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Heat Exchangers and Vapor-Cooled
Cryogenic Current Leads**

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LOW FLOW VELOCITY, FINE-SCREEN HEAT EXCHANGERS
AND VAPOR-COOLED CRYOGENIC CURRENT LEADS

by

W. A. Steyert and N. J. Stone

ABSTRACT

The design, construction, and testing of three compact, low temperature heat exchangers are reported. A method is given for the construction of a small ($\approx 20\text{-cm}^3$ volume) exchanger that can handle 6 g/s helium flow with low pressure drops ($\Delta P/P = 10\%$) and adequate heat transfer ($N_{tu} = 3$). The use of screen for simple, vapor-cooled current leads into cryogenic systems is also discussed.

Nomenclature

a	Distance between lowest two spacers in vapor-cooled lead (cm)
A	Frontal area (cm^2)
A'	Heat transfer surface area (cm^2)
C _p	Specific heat per unit mass (J/g-K)
d	Diameter of screen wire (cm)
d _p	Sphere diameter (cm)
D	Width of screen opening (cm)
D _h	Hydraulic diameter (cm)
f	Friction factor
G	Mass flow rate per unit of frontal area ($\text{g}/\text{cm}^2\text{-s}$)
h	Heat transfer coefficient per unit of heat transfer surface area ($\text{W}/\text{cm}^2\text{-K}$)
ℓ/A	Length-to-cross-section area ratio of conductor in vapor-cooled lead (cm^{-1})
$\mathcal{L}$	Lorentz number, used in Wiedemann-Franz law ($\text{W-}\Omega/\text{K}^2$)
L	Exchanger length (cm)

$\dot{m}$	Mass flow rate (g/s)
M	Number of layers in a vapor-cooled lead
n	Ratio of sections on #2 side of discrete exchanger to sections on #1 side
N	Number of section on #1 side of discrete exchanger
N_{tu}	Number of heat transfer units in balanced flow exchanger (taking into account longitudinal conduction); or, a measure value of heat transfer quality
N_{tu}^0	Number of heat transfer units in balanced flow heat exchanger in the absence of longitudinal conduction
Nu	Nusselt's number, hd_p/κ
Pr	Prandtl number, $C_p\mu/\kappa$
$\dot{Q}$	Heat flow (W)
R	Electrical resistance (Ω)
Re_h	Reynolds number for calculating heat transfer coefficient
Re_p	Reynolds number for calculating pressure drop
RRR	Residual electrical resistance ratio (room temperature to 4 K)
s	Number of screens in a layer in vapor-cooled lead
T	Temperature (K)
v	Superficial fluid velocity (velocity if matrix were absent) (cm/s)
V	Volume of bed of spheres (cm ³)
w	Exchanger width (cm)
α	Porosity of screen or bed of spheres (fraction of void volume in both cases as described in text)
α_f	Fraction projected free frontal area of screen $[D^2/(d+D)^2]$
ΔP	Pressure drop (dyn/cm ² $\approx 10^{-6}$ atm)
ΔP_h	Pressure drop on high-pressure side of exchanger (dyn/cm ²)
ΔP_l	Pressure drop on low-pressure side of exchanger (dyn/cm ²)
ΔT	Temperature drop (K)
ϵ	Actual efficiency of exchanger
ϵ^0	Exchanger efficiency in the absence of longitudinal conduction
κ	Thermal conductivity (W/cm-K)
λ	Dimensionless parameter describing longitudinal conduction
μ	Viscosity (poise = dyn-s/cm ² = g/cm-s)
ρ	Fluid density (g/cm ³)
ρ_e	Electrical resistivity (Ω -cm)

I. INTRODUCTION

The design, construction, and testing of three heat exchangers are described. To minimize the need for heat flow through the poorly conducting fluid, particular attention was paid to achieving low fluid flow velocities, large contact surface areas, and small fluid passages.

The merits of such heat exchangers based on these principles are generally recognized.¹ The objective of this study was to demonstrate that suitably chosen materials and techniques can be used to construct highly efficient and notably compact units to handle fluid flow rates as high as those normally found in helium liquefiers and other cryogenic apparatus.

In two of the three designs tested, the heat transfer medium for each fluid flow was 100-mesh (US Std) copper screen.* In the third exchanger, sintered 200- μ m-diam copper spheres were used in the high-pressure fluid stream. To enhance its thermal conductivity the copper in all exchangers received a high temperature oxygen treatment.² Pressure drops through the exchangers and interflow heat exchange were measured at room temperature and between 4 and 15 K. The quality of the heat exchange performance is discussed in terms of the N_{tu} formalism.³

The calculation of the heat transfer between fluid and heat exchanger is given in Sec. II, as is the calculation of the pressure drop in the fluid as a function of flow path geometry, with specific values being given for the mesh and particle beds used in this work. The N_{tu} formalism is summarized. The design of the three exchangers is discussed in Sec. III, where specific design aspects and pressure drop and heat transfer calculations are given.

Results of pressure drop measurements at room temperature and in the 5-15 K range and the heat transfer measurements are summarized in Sec. IV. In Sec. V we discuss the application of the three exchangers to a superconducting magnet lead design capable of carrying large currents but stable against local heating and with reduced refrigerant loss. Conclusions are given in Sec. VI.

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II. HEAT EXCHANGER DESIGN

Pressure Drop and Heat Transfer in Flow Through Screen and Beds of Spheres

Pressure drop in 100-mesh screen. For a rectangular wire mesh, the dimensionless friction factor⁴ f , relating pressure drop ΔP to screen dimensions and flow rate, is given by

$$f = \frac{2\Delta P \alpha_f^2 \rho}{(1 - \alpha_f^2) G^2} \approx 0.4 + \frac{90}{Re_p} \quad , \quad (1)$$

where the Reynolds number for pressure calculation is

$$Re_p \equiv GD/(\alpha_f \mu) \quad . \quad (2)$$

ρ is the fluid density (g/cm^3); μ is the fluid viscosity (poise), and D is the projected width of the screen opening. Equation (1) represents one empirical fit to the experimental data to within $\pm 20\%$.^{4,5}

For 100-mesh screen (100 wires per linear inch), the wire diameter d is 0.011 cm and the diameter of the screen opening D is 0.014 cm. Therefore, the fraction of projected free frontal area α_f is 0.31 and the pressure drop through one layer of 100-mesh screen is

$$\Delta P(\text{dyn/cm}^2) = (0.4 + \frac{1990\mu}{G})(4.7 \frac{G^2}{\rho}) \quad . \quad (3)$$

Heat transfer in 100-mesh screen. For fine screens the heat transfer coefficient h (heat flow per unit heat transfer area A' and per unit screen-gas temperature difference ΔT) is so large that accurate measurement is difficult. Published results differ by as much as a factor of 2 (Refs. 5 and 6). Taking the mean of these results, we find

$$h Pr^{2/3} \alpha / (GC_p) = 0.63 Re_h^{-0.38} \quad , \quad (4)$$

where

$$Re_h = (1.27 D + 0.27 d)G/(\alpha\mu) \quad . \quad (5)$$

Note that the empirical fit for h uses a different definition of the Reynolds number than does the empirical fit for ΔP in (2). Pr is the Prandtl number, $C_p\mu/\kappa$. (C_p is the fluid specific heat per unit mass, and κ is the fluid thermal conductivity.) Here the porosity α is $1 - \pi d/4(D + d)$. Use of the empirical relation (4) gives the heat transfer coefficient to 100-mesh screen:

$$h(W/cm^2 - K) = 3.6 G^{0.62} C_p^{1/3} \kappa^{2/3} / \mu^{0.29} \quad . \quad (6)$$

