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MTF: A COMPUTER PROGRAM FOR CALCULATING
THE MARS MODE OPERATING PARAMETERS FOR
THE AXICELL MTF-B

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MFTF: A COMPUTER PROGRAM FOR CALCULATING THE MARS MODE OPERATING PARAMETERS
FOR THE AXICELL MFTF-B

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ABSTRACT

We describe the models used to calculate the equilibrium operating point for the MARS mode of the axicell MFTF-B, and present the model equations necessary to calculate the plasma and potential parameters in the central-cell, axicell, and anchor regions. In addition, we calculate the neutral beam, rf power, and gas inputs required to sustain the plasma and maintain the ion and electron-particle energy balance. After a brief description of the MFTF code, we present the results of a sample calculation using MFTF.

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INTRODUCTION

The latest models used to calculate the operating point for the MARS mode of the axicell MFTF-B have changed somewhat since they were first described.^{1,2} In this paper we present the relevant model equations and any improvements to the models still required.

We calculate the plasma and potential parameters of the central cell, axicell, and thermal barrier region of the anchor; the warm electron parameters in the potential peak region of the anchor; the anchor sloshing beam and pump beam requirements, and the microwave power necessary to produce the thermal barrier and potential peak.

We also describe the central-cell particle and energy balance models as well as the iteration procedure used in these models.

The computer program MFTF uses the models to calculate the steady-state operating point. We present the results of a sample problem from the program calculations.

PHYSICS MODELS

CENTRAL-CELL PARAMETERS

To calculate the central-cell parameters, we chose a design point based on a set of central-cell parameters near the limit of magnetohydrodynamic (MHD) stability for our magnet design.³ In this case, for a peak anchor beta of $\hat{\beta}_b = 0.55$, we chose a peak central cell $\hat{\beta}_c$ of 0.4 with a peak axicell $\hat{\beta}_x$ of 0.1. To maximize the fusion power that would be achievable from a 50-50 D-T mix, we chose a thermal ion temperature in the central cell of $T_{iwc} = 15$ keV. The electron temperature is estimated to be about 60% of this value (as in Ref. 4), and hence, $T_{ec} \approx 9$ keV. The desired ion Pastukhov confinement parameter is chosen as $(n\tau)_{iwc} \approx 5 \times 10^{13}$, and this determines the required ion confining potential ϕ_c . This value of ϕ_c is near the maximum that can be generated with 80-kV beams and electron-cyclotron resonant heating (ECRH).

We initially calculated the electron confining potential ϕ_e to produce an electron Pastukhov $(n\tau)$ equal to the ion value. This tends to give an overestimate of ϕ_e because the presence of secondary electrons flowing in from the end walls would produce an electron confinement time less than the ion Pastukhov confinement time. However, this estimate for ϕ_e is a satisfactory

starting value that can be easily refined in several iterations. The nomenclature used in this report follows the axicell field and potential profiles shown schematically in Fig. 1.

To calculate the ϕ_c value appropriate for the chosen ion Pastukhov confinement parameter, we set $(n\tau)_{iWC}$ equal to the desired value and use

$$(n\tau)_{iWC} = \frac{\sqrt{\pi}}{4} \frac{(n\tau)_{ii} G_i}{\xi_i I \left(\frac{T_{iWC}}{\phi_c} \right)} \frac{\phi_c}{T_{iWC}} \exp \left(\frac{\phi_c}{T_{iWC}} \right), \quad (1)$$

where

$$(n\tau)_{ii} = 4.56 \times 10^{11} (T_{iWC})^{3/2} / \ln \Lambda_{ii},$$

$$\xi_i = \frac{\sum_j z_j^2 n_j \ln \Lambda_{ij}}{2 \sum_j \left(\frac{m_i}{m_j} \right) z_j^2 n_j \ln \Lambda_{ij}} = \frac{1}{2} \text{ when } i = j,$$

$$I(T/\phi) = 1 + \frac{1}{2} \sqrt{\pi T/\phi} e^{\phi/T} \operatorname{erfc}(\sqrt{\phi/T}),$$

$$G_i = W_i \ln \left(\frac{W_i + 1}{W_i - 1} \right),$$

$$W_i = \left(1 + \frac{1}{\xi_i R} \right)^{1/2},$$

and

$$R = B_{\max}/B_c (1 - 0.5 \hat{\beta}_c)^{1/2}.$$

To determine the initial estimate for ϕ_e , we equate the ion and electron Pastukhov confinement parameters by using Eq. (1) and

$$(n\tau)_{eC} = \frac{\sqrt{\pi}}{4} \frac{(n\tau)_{ee} G_e}{\xi_e I \left(\frac{T_{ec}}{\phi_e} \right)} \frac{\phi_e}{T_{ec}} \exp \left(\frac{\phi_e}{T_{ec}} \right), \quad (2)$$

where

$$(n\tau)_{ee} = 8.19 \times 10^9 T_{ec}^{3/2} / \ln \Lambda_{ee} ,$$

$$\xi_e = \frac{1}{Z} \left(1 + \frac{\sum_j z_j^2 n_j \ln \Lambda_{ej}}{n_e \ln \Lambda_{ee}} \right) \approx 1 ,$$

$$G_e = W_e \ln \left(\frac{W_e + 1}{W_e - 1} \right) ,$$

and

$$W_e = \left(1 + \frac{1}{\xi_e R} \right)^{1/2} .$$

Using the chosen values of $\hat{\beta}_c$, T_{iwc} , and T_{ec} , we can determine a value of ion density using

$$n_i \approx \frac{2.5 \times 10^{15} \hat{\beta}_c B_c^2}{T_{iwc} + T_{ec}} . \quad (3)$$

However, because the central-cell beam and/or ion cyclotron resonant heating (ICRH) would produce a hot ion component in the central cell, we have reduced the warm ion density by assuming a hot ion population at $T_{ihc} \approx 35$ keV and a density of about 10% of the warm ion density. For a $\hat{\beta}_c$ of 0.4, this gives

$$n_{iwc} \approx 3.5 \times 10^{13} \text{ cm}^{-3}$$

and

$$n_{ihc} \approx 3.6 \times 10^{12} \text{ cm}^{-3} .$$

Furthermore, because of the presence of the hot ions, we have increased the fusion reaction rate by an arbitrary 15% over the value of $2.65 \times 10^{-16} \text{ cm}^3/\text{s}$ appropriate for a Maxwellian D-T mixture at $T_{iwc} = 15$ keV. This enhanced $\langle \sigma v \rangle_f$ generates a larger central-cell fusion power than if we had assumed only warm ions at the density given by Eq. (3) above. Fokker-Planck

Figure 1. Magnetic field and potential profiles for MFTF-B axicell.

calculations of the central cell are currently being modeled to determine the appropriate proportion of warm and hot ions.

AXICELL PARAMETERS

The plasma density and potential profile in the axicell as well as the neutral beam required are calculated in a manner similar to that in Ref. 4. The potential rise from the central cell to the axicell midplane, $\Delta\phi_{pc}$, is calculated from

$$\Delta\phi_{pc} = T_{ec} \ln \left(\frac{n_{ex}}{n_{ec}} \right), \quad (4)$$

where the electron density at the axicell midplane is determined from

$$n_{ex} = n_{iwX} + n_{ihX}. \quad (5)$$

The warm ion density is assumed to be Maxwellian. Hence,

$$n_{iwX} = n_{iwC} e^{-\Delta\phi_{pc}/T_{iwC}}. \quad (6)$$

The hot ion density is determined by assuming that the perpendicular $\hat{\beta}_X$ in the axicell is given by the MHD-stability limit. Then,

$$n_{ihX} \approx \frac{2.5 \times 10^{15} \hat{\beta}_X^2 B_X^2 - n_{iwX} (T_{iwC} + T_{ec})}{(0.9 E_{ihX} + T_{ec})}. \quad (7)$$

The perpendicular hot-ion energy is assumed to be given by $0.9 E_{ihX}$, accounting for the anisotropy in the ion distribution. The solutions of Eqs. (4) through (7) give $\Delta\phi_{pc}$ and the plasma density at the axicell midplane.

The potential drop from the axicell midplane to the outboard mirror is calculated from

$$\Delta\phi_{pb} = T_{ec} \ln \left(\frac{n_{ex}}{n_{ex}(m)} \right). \quad (8)$$

The electron density at the outboard axicell mirror, $n_{ex}(m)$, is given by

$$n_{ex}(m) = g_b(m) n_{iX}^{pass}(m),$$

where Campbell⁵ modeled $g_b(m)$ as

$$g_b(m) = 1 + 0.3 \frac{\Delta\phi_{pc}}{T_{iwC}} .$$

The cold passing density at the axicell outer mirror is mapped from the axicell midplane,⁶ and is given by

$$n_{iX}^{pass}(m) = F(B_{mX0}, B_{xp}, \Delta\phi_{pb}, 0, B_{mX0}/B_{xp}, 1, n_{iwx}) \\ - F(B_{mX0}, B_{ap}, \phi_1, 0, B_{mX0}/B_{ap}, 1, n_{iwa}) , \quad (9)$$

where the function F is defined in Appendix A. Here, we have used

$$B_{xp} = B_X (1 - 0.5 \hat{\beta}_X)^{1/2} ,$$

$$B_{ap} = B_a (1 - 0.5 \hat{\beta}_a)^{1/2} ,$$

and

$$\phi_1 = \phi_C - \Delta\phi_{pc} + \Delta\phi_{pb} .$$

The plasma lifetime is calculated using a modified Logan-Rensink model. Assuming that the hot ion lifetime is determined by scattering losses, ion drag, and electron drag, and also assuming that the charge-exchange losses are negligible because of re-ionization, the hot-ion confinement parameter for the axicell is given by

$$(n\tau)_X = \left\{ (n\tau)_S^{-1} + \left[(n\tau)_{De} \ln(E_{inj}/E_L) \right]^{-1} \right. \\ \left. + C1 / \left[(n\tau)_{Di} \ln \left(\frac{E_{inj}}{3T_{ec}} \right) \right] \right\}^{-1} , \quad (10)$$

where C1 is a coefficient to be described below, where⁴

$$(n\tau)_s = \frac{3.9 \times 10^{12} (E_{inj})^{3/2}}{\ln A_{ii}} \log_{10}(R_{eff}) ,$$

$$R_{eff} = \frac{B_{mxi}/B_x}{\left(1 + \frac{\Delta\phi_{pc}}{E_{inj}}\right) (1 - 0.5 \hat{\beta}_x)^{1/2}} ,$$

$$(n\tau)_{De} = \frac{2 \times 10^{13}}{\ln A_{ei}} T_{ec}^{3/2} ,$$

and

$$(n\tau)_{Di} = \frac{3.28 \times 10^{11}}{\ln A_{ii}} (2/3 E_{ihx} + T_{iwc})^{3/2} ,$$

and where E_{inj} is the beam-injection energy. The average energy of the ions escaping from the axicell is $\bar{E}_L$ and is obtained from⁴

$$\frac{\bar{E}_L}{E_{inj}} = \frac{1}{1 + \tau_s/\tau_D} + \frac{E_p (\tau_s/\tau_D)/E_{inj}}{1 + \tau_s/\tau_D} \quad (11)$$

and

$$\frac{\tau_s}{\tau_D} = \frac{0.11}{\sqrt{A}} \left(\frac{E_{inj}}{T_{ec}} \right)^{3/2} \frac{\log(R_{eff})}{\ln(E_{inj}/\bar{E}_L)} \frac{\ln A_{ei}}{\ln A_{ii}} , \quad (12)$$

where A is the ion mass in amu and

$$E_p = \frac{\Delta\phi_{pc}}{\frac{B_{mxi}}{B_x} - 1} .$$

The calculation of $\Delta\phi_{pc}$ and $\Delta\phi_{pb}$ requires a value of E_{ihx} , the hot ion energy. This energy can be calculated by balancing the drag losses of the ions with the net energy injected by the axicell beams. If we neglect charge exchange, the hot ion energy is given by

$$E_{ihX} = C2 \left\{ \frac{n_{ihX} (E_{inj} - E_L)}{(n\tau)_X} + \frac{3/2 T_{ec} n_{eX}}{(n\tau)_{De}} + \frac{3/2 T_{ic} n_{icX}}{(n\tau)_{Di}} \right\} / \left\{ \frac{n_{eX}}{(n\tau)_{De}} + \frac{n_{icX}}{(n\tau)_{Di}} \right\}. \quad (13)$$

