1522

14. 2303

FEBRUARY 1979

PPPL-1522
UC-20g

MASTER

ON A RENORMALIZED WEAK PLASMA
TURBULENCE THEORY

BY

J. A. KROMMES AND R. G. KLEVA

**PLASMA PHYSICS
LABORATORY**

**PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY**

This work was supported by the U. S. Department of Energy
Contract No. EY-76-C-02-3073. Reproduction, translation,
publication, use and disposal, in whole or in part, by or
for the United States Government is permitted.

On a Renormalized Weak Plasma Turbulence Theory

John A. Krummes and Robert W. Fernald

Plasma Physics Laboratory, Princeton University

Princeton, New Jersey 08540

A renormalized Vlasov turbulence theory, derived by neglecting a mode coupling term in the Direct Interaction Approximation, is discussed. The theory reduces correctly to weak turbulence theory; it predicts ion-ion diffusion and polarization effects of the turbulent medium on test particles, as well as the inverse scattering of test particles in the medium. A heuristic, physical algorithm is presented for constructing the equations. The theory of the renormalized dielectric function is reviewed.

NOTICE

the repair was provided as a service to the community. The company's employees were not paid for their work. The company's employees were not paid for their work. The company's employees were not paid for their work.

I. Introduction

There have been many attempts to formulate an appropriate theory of turbulence in fully ionized plasma. We are concerned in this paper with certain aspects of an approximation for Vlasov turbulence based on the so-called Direct Interaction Approximation (DIA). The DIA has been derived in many places¹⁻⁴ and we shall assume that it is well-known to the reader. However, it is useful to provide first a brief discussion of some of the features of the earlier techniques and approximations. We devote Sec. I to this purpose.

The earliest approaches to the turbulence question employed the not unreasonable technique of expanding fluctuations δf of the particle distribution function f as a power series in the fluctuating electromagnetic fields. The small expansion parameter is nominally $\epsilon = \sqrt{\delta}$, where for electrostatic turbulence $\delta \equiv \langle E^2 \rangle / 8\pi nT$. The well-known quasilinear theory (QLT)⁵⁻⁶ truncates the expansion at first order, thus treating the fluctuations as linear. The associated background (statistical mean) distribution function $\langle f \rangle$ consists of two parts:⁷ a resonant part which evolves in time according to a diffusion equation with rate coefficient proportional to the fluctuation intensity, and a nonresonant part which describes the "sloshing" motion of the particles in the waves. In the usual weak turbulence theory (WTT)⁸ the fluctuations are expanded through third order and the fluctuation spectrum is constructed to fourth order by using the random phase approximation (RPA). The associated equation for the evolution of the plasmon density N_k contains the

linear growth (or damping) term due to the Landau resonances $[\omega(\underline{k}) = \underline{k} \cdot \underline{v}]$, the induced scattering and nonlinear Landau damping terms $[\omega(\underline{k}_1) - \omega(\underline{k}_2) = (\underline{k}_1 - \underline{k}_2) \cdot \underline{v}]$ and three-wave interaction terms $[\omega(\underline{k}_3) = \omega(\underline{k}_1) + \omega(\underline{k}_2), \underline{k}_3 = \underline{k}_1 + \underline{k}_2]$. Such equations have been used successfully in describing the evolution and saturation of a number of important plasma instabilities.⁹

In spite of the successes and clear physical interpretation of QLT and WTT, however, it is now recognized that they are deficient in several important respects. QLT is strictly correct only for a continuous spectrum. When the spectrum is discrete, either because of spatial quantization or because the system is immersed in a magnetic field, the steady-state quasilinear diffusion operator involves formally a series of delta functions, at each of the wave-particle resonances. This behavior arises because the diffusion coefficient is the time integral of a time-displaced force correlation function which in linear theory does not decay asymptotically to zero but rather recurs on a time scale $\tau_r \sim [\delta k (\partial \omega / \partial k - v/k)]^{-1}$, where δk is the spacing between modes in k -space. Nonlinearly, however, correlations do decay because of the stochastic instability¹⁰ arising from island overlap,¹¹ which we assume. A proper discussion of this nonlinear decay has been given only recently¹²; we shall not comment further here. For our purposes, the important point is that stochastic instability leads to test particle diffusion in phase space. This diffusion affects both $\langle f \rangle$ and δf . Only the effect on $\langle f \rangle$ is predicted by quasilinear theory.

Diffusion is mathematically a non-analytic process, which cannot be dealt with successfully by finite order truncations of a regular perturbation theory such as WTT. Such approximations are secular. In fact, it is well-known that when there are linearly resonant particles [$\omega(\underline{k}) = \underline{k} \cdot \underline{v}$] in steady state, one term of the Compton scattering amplitude is divergent.¹³ As is well-known,¹⁴ Compton scattering is the process in which a particle absorbs a plasmon $N_{\underline{k}}$, then emits another plasmon $N_{\underline{k}'}$. Classically, emission occurs because the incoming wave $\underline{k}$ jitters the particle about its free orbit; the oscillating particle then radiates the wave $\underline{k}'$. The divergence is associated with the fact that the actual particle orbit is not free, but should more correctly be considered to be the actual, perturbed orbit in the presence of all waves (except the test wave $\underline{k}$). Statistically, such orbit perturbations can be described, in part, by a diffusion process.

Dupree¹⁵ and Galeev¹⁶ were the first to consider corrections due to orbit perturbations. Dupree essentially assumed¹⁷ that each particle was immersed in a white noise bath due to the waves. Since white noise is Gaussian, particles simply diffused away from their unperturbed orbits. This diffusion effected a broadening of the Landau resonances. Alt'shul and Karpman¹⁸ showed that this resonance broadening was a kind of renormalization; i.e., that the effect could be recovered by summing a formal infinite subseries of terms of primitive perturbation theory. We will comment on Galeev's theory in Sec. IV.

The resonance broadening theories were perhaps the single most important advance in plasma turbulence theory. Nevertheless,

they can be criticized.¹⁹ The problem is that by assuming that the particles see white noise one ignores the fact that the particles also participate in the coherent wave motions, so that the motions are not purely Gaussian. The coherent, oscillatory motions in a test wave are responsible for the radiation processes in induced scattering, for example; Dupree's original theory did not describe such processes. This deficiency was recognized by Rudakov and Tsytovich,¹⁹ who provided an improved renormalization which included both resonance broadening and the standard processes of WTT. Later, Tsytovich and co-workers extended their techniques to include the renormalization of the plasmon propagator.²⁰

The Tsytovich technique involves adding an unknown, "effective collision" operator ν_{eff} to both sides of the WTT expansion, then choosing ν_{eff} to cancel secularities to some order in the expansion. The systematology of this approach is not entirely clear (what happens to higher order?) and the method becomes quite tedious when plasmon renormalization is included. Recently, alternative, more physically intuitive and also more powerful methods have been developed for effecting the renormalization.

A fundamental and elegant treatment of classical renormalization was given by Martin, Siggia, and Rose (MSR).² MSR borrowed techniques of quantum field theory²¹ to provide functional relationships between the observables of the nonlinear statistical system. Observables are defined to be statistical averages; they are conventionally taken to be the mean distribution $\langle f \rangle$, the two-point

correlation function $\langle \delta f \delta f \rangle$, a mean response function $\langle \delta f / \delta \hat{\eta} \rangle$ (where $\hat{\eta}$ is a source of probability added to the right-hand side of the Vlasov equation), and a small number of vertex functions Γ_i which describe the three-point correlations in the system. The formally exact²² nonlinear equations relating the observables involve a functional derivative and are thus of infinite order in the vertex functions. Nevertheless, it is esthetically satisfying that it is possible to write such exact equations at all. It is observable quantities which are smooth functions of their arguments and take on definite sizes which can be small; fluctuations can be arbitrarily jagged and large in any given realization. Furthermore, even when the infinite order couplings are reduced to finite order ones by some form of truncation, the equations remain self-consistent for the observables. This is distinct from and superior to, e.g., the standard WTT with the RPA, which determines higher order spectral functions in terms of lower order (unrenormalized) functions and propagators. In the functional approach it is the form of the nonlinear couplings in the equations which is approximated, not the form of the statistical functions themselves.

The natural expansion parameter in the renormalized theory is the so-called skewness parameter (really an operator), which measures the size of the three-point correlations and thus the deviations of the plasma from Gaussianity. If one assumes the skewness to be small everywhere, the functional expansion truncates to the so-called Direct Interaction Approximation (DIA).¹⁻⁴ It

can be shown that DIA is the exact statistical description for certain model problems,³ so DIA is automatically well-behaved. Krommes²³ pointed out that in certain circumstances the DIA can be further expanded in δ and reduced quite simply to a form of WTT. We shall discuss the specifics in Sec. II. Dubois⁴ has set down details of the expansion, which were not given in Krommes' paper.

In the model problems referred to, the nonlinear couplings are randomly phased by construction. In the Vlasov problem of present concern, the nonlinear couplings have constant phase, which implies that specific, non-Gaussian correlations exist to all orders in renormalization. Such phase correlations describe orbit divergence in phase space¹² and the $O(E^4)$ wave-wave and wave-particle interactions. Although each of these effects can be important in certain physical situations, we do not consider them here and shall adopt the DIA as fundamental. It appears that DIA contains, in the proper places, all the nonlinear space and time scales which enter to $O(E^2)$. It describes test particle diffusion. Furthermore, it predicts the self-consistent response of the medium to the diffusing test particles, an important effect necessary for energy conservation but omitted in many simpler theories. We shall be concerned with the nature of this self-consistent response, which is not well understood.

