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IDEAL MHD BETA-LIMITS OF
POLOIDALLY ASYMMETRIC EQUILIBRIA

BY

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Ideal MHD Beta-Limits of Poloidally Asymmetric Equilibria

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Abstract

The ideal MHD stability of poloidally asymmetric equilibria, which are typical of a tokamak reactor design with a single-null poloidal divertor is examined. As with symmetric equilibria, stability to non-axisymmetric modes improves with increasing triangularity and ellipticity, and with lower edge safety factor. Pressure profiles optimized with respect to ballooning stability are obtained for an asymmetric shape, resulting in $\beta_{\text{critical}} \approx 5.7\%$. The corresponding value for an equivalent symmetric shape is $\beta_{\text{critical}} \approx 6.5\%$.

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1. Introduction

Tokamak configurations with a single off-axis poloidal divertor, as opposed to the usual double-null divertor, have recently received serious consideration [1]. With a single-null it is possible to utilize more of the available volume inside the toroidal field coils, or to reduce the required toroidal magnetic field energy in the system. In addition, assuming a magnetic divertor is required, elimination of one of the complex and expensive divertor systems will lead to considerable simplification of the mechanical structure and significant cost reduction for a reactor or next generation tokamak.

A single-null divertor creates poloidal symmetry, but recent PDX and D-III experiments [2,3] have demonstrated that asymmetric plasma equilibria can be maintained with typical machine parameters. In this paper we describe an investigation into the ideal MHD stability of parameterized asymmetric equilibria. Our aim is to demonstrate the existence of asymmetric equilibria whose ideal MHD beta-limits are comparable to established symmetric limits.

There have been several detailed numerical studies of ideal MHD stability in symmetric tokamaks over the last few years [4-8]. These studies have shown that stability to non-axisymmetric modes is improved for shapes with increasing ellipticity and triangularity. Stability also improves with decreasing values of the plasma edge safety factor q_e . Recently, studies of asymmetric equilibria [9] and stability [10] have established a basis for the present investigation.

In Sec. 2 of this paper, we discuss the results of a systematic survey of the effect of parameterized asymmetry on ideal MHD stability. In Sec. 3, we select a particular asymmetric shape and investigate the dependence of

ballooning mode stability on the safety factor and pressure profiles. A procedure is described which constructs pressure profiles that are marginally stable to ballooning at all flux surfaces in the plasma. The results for the asymmetric and an equivalent symmetric shape are compared. In Sec. 4, we assess the significance of the results of the two preceding sections. As a result of the present study, we conclude that ballooning-stable values of $\beta \equiv 2\mu_0 \langle p \rangle / B_0^2 \gtrsim 5\%$ can be supported in significantly asymmetric equilibrium configurations. Here $\langle p \rangle$ is the volume averaged plasma pressure, and B_0 is the vacuum toroidal magnetic field at the geometric plasma center, X_0 .

2. Ideal MHD Stability of Asymmetric Equilibria

The study of the stability properties of tokamaks is complicated by the number of equilibrium quantities and their relation to each other through the nonlinear Grad-Shafranov equilibrium equation. In judging the relative stability merits of different tokamak designs and operating regimes, it is particularly important to isolate the various effects in a consistent and controllable manner [6]. Thus, although it may be more realistic to study free boundary equilibria [9], stability studies of the effect of plasma shaping are best carried out by specifying the shape of the plasma boundary analytically, and then considering sequences of equilibria with identical plasma pressure and safety factor profiles.

Several considerations affected our choice of boundary shape. To enable a direct comparison with equivalent double-null equilibria, we wished to choose a parameterization which encompassed previously studied symmetric

configurations [6]. In addition we wished to complement the work of Ref. [10] which demonstrated that a rigid rotation of symmetric shapes about their principal axes was detrimental to MHD stability. Thus, our parameterization attempts to minimize rotation.