The coefficients C1 and C2 were chosen to match a Fokker-Planck calculation done for a representative axicell density and potential profile.⁷ The values of C1 = 0.12 and C2 = 1.20 give values of $(n\tau)_X$ and E_{ihX} that match the Fokker-Planck values for the sample case where $n_{eX} = 1.0 \times 10^{14}$, $\Delta\phi_{pc} = 9$ keV, $\Delta\phi_{pb} \approx 9$ keV, and the mirror ratio is $R = 1.5$.

Because the hot-plasma confinement parameter $(n\tau)_X$ and, hence, E_{ihX} depend on $\Delta\phi_{pc}$, and because $\Delta\phi_{pc}$ depends on the value of E_{ihX} , it is clear that an iterative procedure is required to solve for the potential profile and the axicell confinement parameter $(n\tau)_X$.

The axicell beam ionization current required to maintain the axicell is equal to the scattering losses

$$I_{X \text{ ion}} = q \frac{n_{ihX} n_{eX} V_X}{(n\tau)_X}, \quad (14)$$

where V_X is the effective volume of the axicell ions.

BARRIER PARAMETERS

With our choice of $\hat{\beta}_b$ (the beta in the midplane of the anchor, determined by MHD stability)³ and our estimated value of ϕ_e , we can solve for the barrier potential ϕ_b as a function of the fraction F_{ec} of the cold electrons at the barrier midplane by applying a quasi-neutrality relation at the barrier midplane. A value of $F_{ec} \approx 0.05$ was found to be stable against the two-ion stream instability and was used in the subsequent calculations. The distribution of the cold-electrons-density in the barrier midplane region is assumed to follow a cut-off distribution

$$n_{ecA}(\phi_b) = g_b(m) n_{iX}^{pass}(m) \left(\frac{\exp \left(-\frac{\phi_b - \Delta\phi_{pb}}{T_{ec}} \right) - \exp \left(-\phi/T_{ec} \right)}{1 - \exp \left(-\phi/T_{ec} \right)} \right), \quad (15)$$

where

$$\phi = \phi_e + \Delta\phi_{pc} - \Delta\phi_{pb}.$$

The cold electron density thus equals the electron density at the 12-T mirror when $\phi_b = \Delta\phi_{pb}$, and goes smoothly to zero when $\phi_b = \phi_e + \Delta\phi_{pc}$.

The quasi-neutrality relation, which is solved for the barrier potential ϕ_b , is then

$$n_{eA}(\phi_b) = n_i^{\text{pass}}(b) G_b F_{ec} \quad , \quad (16)$$

where G_b is the ratio of total ion density to passing ion density at the anchor midplane. The passing ion density $n_i^{\text{pass}}(b)$ at the anchor midplane is mapped from the axicell midplane density by

$$n_i^{\text{pass}}(b) = F(B_{bp}, B_{xp}, \phi_b, \phi_2, B_{bp}/B_{xp}, B_{bp}/B_{mx0}, n_{iwx}) \\ - F(B_{bp}, B_{ap}, \delta\phi_a, \phi_2, B_{bp}/B_{ap}, B_{bp}/B_{mx0}, n_{iwa}) \quad . \quad (17)$$

Here, we have defined

$$B_{bp} = B_b (1 - 0.5 \hat{\beta}_b)^{1/2} \quad ,$$

$$\phi_2 = \phi_b - \Delta\phi_{pb} \quad ,$$

and $\delta\phi_a$ is the potential rise from the barrier minimum to the potential peak

$$\delta\phi_a = \phi_b + \phi_c - \Delta\phi_{pc} \quad . \quad (18)$$

In addition, $\delta\phi_a$ is the confining potential for the warm electron population. Recall that B_b is the vacuum magnetic field at the barrier (anchor) midplane and B_{mx0} is the outer (12-T) mirror of the axicell.

Equation (16) is solved for the barrier potential ϕ_b . Then the passing density at the barrier midplane is given by Eq. (17), while the trapped- and total-ion densities at the barrier midplane are obtained from

$$n_i^{\text{trap}}(b) = [g_b(b) - 1] n_i^{\text{pass}}(b) \quad (19)$$

and

$$n_{iA}(b) = G_b n_i^{\text{pass}}(b) \quad , \quad (20)$$

where $g_b(b)$ is the ratio of total warm-ion-to-passing-ion density at the anchor midplane.

The parameters $g_b(b)$ and G_b were chosen so that the warm ion fraction at the anchor midplane would be sufficient to ensure microstability. Using $G_b = 6.5$ and $g_b(b) = 2.75$ gives $n_{\text{warm}}/n_{\text{hot}} \approx 0.73$, sufficient for microstability.

The hot ion density at $\underline{b}$ is given by

$$n_{ihA}(b) \approx n_{iA}(b) - g_b(b) n_i^{\text{pass}}(b) ,$$

and the hot sloshing ion density at $\underline{a}$ is given by

$$n_{ihA}(a) \approx R_s n_{ihA}(b) ,$$

where R_s is a parameter chosen to give an appropriate sloshing density ratio that will lead to a microstable anchor plasma.

The hot electron energy is determined by assuming that the ions and hot electrons at the barrier midplane supply a known value for the perpendicular beta β_b . Hence, for energy in keV,

$$E_{ehA} = \left(2.5 \times 10^{15} \beta_b B_b^2 - E_{inj}(a) \frac{B_b}{B_A} \frac{(1 - 0.5 \hat{\beta}_b)^{1/2}}{(1 - 0.5 \hat{\beta}_a)^{1/2}} n_{ihA}(b) \right. \\ \left. - T_{iwc} \frac{B_b (1 - 0.5 \hat{\beta}_b)^{1/2}}{B_{mAi}} g_b(b) n_{iWA}(a) \right) / [0.8 n_{iA}(b) (1 - F_{ec})] , \quad (21)$$

where B is in tesla, the density is in cm^{-3} , and $E_{inj}(a)$ is the beam injection energy at point $\underline{a}$ in keV. The 0.8 factor in the denominator accounts for the anisotropy in the electron distribution.

WARM ELECTRON AND CONFINING POTENTIAL PARAMETERS

The peak in the potential profile for the anchor occurs at the position shown as $\underline{a}$ in Fig. 1. The potential and density peak are produced by injecting the sloshing ions at an angle of 90° at the 1.4-T point in the vacuum field. Fokker-Planck calculations⁸ indicate that the potential and density peak occur inboard of the injection point. However, for these calculations, we have neglected this small difference and put the potential and density peaks at the injection point. The potential peak is enhanced using auxiliary electron heating by microwave power applied near the 1.4-T position.

As in Ref. 4, the warm electron density at a is obtained from a quasi-neutrality condition

$$n_{ewA}(a) = n_{iA}(a) - n_{ehA}(a) - n_{ecA}(a) . \quad (22)$$

The ion density at a is taken to be a factor R_s times the hot sloshing-ion density at the barrier midplane. The hot and cold electron densities at a are mapped from the known values at b using

$$n_{ehA}(a) = n_{ehA}(b) \left[\frac{B_{mAO} - B_a (1 - 0.5 \hat{\beta}_a)^{1/2}}{B_{mAO} - B_b (1 - 0.5 \hat{\beta}_b)^{1/2}} \right] \left(\frac{E_{ehII}}{\delta\phi_a + E_{ehII}} \right)^{1/2} \quad (23)$$

and

$$n_{ecA}(a) = n_{ecA}(b) \frac{B_a}{B_b} \left(\frac{T_{eC}}{T_{eC} + \pi\delta\phi_a} \right)^{1/2} \left(\frac{1 - 0.5 \hat{\beta}_a}{1 - 0.5 \hat{\beta}_b} \right)^{1/2} . \quad (24)$$

In this case, E_{ehII} is the parallel component of the hot electron energy at b and is assumed to be about 20% of the total hot electron energy, where we have assumed a four-to-one temperature anisotropy for the hot electrons, consistent with the 0.8 factor in Eq. (20). The confining potential for the warm electrons $\delta\phi_a$ was defined in Eq. (18).

Using the model in Ref. 4 with corrections from Cohen⁹ and Logan,¹⁰ the warm electron temperature can then be determined from

$$\frac{\delta\phi_a}{T_{ewA}} = \ln \left[N_{cw} \frac{(1 + J_{AB}) C_{rf}}{(\gamma + N_{cw} C_{rf} J_{AB})} \right] \quad (25)$$

as a function of the cold-electron ionization source J_e , where

$$J_{AB} = \frac{\sqrt{\pi}}{4q} \left(1 + \frac{B_b (1 - 0.5 \hat{\beta}_b)^{1/2}}{B_a (1 - 0.5 \hat{\beta}_a)^{1/2}} \right) \frac{\tau_{ee} J_e}{n_{ew}(a) C_{rf}} , \quad (26)$$

$$\tau_{ee} = \frac{8.19 \times 10^9 T_{ew}^{3/2} (\text{keV})}{[n_{ewA}(a) + n_{ecA}(a)] \ln \Lambda_{ee}} , \quad (27)$$

and

$$N_{cw} = \frac{n_{ewA}(a)}{n_{ecA}(b)} \left(\frac{T_{eC}}{T_{ew}} \right)^{1/2} . \quad (28)$$

In general, the cold electron source is caused by ionization of the background gas, the sloshing-ion beam, and the axial high-energy pump beam at the position $\underline{a}$. Because the value of J_e will be determined as a function of the sloshing ion and high energy pump beam, we can find the warm electron temperature that is consistent with the neutral beam requirements and the required potential profile. For these calculations, the contribution to J_e from the background gas is negligible compared with the beam contributions.