Because the DIA is quite complicated, further analytic simplification is desirable. We shall discuss a natural reduction⁴ which is basically a renormalized version of the procedure

used to extract WTT from the DIA. Although the derivation proceeds straightforwardly, the result is still complex, with many terms. The main goal of the paper is to provide an intuitive basis for these terms. We begin in Sec. II by reviewing the DIA and deriving from it the conventional and renormalized WTT. The main section of the paper is Sec. III, where we give a simple, heuristic procedure for constructing our renormalized approximation by combining certain linearized forms of the usual WTT expansion for fluctuating quantities with simple test particle motions and a random phase approximation. The symmetries of DIA acquire added meaning in this approach and one can understand more clearly what physics is being omitted in simpler approximations. We discuss our results in Sec. IV. In Appendix A we review the theory of the renormalized dielectric function.

II. Direct Interaction Approximation and Renormalized WTT

We will consider electrostatic Vlasov turbulence. (The extension to the general electromagnetic case is straightforward; proper treatment of particle discreteness is subtle.^{22b}) The Vlasov equation for the probability distribution $f_s(\underline{x}, \underline{v}, t)$ of species s at position $\underline{x}$, velocity $\underline{v}$, and time t is then

$$\partial_t f + \underline{v} \cdot \underline{\nabla} f + \underline{E} \cdot \underline{\partial} f = 0, \quad (1)$$

where $\underline{\nabla} \equiv \partial/\partial \underline{x}$, $\underline{\partial} \equiv (q/m) \partial/\partial \underline{v}$, $\underline{E} = -\underline{\nabla} \phi$, and the system is closed

by Poisson's equation:

$$\nabla^2 \phi = -4\pi \sum_s (nq)_s \int dv f_s . \quad (2)$$

Here n_s is the mean density, q_s is the charge, and m_s is the mass of species s ; we omit species labels when no confusion arises. We can rewrite Eq. (1) in the symmetric form

$$g^{(0)}(1,2)^{-1} f(2) = \frac{1}{2} \bar{U}(1,2,3) f(2) f(3) , \quad (3)$$

where an argument "1" denotes the set $\{x_1, v_1, t_1, s_1\}$; later, it will also include a "spin" index, to be defined. We have adopted a summation-integration convention over repeated indices and have defined

$$g^{(0)}(1,2)^{-1} \equiv (\partial_{t_1} + v_1 \cdot \nabla_1) \delta(1-2) , \quad (4)$$

and

$$\bar{U}(1,2,3) \equiv \nabla_1 \phi(1,2) \cdot \partial_1 \delta(1-3) + (2 \leftrightarrow 3) , \quad (5)$$

where ϕ is that operator which inverts Eq. (2):

$$\phi(1) = \phi(1,2) f(2) .$$

When one can ignore finite spatial boundaries, as we shall assume

throughout this paper, then

$$\begin{aligned}\Phi(1,2) &= 4\pi |\underline{x}_1 - \underline{x}_2|^{-1} (nq)_2 \delta(t_1 - t_2) \\ &= \int \frac{d\underline{k} d\omega}{(2\pi)^4} \exp[i\underline{k} \cdot (\underline{x}_1 - \underline{x}_2) - i\omega(t_1 - t_2)] \frac{4\pi}{k^2} (nq)_2.\end{aligned}$$

As usual, an equation for the evolution of the background distribution $\langle f \rangle$ follows by averaging Eq. (3):

$$(g^{(0)})^{-1} \langle f \rangle - \frac{1}{2} \bar{U} \langle f \rangle \langle f \rangle = \frac{1}{2} \bar{U} \langle \delta f \delta f \rangle. \quad (6)$$

Eq. (6) is, in somewhat disguised notation, just the first member of the Vlasov cumulant hierarchy.²⁴ In this paper we shall not be concerned with this equation explicitly, but rather with approximate equations for the correlation function

$$C(1,2) \equiv \langle \delta f(1) \delta f(2) \rangle,$$

which enters into Eq. (6) and also determines the energy spectrum. To this end, it turns out to be useful to introduce the mean response function

$$R(1;2) \equiv \langle \delta f(1) / \delta \hat{n}(2) \rangle |_{\hat{n}=0}, \quad (7)$$

where $\hat{n}$ is an arbitrary (nonrandom) source added to the right hand side of Eqs. (1) or (3). Let us follow MSR and define the

correlation matrix

$$G(1,2) \equiv \begin{bmatrix} C(1,2) & R(1;2) \\ R(2;1) & 0 \end{bmatrix}$$

$$= \langle\langle \psi(1) \psi(2) \rangle\rangle_+ , \quad (8a)$$

where

$$\psi = \begin{bmatrix} \hat{f} \\ \hat{f} \end{bmatrix} \equiv \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} , \quad (8b)$$

$\hat{f}$ is an operator adjoint to f ,^{22a,25} $\langle\langle \cdot \rangle\rangle$ denotes the cumulant average and in Eq. (8a) "+" denotes time ordering. The arguments of G include matrix or spin indices; noting the convention of Eq. (8b), one writes $G_{++}(1,2)=C(1,2)$, $G_{+-}(1,2)=R(1;2)$, $G_{-+}(1,2)=R(2;1)$, $G_{--}(1,2)=0$. For the case of Gaussian ("normal") initial conditions, one can write a matrix Dyson equation which expresses succinctly the coupled evolution of C and R :

$$\begin{aligned} -i\sigma^{(0)}(1,\bar{1})^{-1} G(\bar{1},1') - \gamma(1,2,\bar{1}) \langle\psi(2)\rangle G(\bar{1},1') - \int (1,\bar{1}) G(\bar{1},1') \\ = \delta(1-1') , \end{aligned} \quad (9)$$

where the spin matrix $i\sigma$ is defined by

$$i\sigma \equiv \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} .$$

The "bare vertex" or nonlinear coupling term γ satisfies

$$\gamma \begin{pmatrix} 1, 2, 3 \\ - + + \end{pmatrix} = \bar{U}(1, 2, 3)$$

and is furthermore symmetric in all indices, including spin. General expressions for the "mass" operator Σ are given by MSR. In the DIA, which can be thought of as the "most Gaussian" approximation consistent with nonlinear interaction,

$$\bar{\chi}(1, \bar{1}) = \frac{1}{2} \gamma(1, 2, 3) G(2, \bar{2}) G(3, \bar{3}) \gamma(\bar{3}, \bar{2}, \bar{1}) \quad (10)$$

We have begun with this formal matrix notation so that we can emphasize the important symmetry of the DIA, manifest in Eq.(10). This symmetry has physical significance, with origins in the symmetric form of the Vlasov equation (3). That symmetry is a statement of self-consistency: the potential is uniquely determined in terms of the particle distribution; a change in ϕ affects f , and vice versa. In many theories, in particular Dupree's original formulation of the resonance broadening problem, this self-consistency is not enforced during the approximations leading to statistical closure (it is, of course, reinstated at the end of the calculation), and the symmetry is lost. This loss generally implies violation of various conservation laws, improper treatment or complete lack of nonlinear wave-particle interactions, etc.

Having stressed the symmetry, we now find that it becomes more convenient to discuss the physics in terms of the components of the matrix equation (9) with (10). The $(-, -)$ component provides an equation for the response function:

$$(\mathbf{g}^{(0)})^{-1} \mathbf{R} + \partial \langle \mathbf{f} \rangle \cdot \tilde{\mathbf{E}} \mathbf{R} - \sum_{-+} \mathbf{R} = \mathbf{1}, \quad (11)$$

where

$$\tilde{\mathbf{E}}(1, \bar{1}) \equiv -\nabla_1 \phi(1, \bar{1})$$

and

$$\sum_{-+}(1, \bar{1}) \equiv \bar{\mathbf{U}}(1, 2, 3) \mathbf{R}(2, \bar{2}) \mathbf{C}(3, \bar{3}) \bar{\mathbf{U}}(\bar{2}, \bar{3}, \bar{1}). \quad (12)$$

Similarly, the $(-, +)$ component gives the evolution of the correlation function:

$$(\mathbf{g}^{(0)})^{-1} \mathbf{C} + \partial \langle \mathbf{f} \rangle \cdot \tilde{\mathbf{E}} \mathbf{C} - \sum_{-+} \mathbf{C} = \tilde{\sum} \mathbf{R}^T, \quad (13)$$

where "T" means transpose and

$$\tilde{\sum}(1, \bar{1}) \equiv \frac{1}{2} \bar{\mathbf{U}}(1, 2, 3) \mathbf{C}(2, \bar{2}) \mathbf{C}(3, \bar{3}) \bar{\mathbf{U}}(\bar{1}, \bar{2}, \bar{3}). \quad (14)$$