The plasma boundary in our cylindrical (X, ϕ, Z) coordinate system is defined in terms of a parametric angle α as

$$\begin{aligned} X &= X_0 + a \cos [\alpha + (\delta_1 + \delta_2) \sin \alpha + (\delta_1 - \delta_2) \sin^2 \alpha] / 2 \\ Z &= Z_0 + a \sin \alpha [\kappa_1 + \kappa_2 + (\kappa_1 - \kappa_2) \sin \alpha] / 2 \end{aligned} \quad (1)$$

Here (X_0, Z_0) is the nominal magnetic axis, a is the nominal minor radius, κ_1 and κ_2 respectively define an upper and lower ellipticity, and δ_1 and δ_2 an upper and lower triangularity. When $\kappa_1 = \kappa_2$, $\delta_1 = \delta_2$, the formulae of Ref. [6] for symmetric shapes are recovered.

Twelve shapes considered in this study are shown in Fig. 1. For the ten cases (shapes 1-10) whose results are presented in this section, $X_0 = 5.2\text{m}$, $\delta_1 = 0.3$, $\kappa_1 = 1.6$, while δ_2 and κ_2 are varied. Thus, near the upper 'x' point ($\alpha = \pi/2$), the plasma shape is conserved for all ten cases, with the lower ($Z < 0$) boundary varying significantly. The total plasma volume is conserved between shapes by adjusting the minor radius a , with $a = 1.3\text{ m}$ for the "equivalent" symmetric shape 7. Shapes 11 and 12 (the INTOR configuration of Ref. [1]) which are used in Sec. 3, belong to a family where $\delta_2 = 0.5$, $\kappa_2 = 1.6$, and δ_1 and κ_1 vary. Shape 10 is also included in this second family.

Solutions for the poloidal magnetic flux Ψ , which satisfy the Grad-Shafranov equilibrium equation

$$\Delta^* \Psi = 2\pi X J_\phi = -4\pi^2 \left(X^2 \frac{dp}{d\Psi} + X_O^2 g \frac{dg}{d\Psi} \right) \quad (2)$$

are obtained using the PPPL axisymmetric equilibrium code [11]. The pressure profile is defined by

$$\frac{dp(\Psi)}{d\Psi} = \beta_J \{ P_1 + P_2 \tilde{\Psi} - (P_1 + P_2) \tilde{\Psi}^{\gamma} \} \quad (3)$$

where $\tilde{\Psi} = (\Psi - \Psi_O) / (\Psi_e - \Psi_O)$, and Ψ_O , Ψ_e are the values of the poloidal flux at the magnetic axis and the plasma edge. In this section, $P_2/P_1 = -0.9$ with P_1 negative, $\gamma = 4$, and a safety factor profile with $q_O = 1$ and $q_e = 3.58$ are used. These profiles, hereafter referred to as p^a and q^a , are shown in Fig. 7, and q^a is determined in the following manner. Using INTOR values for the vacuum toroidal field $B_O = 5.5T$, and the toroidal plasma current $I_O = 5.5MA$, $\beta_J = 1$ and $dg/d\Psi = 0$ in shape 7, the ratio P_2/P_1 was determined to give $q_O = 1$, with the absolute magnitude of P_1 chosen to match I_O . The resultant q^a is very similar to the base safety factor profile of Ref. [6]. Although it has been shown [6,7] that values of q_O which are greater than unity lead to improved stability, experimental evidence for current profile peaking until $q_O \sim 1$ indicates that the choice $q_O = 1$ is a very practical constraint.

For subsequent equilibria and shapes, the toroidal field function,

$$q(\Psi) = X B_\phi / X_O B_O \quad (4)$$

is chosen to satisfy the above forms of $p(\Psi)$ and $q(\Psi)$, with increasing values of β in each stability sequence generated by increasing β_J at

constant $\delta\Psi = \Psi_e - \Psi_0$. Details of our equilibrium and stability procedures, such as growth rate convergence, used in obtaining stability limits with the PEST [2] and BALLOON [13] codes, have been described elsewhere [4,6].

We consider four modes which provide an overall picture of ideal MHD stability in tokamaks. The toroidal mode number $n = 0$ axisymmetric instability and the $n = 1$ kink mode are computed for a plasma surrounded by a vacuum that extends to infinity. Such vacuum cases are often referred to as external modes. The $n = 3$ mode and the $n = \infty$ ballooning modes are referred to as internal, since the plasma extends to a perfectly conducting wall for these cases. Since external modes are often least stable at low- n , while internal modes are least stable at high- n [6], these four cases provide excellent insight into overall MHD stability.

