Measurement of the Spin of a Particle Using Undulator Radiation*

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Abstract
Use of many period wigglers to rotate the spin of particles in accelerators has been proposed [1]. An added advantage of this scheme is that a spectrum of synchrotron radiation will be produced that contains a contribution due to the spin, so that the device can be effectively used as spin polarimeter, with advantages compared to Compton backscattering methods. Results are presented for two high energy proton storage rings: RHIC and the SSC.

I. INTRODUCTION

We have proposed to build spin rotators with two transverse wigglers of many poles with fields perpendicular to each other, longitudinally shifted. Wiggler spin rotators can also be used as beam diagnostic tools, allowing measurements of the beam size and spin polarization from observation of the synchrotron radiation spectrum. Beam diagnostics can be done with the SR spectral continuum from bending magnets, however the spectrum of wigglers being the result of interference presents a characteristic line structure and therefore has a higher specific brightness.

Spin polarization can be measured by Compton backscattering of laser light by the particle beam. The physical mechanism of photon production by wiggler radiation and backscattering is the same, and lead to comparable results. However, also laser backscattering produces a continuum and tR in the integral of Eq. (4) and apply Parseval's Theorem backscattering of laser light by the particle beam. The physical mechanisms of photon production by wiggler radiation and backscattering are lost.

II. ELECTRIC AND MAGNETIC DIPOLE SYNCHROTRON RADIATION

A particle of mass $m$ and charge $e$ is accelerated in a magnetic field $B$. The frame of reference is shown in figure 1. The preferred direction of motion is along $z$ ($k$). The radial constant, in Eq.(3), the particle velocity $\beta$ and it's time derivative are calculated from the equation of motion

$$\frac{d\mathbf{p}}{dt} = \mathbf{E} \times \mathbf{B} ; \quad \frac{d\mathbf{R}}{dt} = R^2 \frac{E^2}{\mu_0 c} ,$$

(1)

with $R$ the distance between charge and field point. If we only retain the "radiation" term in the expression for the field E (i.e. the term $\approx 1/R$), it is

$$RE = \frac{\mu_0 e c |\mathbf{A}(t)|}{4\pi} ; \quad 4\pi \frac{d\mathbf{P}}{d\Omega} = \frac{\mu_0}{4\pi} e^2 c |\mathbf{A}(t)|^2,$$

(2)

$$\mathbf{n} \times \left( \mathbf{n} - \beta \right) \times \frac{d\mathbf{\beta}}{dt} ,$$

$$\mathbf{A} = \frac{\mathbf{n} \times \left( \mathbf{n} - \beta \right) \times \frac{d\mathbf{\beta}}{dt}}{\left[ 1 - \mathbf{n} \cdot \beta \right]^3} ,$$

(3)

where $\mathbf{A}$ is the vector potential,

Fig. 1. The unit vector $n$ points towards the observation point.

and the energy radiated by the particle in a time interval $\Delta t$ is

$$4\pi \frac{dW}{d\Omega} = \frac{\mu_0}{4\pi} e^2 c \int |\mathbf{A}(t)|^2 dt .$$

(4)

To calculate the spectrum, replace $t$ with the retarded time $t_R$ in the integral of Eq. (4) and apply Parseval's Theorem, then, introducing the source current $I [amp]$, obtain for the number of photons emitted per second, per unit solid angle and per unit photon energy interval

$$\frac{d^2 n}{d\Omega d(\hbar \omega)} = \frac{\alpha d}{2\pi e} |\mathbf{A}(\omega)|^2$$

(5)

($\alpha$ is the fine structure constant).