The heat transfer area A' of 100-mesh screen relative to the frontal area A on one side is

$$\frac{A'}{A} = \frac{2\pi d}{d + D} = 2.8 \quad . \quad (7)$$

Pressure drop in beds of 200-mesh-diam spheres. The results of Refs. 5 and 7 are similar:

$$f = \frac{\Delta P d_p \alpha^3 \rho}{LG^2(1 - \alpha)} \approx 1.75 + (150/Re_p) \quad , \quad (8)$$

where d_p is the sphere diameter (0.02 cm), α is the porosity (volume fraction of voids in the particle bed), and L is the length of bed in the direction of fluid flow. Here

$$Re_p \equiv d_p G / [\mu (1 - \alpha)] \quad . \quad (9)$$

Using the measured values of $\alpha = 0.39$ and $d_p = 0.02$ cm, we have

$$\Delta P(\text{dyn/cm}^2) = \frac{LG^2[(2.4 \times 10^6 \mu/G) + 900]}{\rho} \quad (10)$$

Heat transfer in beds of 200- μm -diam spheres. Considerable uncertainty is involved in estimating the heat transfer in this geometry. Reference 5 has results for small Re ; however, measurements are difficult in this range because h/GC_p is so large. Reference 6 provides data only down to $Re_h = 25$ for spheres. Other estimates⁸ give Nusselt's number $Nu \equiv hd_p/\kappa = 2$ as $G \rightarrow 0$; Nu becomes larger as G gets larger. We take the values from Ref. 6 as the best compromise; for $Re > 40$, Refs. 5 and 6 provide nearly the same results and for $Re < 10$, values in Ref. 6 lie between those in Ref. 5 and the $Nu = 2$ values.

From Ref. 6,

$$\frac{h}{G C_p} = 0.21 Re_h^{-0.31} Pr^{-1}, \quad (11)$$

where

$$Re_h = D_h G/(\mu\alpha) \quad (12)$$

Here the best value for the hydraulic diameter D_h is $2\alpha d_p/3(1-\alpha) = 0.0085 \text{ cm}$. Thus

$$h(W/\text{cm-K}) = 1.76 \kappa G^{0.69}/\mu^{0.69} \quad (13)$$

The heat transfer area in a volume $V(\text{cm}^3)$ of 200- μm -diam spheres is given numerically by

$$A'(\text{cm}^2) = 6 V(1-\alpha)/d_p = 183 V \quad (14)$$

N_{tu} Formulation

The calculations and measurements reported here can be interpreted in terms of balanced counterflow heat exchange between two fluids of constant specific heat. After Barron,³ the number of heat transfer units N_{tu}^0 in the absence of longitudinal heat conduction through the exchanger is

$$N_{tu}^0 = \frac{\dot{Q}/\Delta T|_{\text{overall}}}{C_p \dot{m}}, \quad (15)$$

where $\dot{Q}/\Delta T|_{\text{overall}}$ is the heat flow between the two streams per unit of temperature difference between them, and $\dot{m}C_p$ is the capacity rate (W/K) in each stream. Exchanger performance can be described alternatively by the effectiveness ϵ .

$$\epsilon = \frac{\text{actual heat transferred}}{\text{maximum possible heat transfer}} \quad (16)$$

for balanced flow; even in the presence of longitudinal conduction

$$\epsilon = \frac{T_2^i - T_2^e}{T_2^i - T_1^i} = \frac{T_1^e - T_1^i}{T_2^i - T_1^i}, \quad (17)$$

where the subscripts refer to fluid 1 and fluid 2 and the superscripts refer to the incoming fluid i and the exiting fluid e. T is temperature.

In the absence of longitudinal conduction for balanced flow³ we have

$$\epsilon^0 = \frac{N_{tu}^0}{1 + N_{tu}^0}. \quad (18)$$

Calculation of N_{tu} . The first step is to calculate $\dot{Q}/\Delta T$ and include the thermal impedance between each stream of gas and the metal as well as the impedance of the metal itself. N_{tu}^0 for a particular flow rate is then obtained from (15).

Longitudinal conduction clearly reduces the values of ϵ and N_{tu} and is taken into account as follows. The dimensionless longitudinal thermal conduction parameter is calculated, where $\dot{Q}/\Delta T|_L$ is the longitudinal heat flow per

$$\lambda = \frac{\dot{Q}/\Delta T|_L}{\dot{m}C_p} \quad (19)$$

unit of temperature difference between the exchanger ends (W/K). The results of Bahnke and Howard⁹ can be used to obtain values of ϵ , given N_{tu}^0 and λ . To obtain the final predicted values we then use the inverse of (18) as the definition of N_{tu} in the presence of longitudinal conduction:

$$N_{tu} = \frac{\epsilon}{1 - \epsilon} \quad (20)$$

Measurement of N_{tu} . Equation (17) is used to calculate ϵ from measurements of T_1^i , T_2^i , T_1^e , and T_2^e in conditions of balanced flow, and (20) is used to calculate N_{tu} . Some measurements were made with slightly unbalanced capacity flow rates. In such cases we calculated ϵ as defined by Barron³ for unbalanced flow. Figure 3-46 of Ref. 3 then relates the measured values of N_{tu} characteristic of balanced flow to that ϵ .

III. EXCHANGER CONSTRUCTION AND DETAILED PERFORMANCE CALCULATIONS

A. Concentric Heat Exchanger

Construction. Figure 1 is a cutaway drawing of our concentric heat exchanger. The high-pressure gas flows through a copper tube filled with layers of 100-mesh copper screen placed perpendicular to the flow. The low-pressure gas flows through a much larger area of 100-mesh screen. The unit was built by inserting slightly oversize disks into the 0.7-cm-diam tube; the tube was then heat-treated for 15 h in 0.026 Pa (2×10^{-4} torr) of air pressure at 900°C. This treatment should oxidize the impurities in the copper and provide a high residual electric resistivity ratio (RRR) and a correspondingly high thermal conductivity.² At the same time, the copper screen is diffusion-bonded to the copper tube. The measured electrical resistance of the tube and screen was 0.28 m Ω at room temperature, with an RRR to 4 K of 64 (a value surprisingly low for oxygen-treated copper, possibly because the copper tube was so impure). Next, copper screen annuli (16-mm o.d. and 6-mm

Fig. 1

Concentric-tube heat exchanger. A low-pressure gas header is shown before assembly. Active length of exchanger is 10 cm.

i.d.) were slipped over the 0.7-cm tube. The whole unit was again heat-treated in 0.026 Pa of air at 1010°C for 21 h. The final room-temperature resistance was 0.24 mΩ, with an RRR at 4 K of 1470. The increased RRR probably is associated with some diffusion-bonding of the screens to each other as well as with further treatment of the tube at the higher temperature. Metal headers for the low-pressure gas were soldered in place, and a glass cloth-epoxy tube was constructed around the annuli and the headers to contain the low-pressure gas.

Pressure drop $\Delta P|_h$ on high-pressure side. We will calculate $\Delta P|_h$ for 580 torr of helium at room temperature and for 1 standard atmosphere at low temperature to compare with measurements. We will also predict $\Delta P|_h$ for high-pressure helium at low temperatures.