The parameter C_{rf} is a measure of the amount of perpendicular velocity diffusion that results from the application of ECRH at point $\underline{a}$. In earlier calculations¹ this parameter was set to unity, implying no enhanced rf diffusion. For the present calculations, C_{rf} is adjusted to the value that gives a value of $T_{ew} < \delta\phi_a$ as the solution to Eq. (25).

Because $C_{rf} \gg 1$ when $T_{ew} < \delta\phi_a$, we have an effective source for the hot electrons resulting from the diffusion of the warm electrons. Because angle scattering is the only loss mechanism for the hot electrons in this model, there must be a limit on the number of warm electrons that can be diffused into the hot electron population. One estimate of the maximum warm electron source for the hot electrons is given by Eq. (45) in Ref. 4. At present, we are neglecting this constraint on the particle flow. Further Fokker-Planck calculations modeling the ECRH heating of the hot and warm electron populations in the anchor must be performed to determine the electron distribution functions and the value of the parameter C_{rf} , and to ascertain the validity of the model in Eq. (25).

The parameter γ is a measure of the effect of the hot electrons at the potential peak position $\underline{a}$. Cohen⁹ gives it as

$$\gamma = 1 + \exp(-\hat{E}/T_{ec}) \left\{ \frac{T_{eff}}{T_{ec}} - 1 - \frac{T_{eff}}{T_{ec}} \exp\left(\frac{\hat{E}}{T_{eff}} - \frac{\delta\phi_a}{(R_{ba} - 1)T_{eff}}\right) \right\} \quad (29)$$

We have used Cohen's estimate⁹ of

$$\hat{E} = \phi_{bm}/(R_{bm} - 1) \quad (30)$$

where ϕ_{bm} is the potential between the yin-yang inner mirror and the yin-yang midplane. The effective mirror ratios are given by

$$R_{bm} = \frac{B_{\text{mirror}}}{B_b (1 - 0.5 \hat{\beta}_b)^{1/2}} \quad (31)$$

and

$$R_{ba} = \frac{B_a}{B_b} \left(\frac{1 - 0.5 \hat{\beta}_a}{1 - 0.5 \hat{\beta}_b} \right)^{1/2} \quad (32)$$

The factor T_{eff} is the effective hot electron temperature near the loss boundary of the hot mirror-confined electrons. We have used the hot-electron perpendicular energy E_{ehl} as an upper bound estimate of T_{eff} . This is clearly an overestimate, and when combined with Cohen's definition⁹ of $\hat{E}$, gives an overestimate of γ and consequently an overestimate of C_{rf} .

We have used this estimate of $\hat{E}$ and T_{eff} even though Logan¹⁰ has pointed out that it is not consistent with the low fraction of hot electrons assumed in the barrier. He notes that $\hat{E}$ should be the energy at which the hot electron distribution with effective energy T_{eff} joins the bulk electron distribution with temperature T_{ec} . When

$$F_{ec} = \frac{n_{ec}(b)}{n_{tot}(b)} \approx 0.05 \quad ,$$

and assuming that $T_{\text{eff}} = E_{ehl}$ (the hot electron energy perpendicular to B), then

$$\hat{E} \gg \phi_{bm} / (R_{bm} - 1) \quad . \quad (33)$$

Hence, our estimate of γ is large by a factor of 2 to 4. To get consistently lower values of γ using our model, we could either redefine $\hat{E}$ to be a larger value¹⁰ or, alternatively, model $T_{\text{eff}} \ll E_{ehl}$.

While a better model might lower γ and, hence, the value of C_{rf} , the power requirements at a and b will not be affected in our models unless C_{rf} can be lowered to unity (which is unlikely). However, lower C_{rf} values would certainly increase our confidence in the model Eq. (25).

ANCHOR CELL AND PUMP BEAM REQUIREMENTS

The neutral beams injected into the barrier region in the anchor include the high-energy sloshing-ion beam, incident at 90° at the vacuum 1.4-T outboard position, and an axial high-energy pump beam (HEPB). The HEPB is designed to pump out the warm trapped ions, which would otherwise build up in the potential well in the barrier. In addition, the HEPB pumps the hot sloshing ions and maintains the appropriate ratio of peak-to-minimum sloshing-ion density in the anchor R_s .

The ionization current of the sloshing-ion beam in the anchor balances the scattering losses of the hot sloshing ions and the charge-exchange losses off the HEPB. If, in addition, the one-half and one-third energy components of the sloshing beam were not confined when injected at the 1.4-T point, they would act as a charge-exchange pump removing the hot sloshing ions that are confined. The ionization current would then be written

$$I_{\text{slosh}}^{(\text{ion})} = \frac{I_{\text{scat}} + I_{\text{cx}}^{\text{HEPB}}}{1 - F_2} \quad (34)$$

Here, I^{scat} is the scattering-loss current that Logan¹¹ obtained by modeling Fokker-Planck calculations of the scattering loss of the sloshing ions, such that

$$I^{\text{scat}} = \frac{q n_{\text{ihA}}^2(b) v_{\text{slosh}}}{(n\tau)_{\text{slosh}}} \quad (35)$$

$$(n\tau)_{\text{slosh}} = 2.23 \times 10^{10} (\bar{E}_t)^{3/2} \log_{10} \left[\frac{B_{\text{mAO}}/B_a (1 - 0.5 \beta_a)^{1/2}}{1 + \frac{\Delta\phi}{\bar{E}_t}} \right] \quad (36)$$

$$\bar{E}_t = \frac{\sum_{j=1,1/2,1/3} \left[\frac{\langle \sigma v \rangle_{ij}}{v_j} f_j E_j + \frac{\langle \sigma v \rangle_{\text{cx}j}}{v_j} f_j (E_j - \bar{E}_{jn}) \right]}{\sum_{j=1,1/2,1/3} \frac{\langle \sigma v \rangle_{ij}}{v_j} f_j} \quad (37)$$

and

$$\bar{E}_{in} = \frac{\sum_{j=1,1/2,1/3} \frac{\langle \sigma v \rangle_{ij}}{v_j} f_j E_j}{\sum_{j=1,1/2,1/3} \frac{\langle \sigma v \rangle_{ij}}{v_j} f_j} . \quad (38)$$

When the one-half and one-third beam-energy components are not confined, the term F_2 is the ratio of the charge-exchange rate of the one-half and one-third beam-energy components to the ionization rate of the full-energy component of the sloshing beam. I_{cx}^{HEPB} is the charge-exchange loss current resulting from charge-exchange interactions between the sloshing ions and the HEPB ions as they traverse the anchor.

The summations indicated in Eqs. (37) and (38) include the full, one-half, and one-third beam-energy components of the beam; $\langle \sigma v \rangle_{ij}$ and $\langle \sigma v \rangle_{cxj}$ are the total ionization and charge-exchange rates for beam component j , shown in Table 1 along with E_j and f_j (the energy and atomic fraction for each beam component); v_j is the velocity of beam component j , corresponding to the energy E_j ; and $\Delta\phi$ is the potential difference between the outer mirror and the point of beam injection.

The ionization currents from both the sloshing beam and the HEPB deposited in the region of the potential peak provide the cold electrons that form the cold-electron source J_e in Eq. (26). This value of J_e places a limit on the maximum sloshing-ion and HEPB currents that can be used in the anchor. When J_e becomes too large, the model Eq. (25) only has solutions with $T_{eWA} > \delta\phi_a$, a regime where the model becomes inaccurate. In practice, the value of J_e is limited to several microamps per cubic centimeter.

The high-energy pump beam in the anchor is determined by assuming a value for v_{cx} , the charge-exchange rate for the HEPB at the bottom of the barrier. Using the calculated density at several cardinal points in the anchor and also using the effective lengths, we can evaluate the beam attenuation and absorption of the HEPB in each region of the anchor. The value of $v_{cx} = 2.5$ was determined by a Fokker-Planck calculation⁸ of sloshing ion and pump beam injection into the anchor to achieve a peak-to-minimum ratio of sloshing density of $R_s \leq 2.5$.

The high-energy pump beam and passing-particle-barrier beam requirements for the transition region were obtained by scaling from the Fokker-Planck

Table 1. Sloshing ion reaction rates and atomic fractions.

E_{beam} (keV)	f_j	$\langle\sigma v\rangle_{3\text{cx}}$ (cm^3/s)	$\langle\sigma v\rangle_{3\text{ii}}$ (cm^3/s)	$\langle\sigma v\rangle_{3\text{ie}}$ (cm^3/s)
80	0.833	2.7×10^{-8}	4.5×10^{-8}	3.2×10^{-9}
40	0.075	4.0×10^{-8}	4.3×10^{-8}	3.2×10^{-9}
26.7	0.092	5.5×10^{-8}	4.1×10^{-8}	3.2×10^{-9}

calculations of Futch and Leppelmeier¹² and Porter.¹³ The scaling factor was obtained by assuming that the total trapped current to be removed by the pump beams was proportional to the square of the passing density at the bottom of the transition and to the length of the transition and anchor region from the 12-T magnet to the position of the potential peak in the anchor. This density weighting is consistent with the scaling of Futch and LoDestro¹⁴ in their study of collisional trapping rates.

The scaling factor is

$$F = \left(\frac{n_{\text{pass}}^2(B_t)}{10^{24}} \right) \times \left(\frac{L_t + L_{\text{an}}}{1000} \right) \times F_p, \quad (39)$$

where the values of $n_{\text{pass}}(B_t) = 10^{12} \text{ cm}^{-3}$ and $L_t + L_{\text{an}} = 1000 \text{ cm}$ were used in the Futch-Leppelmeier calculations.¹² The parameter $F_p = 1/3$ scales their results to Porter's results¹³ obtained using the new multi-region Fokker-Planck code developed by Matsuda.

Porter¹³ is presently performing calculations that more accurately model the potential and density profiles expected in the transition and anchor regions; these results will be used in subsequent calculations to determine the scaling of the trapping rate in the transition and pumping requirements.

MICROWAVE POWER REQUIREMENTS

The microwave power supplied to both the potential peak region a and the thermal barrier region b in the axicell is calculated in a manner similar to that in ref. 4. The power (in kW) at a is given by

$$P_a = \left\{ J_{\text{in}} R \left[\frac{f \delta \phi_a}{1 - R} + T_{\text{ew}} - T_{\text{eC}} \right] + J_e \left[\frac{\delta \phi_a}{1 - fR} + R T_{\text{ew}} \right] + \frac{J_{\text{in}}}{2} [T_{\text{ew}} - T_{\text{eC}}] \right\} a V_a. \quad (40)$$

In the so-called "weak ECRH" limit of Ref. 4, $f = 0$; otherwise, $f = 1$. Also,

$$J_{\text{in}} = \frac{n_{\text{eCA}}(b) n_{\text{ewA}}(a)}{(n\pi)} \left(\frac{T_{\text{ew}}}{T_{\text{eC}}} \right)^{1/2} e^{\delta \phi_a / T_{\text{ew}}}, \quad (41)$$

$$R = \frac{B_b}{B_a} \left(\frac{1 - 0.5 \hat{B}_b}{1 - 0.5 \hat{B}_a} \right)^{1/2}, \quad (42)$$

and

$$(n\tau) = \frac{\sqrt{\pi}}{4} (1 + R) n_{ewA(a)} \tau_{ee} \left(e^{\delta\phi_a/T_{ew}} - 1 \right). \quad (43)$$

The dominant term in Eq. (40) is the one involving J_e . Hence, the power at a is controlled by the magnitude of the cold-electron source at a.

