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Computer Studies of the Dynamic Strength of Ceramics

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Computer Studies of the Dynamic Strength of Ceramics

Abstract

Using a new constitutive model, we performed computer studies concerning the dynamic yield strength of six ceramics, SiC, TiB₂, AlN, two types of B₄C, and partially stabilized zirconia. The relative importance of the thermomechanical variables, strain, strain-rate, pressure, and temperature, as well as the Bauschinger effect, is demonstrated in determining the time response of ceramics to high-strain-rate deformation. The constitutive model is easy to implement in a hydrodynamic computer code, and it successfully reproduces a variety of data for these six materials.

Introduction

The high-strain-rate behavior of ceramics offers exciting scientific possibilities, for these materials appear to have more individual peculiarities than do metals. Naturally, some constitutive model is required in order to do computer studies of this behavior. If the model in this report fits the reader's needs, so much the better, but the primary purpose of this report is not to propose a new constitutive model. Rather, it is to present the computer studies themselves and what these well-posed and diagnosed "experiments" can tell us.

In the case of metals, a large experimental data base exists that permitted us to construct a constitutive model for metals by using principally non-shock-wave sources (S80, S89). We then used shock-induced, time-resolved wave profiles to test the predictive capability of the model. These experiments are excellent and stringent tests of any model because so many thermomechanical effects take place simultaneously. Hydrodynamic computer-code calculations using this model were very successful in simulating the experimental results.

For ceramics, we are not so fortunate. Because of the dearth of data, I have often had to use shock-wave profiles as the primary source of data. One important result of this study is that it highlights what we don't know and what data we are lacking. Perhaps the most important part of this report is the list of suggested experiments that I believe are necessary to provide a good data base from which any constitutive model will be built, and against which it must be tested.

I have chosen six materials to study: SiC, TiB₂, two types of B₄C, AlN, and ZrO₂, partially stabilized with 3 wt.% Yttria (PSZ) in the tetragonal form. These choices were dictated by the fact that the shock-wave study group at Sandia National Laboratories, Albuquerque, had obtained or were intending to obtain shock-wave profiles on these materials. Another requirement was that these ceramics be near their theoretical density; I felt any study of porosity to be premature until more understanding of the fully dense materials was achieved.

In the following sections, I discuss the basic assumptions regarding my approach to modeling the dynamic strength of ceramics. Following this, I describe the hydrodynamic equation of state (EOS) and the constitutive model. This is followed by a comparison of the data with the computer simulations. The report ends with some general conclusions and suggestions for future experimental work.

Basic Assumptions

Regardless of the actual micromechanical mechanisms, I have assumed that all thermomechanical behavior can be represented through the following macroscopic variables: strain ϵ , strain rate $\dot{\epsilon}$, temperature T , and pressure P . The reason for this is based on my experience with metals. For example, I believe that the S-shaped shock-loading profile, seen in the lowest-stress SiC experiment, shows $\dot{\epsilon}$ effects very strongly. Consequently, the model should include this variable. This assumption also means that compressive fracture is not specifically modeled. Although compressive fracture almost certainly occurs, it is reflected in one of the macroscopic variables used. Further, strain is used only for the special case of B_4C because plastic strains are typically only a few percent and also because the Bauschinger effect appears to be small.

The simplest form of the Cochran-Guinan Bauschinger model is used throughout because I believe it is a better general representation of reality than is simple elastic-plastic behavior (S80). Figure 1 illustrates this assumption. The dashed curve is arrived at by assuming that the slope of the deviator stress vs deviator strain, $dS/d\epsilon$ varies linearly from its value at point A (i.e., at $S = 0$) to zero at point B. This form of the model requires no additional material parameters. The Cochran-Guinan model was originally constructed to describe yield behavior in metals, and it works very well in this context, where the data base consists of compression-tension data. However, no such data exist for ceramics and, because of the high strength of ceramics, it is unlikely that there will be any in the future. The shock-wave profiles that I am modeling may very well be the best data we have.

I have used the Cochran-Banner spall model (C77) throughout with a spall strength of 10 kb. Because spall strengths relative to yield strengths for ceramics are small and the peak stresses in the experiments exceed 200 kb, it is very difficult to quantify spall in these studies. Indeed, various spall models, which use a spall strength of a few kilobars, appear to work equally well in these calculations.