If we assume that the energy of the particle remains constant, in Eq.(3), the particle velocity $\beta$ and its time derivative are calculated from the equation of motion

$$\frac{d\mathbf{\beta}}{dt} = \beta \times \Omega ; \quad \Omega = \frac{eB}{m \gamma} .$$

(6)

In far field (Fraunhofer), the radiation will be calculated along the direction defined by the unitary vector $\mathbf{n}$, with angles $\theta$ and $\phi$. Vector potential components $A_p$, parallel to the horizontal plane and perpendicular to $\mathbf{n}$, and $A_\sigma$, parallel to $\mathbf{n}$ and to $A_p$ are

$$A_p = A \cdot [j \times \mathbf{n}] ; \quad A_\sigma = A \cdot \left( [j \times \mathbf{n}] \times \mathbf{n} \right) .$$

(7)

Explicit expressions are of the form

$$A_p = \frac{1}{D^3} \mathbf{P} \cdot \Omega ; \quad A_\sigma = \frac{1}{D^3} \mathbf{S} \cdot \Omega$$

(8)

with $\mathbf{P}$ and $\mathbf{S}$ some vector expressions, and

$$D = 1 - \mathbf{n} \cdot \beta .$$

(9)

The "usual" synchrotron radiation is the radiation of an oscillating electric dipole in the external magnetic field. If the
particle has a magnetic moment \( \mu \) proportional to the spin \( s \) (in units \( \pm 1/2 \))
\[
\mu = \frac{ge\hbar}{mc^2} s
\]
where \( s \) is the particle spin in the particle rest frame (PRF), magnetic dipole radiation will appear.
In the LAB frame, the spin transforms as follows
\[
S = s + \frac{\gamma^2}{\gamma + 1} (\beta \cdot s) \beta.
\]

The particle spin, in its precession in an external magnetic field, adds to the radiation field of Eq. (3) a contribution
\[
A^{(\sigma)}(t) = \frac{1}{(1 - \beta \cdot n)dt} \left[ n \times (S + n \times (\beta \times S)) \right] \tag{12}
\]
and the total radiation field will be
\[
A^{(T)}(t) = A(t) + \eta A^{(\sigma)}(t),
\]
where \( A(t) \) is the radiation field of Eq. (3) and \( \eta \) is the contribution to the radiation field of Eq. (3) by the particle spin.

Performing the time derivative in Eq. (12) and find
\[
A^{(T)}(t) = d\left( (\beta \cdot s)n \times \beta + (\beta \cdot s)n \times \beta \right) + \frac{d}{dt} \left[ (\beta \cdot s)n \times \beta \right] \tag{15}
\]
where the first term contains the spin and the second its time derivative.

The spin \( s \) precesses in an external magnetic field, according to the BMT equation
\[
\frac{ds}{dt} = C_1 s + \Omega + C_2 (\beta \cdot \Omega)(s \times \beta) \tag{16}
\]
where \( C_1 = 1 + G\gamma \) and \( C_2 = -\frac{G\gamma^2}{1 + \gamma} \) with \( G = g - 2 \).

If only a transverse magnetic field (\( \Omega_x = 0, \Omega_y = 0 \)) is present and the beam has no emittance, for observation on axis
\[
A_p = B\Omega_y, \quad A_{\sigma} = 0
\]
\[
A^{(\sigma)} = 0, \quad A^{(\sigma)} = -\frac{d}{dt} \left[ C_1 \theta_b s_\perp + s_\perp \right] \Omega_y \tag{18}
\]
with the positions
\[
F = 1 + \gamma^2(\Delta \theta^2 + \Delta \phi^2) \quad \Delta \theta = \theta - \theta_b
\]
\[
B = 1 - \gamma^2(\Delta \theta^2 - \Delta \phi^2) \quad \Delta \phi = \phi - \phi_b
\]

Eq. (18) shows that on axis the spin dependent radiation is polarized in a direction parallel to the magnetic field.

The spectrum is obtained by a Fourier transform of the vector potential of Eq. (18). In an undulator with \( N \) periods (lines of width \( 1/N \)), write the magnetic field as
\[
\Omega_0 = \Omega_0 \sin \omega_0 t; \quad \omega_0 = \frac{2\pi c}{\lambda_0},
\]
with \( \lambda_0 \) the undulator period. In this field, an approximate expression for the instantaneous angle of the trajectory is
\[
\theta_b = \frac{k}{\gamma} \sin \omega_0 t; \quad k = \frac{e}{2\pi mc} \lambda_0 \theta_0 \Omega_0 \tag{21}
\]

The first Eq. (18) yields
\[
A'_p(t) = \left( 1 - \gamma^2 \theta_b^2 \right) \Omega = \left( 1 - k^2 \sin^2 \omega_0 t \right) \sin \omega_0 t \sin \omega_0 t \tag{22}
\]
showing that the spin independent radiation field only contains odd harmonics on axis (in this simplification, only 1 and 3).

