Verification of the Classical Theory of Helical Transport in Stellarators

by

H. E. Mynick

Plasma Physics Laboratory, Princeton University

Princeton, New Jersey 08544

ABSTRACT

The apparent discrepancies of the classical theory of helical transport in stellarators, versus two recent numerical studies of stellarator transport, are investigated. Numerical results are presented, verifying the classical theory, when the model for the magnetic field has the simple form assumed by the classical theory. When the helical contribution to the total transport is isolated numerically, and the different energy dependence of the particle distribution is accounted for, the results of one of the numerical studies is brought into substantial agreement with theory. It is argued that the anomalously favorable low collisionality results of the second numerical study are due partly to numerical procedure, and partly to a more complicated spatial dependence of the magnetic field. The latter may enable collisionless helical detrapping to dominate the usual collisional mechanism at low collisionality, thereby controlling transport.

DISCLAIMER

This document is prepared as an account of work sponsored by an agency of the United States Government. Therefore, the United States Government and any agency thereof, for any of their employees, makes no warranty, expressed or implied, or assumes any legal liability or responsibility, for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISSEMINATION OF THIS DOCUMENT IS UNLIMITED

224

I. INTRODUCTION

Recently, numerical results have been obtained^{1,2} which seemed to indicate radial transport in stellarators at interesting values of collisionality much smaller than the transport predicted by earlier analytic theories.^{3,4} These theories considered the contribution D_h to the total diffusion coefficient D coming from particles which are helically trapped (the so-called "superbanana" orbits). The theories predicted that D_h should be inversely proportional to the collision frequency ν , and with size one to two orders of magnitude greater than the neoclassical transport, for a tokamak with like parameters, at collisionality ν_R typical of reactors. In contrast, the numerical results found D to have a much weaker ν dependence, and a magnitude roughly one to ten times the tokamak neoclassical value, depending on the particular device parameters chosen. The present work further investigates this apparent discrepancy between the numerical findings and analytic theory.

The principal finding is that the $1/\nu$ scaling of D_h is observed, with a numerical value in approximate agreement with that predicted theoretically. The weaker ν dependence of D in Ref. 1 arises because in the vicinity of $\nu = \nu_R$, the neoclassical contribution D_{nc} to D is comparable to D_h for the monoenergetic particle distribution used in Ref. 1. Since D_{nc} is proportional to ν , while D_h is proportional to ν^{-1} , their sum has a rather weak ν dependence, as observed. In the present work, the helical contribution D_h to D has been isolated numerically. The magnitudes of the numerical results obtained here (and in Ref. 1) and the analytic predictions of Ref. 4 agree, to within a factor of two, when account is made of the different energy dependences of the distributions used in the two papers.

In Section II, the coordinate system and units in which results will be expressed is described. Then, in Section III, a brief review of the theoretical expressions for the relevant transport coefficients is given. The sense in which neoclassical transport may and may not be expected to be a lower bound on the total transport is commented upon. The numerical procedure and results are presented in Sections IV and V. These results approximately agree with the classical theory, and also serve to reconcile the results of Ref. 1 with that theory.

In contrast to the findings of Ref. 1, those of Ref. 2 are in fundamental disagreement with classical theory. In Sections IV and VI, some possible reasons for this remaining discrepancy are discussed, and their significance for stellarator design commented upon. A somewhat broader summarizing discussion of the implications of the present and previous findings for the viability of stellarators as confinement devices is given in Section VII.

II. COORDINATE SYSTEM, UNITS

It is convenient to use a coordinate system natural to the magnetic geometry, as is done in Ref. 1. As there, we take for our radial coordinate the flux function ψ , which satisfies $\nabla \cdot \nabla \psi = 0$, so that the magnetic field lies in surfaces of constant ψ . Each ψ -surface is parameterized by toroidal angle ϕ and poloidal angle θ , chosen so that the magnetic field lines are straight in the $\phi = 0$ plane; i.e., so that the rotational transform $q^{-1} \equiv d\theta/d\phi$ is a function of ψ only. We will also use a radial variable r with units of length, defined by $\psi = (1/2) B_0 r^2$, where B_0 is the toroidal field strength at the magnetic axis, so that r is as close to the usual radial coordinate in tokamak geometry as a stellarator configuration permits. The guiding-center equations of motion in these coordinates are then

given, once a description of the field strength $B(r, \phi, \theta)$ is specified.¹ We adopt the model

$$B(r, \phi, \theta) = B_0 [1 - \epsilon_t(r) \cos \theta - \epsilon_h(r) \cos \eta] . \quad (1)$$

Here $\eta \equiv \ell\theta - m\phi$, with ℓ and m integers, $\epsilon_t = r/R_0$ is the inverse aspect ratio, and $\epsilon_h = \delta_a (r/a)^k$ gives the modulation of B due to the helical windings. (The value of r at the plasma edge is $r = a$.)