Hydrodynamic EOS

The hydrodynamic EOS used throughout this study is the Mie-Grüneisen equation with a nonlinear shock velocity-particle velocity (U_s-U_p) relationship,

$$U_s = C_0 + S_1 U_p + S_2 \left(\frac{U_p}{U_s} \right) U_p + S_3 \left(\frac{U_p}{U_s} \right)^2 U_p , \quad (1)$$

and a constant, $\gamma\eta$:

$$\gamma\eta = \gamma_0 , \quad (2)$$

where γ is Grüneisen's gamma, and η denotes compression.

Table 1 gives the required parameters for the materials of interest. The values for the bulk sound speeds C_0 , the initial densities ρ_0 , and the initial shear moduli G_0 (see Table 2), are taken from the same Sandia reports as the wave profiles. I calculated the values of γ_0 by using handbook values of specific heat and volume thermal expansion. The sources of Hugoniot data are given in Table 1.

Constitutive Model

Shear Modulus

The shear modulus G is the same form as for metals (S80).

$$G(P,T) = G_0 \left[1 + A \frac{P}{\eta^{1/3}} - B(\Delta T) \right] . \quad (3)$$

Here, $\Delta T = T - 300$ K, subscript 0 refers to the STP state ($P = 0$ and $T = 300$ K), and coefficients A and B are material-dependent parameters:

$$A = \frac{1}{G_0} \frac{dG}{dP} , \quad (4a)$$

and

$$B = \frac{1}{G_0} \frac{dG}{dT} . \quad (4b)$$

For metals, a wealth of data exists for A and B . Unfortunately, almost no such data exist for ceramics. To estimate A , I used a relationship, derived originally for metals, from Guinan and Steinberg [G74; see Eq. (31)]. This equation requires a knowledge of the STP elastic constants, γ_0 and the Hugoniot. Values for $A(\text{est})$ are given in Table 2, as are the actual values of A used in simulating the wave profiles. Except for PSZ, A and $A(\text{est})$ agree as well as I would expect, based on my previous experience with metals. I say more about this during the subsequent discussion of the results for each ceramic.

Since the temperature effect is usually very small for metals, I wanted to see if this was also the case for ceramics. I again used the results of Guinan and Steinberg [G74; see Eq. (A7)] to estimate B . Some data exist for the temperature derivative of Young's modulus E_T for all five ceramics. This information plus the STP elastic constants, the Hugoniot, and the volume thermal expansion coefficient provide the necessary data to determine $B(\text{est})$. These results are given in Table 2. Only for TiB_2 has B been measured (W69), and the result is in very good agreement with $B(\text{est})$.

Calculations of the wave profiles were done, including the temperature effect, with the result that ceramic behavior was very similar to that of metals, i.e., the temperature effect was about an order of magnitude less important than the effect of pressure, or barely discernible compared with experimental uncertainty. For the sake of completeness, the calculations that follow do include the effect of temperature.

Yield Strength

Grady (G85, G88) has developed a model for the brittle fracture stress σ_c vs $\dot{\epsilon}$ for brittle materials in tension that appears to hold experimentally for the yield strength Y in compression as well (L83, L85):

$$\sigma_c = D \dot{\epsilon}^n , \quad (5)$$

where

$$D = (3\rho_0 C_0 K_{IC}^2)^n , \quad (6)$$

and K_{IC} is the critical stress intensity factor. I believe that Eq. (6) is describing macroscopically, and in a way very suitable for hydrodynamic computer codes, the effect on Y of the microscopic process of compressive fracture.

In Grady's work, n is $1/3$. However, Grady has also noted (G90b) that there is no reason why n could not be larger or smaller than this value. I believe the only requirement is that n must be less than unity for the following reason. For metals at high strain rates ($\dot{\epsilon} > 10^4 \text{ s}^{-1}$), Y varies linearly with $\dot{\epsilon}$, due to dislocation drag (S89). In brittle materials, such as ceramics, some of the shock energy goes into making microcracks; therefore, Y cannot increase so rapidly with $\dot{\epsilon}$, (i.e., n will be less than unity).