In the following discussion, we define a quantity F which can be viewed as a measure of the relative merit of an asymmetric equilibrium to the equivalent symmetric case (shape 7 of Fig. 1). We define

$$F = |\omega_0/\omega|, \text{ for } n = 0$$

and

$$F \equiv \beta^n/\beta_0^\infty, \text{ for } n > 0 \quad , \quad (5)$$

where β^n is the critical β value of the particular n mode, and β_0^∞ is the critical ballooning ($n = \infty$) value for the equivalent symmetric case. Since the $n = 0$ mode is always unstable for the cases considered here, we use the different definition for $n = 0$, in which ω_0 is the growth rate of the symmetric case and ω is the growth rate of the shape in question, at $\beta = \beta_0^\infty$.

Typical poloidal plane projections of instability displacement vectors for the $n = 0, 1, 3$ modes are shown in Fig. 2, together with the corresponding equilibrium flux plot. These projections exhibit the same gross behavior as symmetric modes, with some asymmetric distortion [e.g., poloidal mode (1) contributions are localized about the various $l = nq$ surfaces].

Figure 3 shows the $n = 0$ results for shapes 1-10 plotted as a carpet in F. The circled point is the equivalent symmetric shape, $\kappa_r = \kappa_2/\kappa_1$ and $\delta_r = \delta_2/\delta_1$. Only the $\kappa_r = 1$ point is shown for $\delta_r > 1$ (shape 10), since other values of κ_r require impractical external field shaping. This plot demonstrates that either decreasing lower ellipticity (κ_2) or increasing lower triangularity (δ_2) improves axisymmetric stability. It should be recalled that for this case only, improvement is defined by reduced growth rate and not by absolute stability. In order to understand this behavior, we recall that for symmetric shapes increasing ellipticity destabilizes the axisymmetric instability by requiring an equilibrium field with increasingly bad curvature (concave relative to the toroidal axis). Thus, decreasing κ_2 can be understood as reducing the instability driving force. Variation with δ_2 is comparatively minor as we would expect.

For the other three modes, shown in Figs. 4-6, stability improves with increasing κ_2 and δ_2 as for symmetric cases. The dependence on triangularity is in general much weaker, with the rate of improvement most pronounced at the highest values of κ_2 and δ_2 . It should be noted that the degradation in critical β is less than 20% for all cases studied, and that in Ref. [10] a $\pi/9$ rotation results in a similar degradation in β . Undramatically, we conclude that having established a maximum practical ellipticity and triangularity, our asymmetric shape should be as symmetric as possible.

3. Optimization of β Limits with Respect to Plasma Profiles

In the previous section, the value of β_0^∞ corresponding to $F = 1$ for non-axisymmetric modes is 2.3%. This value is considerably lower than the $\beta_m^\infty = 5.7\%$ demonstrated by Manickam et al. [1] for an almost identical geometrical configuration. Clearly the differing pressure and safety factor profiles must account for this large variation.

In order to explicitly separate the roles of pressure and safety factor, we have developed a modified equilibrium code that uses a pressure profile which is marginally stable to ballooning modes at all surfaces in the plasma. The marginally stable pressure profile is obtained in the following manner. Pressure values are specified at eleven equal increments in poloidal flux across the plasma. A cubic spline with boundary conditions $d^3 p/d\psi^3 = 0$ at $\psi = \psi_0$ and $dp/d\psi = 0$ at $\psi = \psi_e$ is used to interpolate $dp/d\psi$ within the equilibrium code. The boundary condition on axis is an arbitrary prescription that has been found in practice to preserve local stability at the magnetic axis. Iteration values for the pressure nodes are determined by examining the ballooning mode growth rates at fifty surfaces in the plasma. In practice, the value of $\delta^2 p(\psi)$ in a divided difference table is raised or lowered depending on the ballooning stability in the vicinity of each node. This is found to be the most effective method of adjusting the pressure gradient driving term so that optimum pressure profiles are obtained in a few equilibrium/ballooning iterations (typically ~ 5 for greater than second digit accuracy in the beta values).