The fourth Eq. (13) for the spin dependent radiation field is
\[
A^{(\sigma)}(t) = -\left( 1 + \gamma^2 \theta_b^2 \right) \left[ C_1 \theta_b s_\perp + s_\perp \right] \Omega_y \tag{23}
\]

After integration of the BMT, e.g. if we assume example that the beam is totally \( x \) polarized at the entrance in the magnetic field, we obtain an expression also containing only odd harmonics.

A first competitor that can make the observation of the spin dependent radiation difficult is the finite emittance of the beam. In this case, a contribution \( A_{\sigma} \) also appears on axis, but in the even spectral harmonics. The ratio is
\[
\frac{A^{(\sigma)}}{A^{(\sigma)}} = \frac{2\eta}{(1 + \gamma^2)} \frac{\beta_x}{\gamma \varepsilon_y} \tag{24}
\]
with \( \varepsilon_x \) and \( \beta_y \) are the emittance and the twiss function. This may impose a limit on beam emittance, and shows that it is convenient to make the beam vertically parallel and wide.

Another competing effect in the odd harmonics and in the \( \sigma \) polarization is due to undulator field imperfections, since a small \( \Omega_0 \) field residual component with the same periodicity of the main field will again produce an unwanted \( A_{\sigma} \) on axis. A signal to noise ratio of the order of one is obtained when
\[
\frac{d\Omega}{\Omega} = \frac{\gamma}{4 \varepsilon \Omega} \tag{25}
\]

This is of the order of \( 10^{-5} \) for protons, thus to measure the proton spin the contribution to the field errors in phase with the field should be very small. Using the random walk argument through \( N \) periods [3], we find an upper limit of the error in phase of the order of
\[
\left( \frac{\Delta \Omega}{\Omega} \right)_{\text{in phase}} = \frac{1}{\sqrt{N}} \left( \frac{\Delta \Omega}{\Omega} \right)_{\text{random}} \quad \tag{26}
\]

III. COMPTON BACK SCATTERING

The fundamental frequency of undulator radiation is
\[
\omega_1 = \omega_0 \left( 1 - \frac{1}{2} k^2 + \gamma^2 (\Delta \theta^2 + \Delta \phi^2) \right) \tag{27}
\]

Compare this with the radiation obtained by scattering of laser light by a charged particle beam (head-on collisions)
\[
\hbar \omega_s \equiv \hbar \omega_L \left( 1 + 2(1 + \beta) \gamma^2 \frac{\hbar \omega_L}{mc^2} + \gamma^2 \theta_b^2 \right) \tag{28}
\]
with \( \omega_s \) and \( \omega_L \), the frequency of the scattered and laser radiation, and \( \theta_b \) the angle of the back scattered photon.

Let us treat the radiation from an undulator as a scattering process of a photon and a charged particle. In the PRF, the
The undulator is seen as an incoming e.m. wave, since Relativity builds up an electric field from the static magnetic field of the undulator. (Note that this equivalent wave does not travel at the speed of light, but at the lower speed \( \beta c \).)

The intensity of the back scattered radiation is related to the luminosity of the process as

\[ n_s = \frac{N_p N_L}{\Sigma \sigma} \]

with \( n_s \) the number of back scattered photons per unit time, \( N_p \) and \( N_L \) the number of particles and primary photons contained in the volume of interaction, \( \Sigma \) the common cross section of the particle and laser beams, \( \sigma \) the frequency of encounters, and \( \sigma \) the scattering cross section.

The radiation cone has a half-aperture defined by

\[ \gamma \theta \approx 1 \]

If the particles have a spin, the scattering cross section can be written as follows [4]

\[ \sigma = \sigma_0 \pm \sigma P_L P_p \cos \phi \]

(31)

where \( \sigma_0 \) is the scattering cs for the unpolarized beam, \( \sigma \) the spin dependent scattering cs, \( P_L \) and \( P_p \) the particle spin and \( \phi \) the scattering angle relative to \( P_p \).