To help determine D and n , there are also the data of Lankford for Y vs $\dot{\epsilon}$ for SiC (L83, L89), PSZ (L85), and AlN (L90b). These data are shown in Figs. 2 through 4. At static conditions, one set of data for SiC was 7 kb lower than the other. I added 7 kb to this set because it is the shape of the Y vs $\dot{\epsilon}$ curve that is of interest here. Using Eq. (5), with $n = 0.366$ in all cases, these data were fit using the values of D given in Table 2. Lankford (L90b) has said that the absolute error for the data at $\dot{\epsilon} = 10^3 \text{ s}^{-1}$ is about 20 to 30%. Eq. (5) fits the data very well within this uncertainty.

Table 2 also gives values for $D(\text{est})$ based on Eq. (6) and the data for K_{IC} . Here, n must be $1/3$ to be dimensionally correct. For PSZ and AlN, D and $D(\text{est})$ are in remarkable agreement, but for SiC, D exceeds $D(\text{est})$ by an order of magnitude. Perhaps Grady's model is not able to account for strain-rate effects in all kinds of ceramic materials.

The yield strength is assumed to be a sum of strain-rate dependent and strain-rate independent terms similar to our approach for metals (S89). In addition, both terms are multiplied by the normalized shear modulus. This scaling of Y on G is also the same as we have done for metals (S80):

$$Y = (D \dot{\epsilon}^n + Y_A) G(P,T)/G_0 . \quad (7)$$

Here, Y_A is a material constant which can be sample dependent, as it is a function of purity, grain-size, previous mechanical history, etc.

The rate-dependent part of Y is limited. That is,

$$D \dot{\epsilon}^n \leq Y_L . \quad (8)$$

The yield strength cannot increase without limit as $\dot{\epsilon}$ increases. We know, for example, that the Hugoniot elastic limit (HEL) does not increase as a material is shocked to higher and higher stress. The limit on $\dot{\epsilon}$ effects, Y_L , should therefore be the difference between the yield strength at the HEL and Y_A . The effect of Y_L on the calculations is evident only when Y increases very rapidly with $\dot{\epsilon}$ (i.e., D is large) and the peak stress is very high. Such is the case only for SiC, where $Y_L = 19 \text{ kb}$.

Eqs. (3) and (7) are used to describe G and Y for SiC, TiB_2 and PSZ. It is hard to conceive of a simpler model. Basically, there are only four important parameters: the scaling parameters G_0 and Y_A , the pressure-dependence parameter A , and the strain-rate-dependence parameter D . All but Y_A can, in practice, be determined without recourse to shock-wave data. The value of Y_A is determined by means of normalization at the HEL to one wave-profile experiment.

In the following sections, I compare the experimental shock-wave profiles with the computer simulations for each ceramic in turn. I also describe any problems or modifications in the model that are peculiar to each material.

Results and Discussion

SiC

Three shock-wave profile experiments done by Kipp and Grady (K89 and G90b) are compared with the computer simulations in Figs. 5 through 7. Additional results are tabulated in Tables 3 and 4. All calculations were done using the values in Table 1 and A , $B(\text{est})$, D , n , Y_A , and G_0 in Table 2.

The effect of removing Y_L (i.e., Y_L is infinite) is illustrated in Fig. 8. It is apparent that the use of Y_L sharpens the shock front, in agreement with the data, but even so, it hardly plays a dominant or decisive role in the calculations. Limiting Y makes it independent of $\dot{\epsilon}$ for most of the shock-loading time in the two highest-stress experiments. As far as the hydrodynamic code is concerned, only the Von Neumann Q is then left to provide any viscosity. Apparently, this approach segues nicely from a rate-dependent model to one where Q alone dominates.

In Figs. 5 through 7, the data normalizations are made at peak velocity and are within the absolute error of the VISAR. The biggest difference between experiment and calculation is in Fig. 6 at approximately 1.3 μs . The calculation at this time can be affected by the EOS of the foam backing behind the flyer. The calculated HEL is 174 kb, which is an exceptionally large value.

Figures 9 and 10 show what happens if the pressure dependence is turned off. Because the total energy in the system does not change, the biggest difference is in timing, not amplitude. The good agreement in release time shows that the value for $A(\text{est})$ was very good. However, it also shows the importance of measuring $G(P)$ independently of the shock-wave data.

