of translating the (r,v) curve horizontally to give the best match to the $(RW^{1/3}, V)$ curve. The minimization procedure gives the result

$$R_0 = \frac{1}{n} \sum_{i=1}^n (r_i - R_1 W^{1/3}) . \quad (88)$$

In principle, this procedure gives the best values for $W^{1/3}$ and for R_0 . This procedure has not been shown to be mathematically rigorous, but appears valid for all reasonable sets of slifer data. It is prudent, however, to plot the $(t + t_0, r + R_0)$ curve and to check it against the reference $(W^{1/3}T, W^{1/3}R)$ curve.

If the two quantities W and t_0 rather than W and R_0 are unknown, the procedure presented here can be applied almost unchanged except that the roles of r and t are reversed. The shock velocity is expressed as a function of distance and a first determination of W is made. The value for t_0 is then determined by a linear transposition of the (t, v) curve to obtain the best match to the $(TW^{1/3}, V)$ curve.

UNKNOWN W , R_0 , AND t_0

If all three quantities W , R_0 , and t_0 are unknown, a more elaborate (and less accurate) method must be used to solve for these terms. As before, the shock velocity can be determined by differentiating the (t, r) curve with respect to time:

$$v = \frac{\partial r}{\partial(t + t_0)} . \quad (89)$$

In Eq. (89), the presence of the unknown constant t_0 term does not affect the result. A second differentiation can then be made:

$$\frac{\partial v}{\partial t} = \frac{\partial^2 r}{\partial(t + t_0)^2} . \quad (90)$$

Elimination of this time variable by combining Eqs. (89) and (90) gives the $(v, \partial v / \partial t)$ curve.

The velocity incorporates no scaling factor, but the term $\partial v / \partial t$ contains a scaling factor of $W^{-1/3}$. Consequently, the experimental data can be processed to this pair of variables and by varying the value of the scaling factor $W^{-1/3}$, can be matched to the reference curve (also expressed in this set of variables). It is desirable here to match the curves in terms of $\log(-\partial v / \partial t)$ rather than $\partial v / \partial t$ since the scaling factor $W^{-1/3}$ appears as a multiplying factor. The negative sign is used because the velocity ordinarily decreases with time. If, over some intervals, the numerical data lead to increasing velocities with time, these data must be smoothed out so that the velocity decreases with time over the entire data span.

On the plot of the curves $[\log v, \log(-\partial v / \partial t)]$ and $[\log V, \log(-\partial V / \partial T)]$, the curve for the experimental data should lie below the reference curve if the energy exceeds 1 kt. The distance of displacement is $\log W^{1/3}$, and the minimization procedure gives the result

$$\log W^{1/3} = \frac{1}{n} \sum_{i=1}^n \left[\log \left(-\frac{\partial V}{\partial T} \right)_i - \log \left(-\frac{\partial v}{\partial t} \right)_i \right] . \quad (91)$$

Once the energy W has been determined, the term R_0 can be found by matching the $[(r_1 + R_0), v_1]$ plot to the $(W^{1/3}R_1, V_1)$ plot by horizontal translation. The result is, as before,

$$R_0 = \frac{1}{n} \sum_{i=1}^n (r_i - R_1 W^{1/3}) .$$

In a similar manner, t_0 is found from the $[(t_i + t_0), v_i]$ and $(W^{1/3}T_i, V_i)$ curves. The result is

$$t_0 = \frac{1}{n} \sum_{i=1}^n (t_i - T_i W^{1/3}) .$$

In principle, this procedure solves the problem, but in practice, because differentiation is involved, the data may not be good enough to permit a realistic $(\partial r / \partial t, \partial^2 r / \partial t^2)$ curve to be obtained. An alternate iterative method, in which the difficulties are less obvious, can be used, but it may possibly lead to no real gain of accuracy.

