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PREDICTIONS FOR SYMMETRY BREAKING PROCESSES: A VIEW
FROM THE DKP FORMALISM

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THE EFFECT OF THE CHOICE OF WAVE FUNCTIONS ON THEORETICAL PREDICTIONS FOR
SYMMETRY BREAKING PROCESSES: A VIEW FROM THE DKP FORMALISM*

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Abstract

When considering an elementary particle matrix element, of necessity one must make an assumption, which often goes unnoticed, as to what formalism should be used for the wave functions. A current or interaction Lagrangian-density matrix-element is of the form $V = \bar{\psi}_{out} \Gamma \psi_{in}$, where ψ_{in} and $\bar{\psi}_{out}$ represent the physical ingoing and outgoing particles, and Γ represents the vertex function. A current must have the dimensions of $(\text{length})^{-3} = (\text{mass})^3$ in units of $\hbar = c = 1$. ψ_{in} and $\bar{\psi}_{out}$ must be described in terms of the physical on-shell masses or else one has no phase space. It is only the vertex function which can be symmetric in the internal symmetry under consideration.

The decision as to how much of the matrix element will be taken to be symmetric and how much of the matrix element will be taken to be associated with on-mass-shell wave functions is a fundamental assumption. Depending on how the assumption is made, different results will be predicted.

Normally first-order Dirac wave functions, with dimensions $(\text{length})^{-3/2}$ and second-order Klein-Gordon wave functions with dimensions $(\text{length})^{-1}$ are considered for spin-1/2 fermions and spin-0 bosons, respectively. We will discuss the types of new results which are obtained if, on the contrary, one chooses to consider bosons in the first-order Duffin-Kemmer-Petiau formalism. We will argue that the DKP formalism represents a complementary viewpoint to the spectrum generating approach. Both challenge the standard phenomenology: DKP by changing the wave function, spectrum generating by changing the vertex function.

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I. INTRODUCTION

The work I will discuss today results from a rather large collaboration¹⁻⁷ which proposed that one should consider using the Duffin-Kemmer-Petiau⁸⁻¹⁰ first-order wave equation to describe mesons instead of the second-order Klein-Gordon equation. The main members of the collaboration were Ephraim Fischbach, C. Keith Scott, and myself, with a fourth major contributor being Henry Primakoff. There were also a number of others who joined us in this work, and they are credited in Refs. 1-7. These references and the works mentioned therein provide a guide to the literature.

This talk will differ from what would be my usual presentation since I have been asked to discuss our work with reference to the main topic of this session, "Spectrum Generating Algebras." Thus, the main thrust will be geared to answering the question, "What is the relation between the spectrum generating approach and the use of the DKP formalism?" I will argue that although there is not necessarily a one-to-one calculational equivalence, there is a complementary philosophical connection. The two ideas provide complementary alternatives to the standard ideas on symmetry breaking. The meaning of this assertion will become clear when I return to it in a few minutes. For now let me give a quick introduction into the DKP formalism.

II. DKP FORMALISM

Soon after the success of the original nonrelativistic Schrödinger equation, efforts began to generalize the Schrödinger theory to the relativistic domain. The first serious attempt was what we have come to call the "Klein-Gordon Equation," but which, however, was discovered by a number of people.¹¹

The idea of the KG equation was a direct generalization of the Schrödinger idea that the classical nonrelativistic Hamiltonian equation be changed to an operator wave equation with the operator substitutions

$$E \rightarrow i\hbar \frac{d}{dt}, \quad \vec{p} \rightarrow \frac{\hbar}{i} \frac{\vec{\partial}}{dx}. \quad (2.1)$$

In particular, the KG equation takes the relativistic energy-momentum relation

$$E^2 = p^2 c^2 + m^2 c^4 \quad (2.2)$$

and changes it into a wave equation with the same substitutions, so one ends up with

$$\left[\hbar^2 \frac{\partial^2}{\partial t^2} - \hbar^2 c^2 \left(\frac{\vec{\partial}}{dx} \right)^2 + m^2 c^4 \right] \phi = 0. \quad (2.3)$$

The Coulomb problem can be solved exactly for this system and it was discovered that the energy levels

$$E = mc^2 [1 + \alpha^2/v^2]^{-1/2}$$

$$v = n - (l + \frac{1}{2}) + [(l + \frac{1}{2})^2 - \alpha^2]^{1/2} \quad (2.4)$$

do not correspond to the hydrogen atom. (Later it was realized that the KG equation could not work because of the spin of the electron.)