Figures 11 and 12 show the effect of turning off the rate dependence. The shape of the loading curve is totally wrong. In addition, the amplitude of the elastic reflection in Fig. 12 at 0.9 μs is too low. The simulation of these experiments is remarkably sensitive to the value of n . Originally, I tried keeping $n = 1/3$ but failed to get a simulation that agreed well with the data. Increasing n by only 10% to 0.366 solved the problem. Apparently, the shape of the shock-loading curve is a much more sensitive measure of rate effects than are Hopkinson split bar experiments. Additional wave profiles at lower peak stresses (i.e., different strain-rate histories) would be useful in studying this question.

Figure 13 shows Y as a function of time for all three experiments. The increase in Y is caused by the increase in P and $\dot{\epsilon}$. For the lowest stress experiment, the effects of these parameters at the maximum Y are about equal, but for the highest stress experiment, the effect of $\dot{\epsilon}$ is only about 30% of the total.

Except for Y_A , all the model parameters could be determined from non-shock wave sources. This is very encouraging as it implies that the constitutive model is nearly predictive. However, the large value of Y_A is a problem. For metals, Y_A is almost identical to static values of Y ; here, Y_A exceeds the static value by a factor of 3. The same problem also exists for PSZ. I have no explanation for these results.

Partially Stabilized ZrO_2

Grady has provided me with one wave profile for PSZ, in which the peak stress is below the transition to the monoclinic phase (G90b). Figure 14 compares the simulation and data, and the overall agreement is good. In particular, the calculated shape of the shock-loading curve agrees well with the experiment. For PSZ, then, the wave-profile data, Lankford's data, and Grady's model for Y vs $\dot{\epsilon}$ are all in very good agreement. At the maximum value of Y , the effect of pressure accounts for about two-thirds of the increase and rate effects about one-third.

There are, however, several problems. First, A differs significantly from $A(\text{est})$. PSZ has a tetragonal crystal structure, and tetragonal metals are known to have large values of A . However, for metals, A is proportional to γ_0 , and tetragonal metals have large values of γ_0 , quite unlike PSZ. Again, this points out the crucial importance of measuring $G(P)$. Second, the data were increased by 2.3% by normalizing to the calculated peak stress. This increase is about twice what I believe to be the absolute accuracy of the VISAR. I have no explanation for this difficulty.

Finally, the calculation is higher than the data after spall occurs ($t > 1.3 \mu\text{s}$). One possible explanation is 2-D effects. This target was one-half the diameter of the others. I calculate that 2-D effects should begin at $t \approx 1.8 \mu\text{s}$. However, this calculation assumes a perfect geometry with an exact planar impact and a zero-thickness laser beam aligned on the exact target center.

More wave profiles are obviously needed, especially from thicker targets so that the elastic and plastic waves can be better resolved.

TiB₂

TiB₂ has a number of unusual properties. Gust, Holt and Royce (G73) have suggested a possible phase transition at 300 kb based on their Hugoniot data (see Fig. 15). In addition, all the shock-wave profile data from Sandia (K89, G90b) show two breaks in the loading profiles at stresses of about 60 and 135 kb (see Fig. 16). These have been variously referred to as a "double-yield" or yield plus phase transition. Finally, the shear modulus is greater than the bulk modulus. To the best of my knowledge, the only other material for which this is so is beryllium.

The Hugoniot data of Gust, Holt and Royce (G73) and the bulk sound speed of Kipp and Grady (K89) were fit by using Eq. (1). This fit is shown in Fig. 15, in which the slight oscillation is an artifact of the fit only. The arrow at $U_p = 1.05 \text{ mm}/\mu\text{s}$ shows where experiment 6 of Kipp and Grady would fall.

Figure 16 compares this experiment with two calculations, each assuming the different values of the HEL.* As with SiC and PSZ, the data in Tables 2 and 3 were used in the calculation. Because no Y vs ϵ data exist, D was merely adjusted to give the best fit to the slope and shape of the shock-loading curve. It is about a factor of 3 higher than $D(\text{est})$, which could be considered a good or bad agreement, depending on your point of view. In addition, the measured temperature-dependence parameter B was used instead of $B(\text{est})$. The agreement between experiment and calculation is reasonable, but certain features related to the two yields obviously will not be reproduced. More work and study are certainly needed.