UNKNOWN W, R₀, AND t₀; ALTERNATE METHOD

As before, if the three unknowns are W, R₀, and t₀, either R₀ or t₀ can be eliminated by differentiating with respect to time to obtain the shock velocity. By assuming that time has been eliminated, the remaining curve is of the form $(r + R_0, v)$. Because the distance scales as $W^{1/3}$ while velocity remains unchanged, the experimental curve $(r + R_0, v)$ is to be matched to the reference curve $(W^{1/3}R, V)$. The $r + R_0$ term should be fitted linearly while the $W^{1/3}$ term should be fitted logarithmically. On the linear scale, the deviation δ_i in the horizontal distance is

$$\delta_i = (r_i + R_0) - R_i W^{1/3} \quad (92)$$

and the variance is

$$S_\delta = \frac{1}{n} \sum_{i=1}^n [(r_i + R_0) - R_i W^{1/3}]^2 . \quad (93)$$

Differentiating Eq. (93) with respect to R₀ and setting the result equal to zero leads to the relation

$$R_0 = \frac{1}{n} \sum_{i=1}^n (r_i - W^{1/3} R_i) . \quad (94)$$

On a logarithmic plot, the deviation can be expressed as

$$\epsilon_i = \log (r_i + R_0) - \log (R_i W^{1/3}) ; \quad (95)$$

the variance is

$$S_\epsilon = \frac{1}{n} \sum_{i=1}^n [\log (r_i + R_0) - \log (R_i W^{1/3})]^2 . \quad (96)$$

Differentiating Eq. (94) with respect to $W^{1/3}$ or $\log W^{1/3}$ and setting the result equal to zero gives the result

$$\log W^{1/3} = \frac{1}{n} \sum_{i=1}^n \log \left(\frac{r_i + R_0}{R_i} \right) . \quad (97)$$

In order to solve this problem of unknown W, t₀, and R₀, one would solve Eqs. (94) and (97) iteratively for R₀ and $\log W^{1/3}$. When the successive values for $W^{1/3}$ and for R₀ converge to limiting values, the problem would presumably be solved, but it has neither been shown that the values converge nor that they converge to the correct limits for all sets of data.

EXAMPLE OF ENERGY DETERMINATION

PROBLEM DATA

Goldwire presented a set of slifer data consisting of 150 (t,r) points that was used by LASL and by SLA as a test problem.¹⁴ He added time and distance displacements to the t and r values so that these data could be used for energy determination in the case where W, R₀, and t₀ are unknown. He also supplied the actual values for W, R₀, and t₀ so that, for example, the t₀ value could be added to the t_i values, giving a problem where only W and R₀ are considered unknown. Both R₀ and t₀ values could be reintroduced to give the problem where only W is unknown. Using the derived (t,r) relation, we solve these three cases using the methods described in the preceding sections.

Goldwire supplied the following values:

$$W = 54.2 \text{ kt.}$$

$$R_0 = 4.60 \text{ m.}$$

$$t_0 = 0.061 \text{ ms.}$$

The (t,r) data are given in Table 2.

TABLE 2. Slifer test problem.

Time, ms	Distance, m	Time, ms	Distance, m	Time, ms	Distance, m	Time, ms	Distance, m	Time, ms	Distance, m	Time, ms	Distance, m
0.00	0.000	1.04	8.704	2.04	13.602	3.04	17.834	4.04	21.628	5.04	25.127
0.04	0.643	1.08	8.934	2.08	13.770	3.08	17.981	4.08	21.745	5.08	25.264
0.08	1.296	1.12	8.112	2.12	13.958	3.12	18.118	4.12	21.902	5.12	25.441
0.12	2.076	1.16	9.300	2.16	14.136	3.16	18.294	4.16	22.068	5.16	25.587
0.16	2.818	1.20	9.567	2.20	14.284	3.20	18.431	4.20	22.234	5.20	25.704
0.20	2.878	1.24	9.804	2.24	14.452	3.24	18.578	4.24	22.371	5.24	25.851
0.24	3.016	1.28	9.952	2.28	14.610	3.28	18.714	4.28	22.488	5.28	25.978
0.28	3.184	1.32	10.160	2.32	14.828	3.32	18.910	4.32	22.624	5.32	26.114
0.32	3.639	1.36	10.328	2.36	14.976	3.36	19.076	4.36	22.772	5.36	26.252
0.40	4.620	1.40	10.526	2.40	15.135	3.40	19.213	4.40	22.918	5.40	26.389
0.44	5.026	1.44	10.704	2.44	15.322	3.44	19.369	4.44	23.065	5.44	26.486
0.48	5.253	1.48	10.902	2.48	15.491	3.48	19.467	4.48	23.211	5.48	26.633
0.52	5.540	1.52	11.120	2.52	15.659	3.52	19.643	4.52	23.358	5.52	26.770
0.56	5.777	1.56	11.327	2.56	15.856	3.56	19.809	4.56	23.514	5.56	26.916
0.60	5.995	1.60	11.525	2.60	16.004	3.60	19.936	4.60	23.661	5.60	27.033
0.64	6.331	1.64	11.693	2.64	16.163	3.64	20.122	4.64	23.759	5.64	27.171
0.68	6.668	1.68	11.881	2.68	16.341	3.68	20.249	4.68	23.876	5.68	27.288
0.72	6.954	1.72	12.079	2.72	16.539	3.72	20.405	4.72	24.062	5.72	27.396
0.76	7.192	1.76	12.296	2.76	16.707	3.76	20.601	4.76	24.199	5.76	27.571
0.80	7.439	1.80	12.474	2.80	16.875	3.80	20.728	4.80	24.345	5.80	27.689
0.84	7.666	1.84	12.652	2.84	17.013	3.84	20.894	4.84	24.482	5.84	27.816
0.88	7.874	1.88	12.860	2.88	17.192	3.88	21.021	4.88	24.619	5.88	27.943
0.92	8.131	1.92	13.018	2.92	17.350	3.92	21.187	4.92	24.746	5.92	28.080
0.96	8.339	1.96	13.226	2.96	17.528	3.96	21.354	4.96	24.893	5.96	28.207
1.00	8.516	2.00	13.414	3.00	17.659	4.00	21.471	5.00	25.010	6.00	28.344