By comparing Eqs. (11) and (13), one sees that $\mathbf{R}$ is the Green's function for

$$\mathbf{C} = \mathbf{R} \tilde{\sum} \mathbf{R}^T. \quad (15)$$

Because of the product nature of Eq. (14), $\tilde{\sum}$ is a mode-coupling

term; it can be interpreted as a source of nonlinear ("incoherent") noise which drives the fluctuations C . We claim that R can be interpreted as the propagator for a shielded test particle. To see this, we make use of the specific form (5) of $\bar{U}$ to rewrite Eq. (11) as

$$g^{-1}R + (\partial\langle f \rangle + \partial\delta\bar{f}) \cdot \underline{E}R = 1 \quad , \quad (16)$$

where

$$g^{-1} \equiv (g^{(0)})^{-1} + \Sigma' \quad , \quad (17)$$

$$\begin{aligned} \Sigma'(1, \bar{1}) \equiv & -[\partial_1 R(1; 2) \cdot \langle \delta \underline{E}(1) \delta \underline{E}(2) \rangle \cdot \partial_2 \delta(2 - \bar{1}) \\ & + \underline{E}R(1; 2) \cdot \partial_2 \langle \delta f(1) \delta \underline{E}(2) \rangle \cdot \partial_2 \delta(2 - \bar{1})] \quad , \quad (18) \end{aligned}$$

$$\begin{aligned} \partial \delta \bar{f}(1, 2) \cdot \underline{E}(2, \bar{1}) \equiv & -[\partial_1 R(1; 2) \cdot \langle \delta \underline{E}(1) \partial_2 \delta f(2) \rangle \\ & + \underline{E}R(1; 2) \cdot \partial_1 \partial_2 \langle \delta f(1) \delta f(2) \rangle] \cdot \underline{E}(2, \bar{1}) \quad . \quad (19) \end{aligned}$$

[The operator $-\partial\delta\bar{f}(1, 2)$ is to be thought of as the bracketed term in Eq. (19).] Eq. (16) has now precisely the form of the linearized Vlasov equation if we interpret R as a test particle density plus the linearized self-consistent response, except that here the background distribution has been renormalized to

$$\bar{f} \equiv \langle f \rangle + \delta\bar{f}$$

and the bare propagator $g^{(0)}$ has been renormalized to include turbulent collisions (the positive infinitesimal of linear theory is replaced by Σ'). We can, of course, write

$$R = g[1 - \epsilon^{-1} \partial \tilde{f} \cdot \underline{E} g] \quad , \quad (20)$$

where

$$\epsilon \equiv 1 + \underline{E} g \cdot \partial \tilde{f} \quad . \quad (21)$$

The shielded test particle interpretation will be complete if ϵ can be interpreted as a dielectric function. This is indeed the case, as Thompson and Krommes²⁶ and Dubois⁴ have shown. We review the theory of the renormalized dielectric function in Appendix A.

Although it is possible to continue in x space,⁴ clarity is gained by assuming approximate statistical invariance so that we can Fourier transform with respect to difference variables in space-time. Writing $k \equiv (\underline{k}, \omega_k)$ [mode frequencies will be denoted by $\omega(k)$], we have

$$(g_k^{(0)})^{-1}(1,2) = -i(\omega_k - \underline{k} \cdot \underline{v}_1) \delta(\underline{v}_1 - \underline{v}_2) \delta_{s_1, s_2} \quad , \quad (22)$$

$$\begin{aligned} \hat{L}_k'(1, \bar{1}) = & - \int_q \{ \partial_1 g_{k-q}(1;2) \cdot \langle \delta \underline{E} \delta \underline{E} \rangle_q \cdot \partial_2 \\ & + \epsilon_{k-q}^{-1} [\partial_1 \cdot \underline{E}_{k-q}(\bar{2}) g_{k-q}(\bar{2};2) \langle \delta f(1) \delta \underline{E} \rangle_q \cdot \partial_2 \\ & - \partial_1 \cdot \underline{E}_{-q}(1; \hat{1}) \partial_{\hat{1}} \underline{E}_{k-q}(\hat{1}) \cdot \underline{E}_{k-q}(\bar{2}) g_{k-q}(\bar{2};2) \\ & \cdot \langle \delta \underline{E} \delta \underline{E} \rangle_q \cdot \partial_2] \} \delta(2-\bar{1}) \quad , \end{aligned} \quad (23)$$

$$\begin{aligned}
 \partial \delta \bar{f}_k(1) = & - \sum_q \{ \partial_1 g_{k-q}(1;2) \cdot \langle \delta E(1) \partial_2 \delta f(2) \rangle_q \\
 & + \epsilon_{k-q}^{-1} [\partial_1 \cdot \bar{E}_{k-q}(\bar{2}) g_{k-q}(\bar{2};2) \langle \delta f(1) \partial_2 \delta f(2) \rangle_q \\
 & - \partial_1 g_{k-q}(1;\hat{1}) \partial_1 \bar{f}_{k-q}(\hat{1}) \cdot \bar{E}_{k-q}(\bar{2}) \\
 & \times g_{k-q}(\bar{2};2) \langle \delta E \partial_2 \delta f(2) \rangle_q] \} .
 \end{aligned} \quad (24)$$

We shall study the renormalized approximation which is obtained by neglecting the mode-coupling term $\bar{\Sigma}$ in Eq. (13); we call this the "coherent" approximation to the DIA (DIAC). In WTT this amounts to neglecting three-wave coupling. (This is often justifiable for many kinds of waves except at long wavelengths. However, the approximation must be examined specifically for each problem.) We thus find a relation between $\langle \delta f \delta f \rangle$ and $\langle \delta E \delta f \rangle$:

$$\langle \delta f(1) \delta f(2) \rangle_k^+ = - g_k(1;\bar{1}) \partial_{\bar{1}} \bar{f}_k(\bar{1}) \cdot \langle \delta E \delta f(2) \rangle_k^+ , \quad (25)$$

where "+" denotes the positive frequency part. Equation (25) can be used to express all two-point correlations involving δf in terms of the spectral function $\langle \delta E \delta E \rangle$, e.g.

$$\langle \delta f \delta E \rangle_k^+ = - g_k \partial \bar{f}_k \cdot \langle \delta E \delta E \rangle_k^+ .$$

Let us introduce $I_k \equiv \langle \delta \phi \delta \phi \rangle_k$. One then finds from Eqs. (23) and (24)

$$\bar{\Sigma}' = \bar{\Sigma}^{(d)} + \bar{\Sigma}^{(p)} , \quad (26)$$

where

$$\sum_k^{(d)} (1, \bar{1}) \equiv - \sum_q q \cdot \partial_1 g_{k-q} (1; 2) I_q q \cdot \partial_2 \delta (2 - \bar{1}) , \quad (27a)$$

$$\begin{aligned} \sum_k^{(p)} (1, \bar{1}) \equiv & -i \sum_q \{ q \cdot \partial_1 g_{k-q} (1; 2) (k-q) \cdot \partial_2 \bar{f}_{k-q} (2) + [q \leftrightarrow (k-q)] \} I_q \\ & \times \epsilon_{k-q}^{-1} \phi_{k-q} (\bar{3}) g_{k-q} (\bar{3}; \bar{2}) q \cdot \partial_2 \delta (\bar{2} - \bar{1}) , \end{aligned} \quad (27b)$$

and

$$\bar{f} = \bar{f}^{(d)} + \bar{f}^{(p)} , \quad (28)$$

where

$$\partial \delta \bar{f}_k^{(d)} (1) \equiv \sum_q q \cdot \partial_1 g_{k-q} (1; 2) I_q \partial_2 g_{-q} (2; 3) q \cdot \partial_3 \bar{f}_{-q} , \quad (29a)$$

$$\begin{aligned} \partial \delta \bar{f}_k^{(p)} (1) \equiv & i \sum_q \{ q \cdot \partial_1 g_{k-q} (1; 2) (k-q) \cdot \partial_2 \bar{f}_{k-q} (2) + [q \leftrightarrow (k-q)] \} I_q \\ & \times \epsilon_{k-q}^{-1} \phi_{k-q} (\bar{3}) g_{k-q} (\bar{3}; \bar{2}) \partial_2 g_{-q} (\bar{2}; \bar{1}) q \cdot \partial_1 \bar{f}_{-q} (\bar{1}) . \end{aligned} \quad (29b)$$

The DIAC is then Eqs. (16), (17), and (26) - (29). The superscripts "d" and "p" stand for "diffusion" and "polarization" respectively (see Sec. III).

It is easy to show the relation of DIAC to the conventional unrenormalized WTT. Away from the resonances $\omega_k - k \cdot v = 0$, one can expand

$$g_k \approx g_k^{(0)} - g_k \sum_k' g_k . \quad (30)$$

To order E^2 , the dielectric function (21) becomes

$$\epsilon_k \approx 1 - i \phi_k [g_k^{(0)} k \cdot \partial \langle f \rangle + g_k (- \sum_k' g_k k \cdot \partial \langle f \rangle + k \cdot \partial \delta \bar{f}_k)] , \quad (31)$$

where Σ' and $\delta \bar{f}$ are evaluated with $\bar{f}$ replaced by $\langle f \rangle$. One

shows readily that Eq. (31) is the conventional dielectric of WTT through $O(E^2)$:

$$\epsilon = \epsilon^{(l)} + \epsilon^{(n)} , \quad (32)$$

where

$$\epsilon_k^{(l)} \equiv 1 - i\phi_k g_k^{(0)} k \cdot \partial \langle f \rangle \quad (33)$$

is the linear dielectric function and

$$\begin{aligned} \epsilon_k^{(n)} \equiv & 2 \int_q [\epsilon^{(3)}(k|q, k, -q) - 2\epsilon^{(2)}(k|q, k-q) \epsilon_{k-q}^{-1} \\ & \times \epsilon^{(2)}(k-q|-q, k)] I_q , \end{aligned} \quad (34)$$

$$\begin{aligned} \epsilon^{(3)}(k|k_1, k_2, k_3) \equiv & \frac{1}{2} i\phi_k (1) g_k (1;2) \underline{k}_1 \cdot \partial_2 g_{k-k_1} (2;3) [\underline{k}_2 \cdot \partial_3 g_{k_3} (3;4) \\ & \times \underline{k}_3 \cdot \partial_4 \langle f(4) \rangle + (k_2 \leftrightarrow k_3)] , \end{aligned} \quad (35a)$$

$$\begin{aligned} \epsilon^{(2)}(k|k_1, k_2) \equiv & \frac{1}{2} \phi_k (1) g_k (1;2) [\underline{k}_1 \cdot \partial_2 g_{k_2} (2;3) \underline{k}_2 \cdot \partial_3 \langle f(3) \rangle + (k_1 \leftrightarrow k_2)] . \end{aligned} \quad (35b)$$

Notice that we have retained the full propagator in the last term of Eq. (30) and the full dielectric in the last term of Eq. (34). Retaining the full dielectric is sometimes called "renormalizing the plasmon propagator"; this renormalization appears naturally in our method, but is very difficult to obtain in other methods.²⁰

The $\epsilon^{(3)}$ term is responsible for Compton scattering and arises from the diffusion parts of Σ' and $\delta\bar{f}$. The $\epsilon^{(2)} \epsilon^{-1} \epsilon^{(2)}$ term

generates the nonlinear scattering from the polarization clouds and arises from the polarization parts of Σ' and $\delta\bar{F}$.