Using (3) with $G = 3.19 \text{ m}$, we have for 250 layers

$$\Delta P|_h = 1.2 \times 10^4 (0.4 + 624 \mu/\text{m}) \dot{m}^2 / \rho \quad . \quad (21)$$

For room temperature helium*

$$\Delta P|_h = 0.92 \times 10^8 (0.4 \dot{m}^2 + 0.12 \dot{m}) \quad . \quad (22)$$

At 4 K near 1 atm with a mean temperature of 7.5 K,**

$$\Delta P|_h = 1.7 \times 10^6 (0.4 \dot{m}^2 + 0.011 \dot{m}) \quad . \quad (23)$$

At 4 K and at 12 atm,***

$$\Delta P|_h = 1.0 \times 10^5 (0.4 \dot{m}^2 + 0.022 \dot{m}) \quad . \quad (24)$$

$\Delta P|_l$ on low-pressure side. Again using (3) with $G = 0.61 \dot{m}$, we have

$$\Delta P|_l = 2.5 \times 10^6 (0.4 \dot{m}^2 + 0.63 \dot{m}) \quad (25)$$

at room temperature and 580 torr, and

$$\Delta P|_l = 4.7 \times 10^4 (0.4 \dot{m}^2 + 0.06 \dot{m}) \quad (26)$$

at 7.5 K and 1 atm. These values are in dyn/cm² ($\approx 10^{-6}$ atm). The curves in Figs. 2 and 3 show the pressure drops calculated for this exchanger.

Fluid-metal heat transfer. From Eqs. (6) and (7) and the exchanger parameters given in Fig. 1, we have for the high-pressure side

$$hA'|_h \text{ (W/K)} = 1620 \dot{m}^{0.62} C_p^{1/3} \kappa^{2/3} / \mu^{0.29} \quad , \quad (27)$$

and for the low-pressure side

$$hA'|_l \text{ (W/K)} = 2240 \dot{m}^{0.62} C_p^{1/3} \kappa^{2/3} / \mu^{0.29} \quad . \quad (28)$$

*For₃helium near room temperature we take μ as 194 μ poise and $\rho = 1.3 \times 10^{-4}$ g/cm³ (atmospheric pressure of 580 torr at Los Alamos, New Mexico), $\kappa = 1.52$ mW/cm-K, and $C_p = 5.19$ J/g-K.

**For helium at a mean temperature of 7.5 K and at one standard atmosphere we have $\mu = 18.5$ μ poise, $\rho = 0.0069$ g/cm³, $\kappa = 0.145$ mW/cm-K, and $C_p = 5.6$ J/g-K.

***For helium at 7.5 K and 12 atm we have $\mu = 36$ μ poise, $\rho = 0.115$ g/cm³, $\kappa = 0.28$ mW/cm-K, and $C_p = 6.2$ J/g-K.

Fig. 2

Pressure drop vs mass flow rate for concentric-tube exchanger. Lines are calculated from Eqs. (22 and (25) for room temperature with mean pressure of 580 torr (local atmospheric pressure) of helium. Points are measured values with outlet pressure of 580 torr.

Transverse heat flow through the screens. The screen can be considered a cylindrical volume of conductivity κ and length L into which heat is deposited uniformly. The outside temperature is held at T . Then

$$\dot{Q}/\bar{\Delta T} = 8 \pi \kappa L \quad , \quad (29)$$

where $\dot{Q}$ is the total heat being deposited and $\bar{\Delta T}$ is the volume average value of $T(r) - T$; $T(r)$ is the temperature of the cylinder at a distance r from its center.

Fig. 3

Pressure drop vs mass flow rate for concentric-tube exchanger. Lines are calculated from Eqs. (23) and (26) for a mean temperature of ≈ 7.5 at 1 atm. Points are measured values with 580-torr outlet pressure on the low-pressure side (triangles) and slightly above 580-torr outlet pressure on the high-pressure side (circles).

In the high-pressure side we are dealing with 250 layers of screen heated uniformly by the flowing gas. The volume of copper is given by

$$V = \frac{\pi}{4} \times (0.63)^2 \times 250 \times \frac{1}{2} \times \frac{\pi}{4} \times \frac{4 d^2 (d + D)}{(d + D)^2} \quad (30)$$

Equating (30) to $\pi(0.63)^2 L/4$, where L is the effective length of the screen, gives $L = 1.9$ cm.

At room temperature we take $\kappa = 4.1$ W/cm-K and (29) becomes (in W/K)

$$\dot{Q}/\Delta T|_h = 196 \quad . \quad (31)$$

At low temperatures, we use the Weidemann-Franz law to estimate κ .

$$\kappa = \frac{f T}{\rho_e} \quad , \quad (32)$$

where f is the Lorentz number, approximately 1.9×10^{-8} W- Ω /K² near 7.5 K for copper.¹⁰ Taking $T = 7.5$ K as a mean value, and $\rho_e = 1.7 \times 10^{-6}/1470$, we have $\kappa = 120$ /cm-K or, at low temperatures,

$$\dot{Q}/\Delta T|_h = 5720 \quad . \quad (33)$$

In the low-pressure side heat is being deposited uniformly in a thick-walled cylindrical shell at constant T on the inside wall and with no heat loss at the outside wall. Then

$$\dot{Q}/\Delta T = \frac{8 \pi \kappa L (\mathcal{D}^2 - 1)}{4 \mathcal{D}^4 (\ln \mathcal{D}) (\mathcal{D}^2 - 1)^{-1} - (3 \mathcal{D}^2 - 1)} \quad , \quad (34)$$

where $\mathcal{D}$ is the ratio of the inner to the outer cylinder diameter. With the effective $L = 1.4$ for 185 screen layers, $\mathcal{D} = 2.3$, $\dot{Q}/\Delta T = 22 \kappa$. Thus, with the above values of κ at room temperature,

$$\dot{Q}/\Delta T|_l = 90.2 \quad , \quad (35)$$

whereas at low temperatures

$$\dot{Q}/\Delta T|_l = 2640 \quad . \quad (36)$$

Calculation of N_{tu}^0 . N_{tu}^0 , which neglects longitudinal conduction, can be calculated from the following approximate relation

$$\dot{Q}/\Delta T|_{\text{overall}} \approx \left\{ (hA'|_h)^{-1} + (hA'|_c)^{-1} + (\dot{Q}/\Delta T|_h)^{-1} + (\dot{Q}/\Delta T|_c)^{-1} \right\}^{-1} \quad (37)$$

and the definition of $N_{tu}^0 = \dot{Q}/\Delta T|_{\text{overall}}/C_p \dot{m}$ is calculated by using (27), (28), (31) or (33), and (35) or (36) and using the properties of the fluids given in footnotes here*,** and on p. 10.

With the Weidemann-Franz law, the dimensionless longitudinal conduction parameter is***

$$\lambda = \frac{\ell \bar{T}/R}{C_p \dot{m}}, \quad (38)$$

where R, the electrical resistance, is 0.24 m Ω and 0.24/1440 m Ω at room temperature and at 4 K, respectively.

$$\lambda = \frac{2.5 \times 10^{-8} \times 293/0.24 \times 10^{-3}}{C_p \dot{m}} = \frac{0.030}{C_p \dot{m}} \quad (39)$$

*For water near room temperature we have $\mu = 10$ mpoise, $\rho = 1.0 \text{ g/cm}^3$, $\kappa = 6$ mW/cm-K, and $C_p = 4.18 \text{ J/g-K}$.

**For nitrogen at room temperature we have $\mu = 174$ μ poise, $\rho = 8.3 \times 10^{-4} \text{ g/cm}^3$, $\kappa = 0.26 \text{ mW/cm-K}$, and $C_p = 1.04 \text{ J/g-K}$.