The great idea of Dirac was to extract the square-root of the energy-momentum relation (2.2). Dirac sought operators γ_μ which obey

$$\begin{aligned} (\vec{\gamma} \cdot \vec{p} - \gamma_0 p_0)^2 &= (\gamma \cdot p)^2 = -m^2, \\ m &= \pm [-(\gamma \cdot p)^2]^{1/2}. \end{aligned} \quad (2.5)$$

(I will now usually take $\hbar = c = 1$.) The solutions are the now well-known Dirac matrices satisfying

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2 \delta_{\mu\nu} \quad (2.6)$$

and which yield the first-order Dirac equation

$$[\partial \cdot \gamma + m]\psi = 0 \quad (2.7)$$

For the Coulomb problem, the solution to this equation yields the energy levels of the relativistic (but not second-quantized) hydrogen atom,

$$E = mc^2 \left\{ 1 + \frac{\alpha^2}{\{n - j - \frac{1}{2} - [(j + \frac{1}{2})^2 - \alpha^2]^{1/2}\}^2} \right\}^{-1/2}. \quad (2.8)$$

Also due to its later successes, the use of this first-order formalism for electrons (and spin-1/2 particles in general) has gone on triumphantly for fifty years.

As to spin-0 particles, the story has been somewhat different. At the end of the 1930's, Duffin, Kemmer, and Petiau proposed what is a direct first-order generalization of the Dirac idea. (I refer the reader to Ref. 12 for a historical account of the work of these and other investigators.)

The DKP proposal can be put in terms of the following question: Are there operators β_μ which satisfy the energy-momentum relation

$$(p \cdot \beta)^3 = (p \cdot \beta) p^2 = -m^2 (p \cdot \beta) \quad (2.9)$$

The answer is, "Yes," and they have the algebra

$$\beta_\mu \beta_\lambda \beta_\nu + \beta_\nu \beta_\lambda \beta_\mu = \beta_\mu \delta_{\lambda\nu} + \beta_\nu \delta_{\mu\lambda} \quad (2.10)$$

In particular there exists a (reducible) 16×16 representation of (2.10), which is given by the symmetric product of two Dirac spaces:

$$\beta_\lambda = \frac{1}{2} [I^{(1)}_\lambda \gamma^{(2)} + \gamma^{(1)}_\lambda I^{(2)}] \quad (2.11)$$

One can see, therefore, that in some sense one is combining internally two spin-1/2 (quark?) spaces. Spin-0 and spin-1 should come out, and indeed the 16×16 reducible representation can be decomposed into a trivial 1×1 , a spin-0 5×5 , and a spin-1 10×10 representation.

The spin-1 representation can be thought of as the equations for massive electrodynamics.¹³ Six of the fields correspond to the $\vec{E}$ and $\vec{B}$ fields and four correspond

to the physical A_μ fields (physical because the photon mass is not zero). For spin-1, this formalism is to be contrasted with the second-order formalism of Proca.

For spin-0 a representation of the β 's is⁹

$$\begin{aligned} \beta_1 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} & \beta_2 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \\ \beta_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} & \beta_4 &= \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \quad (2.12)$$

The free particle solutions to the DKP spin-0 equation

$$(\partial \cdot \beta + m) \psi = 0 \quad (2.13)$$

are

$$\psi = \left[\frac{m}{p_0 V} \right]^{1/2} u(p) e^{ip \cdot x} \quad (2.14)$$

$$u = (2m^2)^{-1/2} \begin{pmatrix} 1 & p_0 \\ 1 & p_1 \\ 1 & p_2 \\ 1 & p_3 \\ -m \end{pmatrix} \quad (2.15)$$

The adjoint solutions are

$$\bar{\psi} = \psi^\dagger \eta_4 = \psi^\dagger (2\beta_4^2 - 1) \quad (2.16)$$

The dimensions of $(\text{mass})^{3/2} = (\text{length})^{-3/2}$ associated with (2.14) come from the Lagrangian derivable from (2.13). (Note that for the Dirac equation $\eta_4 = \gamma_4$, whereas here it is a polynomial of order two. This is a direct result of the fact that the algebra is three-fold vs. two-fold.)