Both DIAC and WTT are completed by deriving the wave kinetic equation. One finds an equation for the spectrum I_k by applying the potential operator Φ_k from both the left and right to Eq. (15) and using approximation (25) to simplify $\tilde{\Sigma}$:

$$\epsilon_k I_k = 2 \sum_{k_1+k_2=k} \frac{|\epsilon^{(2)}(k|k_1, k_2)|^2}{\epsilon_k^*} I_{k_1} I_{k_2} . \quad (36)$$

Taking the real part of Eq. (36) gives us to lowest order

$$I_k = 2\pi I_k \delta[\omega_k - \omega(k)] ,$$

where

$$\epsilon_r[k, \omega(k)] = 0 .$$

(Subscripts "r" and "i" stand for Re and Im respectively.) If we now introduce the plasmon density

$$N_k \equiv \frac{k^2}{8\pi} \frac{\partial \epsilon_r(k, \omega)}{\partial \omega} \bigg|_{\omega(k)} I_k ,$$

take the imaginary part of Eq. (36) and also generalize to the case of a medium weakly inhomogeneous in space-time, we arrive at the well-known wave kinetic equation

$$\frac{\partial N_k}{\partial t} + \nabla \cdot (\mathbf{v}_g N_k) - \frac{\partial}{\partial k} \cdot [(\nabla \omega_k) N_k] = 2\gamma_k N_k + \left(\frac{\partial N_k}{\partial t} \right)_{m.c.} ,$$

where

$$\underline{v}_g \equiv \frac{\partial \omega(\underline{k})}{\partial \underline{k}} ,$$

$$\gamma_{\underline{k}} \equiv - \left(\frac{\epsilon_j(\underline{k}, \omega)}{\partial \epsilon_r(\underline{k}, \omega) / \partial \omega} \right)_{\omega(\underline{k})} ,$$

and

$$\left(\frac{\partial N_{\underline{k}}}{\partial t} \right)_{m.c.} \equiv \int \frac{d\underline{k}_1 d\underline{k}_2}{(2\pi)^6} M(\underline{k} | \underline{k}_1, \underline{k}_2) N_{\underline{k}_1} N_{\underline{k}_2} ,$$

$$M(\underline{k} | \underline{k}_1, \underline{k}_2) \equiv (16\pi) (2\pi)^4 \delta(\underline{k} - \underline{k}_1 - \underline{k}_2) \delta[\omega(\underline{k}) - \omega(\underline{k}_1) - \omega(\underline{k}_2)] \\ \times \frac{\underline{k}^2}{\underline{k}_1 \underline{k}_2} \frac{|\epsilon^{(2)}(\underline{k} | \underline{k}_1, \underline{k}_2)|^2}{\left(\frac{\partial \epsilon_r}{\partial \omega(\underline{k})} \right) \left(\frac{\partial \epsilon_r}{\partial \omega(\underline{k}_1)} \right) \left(\frac{\partial \epsilon_r}{\partial \omega(\underline{k}_2)} \right)} .$$

For the DIAC, it is strictly inconsistent to retain the mode coupling term $(\partial N / \partial t)_{m.c.}$ because it stems from $\tilde{\Sigma}$ which was neglected in deriving Eq. (25). However, the form of the term presented here should be adequate for one to determine the size of the mode coupling effects and thus to understand when they can be safely neglected and the DIAC can be justified.

III. Heuristic Derivation of the DIAC

When there are primary resonances, $\omega_{\underline{k}} - \underline{k} \cdot \underline{v} \approx 0$, the simple WTT is invalid and we must return to the DIAC. We now give a heuristic derivation of this approximation. The procedure is a simple variant of the well-known iteration-renormalization algorithm²⁷

for the DIA itself.

Let one extra particle be inserted into the turbulent plasma at $(\underline{x}', t')$ with velocity $\underline{v}'$. The governing equation, Fourier transformed with respect to $(\underline{x} - \underline{x}', t - t')$, is

$$(\underline{g}_k^{(0)})^{-1} F_k + \sum_{\underline{q}} \hat{\underline{E}}_{k-\underline{q}} \cdot \partial F_{\underline{q}} = \frac{1}{n} \delta(\underline{v} - \underline{v}') \quad , \quad (37)$$

where F is the total distribution and $\hat{\underline{E}}$ is the associated field. Our procedure will be to find the mean change $\langle \Delta f_k \rangle$ induced into the system by the test particle and to relate this to the response function R . To this end, we expand the distribution function in powers of the field amplitude $\epsilon \equiv \sqrt{\delta}$:

$$F = \sum_{n=0}^{\infty} \epsilon^n F^{(n)} \quad . \quad (38)$$

We also write

$$F = f + \Delta f \quad , \quad \hat{\underline{E}} = \underline{E} + \Delta \underline{E} \quad (39)$$

where f describes the plasma in the absence of the test particle [solves Eq. (37) with the right-hand side set to zero]. We take

$$f_k^{(0)} = \langle f \rangle \delta_{k,0} \quad ,$$

$$f_k^{(n)} \equiv \delta f_k^{(n)} \quad , \quad \langle \delta f_k^{(n)} \rangle \equiv 0 \quad (n > 0) \quad ,$$

$$\underline{E}_k^{(0)} = 0 \quad ,$$

$$\underline{E}_k^{(n)} \equiv \delta \underline{E}_k^{(n)} \quad , \quad \langle \delta \underline{E}_k^{(n)} \rangle \equiv 0 \quad (n > 0) \quad . \quad (40)$$

We will work to second order in the amplitude ϵ of the turbulent fluctuations and to linear order in the strength of the test particle ($1/n$ is small; the test particle is weakly coupled to the turbulent bath). To zeroth order in ϵ we have

$$(g_k^{(0)})^{-1} F_k^{(0)} + \sum_q \hat{E}_{k-q}^{(0)} \cdot \partial F_q^{(0)} = \frac{1}{n} \delta(\underline{v} - \underline{v}') \quad (41)$$

The change $\Delta f^{(0)}$ in the zeroth order distribution due to the test particle is then, with Eqs. (40),

$$(g_k^{(0)})^{-1} \Delta f_k^{(0)} + \Delta E_k^{(0)} \cdot \partial \langle f \rangle = \frac{1}{n} \delta(\underline{v} - \underline{v}') \quad (42)$$

which can be trivially averaged to give

$$(g_k^{(0)})^{-1} \langle \Delta f_k^{(0)} \rangle + \langle \Delta E_k^{(0)} \rangle \cdot \partial \langle f \rangle = \frac{1}{n} \delta(\underline{v} - \underline{v}') \quad (43)$$

Clearly, one can identify $n \langle \Delta f_k^{(0)} \rangle$ with the lowest order response function:

$$n \langle \Delta f_k^{(0)} \rangle = R_k^{(0)} \quad .$$

The algorithm maintains this correspondence to all orders:

$$R_k = n \langle \Delta f_k \rangle \quad (44)$$

The first-order fluctuations of the perturbed system obey

$$(g_k^{(0)})^{-1} F_k^{(1)} + \sum_q \{ \hat{E}_{k-q}^{(0)} \cdot \partial F_q^{(1)} + \hat{E}_{k-q}^{(1)} \cdot \partial F_q^{(0)} \} = 0 \quad (45)$$

and the change induced by the test particle obeys

$$\begin{aligned} (g_k^{(0)})^{-1} \Delta f_k^{(1)} + \Delta E_k^{(1)} \cdot \partial < f > \\ + \sum_q \{ \Delta E_{k-q}^{(0)} \cdot \partial \delta f_q^{(1)} + \delta E_{k-q}^{(1)} \cdot \partial \Delta f_q^{(0)} \} = 0 \end{aligned} \quad (46)$$

The terms involving $\Delta E^{(0)}$ of $\Delta f^{(0)}$ vanish under statistical averaging because the test particle is not correlated with the background plasma, so that

$$(g_k^{(0)})^{-1} \langle \Delta f_k^{(1)} \rangle + \langle \Delta E_k^{(1)} \rangle \cdot \partial < f > = 0, \quad (47)$$

which is a trivial correction to Eq. (43).