Figure 7 shows the optimum profiles of shapes 11 and 12 for the safety factor profile q^A . Although we see a significant improvement to $\beta^\infty = 3.88\%$,

this is still considerably lower than β_m^∞ . Next, the safety factor profile of Ref. [1], which has $q_0 = 1$ and $q_e = 2.03$, was generated in shape 12. A new pressure profile where $p_2/p_1 = 0$, and $\gamma = 4$ in Eq. 3, with $I_0 = 10.8$ MA, $\beta_J = 0.5$, and $g \, dg/d\psi = (1/\beta_J - 1) \, dp/d\psi$ was required for the generating equilibrium. The results of the optimization for this safety factor profile, denoted q^b , are shown in Fig. 8 to yield $\beta^\infty = 6.5\%$. Also shown in Fig. 8 is the optimized pressure profile for the asymmetric equilibrium, 12, computed by the same technique.

4. Discussion of Results

The results of Sec. 2 clearly indicate that, as with symmetric equilibria, ideal MHD β limits for non-axisymmetric modes increase with elongation and triangularity. This was the motivation behind the selection of the mildly asymmetric shape 11 as the representative asymmetric configuration for the optimization studies of Sec. 3. The practicality of such a shape is based on two assumptions. First, the similarity of this shape to concern free boundary equilibria of Ref. [9] suggests that it can be supported by a realistic equilibrium field coil system. Second, it is assumed that axisymmetric ($n = 0$) instabilities can be either passively [14,15] or actively controlled [16].

The results of Sec. 3 are summarized in Table 1, which shows that for a given q profile the gain in β from pressure optimization is not sensitively dependent on the shape, and that the safety factor profile is the principal reason for the low values of β_0^∞ in Sec. 2. Specifically, comparing lines 1 and 5, and 3 and F, we can see that the β^∞ of our chosen asymmetric shape 11 and symmetric shape 12 increase by slightly more than a factor of two between

profiles "a" and "b." This relative increase is larger than one would expect from previous studies [6,7]. To explain this apparent discrepancy it is necessary to isolate the role played by the pressure profile in determining the stability limits.

In the process of determining its stability to ballooning modes, an unoptimized profile would be classed as unstable whenever the local pressure gradient, the principal driving force, exceeds the optimized (i.e., marginally stable) pressure gradient at a single surface in the plasma. Figure 8 shows that by change, p^b matches the optimized gradient for q^b over a wide range of the outer and middle plasma cross section. On the other hand, the pressure profile used in Sec. 2 with q^a , is unstable near the magnetic axis, and clearly it does not follow the corresponding optimized profile. As a result, gains in β^{∞} from optimization of the "a" profiles are more than a factor of two larger than those for the b profiles, as shown in the last column of Table 1.

In general, since one does not know in advance whether a given analytically chosen pressure profile is indeed close to the optimized profile, it can be somewhat misleading to compare β values for different safety factor profiles on such a basis. A comparison of the merits of different safety factor profiles for equilibria whose pressure profiles have been optimized in the above way is the appropriate technique to obtain unambiguous results.

In Table II we again present the results of Table 1, comparing only the optimized pressure profiles. The lower q_e of the q^b profile still gives considerably higher values of β^{∞} . The relative increase between the profiles is approximately 60%, a value, incidently, which is more consistent with the results of Ref. [6]. We note that this improvement occurs for both shapes 11 and 12 and that, by examination of Table 1, it is larger than the gain from any pressure optimization at constant q .

5. Summary

We have described an investigation of the ideal MHD stability of a wide class of poloidally asymmetric equilibria. Our results confirm that beta limits increase with increasing ellipticity and triangularity of the plasma cross section and with decreasing q_0 . By optimizing the pressure profile to obtain equilibria which are marginally stable to ballooning modes, we conclude that a reactor device with poloidal asymmetry consistent with a single-null divertor design can reasonably anticipate $\beta_c \sim 5 - 6\%$ based on the ideal MHD model. This value is approximately 15% less than an equivalent symmetric design.

In concluding it is useful to indicate the limitations of this study. Although the shaping comparisons were performed on the basis of considering modes with both small and large toroidal mode number, the pressure optimization was based on the large- n limit only. Such limits are probably pessimistic since kinetic effects significantly stabilize the ballooning branch for large n [8,17]. (This argument does, however, also omit any considerations of the effect of β on the drift mode branch [17].) Thus, while such studies are still being carried out, the results given here should only be interpreted as a qualitative indication of the comparative merits of different MHD equilibria. At the present time they do, nevertheless, give the best available theoretical numbers for design purposes.