The microwave power needed to produce the hot electrons is modeled as the sum of scattering and synchrotron radiation losses. The scattering loss (in kW) is

$$P_{SCATT} = \frac{q n_{iA}^2(b) (1 - F_{ec}) (E_{ehA}/2)}{n\tau} v_{eh}, \quad (44)$$

where

$$n\tau = 1.65 \times 10^8 E_{ehA}^{3/2} \log_{10} (R_{eff}). \quad (45)$$

The numerical coefficient in Eq. (45) is low by comparison with recent electron Fokker-Planck calculations in this geometry,¹⁶ which are best fitted with the coefficient 2.8×10^8 . However, the Fokker-Planck calculations include only the hot electron population and neglect drag effects between the hot electrons and the other electron populations whose spatial distributions overlap. Depending on the details of the spatial distribution, the drag effects might add as much as 50 to 60% to the power estimate for the hot electrons at b. Thus, retaining the lower coefficient in Eq. (45) is tantamount to correcting for these drag effects, giving a more realistic modeling of the total microwave power needed at b. However, because some of this drag power is transferred to the warm electrons at a, we would expect the power estimate P_a given by Eq. (40) to be higher than necessary. In the future, the drag could be included using a model similar to the model in Ref. 16.

The value of R_{eff} is taken as the smaller of R_{eff} (inner) and R_{eff} (outer) for losses to the inner-mirror point or the outer-mirror point, where

$$R_{\text{eff}} (\text{inner}) = \frac{B_{mAi}/B_b (1 - \hat{\beta}_b)^{1/2}}{1 + \phi_b/E_{ehA}} \quad (46)$$

and

$$R_{\text{eff}} (\text{outer}) = B_{mA0}/B_b (1 - \hat{\beta}_b)^{1/2} . \quad (46a)$$

Here we assume that the potential at the outer-mirror point is the same as the potential at the bottom of the thermal barrier.

The warm and hot electron volumes are given by

$$V_a = \frac{\pi r_c^2}{3} L_a(\text{eff}) \frac{B_c}{B_a} \left(\frac{1 - 0.5 \hat{\beta}_c}{1 - 0.5 \hat{\beta}_a} \right)^{1/2} \quad (47)$$

and

$$V_{eh} = \frac{\pi r_c^2}{3} L_b(\text{eff}) \frac{B_c}{B_b} \left(\frac{1 - 0.5 \hat{\beta}_c}{1 - 0.5 \hat{\beta}_b} \right)^{1/2} , \quad (48)$$

where $L_a(\text{eff})$ and $L_b(\text{eff})$ are the effective lengths (n^2 weighting) for the warm and hot electrons, respectively. The warm electrons are assumed to have a parabolic axial profile centered at point a and extending to the anchor midplane at b and to the anchor outer mirror. The hot electrons are assumed to vary as

$$n(B) = n_{ehA}(b) \left(\frac{B_m - B}{B_m - B_b} \right) , \quad (49)$$

where B_m is the vacuum magnetic field at point a.

The one-third factor in the expressions for the effective volumes V_a , V_{eh} , and V_c (to appear later) arise from the assumption of a parabolic radial profile and an n^2 weighting for the volume.

The synchrotron radiation loss (in kW) is given by¹⁶

$$P_{\text{synch}} = 1.59 \times 10^{-17} V_{eh} n_{ehA}(b) B_a^2 (1 - 0.5 \hat{\beta}_a) \left[\left(1 + \frac{E_{eh}}{511} \right)^2 - 1 \right] . \quad (50)$$

While the value of V_{eh} in Eq. (50) should correctly be the n -weighted volume, for simplicity we have used the n^2 -weighted volume of Eq. (48). Because P_{synch} is small compared to P_{scatt} , the error in the total ECRH power needed at $\underline{b}$ is small.

CENTRAL-CELL ION PARTICLE AND ENERGY BALANCE

The model assumes that all the central-cell ions are thermal ions with a temperature T_{iwc} . The rate equation describing the thermal ion particle balance is

$$\begin{aligned} I_i^{loss} + 2I_{trap}^{P2B2} + 2I_{abs}^{P2B2} f_{cx}^{P2B2} \\ = I_{C\ ion}^{gas} + 2I_{abs}^{P2B2} f_{trap}^{P2B2} + I_{C\ ion} + 2I_{X\ ion} \end{aligned} \quad (51)$$

where I_i^{loss} is the axial loss current of the warm central-cell ions given by

$$I_i^{loss} = q \frac{n_{iwc}^2 V_c}{(n\tau)_{iwc}} \quad (52)$$

where $(n\tau)_{iwc}$ is the ion Pastukhov confinement parameter for the warm ions; I_{trap} is the current of ions from the central cell that traps in the transition and anchor regions in each end; I_{abs}^{P2B2} is the total passing-particle barrier-beam (P2B2) current absorbed in the transition region; and f_{cx}^{P2B2} is the fraction of the absorbed P2B2 that charge-exchanges on the passing ions in the transition region.

The right-hand side of Eq. (51) has a term for each of the four sources of ions: ionization of the gas; trapping of a fraction f_{trap}^{P2B2} of the ions from the passing-particle-barrier-beam in the central cell; ionization of the central cell; and ionization of the axicell beams. In the most general case, all four would supply ions to the central cell. The formulation, however, is general enough to handle cases where the gas or the central-cell beam would be turned off. The factors of two in Eq. (51) account for the particle losses and inputs from each end of the system. In Eq. (52) V_c is the n^2 -weighted central-cell volume given by

$$V_c = \frac{1}{3} \pi r_c^2 L_c(eff) \quad .$$

The ion-energy equation is obtained by balancing the energy carried away by each ion with the energy input of each source ion. The difference in potential between the point of ion production and the central cell is also considered in the energy bookkeeping. The ion energy-balance equation is similar to the particle balance equation except that each ion source or loss term is multiplied by the appropriate energy gain or loss. In addition, several terms that do not entail particle loss or gain but do provide energy loss or gain are included. These terms model the ICRH power input, electron drag losses, and energy losses due to charge-exchange off the gas. The equation is

$$\begin{aligned}
 & I_C \text{ ion } f_{ii} \bar{E}_{\text{trap}} + P_C^{\text{ICRH}} + 2I_{\text{abs}}^{\text{P2B2}} (E_{\text{P2B2}} - \delta\phi_L + \Delta\phi_{\text{pc}}) f_{\text{trap}}^{\text{P2B2}} \\
 & + 2I_{\chi \text{ ion}} (E_{\text{loss}} + \Delta\phi_{\text{pc}}) \\
 & + 2I_{\chi \text{ ion}} (E_{\text{loss}} + \Delta\phi_{\text{pc}} - E_{\text{iwc}}) \frac{\langle\sigma v\rangle_{\text{cx}}}{\langle\sigma v\rangle_{\text{ion}}} \frac{n_{i\chi}^{\text{pass}}}{n_{i\chi}} \\
 & = I_i^{\text{loss}} (T_{\text{iwc}} + \phi_C) + 2I_{\text{trap}} E_{\text{iwc}} \\
 & + \frac{\langle\sigma v\rangle_{\text{cx}}^{\text{gas}}}{\langle\sigma v\rangle_{\text{ion}}^{\text{gas}}} I_C^{\text{gas}} \text{ ion } E_{\text{iwc}} + 2f_{\text{cx}}^{\text{P2B2}} I_{\text{abs}}^{\text{P2B2}} E_{\text{iwc}} \\
 & + \frac{q n_{\text{iwc}}^2 v_c}{(\pi\tau)_{\text{drag}}} (E_{\text{iwc}} - E_{\text{eC}}) , \tag{53}
 \end{aligned}$$

where $\bar{E}_{\text{trap}}$ is the mean beam energy trapped per beam neutral ionized, and f_{ii} is the fraction of this energy deposited in the ions. The values of $\bar{E}_{\text{trap}}$ and f_{ii} are calculated with the formalism of Ref. 4 using the atomic fractions, $f_1 = 0.72$, $f_{1/2} = 0.11$, and $f_{1/3} = 0.17$.

These atomic fractions are calculated assuming a 90:5:5 molecular mix in the arc chamber and then accounting for the difference in neutralizer efficiency for each beam energy component. The values differ from those shown in Table I for the sloshing ion beams because the numbers in the table also include a correction for the difference in beam divergence for each energy

component. This correction is not applied to the central-cell beam because the whole beam would intersect the plasma in the central cell.

The absorbed fraction of P2B2 supplies the central cell with an energy per particle of $(E_{P2B2} - \delta\phi_L + \Delta\phi_{pc})$, where E_{P2B2} is the average beam energy and $\delta\phi_L$ is the average potential drop between the potential peak in the axicell and the location where the P2B2 ion is born. The neutral beam in the axicell contributes energy equal to its loss energy [calculated in Eqs. (11) and (12)] from the axicell plus the change in potential $\Delta\phi_{pc}$. Moreover, because the passing ions from the central cell can charge-exchange with the axicell beam, there is a second source term in the energy balance that depends on the axicell ionization current.

The loss terms in the energy balance equation include the axial energy carried away by the ions, the energy lost by ion trapping in the transition and anchor regions, the charge-exchange losses caused by interactions with gas in the central cell and by passing ions in the transition, and also the energy lost by electron drag.

In the general problem of particle and energy balance, we solve Eq. (51) for the gas current (for a given ion-confining potential) and substitute into Eq. (53). This gives the rf power P^{ICRH} as a function of the central-cell beam-ionization current $I_{C\ ion}$, or vice versa. By choosing appropriate values of P^{ICRH} , we can easily determine the beam and gas inputs for zero P^{ICRH} or zero central-cell beam.

CENTRAL-CELL ELECTRON PARTICLE AND ENERGY BALANCE

The electron particle losses are all assumed to be axial losses, so that the electron-loss current is written as

$$I_e^{Loss} = \frac{q n_{eC}^2 v_c}{(n\tau)_{eC}} \quad (54)$$

Here, $(n\tau)_{eC}$ is the Pastukhov confinement for electrons in the confining potential ϕ_e .