B₄C

Description of the Data and Material Characteristics

The data for B₄C show several remarkable characteristics (see Figs. 17-20). The first are the sharp oscillations with a frequency of a few tens of nanoseconds. At first glance, one might think that the data were in error or at least that the experiments suffered from electronic noise. With four such experiments, however, this seems unlikely.

Guinan (G90a) has suggested that these observations could be explained if there is elastic precursor decay. This is not the decay of the leading edge of the wave as it passes through a material, but rather that, at any time, the strength of the wave is greatest at its leading edge and gradually falls off behind the leading edge. This is reminiscent of a Taylor wave in high explosives. When the precursor reflects from the lower-impedance LiF window, it will see a progressively lower elastic stress as it moves back into the shocked material. If the difference between the release wave and the oncoming part of the elastic wave exceeds the spall strength of the material, then a gap or gaps can open, and the measured velocity will drop. As these waves and the main shock wave continue to interact, these gaps can be closed and reopened, causing the signal to alternately rise and fall until the main shock finally closes them.

More structure exists in the data for the Dow material than for the Eagle-Picher material. This is because there is more time between the elastic and plastic waves for the former, allowing more wave interactions. This is consistent with the higher HEL for the Dow material, and hence the shallower Rayleigh line. No attempt is made in this report to actually calculate these oscillations.

Another unusual characteristic of the wave profiles is the very long time between the arrivals of the elastic and plastic waves. Simple elastic-plastic behavior, with a constant offset between the Hugoniot

* Experiment 5, also on TiB₂, is in error and is not discussed here.

and the hydrostat, would imply a very much smaller time difference. This behavior is also shown in the Hugoniot data of Gust and Royce (G71) in Fig. 21. Just above the HEL, the slope of the Rayleigh line becomes very shallow, implying a very low shock speed relative to the elastic-wave speed. The four arrows in Fig. 21 point to where the four wave-profile experiments fall on this Hugoniot.

Other unusual properties of boron-rich compounds have been described by Emin (E87). The crystal structure of B_4C is highly unusual. This structure is illustrated in Fig. 22. The basic structure is rhombohedral, with an icosahedral structure unit occupying each vertex of the rhombohedron. Each icosahedron contains 12 boron atoms, one occupying each vertex. In the middle of this structure is a chain of three carbon atoms, the ends of which are bound to three adjacent icosahedra. To complicate matters further, carbon atoms can occupy one end site, both end sites, or all sites in the intericosahedral chain, as well as one of the sites within each icosahedron. Therefore, even perfect stoichiometry ($B_{12}C_3$), does not mean a uniform material.

A final property of boron-rich compounds is that they do not exhibit the usual two-center electronic bonding, but rather three-center or triangular bonding, where a pair of electrons is shared among three atoms. I believe that these icosahedra are the antithesis of a close-packed structure and that their catastrophic collapse is what causes the sudden drop in the material strength. In any event, the problem is how to reasonably represent the observed dramatic decrease in strength.

In Eqs. (3) and (7), the only way to lower Y , and hence decrease the slope of the Rayleigh line, is with the T dependence. However, as I have already noted, this effect is far too small to account for the observed results. To accomplish this reduction, I have replaced Y_A by $Y_A \exp(-C\epsilon/kT)$, where k is Boltzmann's constant and C is a material-dependent parameter. The basic Arrhenius form was prompted by the fact that Curran, Seaman, and Shockey (C87) have used it in a model for dynamic failure of solids. The choice of ϵ as the independent variable was made principally on practical grounds. It is the only thermomechanical variable in the code that can never decrease, and I did not want to have the strength increase again when the release wave arrived. Not only did an increase seem unphysical, but the agreement with the data was very poor on release when this was permitted. What studies were done with other variables, principally energy, showed that the code complications of preventing Y from ever increasing again were not worth the effort.

Finally, a number of estimates of the activation energy C were made with the result that $C(\text{est})$ lies between 0.7 and 1.1 eV. These estimates are in reasonable agreement with the values of C used in calculating the data for the two types of B_4C . It is, of course, easy to say that Arrhenius relationships can be used to explain away many things we don't understand, and besides, all activation energies are on the order of 1 eV. At this point then, we must turn to the data to see how well the model works.