KNOWN R_0 AND t_0 , UNKNOWN W

The SOCKITTOME code¹⁵ was used to perform the necessary data manipulations (Appendix). Figure 34 presents the original data reported by Goldwire. At early times, a spurious variation was apparent so that we deleted the first nine points. The R_0 and t_0 displacements given by Goldwire were added to the r_i and t_i values to give the true (t, r) curve. From these data, the $[\log(t_i/r_i), \log(t_i/r_i)]$ curve was constructed.

Because the data indicated a moderately large energy input, we assumed that the shock transmission medium was saturated tuff since the larger events at the Nevada Test Site are usually emplaced below the water table. The derived (T, R) curve for saturated tuff was therefore also expressed in the form $[\log(T/R), \log(TR)]$. The y-coordinate values of this curve were subtracted from the corresponding values of the $[\log(t_i/r_i), \log(t_i/r_i)]$ curve at the points i corresponding to the original experimental data points, and this difference curve was averaged over the number of points used. The result was

$$2 \log_e W^{1/3} = 2.8530$$

$$\text{Standard deviation} = 0.029527.$$

Solving for W gives $W = 72.2 \pm 3\%$ kt. The calculated energy value was higher than the actual value of 54.2 kt, probably because the constants used for the reference curve were not exact for the actual medium. The error value of $\pm 3\%$ refers only to the accuracy of fit of the experimental data to the reference curve in logarithmic and linear form (Figs. 35 and 36, respectively).

The same calculation without deletion of the first nine points resulted in an energy of 73.46 kt with an error in fit of 7.6%. This energy value very closely matched the value 72.2 kt calculated without these early points. The resulting (t, r) curves are shown in Figs. 37 and 38.

FIG. 34. Experimental (t, r) data from test problem supplied by Goldwire.

FIG. 35. Fit of (t, r) data from test problem to 72.2-kt wet-tuff curve (logarithmic plot); irregular data points have been deleted from the calculation.

FIG. 36. Fit of (t,r) data from test problem to 72.2-kt wet-tuff curve (linear plot); irregular data points have been deleted from the calculation.

FIG. 37. Fit of complete (t,r) data from test problem to 73.46-kt wet-tuff curve (logarithmic plot).

FIG. 38. Fit of complete (t,r) data from test problem to 73.46-kt wet-tuff curve (linear plot).

KNOWN t_0 , UNKNOWN W AND R_0

To solve this problem, we used the (t_i, r_i) file, deleting the first eight points and applying the $t_0 = 0.061$ correction to the remaining t_i points. The velocity was calculated by numerical differentiation, and the results were expressed in the form $(\log v_i, \log t_i)$; the derived curve for wet tuff was written in the similar form $(\log V, \log T)$. These curves are shown in Fig. 39. The y-values of the derived curve were subtracted from the y-values of the (v_i, t_i) curve at the i points corresponding to the original corrected-time values. Because the differentiation operation averaged over groups of points, the number of points was actually less than the original number and velocities were not determined near the ends of the (t, r) curves. The curve resulting from the subtraction was of the form $(\log v, \log W^{1/3})$ shown in Fig. 40; averaging the y-values of this curve over the i values gave

$$\log_e W^{1/3} = 1.4907,$$

$$\text{Standard deviation} = 0.0505.$$

FIG. 39. Comparison of $(\log v, \log t)$ curve from test problem to $(\log V, \log T)$ wet-tuff reference curve with t_0 given as 0.061 ms, W and R_0 unknown, and velocity in m/ms.