Before proceeding to a comparison of the DKP and KG formalisms, let me mention that the procedure of DKP can be generalized, looking for operators satisfying a higher power energy-momentum relation. This was the idea of Bhabha, Lubanski, and Mahadvarao, which resulted in what are known as the Bhabha first-order wave equations for arbitrary spin.^{12,14} The matrices α_μ in the first-order wave equation

$$(\partial \cdot \alpha + \chi) \psi = 0 \quad (2.17)$$

are various representations (S, S) of the algebra $so(5)$. A beauty is that the lowest representations, $(\frac{1}{2}, \frac{1}{2})$ and $(0, 0)$, $(1, 0)$ and $(1, 1)$, are the Dirac and three DKP

representation matrices. Thus, the Dirac and DKP equations are all special cases of the Bhabha set of equations,

These Bhabha equations are by no means the only first-order wave equations possible. However, since higher-spin equations would only give a detailed generalization of the physical points to be made today, this goes afield from the main topic under discussion. Therefore let me refer the interested members of the audience to Refs. 12, 14 and the review by Ginzburg and Man'ko¹⁵ for references and an introduction to the subject.

III. DKP AND KG IN THE SYMMETRY LIMIT

Having obtained the formalism, one can ask if the two formalisms yield the same results. Since our answer is, "Not always," let's start by looking at where they do yield identical results.

a) Free particles. By deep theorems this must be true. Specifically, let

$$\psi_5 \equiv \phi, \quad \psi_{1,2,3,4} \equiv \phi_{4,1,2,3} \quad (3.1)$$

Putting this into Eq. (2.13) with the β -matrices of (2.12),

$$\begin{aligned} m\phi &= -\partial_\mu \phi_\mu \\ \partial_\lambda \phi &= -im\phi_\lambda \end{aligned} \quad (3.2)$$

which when combined yield

$$(-\square^2 + m^2)\phi = 0 \quad (3.3)$$

Thus, up to a (here) not important normalization, the two solutions are the same.

b) Pi-mesic atom. By the same procedure, one finds that the first-order DKP equation can be made into a second-order equation on ψ_5 which is identical to the KG equation for this problem. Thus, the same energy levels of Eq. (2.4) result.

c) QED. One can also show that 2nd-quantized QED processes involving mesons, such as $e^-e^+ \rightarrow \pi^-\pi^+$ and $e\pi$ scattering, yield identical results in the two formalisms. In fact, a lovely discussion of the QED of pi-mesons in the DKP formalism is contained in the book by Akhiezer and Berestetskii.¹⁶ For people who like to play with the traces in Dirac QED, it can be a lot of fun to do these calculations. They are very similar to Dirac QED (except that there are modifications of the trace theorems). For example, the two diagrams in the DKP description of $e\pi$ scattering yield the same results as the three diagrams in the KG description.

d) πNN coupling. Again the two formalisms agree, up to a normalization which can be absorbed in the coupling constant. This means that one will obtain the same predictions for πN scattering.

In the 1940's and 1950's a great deal of effort was expended proving the above results, and as a result of these efforts the KG formalism came to be preferred to the

DKP formalism, partially because of there not being a 5×5 matrix algebra associated with it.

However, our collaboration began with the realizations that the above discussions of the "equivalence" of the DKP and KG formulations were all done in the context of what we would now call the symmetry limit (before the discovery of the K meson), and that it was "folk lore" that this equivalence had been shown in the symmetry breaking case.