Comparison of Experiment and Calculation

The lower stress experiment for the Eagle-Picher material was used to normalize the code. The parameters A , C , D , and Y_A were adjusted to give the best calculational agreement with the data. As before, A was chosen to reproduce the arrival time of the elastic release, and Y_A was adjusted to give the observed HEL. Because there are no Y vs ϵ data, D was adjusted to give the observed slope of the wave profile at the first elastic reflection. Finally, C was adjusted so that this elastic reflection arrived at the observed time. While A and $A(\text{est})$ are in good agreement, D and $D(\text{est})$ disagree by a factor of about 5. Again, this highlights the need for high-pressure ultrasonic data and more Y vs ϵ data.

Figure 17 compares this adjusted calculation and the data. The release wave is calculated somewhat low; probably the yield strength is still too large. A good test of the model is now to calculate the higher-pressure experiment by using the same parameters. This comparison is shown in Fig. 18. The agreement is remarkably good. Note that all the elastic reflections arrive at the correct time on shock loading. The agreement on release is excellent; the code correctly calculates the gradual falloff of the signal. From this comparison, it is apparent that the choice of A , C , D , and Y_A are very good.

Using the same values of A and D , we adjusted C and Y_A again to fit the lower-stress experiment for the Dow material, which appears to have an HEL that is about 10% higher than the Eagle-Picher material. Because of this, the time difference between the elastic and plastic waves is greater. Therefore, C for this material is also greater than for the Eagle-Picher material. The comparison of experiment and calculation is shown in Fig. 19. The agreement is good, even for the release wave. As was done for the first material,

the higher-stress experiment was used to test the model. This comparison is shown in Fig. 20. The agreement is excellent. Again, all the timing is correctly reproduced, as is the gradually sloping release curve.

An additional test of the model is to compare a set of calculated Hugoniot final states with the data of Gust and Royce (G71). These results are shown in Fig. 21 for the Eagle-Picher material. The solid line is the calculated locus of Hugoniot states, and it is in excellent agreement with the data both above and below the stress levels corresponding to the measured wave profiles. An interesting result of the calculations is that above about 350 kb, the calculated ratio of ϵ/T is roughly constant. This implies that Y does not approach zero, but rather remains at a constant value which is about 30% of the value at the HEL for the Dow material, and about 45% of the value for the Eagle-Picher material. Higher-stress experiments would help to check this aspect of the model.

In summary, the calculations with the Arrhenius relationship do a remarkable job of simulating the data, in particular the Hugoniot and the shock-loading profiles. In the course of time, I hope to put this concept on a firmer physical foundation.

In Fig. 20, the stress corresponding to the sharp peak at the HEL is 207 kb. This exceeds by 46 kb the stress corresponding to the minimum in velocity that follows about 50 ns later. This drop in stress well exceeds the measured spall stress in B_4C (G90b). This information is in accord with the precursor decay hypothesis previously described.

Figure 23 shows only the calculated loading curve with no rate dependence for experiment CE3. This calculation was done to clearly delineate the many wave interactions and how well they correlate with the data. In particular, note the calculated increase at $t = 1.04 \mu s$ and how well it corresponds with the abrupt change from a decreasing to an increasing velocity in the data. I believe that this is the most striking example of wave interactions—in this case, the closing of a spall-induced gap as a result of the arrival of a new shock reverberation.

Figures 24 and 25 illustrate the importance of rate effects and the Bauschinger model as they affect the release curve. In Fig. 24, I compare the data with and without the Bauschinger model. Figure 25 shows the effect of first removing the rate dependence and then also removing the Bauschinger model. It is quite evident that both play a role in smoothing and shaping the release curve. Along with Figs. 9 and 10, these computer studies show how all parts of this model work together to shape the calculated the wave profiles.

AlN

The Sandia group is in the process of obtaining shock-wave profile data for this material. Computational difficulties are expected here also, as the Hugoniot data of Kondo et al. (K82) show a possible phase transformation.

Suggested Experiments to Improve the Data Base

To construct any constitutive model requires an ample data base. I suggest that the following experiments would be of considerable importance.

- Measure the shear and longitudinal sound speeds vs pressure.
- Measure Y vs ϵ for B_4C and TiB_2 .
- Diamond cell studies of the structural changes in TiB_2 .
- Hugoniot measurements on non-porous samples, particularly SiC , PSZ , and AlN .
- Shock-wave profiles, particularly AlN , and PSZ at lower pressure.