Solving for W gives $87.54 \pm 5.2\%$ kt. The error is greater here than in the calculation where R_0 was known. As before, the error value refers only to the closeness of fit between the experimental and reference curves.

Further calculations were made to obtain R_0 using $W = 87.54$. The distance values of the derived (V, R) curve were multiplied by $W^{1/3}$ and subtracted from the corresponding r_1 values of the experimental (v_1, r_1) curve in linear form (Fig. 41) to give

$$R_0 = 5.36 \pm 2.09 \text{ m.}$$

Figures 42 and 43 compare the (t_1, r_1) curves, calculated using the corrections $t_0 = 0.061$ and $R_0 = 5.36$, with the derived $(TW^{1/3}, RW^{1/3})$ curves for $W = 87.54$ in logarithmic and linear form, respectively. The fits appear to be very good. This agreement implies that the values used for a and g were good, but that some of the constants relating W to g in Eq. (14) were incorrect. Errors could be present in the values used for b , ρ_0 , Γ , or f .

UNKNOWN W , R_0 , AND t_0

To solve this problem, we used, as before, the abridged set of (t, r) data, but no R_0 or t_0 corrections could be applied. The data set was differentiated numerically to give the (t, v) curve and the resulting data set differentiated again to give the $[t, (\partial v / \partial t)]$ curve. Elimination of the time variable gave the $[v, (\partial v / \partial t)]$ curve. Because velocity decreases with time, the negative value of $(\partial v / \partial t)$ was used to form the $[\log v, \log (-\partial v / \partial t)]$ curve. A similar procedure was used on the derived (T, R) curve for wet tuff to give the $[\log V, \log (-\partial V / \partial T)]$ curve. These curves are shown in Fig. 44. Subtraction of $\log (-\partial v / \partial t)$ from $\log (-\partial V / \partial T)$ at corresponding values of v gave the $(\log v, \log W^{1/3})$ curve, shown in Fig. 45. The averaged value of the y -coordinates gave

$$\log W^{1/3} = 1.5786,$$

$$\text{Standard deviation} = 0.10799.$$

Solving for W gives $W = 113.95 \pm 11\%$ kt. This calculated energy considerably exceeds the observed value of 54.2 kt.

The values for R_0 and t_0 , determined by comparing the (v, r) and $(V, RW^{1/3})$ curves and the (v, t) and $(V, TW^{1/3})$ curves, respectively, were:

$$R_0 = 7.45 \text{ m,}$$

$$t_0 = 0.334 \text{ ms.}$$

FIG. 40. Curve of $(\log v, \log W^{1/3})$ for test problem with t_0 given as 0.061 ms, W and R_0 unknown, W in kt, and velocity in m/ms.

FIG. 41. Comparison of (v, r) curve for test problem with derived $(V, RW^{1/3})$ curve for wet tuff; t_0 given as 0.061 ms, W calculated as 87.54 kt, and R_0 unknown.

FIG. 42. Fit of complete corrected (t, r) data from test problem with derived $(TW^{1/3}, RW^{1/3})$ curve for wet tuff; t_0 given as 0.061 ms, W calculated to be 87.54 kt, velocity in m/ms, and R_0 calculated to be 5.36 m (logarithmic plot).

FIG. 43. Fit of complete corrected (t, r) data from test problem with derived $(TW^{1/3}, RW^{1/3})$ curve for wet tuff; t_0 given as 0.061 ms, W calculated to be 87.54 kt, and R_0 calculated to be 5.36 m (linear plot).

These results do not agree well with the given values $R_0 = 4.6$ m and $t_0 = 0.061$ ms; these results are much better, however, than those obtainable with the use of the LASL (t, r) formula.