The effects of turbulence enter at second order. We have

$$(g_k^{(0)})^{-1} F_k^{(2)} + \sum_q \{ \hat{E}_{k-q}^{(0)} \cdot \partial F_q^{(2)} + \hat{E}_{k-q}^{(2)} \cdot \partial F_q^{(0)} \} = - \sum_q \hat{E}_{k-q}^{(1)} \cdot \partial F_q^{(1)}, \quad (48)$$

so the variation due to the extra particle obeys

$$\begin{aligned} (g_k^{(0)})^{-1} \Delta f_k^{(2)} + \Delta E_k^{(2)} \cdot \partial < f > + \sum_q \{ \Delta E_{k-q}^{(0)} \cdot \partial \delta f_q^{(2)} + \delta E_{k-q}^{(2)} \cdot \partial \Delta f_q^{(0)} \} \\ = - \Delta \sum_q \hat{E}_{k-q}^{(1)} \cdot \partial F_q^{(1)} \equiv \Delta G \end{aligned} \quad (49)$$

To evaluate the term on the right-hand side of Eq. (49), we "solve" Eq. (45) for $F_q^{(1)}$:

$$F_q^{(1)} = -g_q^{(0)} \sum_l \{ \hat{E}_{q-l}^{(0)} \cdot \partial F_l^{(1)} + \hat{E}_{q-l}^{(1)} \cdot \partial F_l^{(0)} \} \quad (50)$$

We omit an initial condition term, which will not contribute.

Computing ΔG ,

$$\begin{aligned} \Delta G = & - \sum_{q, \ell} (k-q) \cdot \partial g_q^{(0)} (q-\ell) \cdot \partial \left[\Delta f_\ell^{(0)} \delta \phi_{k-q}^{(1)} \delta \phi_{q-\ell}^{(1)} + \delta f_\ell^{(1)} \delta \phi_{k-q}^{(1)} \Delta \phi_{q-\ell}^{(0)} \right] \\ & - \sum_q \left[(k-q) \cdot \partial g_q^{(0)} q \cdot \partial \langle f \rangle + q \cdot \partial g_{k-q}^{(0)} (k-q) \cdot \partial \langle f \rangle \right] \Delta \phi_{k-q}^{(1)} \delta \phi_q^{(1)} . \end{aligned} \quad (51)$$

The term in $\delta f_\ell^{(1)}$ can be rewritten using Eq. (50):

$$\delta f_\ell^{(1)} = i g_\ell^{(0)} \delta \phi_\ell^{(1)} \ell \cdot \partial \langle f \rangle . \quad (52)$$

Also, the term in $\Delta \phi_{k-q}^{(1)}$ can be rewritten using Eq. (46):

$$\epsilon_{k-q} \Delta \phi_{k-q}^{(1)} = i \phi_{k-q} c_{k-q}^{(0)} \sum_\ell (k-q-\ell) \cdot \partial \left[\Delta f_\ell^{(0)} \delta \phi_{k-q-\ell}^{(1)} + \delta f_\ell^{(1)} \Delta \phi_{k-q-\ell}^{(0)} \right] . \quad (53)$$

Using Eqs. (52) and (53) in (51), we can now average Eq. (49):

$$\begin{aligned} (g_k^{(0)})^{-1} \langle \Delta f_k^{(2)} \rangle + \partial \langle f \rangle \cdot \langle \Delta E_k^{(2)} \rangle &= \sum_q g \cdot \partial g_{k-q}^{(0)} \frac{1}{q} \left[q \cdot \partial \langle \Delta f_k^{(0)} \rangle \right. \\ &+ i k \cdot \partial g_{-q}^{(0)} q \cdot \partial \langle f \rangle \langle \Delta \phi_k^{(0)} \rangle \left. \right] + i \sum_q [I] I_q \epsilon_{k-q}^{-1} \phi_{k-q} g_{k-q}^{(0)} \left[q \cdot \partial \langle \Delta f_k^{(0)} \rangle \right. \\ &+ i k \cdot \partial g_{-q}^{(0)} q \cdot \partial \langle f \rangle \langle \Delta \phi_k^{(0)} \rangle \left. \right] , \end{aligned} \quad (54)$$

where $[I]$ is the bracketed term in Eq. (51). The algorithm is completed by writing $\langle \Delta f_k \rangle = \sum_{i=0}^2 \langle \Delta f_k^{(i)} \rangle$, adding Eqs. (43), (47), and (54), and replacing all zeroth-order quantities on the right-hand side of Eq. (54) by renormalized ones: $g^{(0)} \rightarrow g$, $\langle \Delta f^{(0)} \rangle \rightarrow \langle \Delta f \rangle$, and $g_k \partial \langle f \rangle + g_k \partial \bar{f}_k$. With Eq. (44) the result is easily seen to be

identical to the DIAC. In Eq. (54), the terms in $\langle \Delta f^{(0)} \rangle$ become $-\Sigma'R$, the terms in $\langle \Delta \phi^{(0)} \rangle$ become $-\partial \bar{f} \cdot \bar{E}R$.

We can interpret each of the terms in Eq. (54). The first term $(-\Sigma^{(d)})$ represents diffusion of the shielded test particle due to stimulated emission and absorption of fluctuations q , or resonant scattering of the test particles by the plasmons N_q . The second term $(-\delta \bar{f}^{(d)})$ is a "ponderomotive" renormalization of the background distribution due to the fluctuations emitted (or absorbed) in the process just discussed. The third term $(-\Sigma^{(p)})$ is a "drag" term due to the polarization process associated with the stimulated emission and absorption, and the fourth term $(-\delta \bar{f}^{(p)})$ is the associated ponderomotive effect. In the so-called stochastic acceleration problems where the potential is specified externally, and thus in the simplest orbit diffusion or resonance broadening theories, only the first, diffusion term remains.

To be more specific, we summarize in Table 1 the interactions represented by Eq. (54). Each of the four effects is associated with a second-order change in the distribution of test particles arising from a product of first-order disturbances mediated by the test particle itself. We write $\partial_t \Delta f_k^{(2)} \sim \sum_q \hat{E}_q \cdot \partial \hat{f}_{k-q}$, where $\hat{f}$ is either $\delta f^{(1)}$ or $\Delta f^{(1)}$, and similarly for $\hat{E}$. The table shows the origin of the first-order fluctuations $\hat{E}$ and $\hat{f}$. (Although it may be convenient to think of the background fluctuations δf , δE as "waves," they need not be normal modes of the plasma.) Thus, in the diffusion process $\Sigma^{(d)}$, the background fluctuation $\delta E_{-q}^{(1)}$ accelerates the test particles, creating a first-order fluctuation in test particle density at $k-q$. The process is then repeated,

with $\delta E_q^{(1)}$ accelerating the first-order distribution at $k-q$, giving rise to a net (averaged) second-order response in the density of test particles at k . (Compare the Compton scattering process in nonresonant WTT.) In the polarization process $\Sigma^{(p)}$ the first-order fluctuation discussed above is shielded by the medium, producing a shielded first-order field $\hat{E}_{k-q}$ which couples with the background particle fluctuation $\delta f_q^{(1)}$ to produce a net second-order test particle response at k . (Compare the nonlinear scattering process in WTT.) In the $\bar{f}$ processes, the (shielded) field of the test particles accelerates the background distribution $\delta f_{-q}^{(1)}$, producing a first-order background response at $k-q$; the remainder of the interpretations then proceed as before.

IV. Discussion

Previous authors¹⁹ have considered an approximation in which the term $\Sigma^{(p)}$ never appears in the propagator. We can recover this approximation⁴ by defining a "diffusive" propagator $g^{(d)}$ according to

$$(g^{(d)})^{-1} = (g^{(o)})^{-1} + \Sigma^{(d)},$$

then expanding the full propagator in powers of $\Sigma^{(p)}$:

$$g = g^{(d)} - g^{(d)} \Sigma^{(p)} g^{(d)}.$$

The resulting dielectric function then contains intact the nonlinear scattering term $\sim \epsilon^{(2)} \epsilon^{-1} \epsilon^{(2)}$ but has only one-half of the Compton

scattering term $\sim v^{(3)}$. The consistency of this procedure is unclear. As is well known, the diffusion and polarization terms tend to cancel at long wavelengths. For resonant particles it would seem that one must keep Σ' intact and deal with the dielectric in the form (21). For nonresonant particles, the entire Σ' can be treated as small and all of the induced scattering processes are recovered intact, as we have seen.

Galeev^{16,28} has discussed an approximation which amounts to neglecting both polarization terms $\Sigma^{(p)}$ and $\delta\bar{f}^{(p)}$, but which does retain $\delta\bar{f}^{(d)}$ in addition to the usual orbit diffusion term $\Sigma^{(d)}$. Although certain of the further practical approximations Galeev makes are questionable, we believe that his approximation is the best motivated of all the renormalized plasma theories to date which have been applied to practical problems.

Clearly the DIAC discussed here for the Vlasov equation can be written down for related equations of similar structure. A problem of considerable practical importance to the toroidal confinement program is that of describing the self-consistent evolution of electromagnetic instabilities with sufficient magnetic component to destroy flux surfaces and cause stochastic diffusion of magnetic field lines.²⁹ If one can show that the modal amplitudes grow past the stochasticity threshold, then the present techniques can be applied to the relevant drift-kinetic equation (DKE). In preliminary work on the DKE, we have shown³⁰ that the " β " term added by Dupree and Tetreault³¹ to the simple resonance broadening theory of the DKE in order to maintain energy conservation is just the $\delta\bar{f}^{(d)}$ term. In a related development, we have used³⁰ the $\delta\bar{f}^{(d)}$ term to give a systematic derivation of Catto's prescription³² of "resonance-broadening

the nonadiabatic part of the distribution." Further work is in progress.

In summary, we have discussed a certain renormalization of the Vlasov equation, appropriate for weak turbulence, from two points of view: (1) by a simple ("coherent") approximation to the Direct Interaction Approximation; (2) by means of a heuristic algorithm based on the notion of a shielded test particle interacting weakly with the turbulent background medium. The theory predicts both stochastic diffusion and polarization effects of the turbulent medium on the test particles as well as the inverse effect of the test particles on the medium. We believe that it provides a convenient starting point for systematic derivations of practical approximations for turbulence.