***We use a simple extension of the Weidemann-Franz law [Eq. (32)] to calculate $\dot{Q}/\Delta T|_L$ in Eq. (19); $\dot{Q}/\Delta T|_L = \kappa A/\ell = \ell T/(\rho \ell/A) = \ell T/R$, where R is the longitudinal resistance of the exchanger.

at room temperature. At low temperatures,

$$\lambda = \frac{1.9 \times 10^{-8} \times 7.5 \times 1440 / 0.24 \times 10^{-3}}{C \dot{m}_p} = \frac{0.86}{C \dot{m}_p} \quad (40)$$

Using the calculated values of λ to correct N_{tu}^0 , as detailed in Ref. 9, we calculate the values of N_{tu} given in Fig. 4.

B. Sintered-Copper Discrete Screen Exchanger

Construction. Figure 5 is a cutaway drawing of our discrete screen exchanger. The high-pressure gas flows through sintered 200- μ m-diam spheres.

Fig. 4

Number of heat transfer units vs mass flow rate in concentric-tube exchanger for balanced flow. Curves are calculated in text; points are measured values. At room temperature, water in the high-pressure side was measured against nitrogen gas in the low-pressure side as was nitrogen in the high-pressure side vs helium in the low-pressure side. $\dot{m}$ is the nitrogen flow rate for the room-temperature measurements. Incoming fluid temperatures differed by about 40 K. N_{tu} for helium at a mean temperature of 7.5 K also was measured and calculated. Incoming fluid temperatures here differed by about 5 K.

Fig. 5

Sintered-copper discrete heat exchanger. Low-pressure gas headers (Fig. 1) are not shown. Solder is 99.9999% pure indium, and the outer tube is glass cloth-epoxy. The low-pressure gas weaves up and down through the screen.

The spacers shown in Fig. 5 force the low-pressure gas to weave up and down through copper screens mounted longitudinally within the exchanger. The gas must pass through 6 sections, each 1.6 cm long and containing 10 layers of screen. Five layers of screen were soldered to each side of the central tube with 99.999% pure indium. Headers were installed and the unit was wrapped in glass cloth-epoxy. Before assembly, the tube was treated for 15 h in 0.026 Pa of air at 900°C to sinter the loosely packed copper spheres (measured filling factor of 0.61) and to improve the thermal conductivity at low temperatures. The tube had a 0.24-m Ω resistance at room temperature, with an RRR = 295. The copper screen was oxygen-treated at 980°C for 18 h in 0.026 Pa of air pressure. The RRR of the screen was 900. After assembly, the resistance

of the unit was 0.10 m Ω and RRR was 286 (measured slightly above 4 K so that the indium was a normal conductor).

$\Delta P|_h$ on high-pressure side. Using Eq. (10), with $G = 3.2 \dot{m}$ (from the exchanger dimensions given in Fig. 5) and the gas parameters given in footnotes, p. 10, we have for room-temperature helium at 580 torr the pressure drops (in dyn/cm²)

$$\Delta P|_h = 7.8 \times 10^5 (900 \dot{m}^2 + 146 \dot{m}) \quad . \quad (41)$$

For 7.5 K helium at 1 atm,

$$\Delta P|_h = 1.48 \times 10^4 (900 \dot{m}^2 + 13.9 \dot{m}) \quad , \quad (42)$$

and for 7.5 K helium at 12 atm,

$$\Delta P|_h = 8.9 \times 10^2 (900 \dot{m}^2 + 22 \dot{m}) \quad . \quad (43)$$

$\Delta P|_l$ on low-pressure side. In the low-pressure side, with $G = 0.45 \dot{m}$ and using (3) for room-temperature helium at 580 torr, we find the pressure drops (in dyn/cm²)

$$\Delta P|_l = 4.38 \times 10^5 (0.4 \dot{m}^2 + 0.86 \dot{m}) \quad , \quad (44)$$

and for 7.5 K helium at 1 atm,

$$\Delta P|_l = 8.28 \times 10^3 (0.4 \dot{m}^2 + 0.082 \dot{m}) \quad . \quad (45)$$

These expressions were used to calculate the curves shown in Figs. 6 and 7.

Fluid-metal heat transfer. From (13) and (14) and the dimensions given in Fig. 5, we calculate

$$hA'|_h = 2250 \kappa \dot{m}^{0.69} / \mu^{0.69} \quad . \quad (46)$$

For the low-pressure side, using (6) and (7), we calculate

$$hA'|_l = 826 \dot{m}^{0.62} c_p^{1/3} \kappa^{2/3} / \mu^{0.29} \quad . \quad (47)$$

Later in the calculations we allow for the discrete nature of the low-pressure side.

Transverse heat flow through the exchanger. We again use (29) for the high-pressure side, and take κ as 1/6 that of the solid metal, on the basis of measurements made at room temperature on another sintered-copper sample.

With $\kappa = 0.68$ at room temperature and $\kappa = 3.84$ at 7.5 K, we have at room temperature

$$\dot{Q}/\Delta\bar{T}|_h = 175 \quad , \quad (48)$$

and, at 7.5 K,

$$\dot{Q}/\Delta\bar{T}|_h = 960 \quad . \quad (49)$$

Fig. 6

Pressure drop vs mass flow rate for sintered-copper discrete heat exchanger. Lines are calculated from Eqs. (41) and (44) for helium at 580 torr and at room temperature. Points are measured values with 580-torr outlet pressure.

Fig. 7

Pressure drop vs mass flow rate for sintered-copper discrete heat exchanger. Lines are calculated from Eqs. (42) and (45) for helium at 7.5 K and 1 atm. Points are measured values with 580-torr outlet pressure on the low-pressure side and slightly above 580 torr on the high-pressure side.

In the low-pressure side, for heat uniformly deposited in the screen fins of length L (normal to direction of heat flow), thickness t , and width w (along the direction of heat flow), an elementary calculation shows

$$\dot{Q}/\overline{\Delta T}|_s = 3 \kappa t L/w = 3 \kappa \frac{(\pi/4)d^2}{d + D} L/w, \quad (50)$$

where $\overline{\Delta T}$ is the average $T(x) - T$ when one edge is held at T . For the screen, taking $w \approx 0.8$ (heat must travel a little way into the solder joint),

$$\dot{Q}/\overline{\Delta T}|_s = 2.8 \kappa. \quad (51)$$

$L = 200$ cm because there are fins on two sides of the exchanger and there are 10 layers of screen, each 10 cm long. At room temperature, $\kappa = 4.1$ W/cm-K and at 7.5 K. Use of the Weidemann-Franz law and the measured RRR gives $\kappa = 72$ W/cm-K. Thus at room temperature,

$$\dot{Q}/\overline{\Delta T}|_s = 11.5, \quad (52)$$

and, at 7.5 K,

$$\dot{Q}/\overline{\Delta T}|_s = 202. \quad (53)$$

We must also consider the magnitude of ΔT across the screen-tube solder joint. We measured the electrical resistance across a number of 10-layer screen-solder joints 6 mm^2 in area. The average resistance for 10 samples was $8.4 \mu\Omega$ and the RRR was 90. On the average, heat has to travel through 2.5 layers of screen; thus $R = 12.6 \mu\Omega$ for a 1-mm^2 area. Using the Weidemann-Franz law, we expect the conductance at room temperature to be 0.58 W/K-mm^2 , whereas at 7.5 K the conductance should be 1 W/K-mm^2 . There are roughly 1400 mm^2 of solder area, so temperature drops through the solder can be neglected as opposed to the other ΔT 's in the exchanger [for example, Eqs. (48)-(53)].

We use

$$\dot{Q}/\overline{\Delta T}|_{h,\text{overall}} \approx \left\{ (hA'|_h)^{-1} + (\dot{Q}/\Delta T|_h)^{-1} \right\}^{-1} \quad (54)$$

and a similar relation for $\dot{Q}/\overline{\Delta T}|_{\ell,\text{overall}}$ from the appropriate equations (46)-(53).