Electrons are produced by ionization of the gas, the central cell and axicell beams, and the HEPB, P2B2, and sloshing-ion beams in the anchor and transition regions. In addition, a source of secondary electrons, I^{sec} , flows in from the end walls. This current is given in terms of the primary

electron loss current by¹⁷

$$I_{\text{sec}} = \lambda I_{\text{e}}^{\text{Loss}}, \quad (55)$$

where λ is the secondary-emission coefficient. In terms of the total ion loss out the ends of the system, I_{tot} ,

$$\lambda = \frac{I_{\text{sec}}}{I_{\text{sec}} + I_{\text{tot}}}, \quad (56)$$

where

$$I_{\text{tot}} = \frac{q n_{\text{iwc}}^2 v_{\text{c}}}{(n\tau)_{\text{iwc}}} + 2 \left(I_{\text{A ion}}^{\text{slosh}} + I_{\text{ion}}^{\text{HEPB}} + I_{\text{ion}}^{\text{P2B2}} + I_{\text{X ion}} \right). \quad (57)$$

Thus, the electron-particle balance relation that we solve for the secondary current is

$$I_{\text{sec}} + I_{\text{C ion}}^{\text{gas}} + I_{\text{C ion}} + 2 \left(I_{\text{A ion}}^{\text{slosh}} + I_{\text{ion}}^{\text{HEPB}} + I_{\text{ion}}^{\text{P2B2}} + I_{\text{X ion}} \right) = \frac{q n_{\text{ec}}^2 v_{\text{c}}}{(n\tau)_{\text{ec}}}. \quad (58)$$

For the central-cell electron energy balance, the position of ionization is taken into consideration. The potential change between this point and the central cell is then added or subtracted appropriately, depending on whether the electron is a net energy source or a sink to the central-cell electron population. The rate equation is then

$$\begin{aligned} & \frac{q n_{\text{iwc}}^2 v_{\text{c}}}{(n\tau)_{\text{drag}}} (E_{\text{iwc}} - E_{\text{ec}}) + I_{\text{sec}} \phi_{\text{e}} + 2(P_{\text{a}} + P_{\text{b}}) + P_{\text{C}} + I_{\text{C ion}} (1 - f_{\text{ii}}) \bar{E}_{\text{trap}} \\ & + 2 I_{\text{ion}}^{\text{HEPB}} \bar{\phi} + I_{\text{ion}}^{\text{HEPB}} (T) \bar{\phi}_{\text{T}} + I_{\text{ion}}^{\text{P2B2}} (\delta\phi_{\text{L}} - \Delta\phi_{\text{pc}}) \\ & = \frac{q n_{\text{ec}}^2 v_{\text{c}}}{(n\tau)_{\text{ec}}} (\phi_{\text{e}} + T_{\text{ec}}) + 2 I_{\text{A ion}}^{\text{slosh}} \phi_{\text{c}} + 2 I_{\text{X ion}} \Delta\phi_{\text{pc}}. \end{aligned} \quad (59)$$

The electron energy sources are represented by the terms on the left side of Eq. (59). The first term accounts for the drag power from the warm ions. In this model, the secondary electrons bring into the central cell the energy ϕ_{e} obtained from the potential drop from the wall. We assume that all the microwave power that produces the thermal barrier at $\underline{b}$ and the

potential peak at $\underline{a}$, $(P_a + P_b)$, eventually ends up in the central cell. This tends to overestimate the central-cell power input, since some of the hot, magnetically confined electrons that form the thermal barrier are lost to the end walls, rather than to the central cell.

The term P_C allows us to introduce power for direct electron heating of the central-cell electrons. A portion $(1 - f_{ij})$ of the trapped energy from the central-cell neutral beams goes directly into the electrons. The electrons produced by ionization of the HEPB in the anchor, I_{ion}^{HEPB} , and in the transition, $I_{ion}^{HEPB}(T)$, and the electrons produced by ionization of P2B2 in the transition carry in energy obtained when they change potential after entering the central cell.

The electron losses are represented by the terms on the right of Eq. (59). The electron axial losses carry out the energy $\phi_e + T_{ec}$ and the electrons produced by ionization of the sloshing ion beam in the anchor must climb a potential approximately equal to ϕ_c to reach the central cell, thus appearing as an energy loss term in the central cell. Similarly, the axicell electrons must climb the potential $\Delta\phi_{pc}$ to reach the central cell.

Along with the secondary electron current, we can calculate, using Eq. (56), the value of the secondary emission coefficient required to maintain electron particle balance. We can then compare it with the value of the coefficient when the secondary electron current is assumed to be limited by space-charge effects. Logan¹⁸ approximated the space-charge-limited value of the secondary emission coefficient as

$$\lambda_{sc1} = \left[0.15 \ln \left(\frac{T_{ec}}{\phi_e + \phi_c + T_{iw}} \right) + 0.656 \right]^{1/4}. \quad (60)$$

If $\lambda < \lambda_{sc1}$, then clearly some suppression scheme would have to be invoked to get I_{sec} below the space-charge limited value. However, in most calculations to date, the calculated value of λ has been larger than λ_{sc1} .

ITERATION PROCEDURE

The solution of the two rate Eqs. (58) and (59) gives us the secondary electron current and a new value for the electron confining potential ϕ_e . Recall that we initially set ϕ_e by requiring that the ion and electron Pastukhov confinement parameters be equal. When we now introduce the secondary

electrons, the electron confinement parameter and ϕ_e are appropriately smaller. This new value of ϕ_e is used to recalculate the axicell, anchor, and barrier parameters, as well as the beam requirements and microwave power. Using the electron-balance equations with these revised parameters gives the next iteration value for ϕ_e . Experience has shown that ϕ_e converges rapidly in several iterations.

When we have obtained the converged value of ϕ_e we proceed to evaluate the central-cell Q_c and overall Q and also calculate the potential and passing density profiles in the transition region.

Q CALCULATION

The term Q is defined as

$$Q \equiv \frac{\text{total fusion power}}{\text{total absorbed power}} .$$

The fusion power includes contributions from the axicell and anchor, so that in kW,

$$P_{\text{fus}} = \frac{q}{4} E_f \left[V_c \langle \sigma v \rangle_f^{(1)} n_c^2 T + 2V_{\text{slosh}} \langle \sigma v \rangle_f^{(2)} n_{ih}^2(b) + 2V_x \langle \sigma v \rangle_f^{(2)} n_{ihx}^2 \right] . \quad (67)$$

Here, E_f is the energy per fusion in keV; V_c , V_{slosh} , and V_x are the effective central-cell, sloshing-ion, and axicell ion volumes; $\langle \sigma v \rangle_f^{(1)}$ and $\langle \sigma v \rangle_f^{(2)}$ are the fusion reaction rates for warm and hot ions; and T is an enhancement factor to account for the central-cell fusion power produced by the 10% or so hot ions in the central cell. (We used $T = 1.15$.)

The total absorbed power includes contributions from the central-cell neutral beam and/or ICRH, the axicell neutral beam, the pump beams, the anchor sloshing ion beams, and the anchor ECRH. The beam powers are obtained by multiplying the absorbed current by the appropriate average beam energy.

At present, the ECRH power input at the potential peak position a dominates the power absorbed and has the largest influences on Q . Improvements in the models for calculating this power at a would thus have the greatest potential for increasing the overall Q value.

The central-cell Q_c value is defined in Ref. 4 as

$$Q_c \equiv \frac{\text{central-cell fusion power}}{\text{central-cell energy losses}}.$$

Thus, we have

$$Q_c = \frac{(n\tau)_{iwC}}{4} \frac{\langle \sigma v \rangle_f^{(1)} T E_f}{(\phi_e + \phi_c + T_{iwC} + T_{eC})} (1 + F_{TB})^{-1}, \quad (62)$$

where F_{TB} is the ratio of energy loss out the thermal barrier to energy loss out the ends

$$F_{TB} = \frac{2I_{\text{trap}}^{(3/2)} T_{iwC}}{I_i^{\text{loss}} (\phi_e + \phi_c + T_{iwC} + T_{eC})}. \quad (63)$$

TRANSITION DENSITY AND POTENTIAL PROFILES

From the 12-T mirror of the axicell (magnet A2) to the inner 3-T mirror of the anchor (magnet M1), we assume we have only warm ions: the passing ions that flow over the 12-T mirror and the trapped ions that collisionally trap in this region. For any assumed value of $g_b(B)$ --the ratio of total warm-ion density to passing ion density at field position B--we can calculate the potential drop from the axicell midplane and the passing and trapped ion densities as a function of the magnetic field B. The quasi-neutrality relation at any field B is

$$n_{ec}(B) = g_b(B) n_{\text{pass}}(B). \quad (64)$$

The cold electrons are assumed to have a cutoff distribution identical in form to that given in Eq. (15). To obtain $n_{ec}(B)$ from Eq. (15), we replace ϕ_b by $\Delta\phi_B$, the drop in potential from the axicell midplane to field position B.

The passing ion density at B is given by

$$n_{\text{pass}}(B) = F(B, B_{xp}, \Delta\phi_B, \Delta\phi_B - \Delta\phi_{pb}, B/B_{xp}, B/B_{mx0}, n_{iwx}) \\ - F(B, B_{ap}, \Delta\phi_B + \phi_c - \Delta\phi_{pc}, \Delta\phi_B - \Delta\phi_{pb}, B/B_{ap}, B/B_{mx0}, n_{iwa}). \quad (65)$$

The solution of Eq. (64) for a given value of $g_b(B)$ gives $\Delta\phi_B$. The passing density can be calculated from Eq. (65) and the trapped warm-ion density is then obtained from

$$n_{\text{trap}}(B) = [g_b(B) - 1] n_{\text{pass}}(B) \quad . \quad (66)$$

MFTF COMPUTER PROGRAM

The MFTF program is an interactive code that relies on the operator to specify the order in which the operating point calculation is to proceed. Appendix B contains a detailed description of the optimum order of the calculation.

In general, we make assumptions on the overall system performance desired and then, starting in the central cell, we calculate plasma and potential values at cardinal points in the central-cell, axicell, transition, and anchor regions. The microwave power and absorbed neutral beams necessary to support the equilibrium are then calculated. The calculation requires several iterations to obtain the value of electron confining potential ϕ_e consistent with the secondary electrons flowing in from the ends of the system.

The program uses a root-finding subroutine obtained from the MFECN Nag library, entitled COSACF.

The complementary error functions and Dawson's integrals¹⁹ occurring in the expressions for the passing ion density were evaluated using a program for the plasma dispersion function.²⁰

SAMPLE CALCULATION

Using the sample input file shown in Appendix B, we have calculated the operating point, also summarized in Appendix B. In Tables 2 through Table 6 we detail some of the more important parameters of each plasma region and give the absorbed beam currents and rf powers necessary to sustain the plasma.

Table 2. Central-cell plasma parameters.

B_c (T)	1	P_c^{fusion} (kW)	312
ϕ_e (kV)	36.2	$P_c^{\text{ICRH trapped}}$ (kW)	211
ϕ_c (kV)	30.3	$P_c^{\text{beam trapped}}$ (kW)	0
$\hat{\beta}_c$	0.4	$I_c^{\text{gas trapped}}$ (A)	1.8
n_{iwc} (10^{13} cm^{-3})	3.5	r_c (cm)	30
T_{eC} (keV)	9	$L_c^{\text{effective}}$ (cm)	1260
T_{iwc} (keV)	15	Q_c	0.62
$(n\tau)_{eC}$ ($10^{13} \text{ cm}^{-3} \cdot \text{s}$)	0.47	Q_{total}	0.23
$(n\tau)_{iwc}$ ($10^{13} \text{ cm}^{-3} \cdot \text{s}$)	5.0		

Table 3. Axicell plasma parameters.