ACKNOWLEDGMENT

This work was jointly supported by the United States Air Force Office of Scientific Research Contract #F 44620-75-C-0037 and the United States Department of Energy Contract #EY-76-C-02-3073.

We are grateful to Philippe Similon for stimulating discussions and for checking the equations.

APPENDIX: Theory of the Renormalized Dielectric Function

In the linear theory of Vlasov plasma, a central role is played by the linear plasma dielectric function $\epsilon_k^{(l)}$ which characterizes the (dynamically linear) response of the plasma to a small external electric field $E^{(e)}$. We consider only the electrostatic approximation. Then the total (external plus induced) electric field E in the plasma becomes to linear order $E^{(e)}$

$$E_k = [\epsilon_k^{(l)}]^{-1} E_k^{(e)} . \quad (A1)$$

Of course, the zeros of ϵ give the collective wave response $\omega = \omega(k)$.

The definition and form of the dielectric function must be modified when turbulence is present. In this case, the detailed microscopic form of the plasma field is of no interest because of its extreme sensitivity to the initial phases of the waves and particles. In order to maintain the conventional interpretation of the dielectric function as a macroscopic, observable property of the medium, it is necessary to modify definition (A1) to read

$$\langle E_k \rangle = \epsilon_k^{-1} E_k^{(e)} + O\left(E^{(e)2}\right) ,$$

where $\langle \cdot \rangle$ denotes the appropriate average over the statistics of the turbulence. We call ϵ the "nonlinear" or "renormalized" dielectric function; it gives the mean response of the nonlinear plasma correct to linear order in $E^{(e)}$.

In self-consistent calculations of plasma turbulence, no external field $\underline{E}^{(e)}$ need appear explicitly. However, a fluctuation $\delta \underline{E}$ of the turbulent system itself can play the role of $\underline{E}^{(e)}$. Because this fluctuation is in general of finite, not infinitesimal, amplitude and is of random amplitude and phase, the system response to this fluctuation will not be of the form $c^{-1} \delta \underline{E}$. However, just as the self-consistent renormalized theories of quite general nonlinear equations² can be successfully constructed in terms of the mean infinitesimal response function, it is not unreasonable to introduce ϵ as a fundamental building block in the self-consistent description of Vlasov turbulence.

Dubois and Espedal⁴ have previously given the explicit formula for ϵ in the DIA. We will give here an alternative (but closely related) derivation which uses the MSR formalism. The advantage of this formalism is that ϵ can be expressed, to arbitrary order in renormalization, entirely in terms of the observables. In the DIA it is possible to give an explicit representation in terms of $\langle f \rangle$, C , and R . The renormalized theory of the dielectric beyond DIA is not well understood. (Of course, it is straightforward to compute ϵ to arbitrary order in perturbation theory, but such expressions are generally divergent.)

Let random quantities be denoted by tildes. The random electrostatic Vlasov equation with nonrandom external field $\underline{E}^{(e)}$ is

$$\partial_t \tilde{f} + \underline{v} \cdot \nabla \tilde{f} + (\tilde{\underline{E}} + \underline{E}^{(e)}) \cdot \nabla \tilde{f} = 0 \quad . \quad (A2a)$$

The self-consistent field $\tilde{\underline{E}}$ is the solution of Poisson's equation:

$$\tilde{\underline{E}} = \underline{E} \tilde{f} \quad . \quad (A2b)$$

The solution $\tilde{f}$ of Eqs. (A2) is a functional of $\underline{E}^{(e)}$; $\tilde{f} = \tilde{f}\{\underline{E}^{(e)}\}$. This may be written as a functional power series around $\underline{E}^{(e)} = 0$:

$$\tilde{f}\{\underline{E}^{(e)}\} = \tilde{f}\{0\} + \left(\frac{\delta \tilde{f}}{\delta \underline{E}^{(e)}} \right)_0 \cdot \underline{E}^{(e)} + O\left\{ \underline{E}^{(e)2} \right\}. \quad (A3)$$

The mean first-order distribution follows from Eq. (A3):

$$\langle \tilde{f}^{(1)}(\underline{1}) \rangle = \left\langle \left(\frac{\delta \tilde{f}^{(1)}}{\delta \underline{E}^{(e)}(\underline{2})} \right)_0 \cdot \underline{E}^{(e)}(\underline{2}) \right\rangle,$$

where the underscore signifies no velocity dependence. The total field $\tilde{\underline{E}}(t)$ in the plasma,

$$\tilde{\underline{E}}(t) \equiv \underline{E}^{(e)} + \tilde{\underline{E}}^{(1)},$$

thus satisfies

$$\langle \tilde{\underline{E}}^{(1)}(\underline{1}) \rangle = \underline{\epsilon}^{-1}(\underline{1}, \underline{2}) \cdot \underline{E}^{(e)}(\underline{2}),$$

where, if we define

$$\tilde{\underline{H}}(\underline{1}, \underline{2}) \equiv (\delta \tilde{f}^{(1)}(\underline{1}) / \delta \underline{E}^{(e)}(\underline{2}))_0,$$

we have

$$\underline{\epsilon}^{-1}(\underline{1}, \underline{2}) = \delta(\underline{1} - \underline{2}) \underline{1} + \underline{E}(\underline{1}, \bar{\underline{1}}) \langle \tilde{\underline{H}}(\bar{\underline{1}}, \underline{2}) \rangle.$$

The equation for $\tilde{\underline{H}}$ follows from Eq. (A2a):

$$\begin{aligned} (\partial_t + \underline{v} \cdot \underline{\nabla}) \tilde{\underline{H}}(\underline{1}, \underline{2}) + [\partial_1 \tilde{f}^{(1)}] \cdot \underline{E}(\underline{1}, \bar{\underline{1}}) \tilde{\underline{H}}(\bar{\underline{1}}, \underline{2}) + \tilde{\underline{E}}(\underline{1}) \cdot \partial_1 \tilde{\underline{H}}(\underline{1}, \underline{2}) \\ = - \delta(\underline{1} - \underline{2}) \partial_1 \tilde{f}^{(1)} \end{aligned} \quad (A4)$$

where now $\tilde{f}(1)$ is the solution of Eq. (A2) in the absence of $E^{(e)}$. Equation (A4) exhibits the closure problem: because products of random quantities appear, it is not immediately possible to derive a closed equation for $\langle \tilde{H} \rangle$ by averaging Eq. (A4). However, we can write the solution of Eq. (A4) in terms of the random infinitesimal response function $\tilde{R}$, defined by

$$\tilde{R}(1;2) \equiv [\delta \tilde{f}(1) / \delta \hat{\eta}(2)]_{\hat{\eta}=0},$$

where $\hat{\eta}$ is a nonrandom source term added to the right-hand side of the $E^{(e)} = 0$ version of Eq. (A2). One verifies readily that $\tilde{R}$ is the Green's function for the operator on the left-hand side of Eq. (A4), so

$$\tilde{H}(1,2) = -\tilde{R}(1,\underline{2}) \delta(\underline{2}-\underline{2}) \frac{\partial}{\partial \underline{2}} \tilde{f}(\underline{2})$$

or

$$\langle \tilde{H}(1,\underline{2}) \rangle = - \int dv_2 \frac{\partial}{\partial \underline{2}} \langle \tilde{R}(1;2) \tilde{f}(3) \rangle \Big|_{\underline{3}=\underline{2}}. \quad (A5)$$

In Eq. (A5) there is no integration over the coordinates $\underline{2}$. This convention will be followed throughout when $\int dv_2$ appears explicitly.

To evaluate the correlation function in Eq. (A5), we write

$$\tilde{f} = \langle f \rangle + \delta \tilde{f}$$

so that

$$\langle \tilde{R}(1;2) \tilde{f}(3) \rangle = R(1;2) \langle f(3) \rangle + \langle \tilde{R}(1;2) \delta \tilde{f}(3) \rangle. \quad (A6)$$

We have

$$\langle \tilde{R}(1;2) \delta \tilde{f}(3) \rangle = G_3(1+, 2-, 3+) ,$$

where $G_3(+,-,+)$ is one of the triple correlation functions discussed by MSR. The dielectric function is thus expressed as

$$\epsilon^{-1}(\underline{1}, \underline{2}) = \delta(\underline{1}-\underline{2}) - \underline{\underline{E}}(\underline{1}, \underline{1}) \int d\underline{v}_2 \left[R(\underline{1}; \underline{2}) \underline{\partial}_2 \langle f(\underline{2}) \rangle + \underline{\partial}_3 G_3(\underline{1}+, \underline{2}-, \underline{3}+) \right]_{\underline{3}=\underline{2}} . \quad (A7)$$

The MSR formalism relates triple correlations G_3 to the vertex functions Γ . Suppressing spin indices, we have

$$G_3(1, 2, 3) = G(1, \bar{1}) G(2, \bar{2}) G(3, \bar{3}) \Gamma(\bar{1}, \bar{2}, \bar{3}) . \quad (A8)$$

Because the renormalized vertices can be determined self-consistently in terms of the observables $\langle f \rangle$, C , and R by means of renormalized perturbation theory,² Eq. (A8) together with Eq. (A7) gives in principle a complete description of the dielectric function in terms of $\langle f \rangle$, C , and R . To arbitrary order in renormalization, this procedure is highly implicit. However, in the most useful, lowest-order case, the DIA, the renormalized vertices are trivial and one can write an explicit expression for ϵ in terms of $\langle f \rangle$, C , and R .