In the Appendix, the discrete nature of the low-pressure side of the exchanger is taken into account to allow the calculation of N_{tu}^0 by (A-12). With the measured values of $R = 0.1 \text{ m}\Omega$ at room temperature and $0.1/286 \text{ m}\Omega$ at 4 K and by a calculation similar to that of (39) and (40), we get $\lambda = 0.073/\dot{m}C_p$ at room temperature and $\lambda = 0.408/\dot{m}C_p$ at low temperatures. Again, the calculated N_{tu}^0 values were corrected for λ as detailed in Ref. 9. The results are given in Fig. 8.

Fig. 8

N_{tu} vs mass flow rate in sintered-copper discrete tube exchanger for balanced flow. Curves are calculated in text; points are measured values. At room temperature, nitrogen was in the high-pressure side and helium in the low-pressure side; the temperature of the incoming gases differed by about 40 K. With helium at an average temperature of 7.5 K, the incoming fluid temperatures differed by 5 K.

C. Flat Exchanger

Construction. Figure 9 shows several views of our flat exchanger. The fluid is forced to flow back and forth through the 100-mesh screen. In the high-pressure side the gas makes 24 such excursions at a relatively high G . In the low-pressure side the gas executes only 6 such excursions, thereby dropping G by a factor of 4. The design philosophy of the exchanger was to provide the lowest possible G (mass flow rate per unit of frontal area), hence the lowest ΔP in the low-pressure side. The dimensions of the spacers were

Fig. 9

Flat heat exchanger designed to provide a large area of screen for the gas, thereby keeping G as small as possible. High-pressure layers of screen are physically close to the low-pressure layers so that the conduction path through the overlapping copper spacers is short. Inlet and outlet ports are not shown in detail.

chosen to provide a thermal impedance between the high-pressure and low-pressure screens lower than the impedances between the screens and the fluid streams.

The unit was first assembled with glue and placed in a vacuum furnace under a 10-kg weight for 5 h at 1030°C. This treatment diffusion-bonded the components together. The unit was then treated in 0.026 Pa of air at 990°C for 21 h to improve the copper conductivity (the air also helped to remove any glue left after the vacuum treatment). Headers were installed and the sides of the exchanger were vacuum-sealed with lead-tin solder.

The measured electrical resistance of the exchanger at room temperature was 55 $\mu\Omega$. Because at low temperatures the superconducting lead-tin solder precludes a meaningful resistance measurement, we take RRR as 500, a value typical of the heat-treated copper used in the exchanger, thus $R \approx 0.11 \mu\Omega$ at 7.5 K.

Pressure drops. In the high-pressure side, we take as the area of each crossing $\frac{0.85 \text{ cm} \times 1.0 \text{ cm} - 0.7 \times 0.2}{2} = 0.36 \text{ cm}^2$ (allowing 0.05 cm near each spacer as an ineffective area because of excessive screen compression). We have $G = 2.82 \dot{\text{m}}$ and, using Eq. (3) and the values given in footnotes, p. 10, and allowing for 240 screen-layer crossings, we calculate the pressure drops (dyn/cm^2) for room-temperature helium at 580 torr:

$$\Delta P|_h = 6.9 \times 10^7 (0.4 \dot{\text{m}}^2 + 0.14 \dot{\text{m}}) \quad . \quad (55)$$

For 7.5 K helium at 1 atm,

$$\Delta P|_h = 1.3 \times 10^6 (0.4 \dot{\text{m}}^2 + 0.013 \dot{\text{m}}) \quad (56)$$

and, for 7.5 K helium at 12 atm,

$$\Delta P|_h = 7.8 \times 10^4 (0.4 \dot{\text{m}}^2 + 0.025 \dot{\text{m}}) \quad . \quad (57)$$

Similarly, in the low-pressure side for room-temperature helium at 580 torr,

$$\Delta P|_l = 1.0 \times 10^6 (0.4 \dot{\text{m}}^2 + 0.57 \dot{\text{m}}) \quad , \quad (58)$$

and, for 7.5 K helium at 1 atm,

$$\Delta P|_2 = 1.94 \times 10^4 (0.4 \dot{m}^2 + 0.055 \dot{m}) \quad (59)$$

Expressions (55) through (59) were used to calculate the curves shown in Figs. 10 and 11.

Fluid-metal heat transfer. From (6) and (7), taking the unobstructed frontal area of the screen to be 86.4 cm^2 in the high-pressure side and 87.6 cm^2 in the low-pressure side, we have

$$hA'|_h = 1660 \dot{m}^{0.62} C_p^{1/3} \nu^{2/3} \mu^{0.29} \quad (60)$$

Fig. 10

Pressure drop vs mass flow rate for flat exchanger at room temperature. Lines are calculated from Eqs. (55) and (58). Points are measured values with 580-torr outlet pressure.

Fig. 11

Pressure drop vs mass flow rate for flat exchanger. Lines are calculated from Eqs. (56) and (59) for 1 atm of helium at 7.5 K. Points are measured values with 580-torr outlet pressure in low-pressure side (triangles) and slightly above 580 torr in high-pressure side (circles).

and

$$hA|_l = 702 \dot{m}^{0.62} c_p^{1/3} \kappa^{2/3} / \mu^{0.29} \quad (61)$$

Transverse heat flow. In each side of the exchanger, the maximum length of screen that heat must travel to reach a spacer is 0.15 cm. With an analysis similar to that of (50) and (51) we obtain

$$\dot{Q}/\Delta T = 38 \kappa \quad (62)$$

For heat flow through the metal spacers on each side (the total area of spacers is 3.4 cm^2 ; they are 0.2 cm thick), we have

$$\dot{Q}/\Delta T = \frac{3.4 \kappa}{0.2} = 17 \kappa \quad (63)$$

We take $\kappa = 4.2 \text{ W/cm-K}$ at room temperature and, from (32), find $\kappa = 42 \text{ W/cm-K}$ at 7.5 K.

In addition, we must allow for the impedance of the diffusion bond between the screen and the spacer. Across 10 layers of diffusion-bonded screen into a copper spacer we measure $34.5 \mu\Omega$ for a 1-mm^2 area at room temperature, with $\text{RRR} = 150$. If we divide ΔT by 2 because, on the average, heat flows only halfway across the 10 layers, and use the Weidemann-Franz law again, we have $\dot{Q}/\Delta T A = 0.42 \text{ W/K-mm}^2$ at room temperature and 1.2 W/K-mm^2 at 7.5 K. With 350 mm^2 of contact area in both the high- and low-pressure sides,

$$\dot{Q}/\Delta T = 147 \text{ W/K} \quad (64)$$

at room temperature and

$$\dot{Q}/\Delta T = 420 \text{ W/K} \quad (65)$$

at 7.5 K. Thus the complete heat path through the metal is

$$\dot{Q}/\Delta T = 36 \text{ W/K} \quad (66)$$

at room temperature and

$$\dot{Q}/\Delta T = 227 \text{ W/K} \quad (67)$$

at 7.5 K. The sum of reciprocals of (66) or (67) with (60) and (61) yields $Q/\Delta T|_{\text{overall}}$ for the high- and low-pressure sides. When inserted in (A-11) these values yield values of N_{tu}^0 for the exchanger.

We measure $40 \mu\Omega$ at room temperature to get the exchanger's longitudinal resistance. A meaningful R cannot be measured at low temperatures because of the high superconducting transition of the lead-tin solder on the exchanger. We take RRR of 500 as a typical mean value for the oxygen-treated copper. Using the Weidemann-Franz law, we have $\lambda = 0.18/mC_p$ at room temperature and $\lambda = 1.8/mC_p$ at 7.5 K. Now N_{tu} can be calculated according to Ref. 9.