It is my understanding that some of this work is either planned or started at various laboratories in the United States. These include sound speed vs pressure measurements on AlN , Y vs ϵ for B_4C , diamond cell studies of TiB_2 , and wave-profile measurements on AlN .

Conclusions

The choice of materials in this work was dictated by the availability of data. However, it now appears that these materials represent a most interesting cross section of ceramics. The majority of the work presented is for two carbides that could not be more different. SiC, with its large thermal conductivity, appears to be rather metal-like. On the other hand, B₄C is probably as exotic a material as one can find, and quite unlike SiC. TiB₂ has interesting structures in its wave profiles that might or might not be caused by a phase change. In addition, at least four different crystal structures are represented.

In spite of these differences, certain themes seem to run throughout all the studies. One is the importance of pressure and strain rate in determining the yield strength. While this point has been made by others (R88), this study quantifies these effects. Another point is the importance of the thermomechanical variables, as well as the Bauschinger effect, in determining the total time-response of ceramics to high-strain-rate deformation.

Quite possibly, both tensile fracture and compressive yield occur at about the same stress level. Lankford's results and the results of this study would seem to point in this direction. In addition, any compressive fracture would seem to be simply microcracking with no bulking or dilation occurring during the initial stress-loading and unloading. If bulking or dilation were to occur, then the added porosity would decrease the longitudinal sound speed C_l and so increase the time when the measured elastic release arrives. However, the data show that the release wave arrives early, implying that C_l must be greater than it is at STP conditions. This increase in C_l can be accomplished by making the shear modulus a function of pressure as I have done.

The constitutive model presented here is quite easy to implement in a hydrodynamic computer code. Methods have been given to estimate the required parameters when adequate data are not available to determine them. The model does successfully reproduce a variety of data on six ceramics. Finally, I have suggested a number of straightforward experiments to increase our data base. I strongly believe that these data are necessary for the success of any constitutive model.

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Table 1. Parameters for the hydrodynamic EOS.

Material	ρ_0 (g/cm ³)	C_0 (cm/ μ s)	S_1	S_2	S_3	γ_0	References
SiC	3.177	0.819	0.88	—	—	1.16	G73, M79
PSZ	6.028	0.567	1.0*	—	—	1.57	—
B ₄ C	2.516 (E.P.) 2.506 (Dow)	0.957 (E.P.) 0.965 (Dow)	1.0	—	—	1.46	G71
TiB ₂	4.452	0.696	5.288	-32.6	71.3	1.39	G73
AlN	3.26	0.783				1.34	K&P

* Estimate

Table 2. Parameters for the constitutive model.

Material	G_0 (Mb)	Y_A (kb)	A (Mb ⁻¹)	$A(\text{est})$ (Mb ⁻¹)	$B(\text{est})$ (10 ⁻⁴ K ⁻¹)	D (kb,s)	$D(\text{est})$ (kb-s ^{1/3})	K_{IC} (MPa-m ^{1/2})	References
SiC	1.869	120	1.2	1.2	1.0	1.2	0.12	4.4	B78, L83
PSZ	0.834	60	3.0	1.6	3.4	0.12	0.19	8.25	S84, L85
B ₄ C	1.993 (E.P.) 1.972 (Dow)	111 (E.P.) 121 (Dow)	1.1	1.4	3.0	0.36	0.075	2.4	B78, L90a
TiB ₂	2.372	48 (Low) 101 (High)	1.2	1.45	1.0	0.36	0.13	4.6	L66, L90a
AlN	1.30			1.5	1.5	0.12	0.12	4.5	L66, S88

Additional information:

1. n is always 0.366.
2. For SiC, $Y_L = 19$ kb.
3. For B₄C, $C = 0.88$ eV (E.P.) and 1.21 eV (Dow); $C(\text{est}) = 0.7$ –1.1 eV.
4. For TiB₂, $B(\text{meas.}) = 0.94 \times 10^{-4}$.
5. References: First entry is for E_T ; second is for K_{IC} .

Table 3. Calculated values of the Hugoniot elastic limits.

Material	HEL (kb)
SiC	174
PSZ	162
B ₄ C (Dow)	169
B ₄ C (Eagle-Picher)	154
TiB ₂ (low)	62
TiB ₂ (high)	138

Table 4. Calculated peak values of various thermodynamic variables.