IV. DKP AND KG IN SYMMETRY BREAKING

Let us begin to make our point with a specific example: the case of non-conserved electrodynamics. With minimal electromagnetic substitution, the KG current is ($\partial_\lambda^\pm = \partial_\lambda \pm ieA_\lambda$)

$$j_\lambda^{KG} = -i[\phi_B^* \partial_\mu^- \phi_A - (\partial_\mu^+ \phi_B^*) \phi_A] \quad (4.1)$$

The DKP current is

$$j_\lambda^{DKP} = i\bar{\psi}_B \beta_\lambda \psi_A \quad (4.2)$$

By using the DKP wave equations one can rewrite (4.2) in terms of a properly normalized KG field ϕ . The result is

$$j_\lambda^{DKP} = -i \left[(m_B^{1/2} \phi_B^*) \left\{ \frac{\partial_\mu^-}{m_A} (m_A^{1/2} \phi_A) \right\} - \left\{ \frac{\partial_\mu^+}{m_B} (m_B^{1/2} \phi_B^*) \right\} (m_A^{1/2} \phi_A) \right] \quad (4.3)$$

That is, the two currents are different when the current is not conserved, i.e., when there is symmetry breaking.

How can we understand this physically? On dimensional grounds, actually. Consider the $K_{\ell 3}$ decay $K \rightarrow \pi \ell \nu$, which usually is discussed in terms of a current-current interaction.

$$\langle \pi \ell \nu | H | K \rangle = \ell_\lambda \mathcal{H}_\lambda \quad (4.4)$$

$$\mathcal{H}_\lambda = g \bar{\psi}(p') \Gamma_\lambda \psi(p) \quad (4.5)$$

where ℓ_λ is the lepton current. The hadronic current $\mathcal{H}_\lambda$ has units of $(\text{length})^{-3} = (\text{mass})^3$, no matter what is the representation.

When one is doing a practical calculation, the wave functions must have physical (non-symmetric) masses or the phase space will be zero. Standardly, what is also done is to make the most important piece of vertex function Γ_λ be symmetric; in this case SU(3) symmetric. Now we come to the crux: IT IS AN ASSUMPTION to say what piece of the current is SU(3) symmetric, and which piece is physical. In this case, if one chooses the KG formulation, the wave functions $(1/2 \text{ EV})^{1/2} \exp(ip \cdot x)$ have dimensions of $(\text{length})^{-1}$ and the vertex function has units $(\text{length})^{-1}$. If one chooses the DKP wave functions of Eq. (2.14), then the vertex function will be dimensionless. In general one should expect different

answers from a wave function of dimensions $(\text{mass})^{3/2}$ which extrapolates to a new mass than from a wave function of dimensions (mass) which does a similar extrapolation.

Also observe that the same considerations apply when comparing Lagrangians of dimensions $(\text{mass})^4$.

V. CONNECTION TO THE SPECTRUM GENERATING APPROACH

Members of the audience may already realize the point I am now trying to make. The DKP approach and the spectrum generating (SG) approach represent complementary viewpoints on the same problem.

Both approaches question the standard phenomenology which is used for mesons, that one should use wave functions of dimensions $(\text{length})^{-1} = (\text{mass})^1$ and standard vertex functions of dimensions $(\text{length})^{-1}$. The way that DKP challenges the standard approach is to change the wave function. The physical wave functions have dimensions $(\text{mass})^{3/2}$, leaving the vertex function dimensionless.

The spectrum generating approach, on the other hand, leaves the wave functions with dimensions (mass) , but then challenges the symmetry of the vertex function: like having the vertex function be changed from p_μ to p_μ/M where M is a mass operator.

An example which makes the similarity between DKP and SG more intuitive is to compare Eq. (4.1) for the KG current and Eq. (4.3) for the DKP current written in terms of KG wave functions. One sees that the current KG current operator ∂_μ^- can be thought of as having been changed in DKP to a more complicated function with ∂_μ^- and mass operators.

Whether a DKP approach will be computationally the same as an SG approach will depend in detail on what DKP interaction and what SG interaction is used. But the common feature they have is that they provide alternatives to the standard KG approach with standard vertex functions.