For added convenience, we shall also follow Ref. 1 in employing a system of units natural to the device being considered. Thus, we assign unit value to length $r = a$, to frequency $\Omega_0 \equiv (eB_0/mc)$ (the gyrofrequency), to mass m of the particle species being considered, and to magnetic field strength B_0 . (Stated another way, we write all quantities in dimensionless form.) Other quantities are then expressed in the appropriate composites of these; e.g., $l = v_0 \equiv a\Omega_0$ for velocity, $l = E_0 \equiv mv_0^2$ for energy, etc. Thus, for the particle energy E one has $E = (1/2) mv^2 = (1/2)(v/v_0)^2 = (1/2)(\rho/a)^2$, where $\rho \equiv v/\Omega_0$ is the particle gyroradius.

III. THEORY

From Eq. (79) of Ref. 4, one reads off the helical contribution $\bar{D}_h(T)$ to ion transport, for a Maxwellian distribution f_m of temperature T (the overbar indicates this average over f_m). In the present system of units, $\bar{D}_h$ is given by

$$\bar{D}_h(T) = \bar{\sigma}_h \epsilon_a^2 \epsilon_h^{3/2} T^2 / v_T , \quad (2)$$

where $\bar{\sigma}_h \approx 12.38$, $\epsilon_a \equiv \epsilon_t(r=a)$, and $v_T \equiv 2^{1/2} \pi n e^4 \ln \Lambda / (m^{1/2} T^{3/2})$ is the ion thermal collision frequency. As in Ref. 1, the numerical results presented here were obtained taking a monoenergetic particle distribution f_0 . The appropriate expression for the helical transport of this distribution is formally similar to Eq. (2),

$$D_h(E) = \sigma_h \epsilon_a^2 \epsilon_h^{3/2} E^2 / \nu(E), \quad (3)$$

but with a different numerical coefficient σ_h , and collision frequency $\nu(E)$. As will be evident momentarily, the transport in $\bar{D}_h$ is dominated by the more energetic ions, satisfying $x \equiv E/T > 1$, for which $\nu(E) = \nu_T x^{-3/2}$. Along with the factor x^2 in the numerator of Eq. (3) and $x^{1/2}$ from the velocity-space integral one applies in going from D_h to $\bar{D}_h$, the factor $x^{3/2}$ from ν yields an overall factor x^4 in $\bar{\sigma}_h / \sigma_h$: $\bar{\sigma}_h / \sigma_h = \int_0^\infty dx x^4 e^{-x} / \int_0^\infty dx x^{1/2} e^{-x} = 24/\sqrt{\pi} \approx 13.54$, hence $\sigma_h \approx 0.9143$.

For purposes of comparison, the neoclassical diffusion coefficient for a tokamak ($\epsilon_h = 0 \neq \epsilon_t$), with distribution f_0 , is¹

$$D_{nc}^t = \sigma_{nc} \epsilon_t^{-3/2} q^2 E \nu(E), \quad (\nu_t \tau_b^t < 1), \quad (4)$$

where $\sigma_{nc} = 2^{1/2} (0.6894) = 0.975$, the superscript "t" stands for "tokamak," or "toroidal symmetry," $\nu_t \equiv \nu/\epsilon_t$ is the detrapping frequency, and τ_b^t is the bounce time of a toroidally-trapped particle.

It is important to note that D_{nc}^t does not provide a lower bound on stellarator transport, despite the greater symmetry of a tokamak over a stellarator. Since helically-trapped particles have a bounce-time τ_b^h which

is typically small compared to τ_b^t , a helically trapped particle in a stellarator will have a banana width which is correspondingly small compared to that of its toroidally-trapped counterpart in a tokamak of like parameters. Thus, in a device where $\epsilon_h > \epsilon_t$, the total transport D can be less than D_{nc}^t , bounded below instead by the neoclassical transport of a straight stellarator ($\epsilon_t = 0 \neq \epsilon_h$),⁵

$$D_{nc}^h = \sigma_{nc} \epsilon_h^{1/2} \hat{\epsilon}_t^{-2} q_h^2 E \nu(E), \quad (\nu_h \tau_b^h < 1), \quad (5)$$

where $q_h \equiv l/m$, $\hat{\epsilon}_t \equiv r/R$, and where the configuration has m periods every distance $2\pi R$ along its axis. Numerical support for this noteworthy situation, in which the transport of a less symmetric configuration is better than that of a more symmetric one, is found in the higher-collisionality results of Ref. 2.