$\Delta\phi_{pc}$ (kV)	6.9
$\Delta\phi_{pb}$ (kV)	11.2
$\hat{\phi}_x$	0.1
n_{ex} (10^{13} cm^{-3})	7.6
$I_{x \text{ ion}}$ (A)	1.3

Table 4. Transition plasma parameters.

ϕ_T (kV)	7.5
n_T (10^{11} cm^{-3})	8.4
B_T (T)	0.849

Table 5. Anchor plasma parameters.

$n_{iA}(b) (10^{12} \text{ cm}^{-3})$	3.0
$\phi_b \text{ (kV)}$	41.2
$E_{eh} \text{ (keV)}$	553
$\hat{\beta}_A$	0.55
$\tau_{ew} \text{ (keV)}$	64.3
$\delta\phi_a \text{ (kV)}$	64.6
$n_{iA}(a) (10^{12} \text{ cm}^{-3})$	4.4

Table 6. Trapped current and rf power.

<u>Neutral beams</u>	<u>Trapped current (A)</u>
Each axicell (80 kV)	1.7
Each anchor (80 kV)	0.66
Each HEPB (80 kV)	1.8
Each P2B2 (40 kV)	0.55
<u>ECRH (per end)</u>	<u>Absorbed power (kW)</u>
Power at <u>a</u>	247
Power at <u>b</u>	82
<u>ICRH</u>	
Central cell	211

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APPENDIX A. PASSING ION CALCULATION

We have mapped the warm passing-ion density from the axicell midplane through the transition from the 12-T outer axicell mirror (A2) to the 3-T inner anchor mirror (M1). For the magnetic field and potential profile shown in Fig. 1, the relevant ϵ - μ plot for the transition is shown in Fig. A-1. The passing ion density is obtained by integrating the assumed Maxwellian distribution over the shaded ① region of Fig. A-1.

This integration was performed over the larger shaded region (① plus ②) in Ref. A-1. In Ref. A-2, we described the method for obtaining the integral over region ① by subtracting the integral over region ② from the result of Ref. A-1.

From Ref. A-1, the results for the integrals over the larger shaded region are given in terms of the complementary error function and Dawson's integral. For $B > B_x$, the total passing density is

$$\begin{aligned} \frac{n_{\text{pass}}^t}{n_{\text{iwx}}} = & \exp(\phi_2/T) \operatorname{erfc} \left[\left(\frac{\phi_2}{T} \right)^{1/2} \right] - 2 \left(\frac{R_0 - 1}{\pi} \right)^{1/2} \left\{ \exp \left(\frac{(\phi_2 - \phi_3) R_m}{T(R_0 - R_m)} \right) \right. \\ & D \left[\left(\frac{\phi_1}{T} \right)^{1/2} \right] - D \left[\left(\frac{\phi_2}{T(R_0 - 1)} \right)^{1/2} \right] \left. \right\} \\ & - (1 - R_m)^{1/2} \exp \left(\frac{\phi_2}{T} + \frac{R_m \phi_3}{T(1 - R_m)} \right) \operatorname{erfc} \left[\left(\frac{\phi^*}{T} \right)^{1/2} \right] \quad . \quad (\text{A-1}) \end{aligned}$$

For $B < B_x$,

$$\begin{aligned} \frac{n_{\text{pass}}^t}{n_{\text{iwx}}} = & \exp(\phi_2/T) \operatorname{erfc} \left[\left(\frac{\phi_2}{T} \right)^{1/2} \right] \\ & - (1 - R_0)^{1/2} \exp \left(\frac{\phi_2}{T(1 - R_0)} \right) \left\{ \operatorname{erfc} \left[\left(\frac{\phi_2}{T(1 - R_0)} \right)^{1/2} \right] \right. \\ & - \operatorname{erfc} \left[\left(\frac{\epsilon^* + \epsilon_1}{T} \right)^{1/2} \right] \left. \right\} \\ & - (1 - R_m)^{1/2} \exp \left(\frac{\phi_2}{T} + \frac{R_m \phi_3}{T(1 - R_m)} \right) \operatorname{erfc} \left[\left(\frac{\phi^*}{T} \right)^{1/2} \right] \quad . \quad (\text{A-2}) \end{aligned}$$

Fig. A-1. The ϵ - μ diagram for ions at magnetic field position B and potential

ϕ in the transition; $\phi_1 = \phi_e - \phi$, $\phi_2 = \phi_e + \Delta\phi_{pc} - \phi$,

$\phi_3 = \phi_e + \Delta\phi_{pc} - \Delta\phi_{pb} - \phi$, and $\phi_4 = \phi_e + \phi_c - \phi$.

In Eqs. (A-1) and (A-2), n_{pass}^t is the total passing density at B; n_{iwx} is the warm ion density at the axicell midplane; and $T = T_{\text{iwc}}$, the warm ion temperature. The potential difference between the axicell midplane and field position B in the transition is

$$\phi_2 = \phi_e + \Delta\phi_{\text{pc}} - \phi ; \quad (\text{A-3})$$

the potential difference between the axicell outer mirror point and the field position B in the transition is

$$\phi_3 = \phi_e + \Delta\phi_{\text{pc}} - \Delta\phi_{\text{pb}} - \phi . \quad (\text{A-4})$$

The term ϕ is the potential at this transition position. The axicell warm ion density n_{iwx} is given in terms of the central-cell warm ion density as

$$n_{\text{iwx}} = n_{\text{iwc}} \exp (-\Delta\phi_{\text{pc}}/T) . \quad (\text{A-5})$$

We have also defined the parameters

$$R_0 = B/B_x , \quad (\text{A-6})$$

$$R_m = B/B_{\text{mx0}} , \quad (\text{A-7})$$

$$\epsilon^* = \frac{R_0 \phi_2 - R_m \phi_3}{R_0 - R_m} , \quad (\text{A-8})$$

$$\epsilon' = \frac{R_0 \phi_2}{1 - R_0} , \quad (\text{A-9})$$

$$\phi^* = \frac{R_m \phi_3}{1 - R_m} + \epsilon^* , \quad (\text{A-10})$$

and

$$\phi' = \frac{R_0 \phi_2}{R_0 - 1} - \epsilon^* . \quad (\text{A-11})$$

We can represent both Eqs. (A-1) and (A-2) by the general equation

$$n_{\text{pass}}^t = F(B, B_x, \phi_2, \phi_3, R_0, R_m, n_{iwx}) , \quad (\text{A-12})$$

where F is given by the appropriate expression (A-1) or (A-2), depending on whether $B > B_x$ or $B < B_x$. Note that both Eqs. (A-1) and (A-2) reduce to the same form when $B = B_x$, i.e.,

$$\begin{aligned} \frac{n_{\text{pass}}^t}{n_{iwx}} = & \exp(\phi_2/T) \operatorname{erfc} \left[\left(\frac{\phi_2}{T} \right)^{1/2} \right] \\ & - (1 - R_m)^{1/2} \exp \left(\frac{\phi_2}{T} + \frac{R_m \phi_3}{T(1 - R_m)} \right) \operatorname{erfc} \left[\left(\frac{\phi_m}{T(1 - R_m)} \right)^{1/2} \right] . \end{aligned} \quad (\text{A-13})$$

For the integration in region ②, Ref. A-2 notes that the general form is identical to the integral over the larger region ① plus ②. Hence, for $B > B_a$ the component of passing density from region ② is

$$\begin{aligned} \frac{n_{\text{pass}}^a}{n_{iwa}} = & \exp(\phi_4/T) \operatorname{erfc} [(\phi_4/T)^{1/2}] - 2 \left(\frac{R_a - 1}{\pi} \right)^{1/2} \left\{ \exp \left(\frac{(\phi_4 - \phi_m) R_m}{T(R_a - R_m)} \right) \right. \\ & \left. D \left[\left(\frac{\phi_a}{T} \right)^{1/2} \right] - D \left[\left(\frac{\phi_4}{T(R_a - 1)} \right)^{1/2} \right] \right\} \\ & - \sqrt{1 - R_m} \exp \left(\frac{\phi_4}{T} + \frac{R_m \phi_3}{T(1 - R_m)} \right) \operatorname{erfc} \left[\left(\frac{\phi_a^*}{T} \right)^{1/2} \right] . \end{aligned} \quad (\text{A-14})$$

For $B < B_a$,

$$\begin{aligned} \frac{n_{\text{pass}}^a}{n_{\text{iwa}}} = & \exp(\phi_4/T) \operatorname{erfc} \left[\left(\frac{\phi_4}{T} \right)^{1/2} \right] \\ & - \sqrt{1 - R_a} \exp \left(\frac{\phi_4}{T(1 - R_a)} \right) \left\{ \operatorname{erfc} \left[\left(\frac{\phi_4}{T(1 - R_a)} \right)^{1/2} \right] \right. \\ & \left. - \operatorname{erfc} \left[\left(\frac{\epsilon_a^* + \epsilon_a'}{T} \right)^{1/2} \right] \right\} \\ & - \sqrt{1 - R_m} \exp \left(\frac{\phi_4}{T} + \frac{R_m \phi_3}{T(1 - R_m)} \right) \operatorname{erfc} \left[\left(\frac{\phi_a^*}{T} \right)^{1/2} \right] . \end{aligned} \quad (\text{A-15})$$

Here, we have defined

$$R_a = B/B_a \quad (\text{A-16})$$

$$\epsilon_a^* = \frac{R_a \phi_4 - R_m \phi_3}{R_a - R_m} , \quad (\text{A-17})$$

$$\epsilon_a' = \frac{R_a \phi_4}{1 - R_a} , \quad (\text{A-18})$$

$$\phi_a^* = \frac{R_m \phi_3}{1 - R_m} + \epsilon_a^* , \quad (\text{A-19})$$

$$\phi_a' = \frac{R_a \phi_4}{(R_a - 1)} - \epsilon_a^* , \quad (\text{A-20})$$

and

$$\phi_4 = \phi_e + \phi_c - \phi . \quad (\text{A-21})$$

Equations (A-14) through (A-21) are identical to (A-1), (A-2), and (A-6) through (A-11), except that ϕ_4 has replaced ϕ_2 and B_a has replaced B_x . The density n_{iwa} has also replaced n_{iwX} . They are related, however, by the equation

$$n_{iwa} = n_{iwX} \exp [- (\phi_4 - \phi_2)/T] \quad . \quad (A-22)$$

Again, the passing density can be written in terms of the general function F as

$$n_{pass}^a = F (B, B_a, \phi_4, \phi_3, R_a, R_m, n_{iwa}) \quad . \quad (A-23)$$

The final result, the passing density at any field point B in the transition, can then be written as

$$n_{pass}^{(B)} = n_{pass}^t - n_{pass}^a \quad (A-24)$$

and, in terms of the F function,

$$\begin{aligned} n_{pass} (B) = & F (B, B_x, \phi_2, \phi_3, R_0, R_m, n_{iwX}) \\ & - F (B, B_a, \phi_4, \phi_3, R_a, R_m, n_{iwa}) \quad . \end{aligned} \quad (A-25)$$

The particular forms for F, (A-1), or (A-2) and (A-14), or (A-15) depend on the value of B compared to B_x and B_a , respectively.