As indicated above, the β -optimization of Sec. 3 is incomplete in the sense that only the stability to large- n ballooning modes was incorporated and the q -profile remained unoptimized. For internal modes this is reasonable, since large- n modes are more unstable. Previous studies [1,6] indicate that the low- n external modes can be the most restrictive. Such modes are

relatively sensitive to the shape of the current profile near the plasma boundary,¹⁸ and optimization will necessarily involve adjustment of the safety factor profile in that region. A study which utilizes the stability to kink modes to optimize the safety factor profile is thus a logical improvement of this work, and it is currently under way.

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q(Ψ) profile	p(Ψ) profile	Shape	$\beta^w(\%)$	β^w Gain from Optimization
a	a	11	2.26	58%
a	Optimized	11	3.58	
a	a	12	2.50	55%
a	Optimized	12	3.88	
b	b	11	4.62	24%
b	Optimized	11	5.72	
b	b	12	5.35	22%
b	Optimized	12	6.50	

Table I. Results of Profile Optimization Study

q(Ψ) Profile	p(Ψ) Profile	Shape	β^{∞}	β^{∞} Gain With q Profile
a	Optimized	11	3.58	60%
b	Optimized	11	5.72	
a	Optimized	12	3.88	
b	Optimized	12	6.50	68%

Table II. Results for Optimized Profiles

Figure Captions

Fig. 1. Shapes used in asymmetric study. κ_2/κ_1 , $\delta_r = \delta_2/\delta_1$. Shapes 1-10 belong to a family with $\delta_1 = 0.3$, $\kappa_1 = 1.6$. Shapes 11 and 12 belong to a family with $\delta_2 = 0.5$, $\kappa_2 = 1.6$, and do not correspond to the indicated δ_r , κ_r . Shape 12 is symmetric, and shape 11 has $\kappa_1 = 1.45$, $\delta_1 = 0.3$.

Fig. 2a. Equilibrium poloidal magnetic flux contours for an inverted shape 11 at $\beta = 8.95\%$.

Fig. 2b. Poloidal plane projection of the axisymmetric $n = 0$ mode displacement vector, for the equilibrium of Fig. 2a.

Fig. 2c. Poloidal plane projection of the $n = 1$ external mode displacement vector, for the equilibrium of Fig. 2a.

Fig. 2d. Poloidal plane projection of the $n = 3$ internal mode displacement vector, for the equilibrium of Fig. 2a.

Fig. 3. Carpet of F for shapes 1-10 of Fig. 1, for the $n = 0$ instability. The circled point is the equivalent symmetric shape 7.

Fig. 4. Carpet of F for the $n = 1$ instability.

Fig. 5. Carpet of F for the $n = 3$ instability.

Fig. 6. Carpet of F for the $n = \infty$ instability.

Fig. 7. Optimum pressure profiles for shapes 11 and 12, the original marginally ballooning stable pressure profile for shape 12, and the safety factor profile q^a , plotted across the plasma midplane.

Fig. 8. Optimum pressure profiles for shapes 11 and 12, the original marginally ballooning stable pressure profile for shape 12, and the safety factor profile q^b , plotted across the plasma midplane.

Fig. 1

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Fig. 2

Fig. 3 Carpet of F for shapes 1-10 of Fig. 1, for the $n = 0$ instability. The circled point is the equivalent symmetric shape 7.

Fig. 4 Carpet of F for the $n = 1$ instability.

Fig. 5 Carpet of F for the $n = 3$ instability.

Fig. 6 Carpet of F for the $n = \infty$ instability.

Fig. 7 Optimum pressure profiles for shapes 11 and 12, the original marginally ballooning stable pressure profile for shape 12, and the safety factor profile q^a , plotted across the plasma midplane.

Fig. 8 Optimum pressure profiles for shapes 11 and 12, the original marginally ballooning stable pressure profile for shape 12, and the safety factor profile q^b , plotted across the plasma midplane.