In the DIA one has

$$\Gamma(1, 2, 3) \approx \gamma(1, 2, 3) . \quad (A9)$$

Using Eqs. (A8) and (A9), one then finds that

$$\begin{aligned} - \underline{\underline{E}}(\underline{1}, \underline{1}) \int d\underline{v}_2 \underline{\partial}_3 G_3(\underline{1}+, \underline{2}-, \underline{3}+) \Big|_{\underline{3}=\underline{2}} &= - \underline{\underline{E}}(\underline{1}, \underline{1}) \int d\underline{v}_2 [R(\underline{1}, \underline{1}) \underline{\partial}_2 C(2, \bar{3}) \\ &+ \underline{\partial}_2 R(2; \bar{1}) C(\underline{1}, \bar{3})] \bar{U}(\underline{1}, \bar{2}, \bar{3}) R(\bar{2}; 2) . \end{aligned} \quad (A10)$$

The second term in the brackets of Eq. (A10) vanishes because R is causal and $\bar{U}$ is local in time. Thus,

$$\begin{aligned} \epsilon^{-1}(\underline{1}, \underline{2}) = & \delta(\underline{1}-\underline{2}) \underline{1} - \underline{E}(\underline{1}, \underline{1}) \int d\underline{v}_2 R(\underline{1}; \underline{1}) [\delta(\underline{1}-\underline{2}) \underline{\partial}_2 \langle f(\underline{2}) \rangle \\ & + \bar{U}(\underline{1}, \underline{2}, \underline{3}) R(\underline{2}; \underline{2}) \underline{\partial}_2 C(\underline{2}, \underline{3})] \quad . \quad (A11) \end{aligned}$$

Equation (A11) is the desired expression for the dielectric function in the DIA. However, the form of Eq. (A11) may appear unfamiliar because it is couched in terms of the total response function R . This object appears naturally because it describes the motion of a shielded test particle. However, since the form of the shielding response is completely determined by the motion of the bare test particle and the dielectric properties of the medium, it is convenient to introduce the bare test particle propagator g as a fundamental object. To do this, consider the DIA for R :

$$\begin{aligned} g^{-1}(\underline{1}; \underline{1}) R(\underline{1}; \underline{1}') + \int d\underline{v}_2 \left[\delta(\underline{1}-\underline{2}) \underline{\partial}_2 \langle f(\underline{2}) \rangle + \bar{U}(\underline{1}, \underline{2}, \underline{3}) R(\underline{2}; \underline{2}) \underline{\partial}_2 C(\underline{3}, \underline{2}) \right] \\ \cdot \underline{E}(\underline{2}, \underline{1}) R(\underline{1}; \underline{1}') = \delta(\underline{1}-\underline{1}') \quad . \quad (A12) \end{aligned}$$

In writing Eq. (A12) we have collected and displayed explicitly all terms proportional to $\underline{E}R$. The renormalized test particle propagator g obeys

$$(\partial_t + \underline{v}_1 \cdot \underline{\nabla}_1) g(\underline{1}; \underline{1}') + \underline{\Gamma}'(\underline{1}, \underline{1}) g(\underline{1}; \underline{1}') = \delta(\underline{1}-\underline{1}') \quad ,$$

where

$$\underline{\Gamma}'(\underline{1}, \underline{1}) \equiv \bar{U}(\underline{1}, \underline{2}, \underline{3}) \langle \delta f(\underline{3}) \delta \underline{E}(\underline{2}) \rangle \cdot \underline{\partial}_2 \delta(\underline{2}-\underline{1}) \quad .$$

If we write the bracketed term in Eq. (A12) symbolically as $\partial \langle f \rangle + \partial \delta \bar{f} \equiv \partial \bar{f}$, then in operator notation Eq. (A12) becomes

$$R = g - g \partial \bar{f} \cdot \bar{\epsilon} R, \quad (A13)$$

$$\bar{\epsilon} R = (\underline{1} + \bar{\epsilon} g \partial \bar{f})^{-1} \cdot \bar{\epsilon} g, \quad (A14)$$

and Eq. (A11) can be rewritten as

$$\bar{\epsilon}^{-1} = \underline{1} - \bar{\epsilon} R \partial \bar{f}. \quad (A15)$$

Using expression (A14) in Eq. (A15), one verifies readily that

$$\bar{\epsilon} = \underline{1} + \bar{\epsilon} g \partial \bar{f}, \quad (A16)$$

so that, from Eqs. (A13) and (A14),

$$R = g - g \partial \bar{f} \cdot \bar{\epsilon}^{-1} \cdot \bar{\epsilon} g \quad (A17)$$

$$\bar{\epsilon} R = \bar{\epsilon}^{-1} \cdot \bar{\epsilon} g. \quad (A18)$$

The dielectric scalar, $\epsilon \equiv \text{Tr} \bar{\epsilon} - 2$, is explicitly

$$\epsilon(\underline{1}, \underline{2}) = \delta(\underline{1} - \underline{2}) + \text{Tr} \bar{\epsilon}(\underline{1}, \underline{1}) g(\underline{1}; \underline{1}) \left\{ \delta(\underline{1} - \underline{2}) \partial_{\underline{1}} \langle f(\underline{1}) \rangle + \int d\underline{v}_2 \bar{U}(\underline{1}, \underline{2}, \underline{3}) R(\underline{2}; \underline{2}) \partial_{\underline{2}} C(\underline{2}, \underline{3}) \right\}.$$

In the statistically steady state, $\bar{\epsilon}$ may be Fourier transformed in the space and time difference variables:

$$\bar{\epsilon}(\underline{k}) = (\underline{1} - \hat{k} \hat{k}) + \epsilon(\underline{k}) \hat{k} \hat{k}.$$

Of course, only the term proportional to $\hat{k}\hat{k}$ contributes in expressions such as Eqs. (A17) or (A18); in such expressions, the tensor $\underline{\epsilon}$ may be replaced by the scalar ϵ .

It is instructive to consider how these results would be modified if we considered, instead of the self-consistent Vlasov equation (A2), the so-called stochastic acceleration problem in which Poisson's equation is not imposed and $\tilde{E}$ is taken to be a statistically specified random variable. (The field $\tilde{E}$ now describes the effects of some external bath.) In this case $\tilde{E}$ does not participate in the functional variation with respect to $\tilde{E}^{(e)}$ indicated in Eq. (A3), so the second term of Eq. (A4) disappears. Also, the symmetry indicated by Eq. (5) is lost, as is the second term in brackets in Eq. (A12). The result has the form of Eqs. (A16) - (A18), but with the $\delta\bar{f}$ term missing and with the collision operator Σ defined with only the explicit term of Eq. (5). This theory is (the non-Markovian generalization of) Dupree's resonance broadening approximation.¹⁵ As is well known, this theory does not conserve energy (as it need not for interactions with an external bath) and does not reduce correctly to weak turbulence theory.

Table 1

Diffusion Processes

	$\hat{E}_{-q}$	$\hat{f}_{k-q}$
$\Sigma^{(d)}$	$\delta E_{-q}^{(1)}$	$-g_{k-q} \delta E_{-q}^{(1)} \cdot \partial \Delta f_k^{(0)}$
$\delta \bar{f}^{(d)}$	$\delta E_{-q}^{(1)}$	$-g_{k-q} \Delta E_k^{(0)} \cdot \partial \delta f_{-q}^{(1)}$

Polarization Processes

	$\hat{E}_{k-q}$	$\hat{f}_q$
$\Sigma^{(p)}$	$\epsilon_{k-q}^{-1} \phi_{k-q} (g_{k-q} \delta E_{-q}^{(1)} \cdot \partial \Delta f_k^{(0)})$	$\delta f_q^{(1)}$
$\delta \bar{f}^{(p)}$	$\epsilon_{k-q}^{-1} \phi_{k-q} (g_{k-q} \Delta E_k^{(0)} \cdot \partial \delta f_{-q}^{(1)})$	$\delta f_q^{(1)}$

Summary of the second-order beat processes involved in the renormalization of the test particle propagator R : $\partial_t \Delta f_k^{(2)} \sim \sum_q \hat{E}_{k-q} \cdot \partial \hat{f}_q$. The background particle fluctuation $\delta f_q^{(1)} \equiv -g_q \delta E_{-q}^{(1)} \cdot \partial \langle f \rangle$.

REFERENCES

- ¹S.A. Orszag and R.H. Kraichnan, Phys. Fluids 10, 1720 (1967).
- ²P.C. Martin, E.D. Siggia, and H.A. Rose, Phys. Rev. A8, 423 (1973).
- ³R.H. Kraichnan, J. Math. Phys. 2, 124 (1961).
- ⁴D.F. Dubois and M. Espedal (to appear in Plasma Phys. 1978).
- ⁵A.A. Vedenov, E.P. Velikhov, and R.Z. Sagdeev, Nucl. Fusion 1, 82 (1961); W.E. Drummond and D. Pines, Nucl. Fusion Suppl. Pt. 3, 1049 (1962).
- ⁶R.Z. Sagdeev and A.A. Galeev, Nonlinear Plasma Theory, edited by T.M. O'Neill and D.L. Book (Benjamin, New York, 1969), Chap. 2.
- ⁷A.N. Kaufman, J. Plasma Phys. 8, 1 (1972).
- ⁸R.Z. Sagdeev and A.A. Galeev, ibid., Chaps. 1 and 3.
- ⁹B.B. Kadomtsev, Plasma Turbulence (Academic Press, New York, 1965); V.N. Tsytovich, Theory of Turbulent Plasma, translated by D.L. Burdick (Consultants Bureau, New York, 1977).
- ¹⁰G.M. Zaslavskii and N.N. Filonenko, Zh. Eksp. Teor. Fiz. 54, 1590 (1968) [Sov. Phys.-JETP 27, 851 (1968)].
- ¹¹G.M. Zaslavskii and N.N. Filonenko, Usp. Fiz. Nauk. 105, 3 (1971) [Sov. Phys.-Usp. 14, 549 (1972)].
- ¹²J.A. Krommes, C. Oberman, and R.C. Kleva, "Stochastic Instability and Quasilinear Theory in a Discrete Spectrum" (in preparation).
- ¹³V.N. Tsytovich, Usp. Fiz. Nauk. 108, 143 (1972) [Sov. Phys.-Usp. 15, 632 (1973)].