REFERENCES FOR APPENDIX A

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APPENDIX B. MFTF PROGRAM INPUT PARAMETERS AND SAMPLE RESULTS

1. OPERATING INSTRUCTIONS

The MFTF program is stored in a Lib file called MARSMODE that can be accessed from the MFECC Cray using the following command:

FILEM READ 30045 .CRAY MARSMODE

MARSMODE contains the source file, a sample input file, and the executable file.

The program requires an input data file with parameters in namelist format. The user specifies the file name when queried by the program and also supplies an output file name and a name for the next data file. The input data file contains the initial values of all the necessary variables, although these values can be changed in the subsequent calculations. A sample data file follows below.

Start the program with $ical=0$. You must input values for t_{ic} , t_{ec} , n_{tauci} and $betac$. The program will calculate values for n_c , $phic$ and $phie$. The $phic$ value is fixed by the value of n_{tauci} . The $phie$ value is obtained by equating the ion and electron pasthukov parameters. The value of $phie$ will be corrected later. The value of n_c may be changed from that value calculated here if the effects of hot ions on $betac$ are to be included.

If you assume values for t_{ic} , t_{ec} , $betac$, n_{tauci} , $phie$, n_c and $phic$, you can start the program with $ical=1$.

Several iterations with $ical=1$ will be required until the values of $dphic$ and $dphib$ converge. The code will do up to a maximum of $maxit$ iterations at a time. If convergence is not achieved, it will tell you and also ask for more input. To continue just input "\$". If convergence is achieved, go on to the next $ical$ value. Skip is the convergence parameter. You should set $mdisk=0$ to stop the extra disk output, unless you really want to see the values of the parameters in the namelist block /d/ after each calculation. Setting $mdisk=1$ increases the amount of printed output to ridiculous amounts.

Next do $ical=4$. Note that this also calculates the potential and passing density at the anchor inner mirror so that $phibm$ can be calculated. Be sure and redo $ical=4$ at least once after the tew/delphia iteration appears to converge because the first $ical=4$ calculation is done with an initial (default) value for $r_{inj} = b_{ap}/b_{ap} = 1.0$ given in a data statement. This affects the value of the hot electron energy, eeh .

If tew is chosen and $dphia$ is to be calculated, then $ical=8$ must come next. If tew is to be calculated for a given $dphia$, then $ical=2$ replaces $ical=8$ in the required ordering.

The $ical=5$ or $ical=6$ calculation is done next, depending on whether the sloshing ion beam is injected at "a" or aprime.

This calculation is followed by $ic=7$ or $ic=11$ (depending on whether we previously did $ic=5$ or $ic=6$). We must iterate between $ic=2$ (or 8), $ic=5$ (or 6) and $ic=7$ (or 11) so that the value of tew (or $delphia$) is consistent with the value of the electron source at "a", je . Be sure to input a "good" value for $dpslsh$ when doing the $ic=7$ (or 11) case. The default is $dpslsh = dphia$ as given by the $ic=2$ (or 8) result. The correct value may be somewhat less than this, but $dphia$ is probably a conservative guess.

This iteration for tew and je can be automated. By setting $mauto=1$ and $mnext=1$ or 2, the sequence $ic=2$ (or 8), 5, 7 or $ic=2$ (or 8), 6, 11 is done automatically. When you think you have convergence, go back and do one $ic=4$ calculation and then do one more $tew/delphia$ iteration to ensure complete convergence. Set $maprt=1$ for the last $ic=2$ calculation so that the final values will get written into the output file.

When this iteration is completed, we proceed to $ic=9$. The value of central cell $icrh$ power ($picrh$) can be varied to get the desired combination of neutral beam and gas (plus $icrh$ power) for central cell ion energy and particle balance. If both $iccn$ and $igas$ are negative with $picrh=0$, or if, more generally, you cannot find a solution with all of $iccn$, $igas$, and $picrh$ positive or zero, then the ion energy and particle balance cannot be satisfied with that particular parameter combination.

Next we proceed to $ic=12$ to calculate the ECRH power needed for the warm and hot electrons in the anchor.

Next we do $ic=10$. This calculation may require several guesses for the value of pcc . By choosing appropriate values for pcc , we get different values for $phie$. The desired operating point would have $pcc=0$. If the value of $phie$ is determined to be much different from our initial guess, we must start another calculation cycle at $ic=1$.

With the converged value of $phie$, we do $ic=13$ to get the overall q . The central cell tail enhancement factor, $tail$, can be varied to give the desired enhancement in fusion power due to the hot ions in the central cell.

To get a summary printout of all the results set $ic=14$. If the summary is absolutely the last thing you want, set $iquit=0$ and you will get a summary and end the job at the same time. If you want a copy of the namelist block /d/ at this time too, then set $mdisk=1$ before you end.

Do the $ic=3$ cases next if the potential and passing density in the transition region are needed. You can change b and $icgbb$ to get the desired profiles for density and potential. The calculation can be automated to do 13 fixed values of b if you set $mauto=1$; you get to vary the value of $icgbb$.

A summary of the appropriate calculations for various values of ical follows:

ical=1,2,3,4...for axicell, tew, transition, or anchor
 ical=5,6,7...for "a" or aprime injection or yybeam ("a" inj)
 ical=8,9,10...for delphia, ion balance, or electron balance
 ical=11,12,13...for yybeam ("a" inj), microwave power, or Q
 ical=14...summary printout of results

II. LIST OF INPUT PARAMETERS

ical...switch for determining the appropriate calculation
 mode...switch to set operating mode; 0 for mars; 1 for kelley/tdf
 mprint...0 for no tty output; 1 for tty output
 mdisk...0 for reduced disk output; 1 for all disk output
 adpap1, adpap2...initial guesses for aprime injection
 b...b field for density and potential scan calculation (ical=3)
 baav...vacuum b field at "a" in the anchor
 bav...vacuum b field at anchor midplane position "b"
 bc...vacuum b field in central cell
 betac...beta in central cell

 betan...beta at anchor midplane position "b"
 betat...beta at the minimum b field point in the transition
 betax...beta at axicell midplane
 binn...average b field in innermost region of anchor
 bmaiv...vacuum b field of anchor inner mirror
 bmaov...vacuum b field of anchor outer mirror
 bmx1...vacuum b field of axicell inner mirror
 bmxo...vacuum b field of axicell outer mirror
 bt...minimum vacuum b field in the transition
 bxv...vacuum b field at axicell midplane

 celtrap...average energy per ionized ion in the central cell
 crf...rf diffusion enhancement parameter for warm electrons
 del1, del2...initial guesses for delphia calculation
 dpap1, dpap2...initial guesses for "a" injection calculation
 dpb1, dpb2...initial guesses for dphipb calculation
 dpc1, dpc2...initial guesses for dphipc calculation
 dpslsh...potential drop from sloshing ion injection point to the anchor outer mirror
 efus...fusion energy (17.56e3 kev)
 ehpb...high energy pump beam energy
 eih...average ion energy in the axicell

 einja...sloshing ion injection energy
 einjc...central cell ion injection energy
 einjx...axicell ion injection energy
 ele1, ele2...initial guesses for electron balance calculation
 elss1, elss2...initial guesses for axicell loss energy calculation
 eps...parameter in root finding subroutine c05acf
 ep2b2...p2b2 beam energy
 eta...parameter in root finding subroutine c05acf
 etrap...energy parameter for sloshing ion ntau calculation
 fa...ratio of plasma area at "a" to hepb area (fa.le.1.)

fan1,fan2...initial guesses for anchor calculation
 fap...ratio of plasma area at aprime to hepb area (fap.le.1.)
 fb...ratio of plasma area at anchor midplane to hepb area
 (fb.le.1.)
 fc1,fc2,fc3...full, half, and one-third central-cell beam
 molecular mix fractions incident on plasma
 fec...fraction of cold electrons at the thermal barrier
 fhote...fraction of ECRH at "b" that goes to central cell
 fii...fraction of the central-cell beam energy that goes
 directly into the ions
 finn...ratio of plasma area in inner anchor region to the hepb
 area (finn.le.1.)
 fncinn...ratio of ion density in inner anchor region to the
 central cell warm ion density
 fp2b2...fraction of p2b2 ions that can trap in the central cell

 fsin1,fsin2,fsin3...full, half, and one-third energy sloshing-
 beam molecular mix incident on plasma
 fsub...fraction of sloshing ionization current that acts as a
 charge-exchange pump in the anchor
 fwarm...fraction of ECRH at "a" that goes to central cell
 f41,f42...initial guesses for transition density and potential
 calculation
 gban...upper case gb at anchor midplane
 icc...central-cell beam ionization current
 ifail...error return parameter for root finding routine c05acf
 ip2b2a...absorbed p2b2 beam current
 ip2b2i...ionized p2b2 beam current
 itrap...trapped current from central cell lost to the barrier
 and transition regions
 iquit...0 to stop the calculations
 la...effective length of potential peak (warm electron) region
 lanch...length from inner anchor mirror to the potential peak
 lap...effective length of conjugate turning point (aprime)
 region
 lb...effective length of hot electron region
 lc...effective length of central cell
 lcgbam...lower case gb at the anchor inner mirror
 lcgban,lcgbap,lcgbb,lcgbi,lcgbp2,lcgbxm...lower case gb at the
 anchor midplane, aprime point, in the transition region,
 the anchor inner region, the p2b2 injection point, and the
 axicell outer mirror. if lcgbxm is .le. 0., it is
 calculated as a function of dhipc in the code
 lcgbt...lower case gb at the minimum b point in the transition
 inner... length of the inner anchor region
 ls...effective length of the sloshing ions
 lt...effective length of transition region
 lx...effective length of the axicell ions
 maprt...0 for skip print when mauto=1; 1 for always print during
 automatic iteration
 mauto...1 for auto iteration of ical=2,5,7 or 2,6,11 ; 0 for
 manual control. be sure and set mnext for "a" or
 aprime injection
 maxit...maximum number of axicell iterations
 mdens...passing density calculation parameter; 0 includes upper
 loss boundary; 1 integrates to infinity
 mdp...drift pumping parameter. 0 for beam pumping; 1 for drift
 pumping
 mlcgb...0 if lcgbxm is calculated: 1 if lcgbxm is set in data