IV. NUMERICAL PROCEDURE

To study helical transport, the Monte Carlo code developed by Boozer and Petravic¹ to generate particle orbits in flux coordinates was equipped with diagnostics which discern when a particle is helically trapped. Thus, in addition to the total radial excursion $(\Delta r)_j$ made by the j th member of an ensemble during its run-time T , the code also obtained the radial excursion $(\Delta r_h)_j$ of particle j , counting only radial motion made while helically trapped. In this way, both the total diffusion coefficient,

$$D(\text{num}) = J^{-1} \sum_{j=1}^J D_j, \quad D_j \equiv (2T)^{-1} (\Delta r)_j^2, \quad (6)$$

and the helical contribution,

$$D_h(\text{num}) = J^{-1} \sum_{j=1}^J D_{hj}, \quad D_{hj} \equiv (2T)^{-1} (\Delta r_h)_j^2, \quad (7)$$

were computed. (Here "num" means the numerical values of D , D_h .)

Each ensemble consisted of J particles of the same energy, started at $r/a = 0.5$, and with randomly-chosen initial values of θ , ϕ , and pitch $\lambda \equiv v_{\parallel}/v$. The model collision operator,¹

$$C(f) = (\nu/2)(\partial/\partial\lambda)(1 - \lambda^2)(\partial f/\partial\lambda),$$

scatters in pitch angle only, leaving the energy of a particle constant. J was taken large enough that a standard deviation of about 20% was obtained.

Two approaches were used to compute the run time T per particle orbit, each with its strengths and weaknesses. The first approach, paralleling the method used in Ref. 1, is to take $T = T_1 \equiv 2/\nu$, inversely proportional to the collision frequency, and large compared to the "retrap time" $\tau_r = \sqrt{\epsilon_h}/\nu$, the typical time between successive helical entrapments, hence between successive steps in the random walk process. From the arguments advanced in Appendix A, one need only take T large compared to $\tau_h \equiv \epsilon_h/\nu$ (the typical time to detrapp from a helical well) in order for expressions (6) and (7) to be valid diffusion coefficients. Since $\tau_r > \tau_h$, taking $T > \tau_r$ also satisfies $T > \tau_h$, and provides some safety margin against counting nondiffusive effects on a time scale comparable to τ_h as part of the diffusive motion. In order to take the same number of radial steps per run, independent of ν , one must take $T \propto \nu^{-1}$.

The difficulty with taking $T = T_1$ is that T_1 becomes unmanageably large at small ν . The alternative, the route taken in Ref. 2, is to take T

independent of v , $T = T_2 \approx 3.75/v_R$, where the choice 3.75 is made to correspond with the 30 msec run time assumed at all v in Ref. 2. This choice makes all v equally accessible computationally. However, for particles having four times thermal energy ($x = 4$), which [as one sees in going from Eq. (3) to Eq. (2)] are the particles contributing dominantly to D , one has $T = \tau_h(E)$ at $v_T/v_R = 1/30$. If, as is done in Ref. 2, one computes the thermal conductivity χ instead of D , the dominantly-contributing particles have $x \approx 5$, and the condition $T > \tau_h(E)$ is violated at still larger v_T/v_R . From Appendix A one sees that for $T < \tau_h$, $D(\text{num})/D(\text{real})$ decreases in proportion to T/τ_h , due solely to numerical effects, as T/τ_h or v/v_R becomes smaller. Moreover, of the sample of 360 particles used in Ref. 2, only a fraction of order $e^{-5} \approx 1/150$ of these will have $x \gtrsim 5$, making the statistics of this crucial subsample questionable. For these reasons, the favorable results at low collisionality in Ref. 2 may be due in large measure to these nonphysical numerical effects.