Sandia ID	Material	Peak values					
		σ (kb)	P (kb)	T (K)	ϵ (%)	$\dot{\epsilon}$ (s ⁻¹)	Y (kb)
CE4	SiC	272	174	506	2.35	1.3×10^4	160
CE5	SiC	361	257	674	4.2	7.8×10^4	170
CE31	SiC	489	77	930	6.7	1.7×10^5	184
CE22	PSZ	288	223	464	3.1	8.0×10^3	107
CE7	TiB ₂ (low)	479	431	556	6.2	5.5×10^4	104
CE3	B ₄ C (E.P.)	229	188	476	3.7	5.2×10^3	61*
CE6	B ₄ C (E.P.)	320	287	579	6.0	2.0×10^4	50*
CE17	B ₄ C (Dow)	239	209	502	4.35	1.0×10^4	44*
CE18	B ₄ C (Dow)	299	275	569	5.85	2.25×10^4	37*

* Y at σ_{max}

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Figure Captions

Figure 1. Deviator stress vs deviator strain. The solid curve is simple elastic-plastic; the dashed curve is the Cochran-Guinan model.

Figure 2. Yield strength vs strain rate for SiC.

Figure 3. Yield strength vs strain rate for partially stabilized zirconia.

Figure 4. Yield strength vs strain rate for AlN.

Figure 5. Comparison of calculation and experiment CE4 for SiC.

Figure 6. Comparison of calculation and experiment CE5 for SiC.

Figure 7. Comparison of calculation and experiment CE31 for SiC.

Figure 8. Comparison of calculation and experiment CE5 for SiC with and without the rate-dependent limit.

Figure 9. Comparison of calculation and experiment CE4 for SiC with and without pressure dependence.

Figure 10. Comparison of calculation and experiment CE5 for SiC with and without pressure dependence.

Figure 11. Comparison of calculation and experiment CE4 for SiC with and without rate dependence.

Figure 12. Comparison of calculation and experiment CE5 for SiC with and without rate dependence.

Figure 13. Calculated yield strength vs time at the center of the target for the three SiC experiments.

Figure 14. Comparison of calculation and experiment CE22 for PSZ.

Figure 15. Hugoniot data for TiB_2 and the nonlinear U_s-U_p fit.

Figure 16. Comparison of calculation and experiment CE7 for TiB_2 for both the high and low HEL.

Figure 17. Comparison of calculation and experiment CE3 for B_4C .

Figure 18. Comparison of calculation and experiment CE6 for B_4C .

Figure 19. Comparison of calculation and experiment CE17 for B_4C .

Figure 20. Comparison of calculation and experiment CE18 for B_4C .

Figure 21. Comparison of the calculated Hugoniot with the data for B_4C . Also shown is the calculated hydrostat.

Figure 22. Crystal structure of B_4C (from E87).

Figure 23. Comparison of a rate-independent calculation and experiment CE3 for B_4C .

Figure 24. Comparison of calculation and experiment CE6 for B_4C , with and without the Bauschinger model.

Figure 25. Comparison of calculation and experiment CE6 for B_4C for (a) the full calculation, (b) no rate-dependence, and (c) no rate-dependence or Bauschinger model.

Figure 1. Deviator stress vs deviator strain. The solid curve is simple elastic-plastic; the dashed curve is the Cochran-Guinan model. The dashed curve is arrived at by assuming that $dS/d\epsilon$ varies linearly from its value at point A (i.e., at $S = 0$) to zero at point B.

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Figure 14. Comparison of calculation and experiment CE22 for PSZ.

Figure 15. Hugoniot data for TiB₂ and the nonlinear U_s-U_p fit. The arrow shows where experiment 6 of Kipp and Grady would fall.

Figure 16. Comparison of calculation and experiment CE7 for TiB_2 for both the high and low HEL.

Figure 17. Comparison of calculation and experiment CE3 for B_4C .

Figure 18. Comparison of calculation and experiment CE6 for B_4C .

Figure 19. Comparison of calculation and experiment CE17 for B_4C .

Figure 20. Comparison of calculation and experiment CE18 for B_4C .

Figure 21. Comparison of the calculated Hugoniot with the data for B_4C . Also shown is the calculated hydrostat. The arrows point to where the four wave-profile experiments fall on this Hugoniot.

Figure 22. Crystal structure of B_4C (from E87).

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