mnxt...parameter for "a" or oprime injection for use with
 auto iteration of tew and rje. 1 for "a" injection;
 2 for oprime injection
 model...0 for no ion drag; 1 for ion drag in the logon rensink
 model for the axicell
 modelt...0 for gamma=1; 1 for gamma>1 in tew and delphia
 calculation
 mtrap...parameter for specifying which sloshing beam energy
 components are confined (1=only full; 2=full+half;
 3=all confined)
 nc...central-cell warm ion density
 ntauci...ion Pastukov confinement parameter
 nucx...charge-exchange pumping rate in the anchor midplane
 pa...ECRH power absorbed at "a"
 pb...ECRH power absorbed at anchor midplane, "b"
 pcc...ECRH power absorbed in the central cell electrons
 pc1,pc2...initial guesses for phic calculation
 pe1,pe2...initial guesses for phie calculation
 phibm...potential difference between the inner mirror and the
 midplane of the anchor
 phic...central-cell ion confining potential
 phie...central-cell potential
 phip2...potential drop from axicell midplane to p2b2 injection
 position
 picrh...central-cell absorbed icrh power
 rbeam...radius of HEPB
 rc...central-cell radius
 rje...cold electron source at "a" in amps per cubic meter
 rs...ratio of sloshing ion density at "a" to that at "b"
 ryoshi...factor for scaling p2b2, itrap and hepb in the
 transition to the matsuda f-p code (ryoshi.le.1)
 scen...factor to scale axicell hot ion energy to match rensink
 fokker planck run
 sentau...factor to scale ion drag part of ntaux to match ntaux
 calculated in rensink fokker planck run
 skip...convergence parameter for axicell iteration
 tail...central-cell fusion power tail enhancement factor
 tcohen...radial loss time in sec. for tic=15. kev ions. the
 default value is 2.
 tec...cold electron temperature
 tew...warm electron temperature
 tic...thermal ion temperature
 thinjo...sloshing ion injection angle
 t1,t2...initial guesses for tew calculation
 exhhb,iihbb...charge-exchange and ionization reaction rates for
 hot ions on hot ions at "b"
 exhcb,iihcb...charge-exchange and ionization reaction rates for
 hot ions on cold ions at "b"
 exhhap,iihgap...charge-exchange and ionization reaction rates
 for hot ions on hot ions at oprime
 exhcop,iihcop...charge-exchange and ionization reaction rates
 for hot ions on cold ions at oprime
 exhcicn,iihcicn...charge-exchange and ionization reaction rates
 for hot ions on cold ions at anchor inner region

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cxhha,iihha...charge-exchange and ionization reaction rates for
                hot ions on hot ions at "a"
iec,ieh,iew...electron ionization rates for cold, hot, and warm
                electrons
sigvii...ionization reaction rate for axicell (1) and central-
                cell (2) ions
sigvcx...charge-exchange rate for axicell (1) and central-cell
                (2) ions
svf...fusion reaction rate for warm ions (1) and hot ions (2)

svgas...ratio of charge-exchange to ionization rate for the
                central-cell gas
svccx1,svccx2,svccx3...charge-exchange rates for full, half
                and one-third central-cell beam energy components
svcii1,svcii2,svcii3...ion-ion ionization rates for full,
                half, and one-third central-cell beam energy
                components
svscx1,svscx2,svscx3...charge-exchange rates for full, half
                and one-third sloshing beam energy components
svsii1,svsii2,svsii3...ion-ion ionization rates for full,
                half, and one-third sloshing beam energy components

```

III. SAMPLE INPUT VALUES

ical	=	C
mode	=	0
mprint	=	1
mdisk	=	1
adpap1	=	1.2
adpap2	=	3.
baav	=	1.4
bav	=	1.
bc	=	1.
betac	=	0.4
betan	=	0.55
betat	=	0.
betax	=	0.1
binu	=	2.5
bmaiv	=	3.
bmaov	=	3.
bmxi	=	6.
bmxi	=	12.
bt	=	0.849
bxx	=	4.
del1	=	1.
del2	=	10.
dpap1	=	1.5
dpap2	=	6.
dpp1	=	0.1
dpp2	=	3.
dpc1	=	0.1
dpc2	=	3.
efus	=	17560.
ehp	=	80.
eh	=	75.

einjd	80.
einjc	= 80.
einix	= 70.
elel	= 3.
ele2	= 6.
elss1	= 0.1
elss2	= 0.9
eps	= 1.e-10
ep2b2	= 40.
eta	= 1.e-10
fan1	= 2.
fan2	= 8.
fc1	= 0.72
fc2	= 0.11
fc3	= 0.17
fec	= 5.e-02
fhote	= 1.
fnclnn	= 0.5
fp2b2	= 1.
fsin1	= 0.833
fsin2	= 7.5e-02
fsin3	= 9.2e-02
fwarme	= 1.
f41	= 0.1
f42	= 6.
gban	= 6.5
iquit	= 1
ld	= 145.
lanch	= 400.
lap	= 50.
lb	= 184.
lc	= 1260.
lcgbm	= 3.07
lcgbn	= 2.75
lcgbp	= 2.75
lcgbp	= 2.75
lcgbn	= 2.75
lcgbp2	= 2.15
lcgbt	= 1.7
linner	= 50.
ls	= 550.
lt	= 723.
lx	= 51.3
mauto	= 1
maprt	= 0
maxit	= 25
mdens	= 0
mdp	= 0
mlcgb	= 0
mnext	= 1
model	= 1
modelt	= 1
mtrop.	= 1
ntauci	= 5.e+13
nucx	= 2.5
pcf	= 1.

```

pc2      = 2.5
pe1      = 3.
pe2      = 6.
picrh    = 0.
rbeam    = 30.
rc        = 30.
rs        = 2.5
ryoshi   = 0.3333
scen     = 1.2
scntau   = 0.12
skip     = 1.e-06
tail     = 1.15
tcohen   = 2.
tec       = 9.
tic       = 15.
thinja   = 90.
t1        = 1.
t2        = 10.
cxhbb    = 3.4e-08
ihhbb    = 5.1e-08
cxhcb    = 6.1e-08
ihhcb    = 3.8e-08
cxhhap   = 2.1e-08
ihhhap   = 6.e-08
cxhcap   = 6.1e-08
ihhcap   = 3.8e-08
cxhcin   = 5.7e-08
ihhcin   = 4.6e-08
cxhha    = 2.1e-08
ihhha    = 6.e-08
iec       = 1.02e-08
ieh       = 7.e-10
iew       = 3.2e-09
sigvii   = 6.e-08 4.38e-08
sigvcx   = 2.1e-08 6.95e-08
svccx1    = 5.2e-08
svccx2    = 8.93e-08
svccx3    = 1.065e-07
svcii1    = 4.99e-08
svcii2    = 3.56e-08
svcii3    = 2.9e-08
svf       = 2.6542e-16 8.7929e-16
svgas     = 2.8
svscx1    = 4.4e-08
svscx2    = 5.3e-08
svscx3    = 5.8e-08
svsii1    = 4.6e-08
svsii2    = 4.5e-08
svsii3    = 4.5e-08

```

IV. SAMPLE RESULTS

***** Results for phic and phie *****

phic = 30.3411 phie = 36.2110 nc = 3.5000e+13
tic = 15.0000 tec = 9.0000 ntauci = 5.0000e+13

***** Results for axicell *****

ihpbta = 0.0907 ihpbti = 0.0253 nihx = 5.3663e+13 nicx = 2.2033e+13
nex = 7.5695e+13 npassx = 1.6508e+13 eih = 61.8733 enloss = 49.9839
ntausc = 1.4762e+13 ntaude = 3.0836e+13 ntaux = 5.4795e+12
ixion = 1.3173 ixabs = 1.7113 ntaudi = 6.1184e+12
nicxm = 1.9214e+13 scen = 1.2000 scntau = 0.1200 betax = 0.1000
ip2b2a = 0.5488 ip2b2i = 0.1526 itrap = 0.3036
dphipc = 6.9423 dhipb = 11.1694 lcgbxm = 1.1388 ryoshi = 0.3333
ecriti = 12.9839 ecrito = 5.3752 nc = 3.5000e+13 phie = 36.2110

***** Results for transition density and potential *****

b = 0.8490 lcgb = 1.7000 delphb/tec = 3.9612
delphb = 35.6512 phiatb = 7.5021
npassb = 4.9353e+11 ncoldb = 8.3900e+11 phimi = 0.0000

b = 3.0000 lcgb = 3.0700 delphb/tec = 2.3660
delphb = 21.2943 phiatb = 21.8590
npassb = 2.1723e+12 ncoldb = 6.6689e+12 phimi = 21.8590

***** Results for anchor/barrier region *****

delpha = 41.2484 phian = 1.9049 eeh = 552.7366 phibm = 19.9541
nehan = 2.8872e+12 necan = 1.5196e+11 nihaa = 4.3834e+12
nihab = 1.7534e+12 npasan = 4.6757e+11 ntotan = 3.0392e+12

bapv = 1.9812 bapp = 1.9461 nihaa = 4.3834e+12
nihap = 6.6304e+12 nicap = 4.0669e+12
neap = 1.0697e+13 npasap = 1.4789e+12 dphiap = 17.6105
betap = 0.0701

***** Results for warm electron temperature *****

je = 8.7346 crf = 4.6200 rs = 2.5000 tew = 64.2913
dphia = 64.6472 newaa = 2.5071e+12 nehao = 1.8290e+12

necaa = 4.7300e+10 gamma = 5.3851 betaa = 3.1086e-01

***** Results for ying-yang beam calculation *****

nucx = 2.5000 ntausl = 1.7657e+12 ision = 0.4934
hepbiona = 0.3364 hepbini = 2.5864 hepbout = 0.7059
je = 8.7346 a/m**3 ntloss = 1.5188e-01 cxloss = 1.6258e-01
isabs = 0.6599 iscx = 0.1665 hpbabs = 1.6697 hpbion = 0.4805
fsub = 0.3627 etrap = 80.0000 old rje = 8.7346

***** Results for central-cell ion balance *****

phic = 30.3411 ntauci = 5.0000e+13 picrh = 210.7000
ip2b2a = 0.5488 fp2b2 = 1.0000 ep2b2 = 40.0000
ixion = 1.3173 iccion = 0.0003 igas = 1.8229 itrap = 0.3036
ipasscx = 0.1437 iloss = 4.6609 edrag = 68.0173
cetrap = 111.8521 fii = 0.8767 phip2 = 21.2943

***** Results for microwave power *****

pa1 = 34.7150 aecrh = 246.9505 pb1 = 62.0933 pb2 = 24.1122
psynch = 20.1870 becrh = 44.2992 rbeffi = 3.4006 rbeffo = 3.5233
crf = 4.6200 je = 8.7346 a/m**3 tew = 64.2913
pa = 246.9505 pb = 82.2803

***** Results for electron balance *****

pcc = 0.0000 pa = 246.9505 pb = 82.2803
phieot = 4.0234 phie = 36.2110 isec = 42.6973
ntauce = 4.7119e+12 lam = 0.8781 lamsl = 0.7553
old phie = 36.2110

***** Results for Q *****

qtot = 0.2298 qc = 0.6232 ccpf = 312.2743
yypf = 1.0352 powin = 1539.9731 pfusion = 353.8489 xpf = 19.7521
central-cell fusion power tail enhancement factor = 1.15

***** Power in kW input per end or cell*****

pa = 246.9505 pb = 82.2803 pcc = 0.0000 picrh = 210.7000
pccbm = 0.0605 paxibm = 119.7917 pyybm = 52.7933
phepb = 140.8371 pp2b2 = 21.9533