A heat exchanger of this design has longitudinal components that contribute to the undesirable longitudinal conductivity but do not contribute to the desirable transverse conductivity. In later versions, these components were gold-plated; subsequent diffusion of the gold into the copper resulted in a roughly 1 at.% gold-in-copper alloy with RRR = 2.6.

IV. EXPERIMENTAL RESULTS

A. Pressure Drop Tests

At room temperature, helium gas at local atmospheric pressure (580 torr) was passed through each side of the exchanger, and the pressure drop and volume flow rate were recorded -- a simple technique similar to that reported in Ref. 5. The results are shown in Figs. 2, 6, and 10.

During the low-temperature heat transfer tests, pressure drops and volume flow rates were also recorded and are shown in Figs. 3, 7, and 11. The pressure drops in the low-pressure side in these tests are subject to considerable uncertainty because the pressure gauges were installed such that the low-pressure side ΔP was the difference between two large pressures.

Large deviations from expected pressure drops appear in the room-temperature and low temperature results for the low-pressure side of the sintered tube exchanger (Figs. 6 and 7). Destructive disassembly of the exchanger showed that the glass cloth-epoxy covering had sagged between the spacers and had blocked a large fraction of the screen area, thus causing the large pressure drop.

At low temperature there is an apparent partial blockage of the low-pressure side of the flat exchanger (Fig. 11).

B. Heat Transfer Measurements

Heat exchange parameters were measured near room temperature, first with nitrogen gas flowing through the high-pressure side and helium gas through the low-pressure side and then with water in the high-pressure side and nitrogen on the low-pressure side. One fluid was cooled to about 5°C with an ice bath; the other was warmed to about 45°C with a hot-water bath. The exchanger was insulated from ambient temperature by a fiber glass wrapping. Inlet and outlet fluid temperatures were measured with a 0.03-cm wire-diameter copper-constant and with thermocouples in the flowing fluids, and the gas flows were measured with dry test meters.

The low temperature measurements were performed by flowing low temperature helium gas (boiling from a pressurized dewar) through the high-pressure side of the exchanger. The gas was then electrically heated and returned through the low-pressure side of the exchanger, finally venting through a dry test gas meter that measured the gas volume during a given time. Gas temperatures were measured in the flowing gas with calibrated* carbon resistance thermometers. Each temperature was measured with two thermometers (installed in different places) and the results from the two were always consistent. The data were examined to compare the heat lost by the warm gas with the heat gained by the cold gas. In general, this energy balance check was correct to better than 10%, showing that heat transfer to the surroundings was small.

In the low temperature measurements the gas flowing through the high-pressure side of the exchanger increased by as much as 400 torr, resulting in a small imbalance in the capacity flow rates of the two gas streams. The imbalance was corrected for by the method shown in Fig. 3-43 of Ref. 3. Thus, we were able to deduce a measure of N_{tu} characteristic of balanced flow. Results are shown for both temperature regions in Figs. 4, 8, and 12.

The concentric-tube exchanger (Fig. 4) did not show the expected large value of N_{tu} at room temperature. We attribute this to leakage of the gas down the length of the exchanger past the edges of the screens, particularly in the low-pressure side, or to the inadequacy of the approximation in (54).

*Calibrations to ± 0.02 K from 4-20 K were made by A. Migliori.

Fig. 12

N_{tu} vs mass flow rate in flat heat exchanger for balanced flow. At room temperature, water was measured against nitrogen on the high-pressure side, as was nitrogen vs helium on the low-pressure side. At room temperature, $\dot{m}$ is the nitrogen gas flow rate; incoming fluid temperatures differed by 40 K. At a mean temperature of 7.5 K, helium was also measured and calculated, $\dot{m}$ here is the helium flow rate; incoming fluid temperatures differed by about 5 K. The curves are calculated in the text.

The higher-than-expected N_{tu} values of the sintered-tube exchanger at room temperature and the lower-than-expected values at 2 K are not understood. Our calculated N_{tu} values may be in error because the longitudinal conduction correction used is applicable strictly to a continuous exchanger and not to the semidiscrete sintered unit.

V. CRYOGENIC VAPOR-COOLED CURRENT LEADS

In this section we discuss the application of the discrete fluid flow geometry described above to a vapor-cooled lead entering a cryogenic system. These leads have not yet been built and tested, and in view of the large body of literature on the current-lead problem we do not give a detailed treatment. We demonstrate that leads can be designed to improve on the performance of the best existing designs in terms of loss rate of refrigerant per unit of current, in terms of thermal contact between the leads and the gas, and in terms of pressure drop across the current lead.

We want to minimize the gas evaporation rate $\dot{m}$ required to self-cool leads of length ℓ and cross-sectional area A . At the same time we want to minimize the pressure drop as the fluid flows through the lead.

Consider an array of screens as shown in Fig. 13a; s 100-mesh screens in each layer have 50 spacers to force the flow to interlace the screens. Because of the large temperature difference between the top at 300 K and the bottom at 4 K, it is best to have the spacers closely spaced at the bottom and widely spaced at the top. At the bottom we want to keep the gas-screen temperature difference ΔT small; at the top we can afford a much larger ΔT .

Also, at the bottom of the exchanger a larger mass flow per unit area is possible for a given pressure drop ΔP because ρ is larger. Without trying to optimize the system too carefully, we arbitrarily take the spacing between flow spacers to be a at the bottom, increasing linearly to $20a$ at the top. The total length L of the exchanger is

$$L = 525 a \quad . \quad (68)$$

We take the width of the exchanger as w . At the bottom, where $T = 4$ K, $\rho = 0.015 \text{ g/cm}^3$, $\mu = 13 \times 10^{-6} \text{ poise}$, $\kappa = 10^{-4} \text{ W/cm-K}$, $C_p = 9.5 \text{ J/g-K}$, and $G = \dot{m}/aw$, using (3) we have

$$\Delta P(\text{dyn/cm}^2) = 313 s \left\{ 0.4 \left(\frac{\dot{m}}{aw} \right)^2 + 0.026 \left(\frac{\dot{m}}{aw} \right) \right\} \quad (69)$$

and, from (6),

$$h(W/\text{cm}^2 - K) = 0.43(\dot{m}/aw)^{0.62} = 21(\dot{m}/Lw)^{0.62} \quad ; \quad (70)$$

At the top, where $G = \dot{m}/(20aw)$, we have

$$\Delta P(\text{dyn/cm}^2) = 90.5 s \left\{ 0.4 \left(\frac{\dot{m}}{aw} \right)^2 + 7.7 \left(\frac{\dot{m}}{aw} \right) \right\} \quad (71)$$

and

$$h(W/\text{cm}^2 - K) = 0.15(\dot{m}/aw)^{0.62} = 7.5(\dot{m}/Lw)^{0.62} \quad . \quad (72)$$

Fig. 13

Vapor-cooled current lead. (a) 1-kA lead ($s = 10$, $L = 20$ cm, and $w = 2$ cm as in text example). (b) 25-kA lead ($s = 10$, $L = 20$, and $w = 5$). Five of the 10 layers of screen are shown. For superconducting applications, a horizontal superconducting current lead could be soldered along the width of the screen at the bottom to gather the current. Heavy copper headers would be needed at the top. To withstand the compressional pinch of the current in 25-kA applications, some kind of mechanical support structure would be needed in addition to the flow separators. Calculations show that spacers a few hundredths of a centimeter thick provide adequate channel widths. Spacers would be more widely separated at the top of the lead than at the bottom.