Figures 46 and 47 compare the (t, r) data, calculated using these time and distance corrections, with the derived (T, R) curve, calculated using the $W^{1/3}$ multiplier for T and R , in logarithmic and linear form, respectively. The fit appears to be quite good, which leads us to conclude that the shape of the (t, r) curve is not a sensitive indicator of the proper values of R_0 and t_0 .

FIG. 44. Comparison of $[\log v, \log (-\partial v/\partial t)]$ for test problems with derived $[\log V, \log (-\partial V/\partial T)]$ curve for wet tuff. The difference curve used to determine energy is also shown. Time is in ms, and velocity is in m/ms.

FIG. 45. Curve of $(\log v, \log W^{1/3})$ for test problem; W , R_0 and t_0 are unknown, W is in kt, and velocity is in m/ms.

FIG. 46. Fit of corrected (t,r) data from test problem with the derived $(TW^{1/3}, RW^{1/3})$ curve for wet tuff; W found to be 113.95 kt, t_0 found to be 0.334 ms, and R_0 found to be 7.45 m (logarithmic plot).

FIG. 47. Fit of corrected (t,r) data from test problem with the derived $(TW^{1/3}, RW^{1/3})$ curve for wet tuff; W found to be 113.95 kt, t_0 found to be 0.334 ms, and R_0 found to be 7.45 m (linear plot).

DISCUSSION

The LASL (t,r) relation has been used extensively for energy determination and has provided good results for many underground detonations. Because it is based solely on empirical data, however, it cannot be used at times and distances beyond the hydrodynamic range.

The BOTE relation is based on some physical concepts, but is unsatisfactory in that the hydrodynamic and sonic effects have not been properly combined in a single equation. The additional variable in this

equation permits fitting to a curved line in the logarithmic plot so that some slifer data can be fitted closely by the BOTE curve using somewhat arbitrary BOTE parameters. Kurtz² and others have attempted to use this BOTE formulation for energy determination.

The derived (t,r) formula described here has not yet been used for general energy determination, but it appears to have good potential because it is directly related to the properties of the medium; in many cases, use of reasonable values has led to very good fits of the calculated results to the slifer data. Further investigation should be made on events in porous media of varying saturation to improve the accuracy and applicability of this formula.

Fair accuracy in energy determination is attainable when both R_0 and t_0 are known. When R_0 is unknown, the accuracy and reliability of the energy determination decrease considerably. When both R_0 and t_0 are unknown, the energy determination becomes quite uncertain, and only when good (t,r) data are available (as in the example considered in this report) can a reasonable determination be made. For poorer data, the random deviations and perturbations from unknown causes probably exceed the ideal values for the second derivative of r with respect to t, making the energy determinations very unreliable. The effects of increases of R_0 on the linear (t,r) plot can be almost cancelled by a corresponding change in t_0 , and the curvature of the (t,r) line is too slight to give a definite indication of the proper values for R_0 and t_0 .

The energy determinations can be made more reliable by extending the measurements to greater distances and longer times. This larger coverage furnishes better curves for the first and second derivatives, and the effect of term a becomes more pronounced.

Other methods of analysis of this problem based on the LASL (t,r) formula have been used by LASL and by SLA, but these approaches have not given reliable results when W, R_0 and t_0 are unknown. Using these methods, a section of the (t,r) curve that appears to be linear on the logarithmic plot is selected to represent the hydrodynamic region. This section is analyzed by varying W, R_0 , and t_0 iteratively until the variation in the values for W is minimized. For some selections of the (t,r) interval, the iterative procedures do not converge to reasonable values. A difficulty encountered using these procedures is that when there is an offset in t or R, curvature appears in the line representing the hydrodynamic region. It may be that the LASL and SLA iterative procedures are valid, but that the LASL (t,r) curve does not describe the actual (t,r) relation properly.

CONCLUSIONS

Although the LASL (t,r) empirical relation has given good results for routine energy determinations, this success was somewhat fortuitous in that the envelopes of the (t,r) curves for the media studied tended to follow a straight line on the logarithmic plot. A methodical study showed that there was a dependence of the (t,r) data on the medium that could not be accommodated by the simple LASL (t,r) description.

The derived formula does give (t,r) curves which fit the experimental data closely in some cases for wet and dry tuff. Further work should be performed to correlate the values for the parameters in the equation with the properties of the shock transmission medium, particularly with respect to the degree of water saturation and the total porosity.