At this point it is appropriate to observe that the questions about symmetry breaking which are raised by this discussion are not limited to mesons. The same type of questions can be raised about fermions. Indeed, if one looks back at the fundamental papers of Feynman on the path integral method¹⁷ he preferred a second-order formalism for fermions. This point of view has been resurrected by Macrae¹⁸ in his work on the unification of fermions and bosons in the fiber bundle picture. One could also change things by keeping a first-order formalism and putting in an SG interaction. In any case, the thing to be remembered is that different results can ensue. Which, if any, of these various possibilities is ultimately to be preferred depends, of course, on physical experiment.

VI. AN ILLUSTRATIVE EXAMPLE

In the references I have quoted,¹⁻⁷ applications have been made to the symmetry-breaking parameters of $K_{\ell 3}$ decay,¹ the Cabibbo angle,² pseudoscalar decays into two photons,⁴ the PS meson-baryon-baryon coupling constants,⁵ and vector-meson annihilation into e^+e^- .⁷ Other processes also have been discussed in the literature.¹⁹

From these I wish to take a specific example and show explicitly how the DKP formalism approach yields deviations from the KG formalism, which deviations disappear in the symmetry limit. The example I will use was one of the results of our first paper. I choose it nonetheless first because it so nicely illustrates the DKP algebra and techniques, and second because the experimental situation is in such chaos.

In the standard discussion of $K_{\ell 3}$ decay, the KG vector current matrix element is

$$\langle \pi(p') | V_\lambda | K(p) \rangle = (4 p_0 p'_0 v^2)^{-1/2} [(p + p')_\lambda f_+(t) + q_\lambda f_-(t)] \quad (6.1)$$

The form factors $f_+(t)$ and $f_-(t)$ are expected to be slowly varying functions of $t = \text{momentum-transfer-squared}$, and if $SU(3)$ is good, one expects the symmetry-breaking parameter

$$\xi \equiv f_-(0)/f_+(0) \quad (6.2)$$

to be small in magnitude compared to unity.

In fact, after first wandering all over the map, the Chounet-Gaillards review²⁰ summarized a growing consensus for a large negative value of order -1 . Then two final grand experiments were done: a SLAC experiment with 1.6 million events which obtained²¹ -0.11 ± 0.03 and a YALE²² experiment with 2.2 million events which obtained -0.655 ± 0.127 , and no one has ever understood the difference. So, taking in mind that a large negative value would be hard to understand, let's look at the DKP description.

The DKP current matrix element is ($m_K = m$, $m_\pi = \mu$)

$$\langle \pi(p') | V_\lambda(0) | K(p) \rangle = i \left(\frac{m\mu}{p_0 p'_0 v^2} \right)^{1/2} \bar{u}(p') [\beta_\lambda g_V(t) + \frac{iq_\lambda}{m+\mu} g_S(t)] u_K(p) \quad (6.3)$$

$$\equiv [4p_0 p'_0 v^2]^{-1/2} [(p + p')_\lambda \hat{f}_+(t) + q_\lambda \hat{f}_-(t)] \quad (6.4)$$

$$\hat{f}_+(t) = \frac{m + \mu}{2(m\mu)^{1/2}} g_V(t) \quad (6.5)$$

$$\hat{f}_-(t) = \frac{m + \mu}{2(m\mu)^{1/2}} g_V(t) \left\{ - \left(\frac{m - \mu}{m + \mu} \right) - \frac{g_S(t)}{g_V(t)} \left[1 - \frac{t}{(m + \mu)^2} \right] \right\} \quad (6.6)$$

This means the apparent "KG" symmetry breaking parameter is

$$\hat{\xi}(0) = \hat{f}_-(0)/\hat{f}_+(0) = - \left(\frac{m - \mu}{m + \mu} \right) - \frac{g_S(0)}{g_V(0)} = -0.57 - \frac{g_S(0)}{g_V(0)} \quad (6.7)$$

Thus, in the DKP formalism where one expects $g_S(0)/g_V(0)$ to be the quantity which is small in magnitude compared to unity, a small negative value would still give the apparent large symmetry breaking parameter ξ when written in the KG formalism. Thus, if the DKP formalism were correct, the apparent large symmetry breaking would only be due to having used the "incorrect" KG formalism. A final comment on the DKP SG analogy is that Böhm²³ recently re-obtained the important factor - 0.57 in Eq. (6.7) from the SG approach.

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