The total pressure drop will be roughly 50 times the average of (69) and (71):

$$\Delta P(\text{dyn/cm}^2) = s \left\{ 1.1 \times 10^9 \left(\frac{\dot{m}}{Lw} \right) + 9.3 \times 10^6 \left(\frac{\dot{m}}{Lw} \right) \right\} . \quad (73)$$

Calculations similar to those of (69) and (71) show that ΔP per screen section near the center of the lead is roughly proportional to the section number, as is required for (73) to be valid. A detailed calculation obviously is needed once a lead temperature profile is decided upon; we took $T = 170$ K near the center of the lead as an overestimate of T , hence ΔP . In any event, ΔP is dominated by the high temperature end of the lead and is not sensitive to these details.

Let us consider what might be reasonable values for Lw and s for a lead to carry 1 ka, for example. For copper with an RRR of 50, the lead length-to-area ratio ℓ/A might be 250 cm^{-1} for a well-designed 1-kA lead:¹¹

$$\ell/A = 250 = L / \left\{ s w \frac{\pi(0.011^2)/4}{0.025} \right\} = 263 \frac{L}{s w} \quad (74)$$

or

$$L/s w \approx 1 . \quad (75)$$

Also, $\dot{m} \approx 0.04 \text{ g/s}$ for a well-designed 1-kA lead.¹¹

Now consider the heat transfer at the top and the bottom of the lead. We will calculate the ΔT between the helium and the screen. Equating the power generation $i^2 R$ over a fraction of length F of the lead to the heat transfer near the bottom of the lead (where the electrical resistivity of the copper is $3.8 \times 10^{-8} \Omega\text{-cm}$), and taking the ℓ/A of the lead to be 250 cm^{-1} and $\dot{m} = 0.04 \text{ g/s}$, we find

$$1000^2 \times 3.8 \times 10^{-8} \times 250 F = 21(\dot{m}/Lw)^{0.62} 2.8 Lws F \Delta T \quad (76)$$

or

$$\Delta T = 1.2 / (L^{0.38} w^{0.38} s) \quad . \quad (77)$$

Similarly, near the top of the exchanger,

$$\Delta T = 150 / (L^{0.38} w^{0.38} s) \quad . \quad (78)$$

If we choose $L = 20$, $s = 10$, and $w = 2$ [satisfying (75)] we have

$$\Delta P = 10^5 \text{ dyn/cm}^2 (\approx 0.1 \text{ atm}) \quad , \quad (79)$$

$$\Delta T = 0.037 \text{ K at the bottom} \quad , \quad (80)$$

$$\Delta T = 3.7 \text{ K at the top} \quad . \quad (81)$$

For a larger lead, say, 25 kA, we could have 25 1-kA leads in parallel ($\dot{m}$ is up by a factor of 25 as well, so G , h , and ΔP are unchanged). Figure 13b shows such a lead. It is reasonably compact: 20 cm long, 5 cm wide, and 1.5 cm thick.

The ΔP is much lower than in most leads, and ΔT is small compared to the usual lead. The main advantage of the lower ΔP is either a saving in helium compressor power needed to refrigerate the helium leaving the apparatus through the lead or a cooler helium bath when a boiling helium bath is used. The smaller ΔT will reduce the lead temperature and the lead resistance. Accordingly, the $i^2 R$ losses in the lead are reduced. For an optimum lead, heat conduction into the top is small and the heat leak out the bottom is small compared to $i^2 R$ (as can be seen by comparing $\dot{Q} = \kappa A (dT/dx) = 4 \text{ W}$ at the top of the 1-kA lead with $\dot{Q} = \dot{m} C_p \delta T = 61 \text{ W}$). Thus, the decrease in $i^2 R$ is directly reflected in a decrease in $\dot{m}$ because

$$\dot{m} C_p \delta T \approx i^2 R \quad . \quad (82)$$

In (82) δT is the increase in temperature of the gas coming through the leads. As another improvement, δT is larger and has better heat exchange, again making $\dot{m}$ smaller, as indicated by (82).

The tight gas-conductor thermal-bonding will make the load more stable against burnout. The effect of thermal contact on stability can be seen if we consider a disturbance that increases the lead temperature by, say, 1%. The i^2R will increase by a few percent because of the increase in R . The heat transfer from the lead, however, will increase manyfold, causing the lead to cool (if $\dot{m}$ is sufficient to carry the extra local heat).

Of course, the small size, simplicity, and lightness of the lead are also important in most applications. These calculations merely outline some of the design considerations, but do show the merits of the heat transfer when screens are used.

VI. CONCLUSIONS

The use of fine-mesh copper screen allows the construction of a compact heat exchanger having low-pressure drops if the geometry is such that the flow velocity through the screen is small. The large heat transfer coefficients of the fine screen and the resulting compactness of the exchanger permit short conduction paths through the metal, and heat treatment of the copper in 0.026 Pa of air makes this screen a good conductor. Diffusion-bonded components make exchanger construction rapid and inexpensive. With the use of detailed experimental data,⁴⁻⁸ electrical resistance measurements, and the Weidemann-Franz law, exchangers can be made with confidence that they will perform as designed (generally with a deviation less than a factor of 2 for properly assembled exchangers).

Table I shows the expected behavior of each side of the three exchangers studied here with a flow rate of 6 g/s and with 12 atm in the high-pressure side and 1 atm in the low-pressure side. This might be the final exchanger in a refrigerator; the exchanger would work between roughly 5 and 10 K. Figures 14 and 15 plot expected performance vs flow rate. In all cases, the longitudinal conduction parameter λ is small for these high flow rates. Because of the large pressure drops in the high-pressure sides, the sintered-copper exchanger is inferior to the designs having screens. Small changes in the number of screen layers and in the exchanger dimensions could further reduce $\Delta P|_h$ for the screen designs without adversely affecting the heat exchange performance. In the low-pressure side, the concentric exchanger would have some overall advantage, especially if the outside diameter were slightly

Fig. 14

Calculated pressure drops in the high-pressure (heavy lines) and low-pressure (light lines) sides of each exchanger. The mean temperature is taken as 7.5 K with 12 atm of pressure in the high-pressure side and 1 atm in the low-pressure side. Curves are taken from Eqs. (24), (26), (43), (45), (57), and (59).

Fig. 15

Expected N_{tu} vs mass flow rate for the three exchangers described in text. Helium at 12 atm is in the high-pressure side and 1 atm in the low-pressure side, and the mean temperature was 7.5 K.

increased to reduce $\Delta P|_L$, but the low-pressure side of all three exchangers would perform well.

These results can be extended to the design of microminiature heat exchangers to be used in small Joule-Thomson refrigerators for low temperature instruments (bolometers, SQUID magnetometers, etc.). The load there is ≈ 10 mW or less in most cases thus a flow rate $\dot{m}$ of a few milligrams per second of nitrogen, hydrogen, and helium would provide the necessary refrigeration if good heat exchangers were used. The highest temperature stage could be a miniature concentric-tube exchanger with three high-pressure tubes (for the three gases) and divided screen disks for the low-pressure side to provide a separate return path for each gas. With such a small $\dot{m}$, the only design problem is to keep the longitudinal conduction small; because copper could not be used in the high-pressure tubes, an appropriate alloy would be needed. Low-pressure screen layers must not be allowed to touch each other during the diffusion-bonding process.