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REFERENCES

1. Heusinkveld, M. and F. Holzer, "Method of Continuous Shock Front Position Measurement," *Rev. Sci. Instr.* **35**, 1105 (1964).
2. Taylor, G. I., "The Formation of a Blast Wave by a Very Intense Explosion, II. The Atomic Explosion of 1945," *Proc. Roy. Soc. (London), Ser. A* **201**, 175 (1950).
3. Mutschlechner, J. P. and Saunders, B. R., "Hydrodynamic Measurements of Yields of Underground Tests During Niblick, Whetstone, and Flintlock," LA-3971 (1968)(title U, report SRD).
4. Rambo, J. T., "Slifers Revisited: A Method for Determining Yields Independent of Radiochemical Measurements," Lawrence Livermore Laboratory, Livermore, Calif., UCID-17292 (1976).
5. Kurtz, S. R., "A Back-of-the-Envelope Approach to Predicting Ground Motion Phenomena," *J. Geophys. Res.* **80**, 4449 (1975).
6. Chernyi, G. D., "The Problem of a Point Explosion," *Dokl. Akad. Nauk. SSR* **112**, 213 (1957). Transl. as Rep. No. MOA TII. T.4871, Ministry of Aviation (Great Britain), 1959.
7. Rice, M. H., R. G. McQueen, and J. M. Walsh, "Compression of Solids by Strong Shock Waves," in *Solid State Physics, Vol. 6* (Academic Press Inc., New York, 1958).
8. Zel'dovich, Ya. B. and Yer. P. Raizer, *Physics of Shock Waves and High Temperature Hydrodynamic Phenomena* (Academic Press, New York, 1966). Vol. 1, pp. 93-106.
9. Perret, W. R. and R. C. Bass, "Free-Field Ground Motion Induced by Underground Explosions," SAND 74-0252, Sandia Corporation, Albuquerque, N. Mex. (1975).
10. Knox, R. J., "Utility Routine CURVE," Lawrence Livermore Laboratory, Livermore, Calif., UR-505 (1973).
11. van Thiel, M., editor, "Compendium of Shock Wave Data," Lawrence Livermore Laboratory, Livermore, Calif., UCRL-50108, Vol. 3 (1977).
12. Bass, R. C., "Additional Hugoniot Data for Geologic Materials," SC-RR-66-548, Sandia Corporation, Albuquerque, N. Mex. (1966).
13. Goldwire, H. C., "Solving the Strong Shock Algorithm for Yield and Origin-Shift," LASL Office memo, J15-OU-77-68, April 7, 1977.
14. Goldwire, H. C., "Notes on Data Sets Prepared for John Bannister, SIA," LASL Office memo, J15-OU-78-27 to D. D. Eilers, January 31, 1978.
15. Derby, W. B., M. E. Hummel, E. C. Lee, and R. D. Neifert, "SOCKITTOME, An Interactive Data Processing Code," Lawrence Livermore Laboratory, Livermore, Calif., UCID-17733 (1977).

APPENDIX

Most of the data manipulations discussed in this report were performed using the SOCKITTOME program developed by members of L-Division at LLL. The material in this appendix refers primarily to this program. Because erroneous results can be obtained when improper calculational methods are used, the individual steps used in the SOCKITTOME subprograms are described here.

The data from many slifer measurements have been stored in the SOCKITTOME format by members of L-Division. These records were inspected both on the TMDS and RJET output with the aid of the PLOTIT routine of SOCKITTOME, and all noisy or otherwise suspect portions of the individual records were deleted with UNOP. All resulting signals for each event were superimposed using PLOTIT, and the composite signal was inspected for internal consistency. If the individual signals did not overlay properly, additional deletions were made to the individual signals until the composite signal appeared to be the optimum obtainable for that event. All purified signals for the event were then joined into a single file using BINOP and given a name associated with the event.

The same number of points per logarithmic interval was maintained for most of these records so that subsequent operations dealing with these events would be uniform. This spacing was produced using UNOP to calculate the logarithms, EVENSAPCE to give the uniform spacing, and UNOP to convert the data to the linear form again. A spacing of $\Delta = 0.02$ (which gives 50 points per e-folding range) was generally used.

The spacing of these points is used in the derivative procedure, and it also forms the weight factor in energy determination. Linear spacing could be used instead of logarithmic spacing if desired. In the sample problem given by Goldwire and presented in the section on energy determination, the actual data points, spaced linearly in time, were carried through the calculations.