It can be shown that the unpolarized cross section is symmetric about the axis of scattering with a maximum there

\[ \sigma_0 \mid_{\text{max}} = 2 r_0^2 \]

\[ r_0 = \frac{\mu_0 c^2}{4 \pi m} \]

(32)

(\( r_0 \) the classical radius of the proton), and that the polarized cross section is anti symmetric about the axis of scattering with maxima at angles determined by

\[ \gamma \theta \approx \frac{1}{\sqrt{2}} \]

(33)

The ratio between maximum polarized cross section to maximum unpolarized is

\[ \frac{\sigma_L \mid_{\text{max}}}{\sigma_0 \mid_{\text{max}}} = \frac{3 \sqrt{3} \gamma}{4} \frac{\hbar \omega_k}{mc^2} \]

(34)

A comparison of the \( \xi \) parameter of Eq. (34) for laser scattering with the \( \eta \) parameter of Eq. (14) shows that the relative intensity of the polarized to unpolarized radiation is of the same order, as it should.

In the scattering of virtual undulator photons, the number of primary photons to enter in Eq. (29) can be calculated by dividing the magnetic energy by the energy of a photon

\[ \frac{eE \gamma B_0^2 V}{2 \hbar \omega_0} \]

(35)

with \( V \) the volume of the particle beam bunch. Both for undulator radiation and BS, the intensity of the observed radiation is inversely proportional to the square of the mass of the particle.

### IV. CONCLUSIONS

The spin polarization of the beam can be measured by synchrotron radiation or by Compton backscattering of laser light. The physical principles are similar for the two modes, with some important differences.

Undulator radiation shows the line structure of a diffraction pattern, with characteristic polarization properties.

Compton BS produces a wide spectrum of radiation, correlated to the observation angle. If the undulator radiation is interpreted as a scattering of virtual photons, the expression for the luminosity shows that the number of undulator photons in the interaction region can be very large, since each photon (of frequency \( \omega_0 \)) is much smaller than a laser photon (of frequency \( \omega_e \)).

Tables 1 and 2 show the order of magnitude of some of the quantities in selected machines. We have considered two proton machines: RHIC and SSC.

### Table 1. Undulator and Laser Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RHIC</th>
<th>SSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_0 ) [eV]</td>
<td>6.20 ( \pm ) 1</td>
<td>2.33</td>
</tr>
<tr>
<td>( r_0^2 ) [m(^2)]</td>
<td>2.36 ( \times ) 10(^{-36})</td>
<td>2.36 ( \times ) 10(^{-36})</td>
</tr>
</tbody>
</table>

### Table 2. Proton Colliders.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RHIC</th>
<th>SSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma ) (= GeV)</td>
<td>2.33</td>
<td>2.33</td>
</tr>
<tr>
<td>emittance ( 10^{-9} \pi \text{ m-rad} )</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>current ( \text{mrad} )</td>
<td>10(^{11})/bunch</td>
<td>7.3 ( \times ) 10(^{9})/bunch</td>
</tr>
<tr>
<td>beta ( \text{m} )</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>undulator radiation ( \hbar \omega_1 ) [eV]</td>
<td>4.4 (5th harm)</td>
<td>5.6 ( \times ) 10(^{3})</td>
</tr>
<tr>
<td>( \frac{d^2n}{d\theta d\phi} ) [1/sec-rad(^2)]</td>
<td>6.5 ( \times ) 10(^{22})</td>
<td>4.1 ( \times ) 10(^{26})</td>
</tr>
<tr>
<td>( \eta ), Eq. (31)</td>
<td>1.8 ( \times ) 10(^{-6})</td>
<td>0.17</td>
</tr>
<tr>
<td>backscattering ( \hbar \omega_s ), max</td>
<td>66 MeV</td>
<td>4.2 GeV</td>
</tr>
<tr>
<td>( ns ) [1/sec]</td>
<td>0.030</td>
<td>7.2 ( \times ) 10(^{3})</td>
</tr>
<tr>
<td>( \xi ), Eq. (81)</td>
<td>2.6 ( \times ) 10(^{-6})</td>
<td>2.1 ( \times ) 10(^{-4})</td>
</tr>
</tbody>
</table>

### IV. REFERENCES

[1] A.Luccio, M.Conte, these Proceedings
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