TABLE I
EXPECTED PERFORMANCE OF EACH SIDE OF THREE KINDS OF EXCHANGER^a

Exchanger	High-Pressure Side ^b			Low-Pressure Side ^b		
	$hA' / \dot{m}C_p$	$\dot{Q} / (\Delta T \dot{m}C_p)$	$\Delta P (\text{atm})$	$hA' / \dot{m}C_p$	$\dot{Q} / (\Delta T \dot{m}C_p)$	$\Delta P (\text{atm})$
Concentric-tube	20	152	1.5	26	78	0.7
Sintered-copper discrete	68	25.6	c	8.6	6	0.12
Flat	21	6.1	1.3	7.3	6.8	0.26

^a Studied at 6 g/s with 12 and 1 atm in high- and low-pressure sides, respectively, and mean temperature of 7.5 K.

^b $hA' / \dot{m}C_p$ = dimensionless ratio of screen-gas heat transfer coefficient to gas capacity flow rate; $\dot{Q} / (\Delta T \dot{m}C_p)$ = ratio of heat transfer coefficient through the metal to the gas capacity flow rate.

^c Pressure drop is so large that 6 g/s could not be maintained with 12 atm of pressure.

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APPENDIX

DISCRETE HEAT EXCHANGERS

The actual N_{tu} value of the discrete exchangers must be related to the N_{tu} values of each side of the exchanger, where each side is treated as a continuous exchanger.

N SECTIONS ON EACH SIDE

First, take N sections on each side of the exchanger (Fig. A-1a). Take the model of each section (Fig. A-1b) where the screen is of such good conductivity that it is isothermal. Fluids enter the section at T_1 and T_2 and leave at T_1' and T_2' after passing through a screen at T_s . Each side of the exchanger has a capacity flow rate $C_p \dot{m}$.

$$T_1' - T_s = (T_1 - T_s) \exp(-hA' / \dot{m} C_p N) \quad (A-1)$$

Fig. A-1

Mathematical model of discrete heat exchangers. (a) Model of complete exchanger made up of N discrete sections, each represented by a fluid-fluid crossing. (b) Detail of fluid-fluid crossing. The screen is considered isothermal (a good heat conductor). (c) More complex model of fluid-fluid crossing. Fluid 1 is split, flows through each of n subsections, and reunites before leaving each fluid-fluid crossing shown in (a). All of fluid 2 flows successively through each screen subsection. Again, each screen subsection is considered isothermal.

and

$$T_2' - T_s = (T_2 - T_s) \exp(-hA|_2 / \dot{m} C_p N) \quad (A-1')$$

hA'/N is the screen heat transfer for the section. Here hA' is approximately the same as $\dot{Q}/\Delta T|_{\text{overall}}$ of (54) when we take into account $\dot{Q}/\Delta T$ in the screen. To simplify the notation, let us define

$$E_1 \equiv \exp(-hA'|_1 / \dot{m} C_p N) \quad (A-2)$$

and

$$E_2 \equiv \exp(-hA'|_2 / \dot{m} C_p N) \quad (A-2')$$

Equations (A-1) and (A-1'), combined with the energy conservation law, $T_1' - T_1 + T_2' - T_2 = 0$, permit solution for T_s . This value of T_s used in (A-1) and (A-1') permits the deduction of

$$\epsilon' = \frac{T_2 - T_2'}{T_2 - T_1} = \frac{T_1' - T_1}{T_2 - T_1} = \frac{(1 - E_1)(1 - E_2)}{2 - E_1 - E_2} \quad (A-3)$$

Here ϵ' is the effectiveness of an exchanger section.

In Fig. A-1a, fluids enter the exchanger at T_1^N and T_2^0 and leave at T_1^0 and T_2^N after passing through the N sections, represented in the figure by crossings. The fluid temperatures inside the exchanger (T_1^1 , etc.) are defined in Fig. A-1a. Using (A-3) we have

$$T_1^1 = (T_1^0 - \epsilon' T_2^0) / (1 - \epsilon') \quad (A-4)$$

and

$$T_2^1 = [\epsilon' T_1^0 + (1 - 2\epsilon') T_2^0] / (1 - \epsilon') \quad ; \quad (A-4')$$

likewise,

$$T_1^2 = (T_1^1 - \epsilon T_2^1)/(1 - \epsilon') = [(1 + \epsilon')T_1^0 - 2\epsilon' T_2^0]/(1 - \epsilon') \quad (A-5)$$

and

$$\begin{aligned} T_2^2 &= [\epsilon' T_1^1 + (1 - 2\epsilon')T_2^1]/(1 - \epsilon') = \\ &[2\epsilon' T_1^0 + (1 - 3\epsilon')T_2^0]/(1 - \epsilon') \quad . \end{aligned} \quad (A-5')$$

Similarly,

$$T_1^3 = [(1 + 2\epsilon')T_1^0 - 3\epsilon' T_2^0]/(1 - \epsilon') \quad (A-6)$$

and

$$T_2^3 = [3\epsilon' T_1^0 + (1 - 4\epsilon')T_2^0]/(1 - \epsilon') \quad ; \quad (A-6')$$

and finally,

$$\begin{aligned} T_1^N &= \{[1 + (N - 1)\epsilon']T_1^0 - N\epsilon' T_2^0\}/(1 - \epsilon') \\ &= T_1^0 - [N\epsilon'/(1 - \epsilon')](T_2^0 - T_1^0) \end{aligned} \quad (A-7)$$

$$\begin{aligned} T_2^N &= \{N\epsilon' T_1^0 - [1 - (N + 1)\epsilon']T_2^0\}/(1 - \epsilon') \\ &+ T_2^0 - [N\epsilon'/(1 - \epsilon')](T_2^0 - T_1^0) \quad . \end{aligned} \quad (A-7')$$

The effectiveness of the complete exchanger is, from (17),

$$\epsilon = \frac{T_1^0 - T_1^N}{T_2^0 - T_1^N} = \frac{N\epsilon'}{1 - \epsilon' + N\epsilon'} \quad (\text{A-8})$$

since $N_{tu} = \epsilon / (1 - \epsilon)$. Using (A-8) and (A-7) we find

$$N_{tu} = N \frac{(1 - E_1)(1 - E_2)}{1 - E_1 E_2} \quad (\text{A-9})$$

N SUBSECTIONS ON ONE SIDE, Nn SUBSECTIONS ON THE OTHER

Figure A-1c shows the model of one section (shown by a fluid crossing in Fig. A-1a); however, the section is itself subdivided. This models the flat exchanger. In side 1 (the low-pressure side), $1/n^{\text{th}}$ of the fluid at T_1 enters each of the n screens. The fluid enters screen i (screen temperature T_s^i), leaves at some temperature T_1^i , and is mixed before it leaves the section. In side 2 all of the fluid goes successively through each screen, entering at T_s^i and leaving at T_s^{i+1} .

For the entire section, this model results in

$$\epsilon' = \frac{(1 - E_1)(1 - E_2)}{(1 - E_1) + n(1 - E_2)} \quad (\text{A-10})$$

where in this case

$$E_2 = \exp(-hA' / \dot{m} C_p Nn) \quad (\text{A-10}')$$

Again, using $N_{tu} = \epsilon / (1 - \epsilon)$ and (A-10) and (A-8) we find

$$N_{tu} = N \left\{ \left[\frac{(1 - E_1) + n(1 - E_2)}{E_2(1 - E_1) + n(1 - E_2)} \right]^n - 1 \right\} \quad (\text{A-11})$$

DISCRETE EXCHANGER ON ONE SIDE, CONTINUOUS ON THE OTHER

In this case, by letting $n \rightarrow \infty$ in (A-11) we have

$$N_{tu} = N \left[\exp \left(\frac{(1 - E_1)E_2}{1 - E_1 + E_2} \right) - 1 \right] \quad , \quad (A-12)$$

where

$$E_2 = hA' | 2 / \dot{m} C_p N \quad . \quad (A-13)$$