To determine the energy W from the t and r values (when the time and distances have been corrected to the true values), it is necessary to establish a table of the values $[(\log t_1 - \log r_1), (\log t_1 + \log r_1)]$. Table 3 lists the set of operations used to calculate these values.

The reference curve, e.g., TTRWT [Theoretical (T,R) curve for Wet Tuff, Fig. 1], is converted to the same form to give the file TT-R\T+RL. [In this discussion, the data file titles are descriptive; a title A\B indicates that the A quantity was the value for the abscissa and the B quantity was the value for the ordinate, which corresponds to the (A,B) convention used in this report. Because commas may not be used as parts of file titles in this computer work, a backward slash (\) has been used. An L designates a logarithmic scale.] This converted reference file is subtracted from the file DT-R\T+RL to give the file DT-R\T+C. In the latter operation, the experimental data file DT-R\T+RL is treated as the foreground file and TT-R\T+RL as the background file so that the identities of the original data points are preserved and the proper weighting is maintained.

This procedure is followed by a simple UNOP AVERAGE operation which computes both the numerical average of the y-coordinate values of C_i over i data points and the standard deviation. This average is equal to $2 \log W^{1/3}$.

When R_0 is unknown, the original (T,R) data must be differentiated with the DERIVATIVE subprogram to calculate W . In most of our work, it was necessary to use 40 points in the Legendre polynomial curve-fitting procedure to minimize the irregular variations that did not seem to be a reasonable part of the velocity curve. Using the sequence of operations listed in Table 4, the data were put into $(\log v, \log t)$ form.

It was necessary to complete the second and third steps in Table 4 to insure that the velocity values corresponded to the original times of the (t,r) data. Because the DTT file covered a longer time span than the DTV file (due to the truncation that occurred in the DERIVATIVE program) the UNOP code was used to delete the artificial values (generated by the DERIVATIVE program) from the ends of the DTV curve.

The corresponding derivative of the reference curve, e.g., TTRWT, is obtained, but manipulation of the spacing of the points with the PARAMELIM steps is not necessary. The output file from this step, designated TVTLWT, is subtracted from DVTL with BINOP; DVTL serves as the foreground file. The resulting file is DVLOGW1/3, and this is averaged with UNOP to give both the value and standard deviation of $\log W^{1/3}$.

When W , R_0 , and t_0 are all unknown, then the derivative of the velocity is obtained by two successive operations of DERIVATIVE on data file TRDATA to give file DT\DV DT. The PARAMELIM subprogram compares the latter file with the file DTV to give the file DV\DV VT. Because DV DT is

TABLE 3. SOCKITTOME procedure to obtain $[\log(t/r), \log tr]$.

Subprogram	Input files	Operation	Output file
UNOP	TRDATA	Log x, log y	DTRL
PARAMELIM	DTRL DTRL	Eliminate y y	DTTL
BINOP	DTTL DTRL	SUBTRACT	DT\T-RL
BINOP	DTTL DTRL	ADD	DT\T+RL
PARAMELIM	DT\T-RL DT\T+RL	Eliminate x x	DT-R\T+RL

TABLE 4. SOCKITTOME procedure to obtain $(\log v, \log t)$.

Subprogram	Input files	Operation	Output file
DERIVATIVE	TRDATA	P 40 1 .5	DTV
PARAMELIM	TRDATA TRDATA	Eliminate y y	DTT
PARAMELIM	DTT DTV	Eliminate x x	DTVI
UNOP	DTVI	Swap, order, delete	DVT
UNOP	DVT	Log x, log y	DVTL

ordinarily negative, it is multiplied by minus one with UNOP to give positive values, and the output file DV\ -DV/DT from this step is converted to logarithmic form. The data points are correlated with the original (t_i, r_i) points as before.

A similar procedure is carried through on the reference file TTRWT, and the difference in y-values is found with BINOP and averaged with UNOP as before. The result of subtracting TV\ -DV/DTL from DV\ -DV/DTL is $-\log W^{1/3}$, since the scale factor is associated directly with t and inversely with $\partial v/\partial t$.

The SOCKITTOME procedures used to solve the following operations are similar to those already described. To obtain the logarithmic form, $\partial v/\partial t$ must be a negative value for all data points and sufficient smoothing must be used to insure this result. Once W has been determined, the values of R_0 and t_0 are obtained by subtraction in a linear rather than a logarithmic form.