- ¹⁴V.N. Tsytovich, Nonlinear Effects in Plasma, translated by J.S. Wood, edited by S.M. Hamberger (Plenum Press, New York, 1970).
- ¹⁵T.H. Dupree, Phys. Fluids 9, 1773 (1966).
- ¹⁶A.A. Galeev, Phys. Fluids 10, 1041 (1967).
- ¹⁷G. Benford and J.J. Thomson, Phys. Fluids 15, 1496 (1972).
- ¹⁸L.M. Al'tshul' and V.I. Karpman, Zh. Eksp. Teor. Fiz. 49, 515 (1965) [Sov. Phys.-JETP 22, 361 (1966)].
- ¹⁹L.I. Rudakov and V.N. Tsytovich, Plasma Phys. 13, 213 (1971).
- ²⁰V.G. Makhankov and V.N. Tsytovich, Nucl. Fusion 10, 405 (1970).
- ²¹C. DeDominicis and P.C. Martin, J. Math. Phys. 5, 14 (1964).
- ²²The equations are exact only for Gaussian initial conditions. Rose has discussed more general cases: a) H.A. Rose, Ph.D. Thesis (Harvard, 1974) (unpublished); b) H.A. Rose, LA-UR-78-1902, 1978 (submitted to J. Stat. Phys.).
- ²³J.A. Krommes, in Proceedings of the Third Kiev Theory Conference (IAEA, Vienna) (to be published).
- ²⁴R.C. Davidson, Methods in Nonlinear Plasma Theory (Academic Press, New York, 1972), Chap. 7.
- ²⁵R. Phythian, J. Phys. A8, 1423 (1975).
- ²⁶H.R. Thompson and J.A. Krommes, Bull. Am. Phys. Soc. 22, 1150 (1977).
- ²⁷R.H. Kraichnan, Fluid Mech. 5, 497 (1959).

²⁸A.A. Galeev, Zh. Eksp. Teor. Fiz. 1361 (1969) [Sov. Phys.-JETP 30, 737 (1970)].

²⁹J.A. Krommes, R.G. Kleva, and C. Oberman, Princeton University Report No. PPPL-1389, 1978 (submitted to J. Plasma Physics).

³⁰J.A. Krommes (in preparation).

³¹T.H. Dupree and D.J. Tetreault, Phys. Fluids 21, 425 (1978).

³²P.J. Catto, Phys. Fluids 21, 147 (1978).

EXTERNAL DISTRIBUTION IN ADDITION TO TIC UC-20

ALL CATEGORIES

R. Askew, Auburn University, Alabama
 S. T. Wu, Univ. of Alabama
 Geophysical Institute, Univ. of Alaska
 G.L. Johnston, Sonoma State Univ., California
 H. H. Kuehl, Univ. of S. California
 Institute for Energy Studies, Stanford University
 H. D. Campbell, University of Florida
 N. L. Oleson, University of South Florida
 W. M. Stacey, Georgia Institute of Technology
 Benjamin Ma, Iowa State University
 Magne Kristiansen, Texas Tech. University
 W. L. Wiese, Nat'l Bureau of Standards, Wash., D.C.
 Australian National University, Canberra
 C.N. Watson-Munro, Univ. of Sydney, Australia
 F. Cap, Inst. for Theo. Physics, Austria
 Ecole Royale Militaire, Bruxelles, Belgium
 D. Paulunbo, Comm. of Eur. Comm., Belgium
 P.H. Sakanaka, Instituto de Fisica, Campinas, Brazil
 C. R. James, University of Alberta, Canada
 T.W. Johnston, INRS-Energie, Varennes, Quebec
 H. M. Skarsgard, Univ. of Saskatchewan, Canada
 Librarian, Culham Laboratory, Abingdon, England
 A.M. Dupas Library, C.E.N.-G, Grenoble, France
 Central Res. Inst. for Physics, Hungary
 R. Shingal, Meerut College, India
 M. Naraghi, Atomic Energy Org. of Iran
 Biblioteca, Frascati, Italy
 Biblioteca, Milano, Italy
 G. Rostagni, Univ. Di Padova, Padova, Italy
 Preprint Library, Inst. de Fisica, Pisa, Italy
 Library, Plasma Physics Lab., Gokasho, Uji, Japan
 S. Mori, Japan Atomic Energy Res. Inst., Tokai-Mura
 Research Information Center, Nagoya Univ., Japan
 S. Shioda, Tokyo Inst. of Tech., Japan
 Inst. of Space & Aero. Sci., Univ. of Tokyo
 T. Uchida, Univ. of Tokyo, Japan
 H. Yamato, Toshiba R. & D. Center, Japan
 N. Yajima, Kyushu Univ., Japan
 R. England, Univ. Nacional Auto-noma de Mexico
 B. S. Liley, Univ. of Waikato, New Zealand
 S. A. Moss, Saab Univas Norge, Norway
 J.A.C. Cabral, Univ. de Lisboa, Portugal
 O. Petrus, A.L.I. CUZA Univ., Romania
 J. de Villiers, Atomic Energy Bd., South Africa
 A. Maurech, Comisaria De La Energy y Recursos
 Minerales, Spain
 Library, Royal Institute of Technology, Sweden
 Cen. de Res. En Phys.Des Plasmas, Switzerland
 Librarian, Fom-Instituut Voor Plasma-Fysica, The
 Netherlands
 V. E. Golant, A.F. Ioffe Physical-Tech. Inst., USSR
 B.B. Kadomtsev, Kurchatov Inst. of Atomic Energy,
 USSR
 The Kharkov Physical-Tech. Inst., USSR
 M. S. Rabinovich, Academy of Sci, USSR
 Bibliothek, Stuttgart, West Germany
 R.D. Buhler, Univ. of Stuttgart, West Germany
 Max-Planck-Inst. fur Plasmaphysik, W. Germany
 Nucl. Res. Estab., Julich, West Germany
 K. Schindler, Inst. Fur Theo. Physik, W. Germany

EXPERIMENTAL THEORETICAL

M. H. Brennan, Flinders Univ. Australia
 H. Barnard, Univ. of British Columbia, Canada
 S. Screenivasan, Univ. of Calgary, Canada
 J. Radet, C.E.N.-B.P., Fontenay-aux-Roses, France
 Prof. Schatzman, Observatoire de Nice, France
 S. C. Sharma, Univ. of Cape Coast, Ghana
 R. N. Aiyer, Laser Section, India
 B. Buti, Physical Res. Lab., India
 L. K. Chavda, S. Gujarat Univ., India
 I.M. Las Das, Banaras Hindu Univ., India
 S. Cuperman, Tel Aviv Univ., Israel
 E. Greenspan, Nuc. Res. Center, Israel
 P. Rosenau, Israel Inst. of Tech., Israel
 Int'l. Center for Theo. Physics, Trieste, Italy
 I. Kawakami, Nihon University, Japan
 T. Nakayama, Ritsumeikan Univ., Japan
 S. Nagao, Tohoku Univ., Japan
 J.I. Sakai, Toyama Univ., Japan
 S. Tjotta, Univ. i Bergen, Norway
 M.A. Hellberg, Univ. of Natal, South Africa
 H. Wilhelmson, Chalmers Univ. of Tech., Sweden
 Astro. Inst., Sonnenborgh Obs., The Netherlands
 N.G. Tsintsadze, Academy of Sci GSSR, USSR
 T. J. Boyd, Univ. College of North Wales
 K. Hubner, Univ. Heidelberg, W. Germany
 H. J. Kaeppler, Univ. of Stuttgart, West Germany
 K. H. Spatschek, Univ. Essen, West Germany

EXPERIMENTAL ENGINEERING

B. Grik, Univ. de Quebec, Canada
 P. Lukac, Komenského Univ., Czechoslovakia
 G. Horikoshi, Nat'l Lab for High Energy Physics,
 Tsukuba-Gun, Japan
 V. A. Glukhikh, D.V. Efremov Sci.
 Res. Instit. of Elect. App., USSR

EXPERIMENTAL

F. J. Paoloni, Univ. of Wollongong, Australia
 J. Kistemaker, Fom Inst. for Atomic
 & Molec. Physics, The Netherlands

THEORETICAL

F. Verheest, Inst. Vor Theo. Mech., Belgium
 J. Teichmann, Univ. of Montreal, Canada
 T. Kahan, Univ. Paris VII, France
 R. K. Chhajlani, India
 S. K. Trehan, Panjab Univ., India
 T. Namikawa, Osaka City Univ., Japan
 H. Narumi, Univ. of Hiroshima, Japan
 Korea Atomic Energy Res. Inst., Korea
 E. T. Karlson, Uppsala Univ., Sweden
 L. Stenflo, Univ. of UMEA, Sweden
 J. R. Saraf, New Univ., United Kingdom