Nevertheless, there is the regime of intermediate collisionality ($1/30 < v_T/v_R \lesssim 1$) in which $T > \tau_h$ is satisfied for $x = 4 - 5$, and yet where $D(\text{num})$ is appreciably below the theoretical value $D_h(\text{th})$. In this range, one may expect the v -dependence of the number of steps taken to distort the numerical results somewhat, but not drastically to alter the magnitude of the results. (This expectation appears to be supported by the results presented in the following section.) The remaining discrepancy in Ref. 2 in this collisionality regime may thus be due to a real physical effect. In Section VI a possible such effect is discussed.

V. NUMERICAL RESULTS

In Fig. 1 are shown the results obtained using $T = T_1$, for a device

having $\ell = 2$, $m = 6$, $\epsilon_a^{-1} = 7$, $\delta_a^{-1} = 9.43$ and a constant $q^{-1} \equiv d\theta/d\phi$ profile, $q^{-1} = 0.5$. Particles were given $E = 10^{-6}$, corresponding, for example, to hydrogen ions of atomic weight 2.5, energy 10 keV, in a device having $B_0 \approx 5$ Tesla, $a \approx 3.2$ meters. The dashed guidelines labeled $D_h(th)$ and $D_{nc}(th)$ are Eqs. (3) and (4), respectively, for the theoretical value of the helical and low- ν tokamak neoclassical contributions to transport. The solid lines connect numerically-obtained points. The bottom line [$D_{nc}(num)$] was obtained for a tokamak ($\delta_a = 0$) with otherwise identical parameters. One sees good agreement with $D_{nc}(th)$ at low ν/ν_R , bending over for $\nu \gtrsim \nu_R$, as the plateau regime is entered. The lines $D(num)$ and $D_h(num)$ show the numerical results for the total and helical transport, respectively. One sees that at lower ν/ν_R , $D \approx D_h$, as one would expect for $D_{nc} \ll D_h$. As one moves toward larger ν/ν_R , and D_{nc} becomes comparable to D_h , D_h drops below D . Thus, while $D \approx D_h + D_{nc}$ displays the weaker ν dependence observed in Ref. 1, one sees that D_h follows the predicted $1/\nu$ scaling quite well, with magnitude about 1.5 times the theoretical value.

Similar results appear in Fig. 2, for which the same device parameters were taken, but a somewhat larger energy $E = 1.57 \times 10^{-6}$, and more importantly, $T = T_2$ was used. Again, the numerical curve for D_h lies near the theoretical one, and in particular the strong inverse dependence on ν is evident. We do not regard the turnover of $D_h(num)$ toward lower ν as manifesting the onset of the plateau expected by theory²; this plateau should occur at $1 \approx \tau_h a / v_d$ (v_d is the grad-B drift velocity), hence when $\nu/\nu_R \approx \epsilon_h \epsilon_a E = 0.596 \times 10^{-2}$, about 6.2 times smaller than the lowest collisionality computed in Fig. 2. Instead, we consider this an artifact of using T_2 instead of T_1 , as discussed in the preceding section.

Figure 3 shows the corresponding results for a quite different

configuration, having $\ell = 2$, $m = 40 = \epsilon_a^{-1}$, $\delta_a^{-1} = 4$, $q^{-1} = 0.25$, with energy $E = 1.57 \times 10^{-6}$, and taking $T = T_1$. The strong inverse scaling of $D_h(\text{num})$ with v is again evident, with magnitude about twice the theoretical prediction. For these parameters, the number $n \equiv |mq - \ell|$ of helical wells encountered in following a field line once around poloidally, is quite small, $n = 2$. For this reason, the code diagnostics section has greater difficulty separating neoclassical from helical effects, particularly at higher collisionality, where rapid scattering in and out of helical wells further blurs the distinction between helical and toroidal trapping. To this we attribute the somewhat larger than predicted value of $D_h(\text{num})$ at higher collisionality, as well as the closeness, at larger v/v_R , of $D_h(\text{num})$ to $D(\text{num})$.

VI. DISCUSSION

The foregoing numerical findings are in approximate agreement with the classical theory, and also provide a reconciliation of that theory with the numerical results of Ref. 1. That is, when the theoretical expression for a monoenergetic distribution [Eq. (3)] is employed, and the helical contribution to the transport is isolated from the total transport, the two approximately agree.

The same cannot be said, however, for the results of Ref. 2. Here, some possible reasons are offered for the remaining discrepancy between those results at low collisionality, versus the present results and the theoretical expectations.

A first possibility, already mentioned in Section IV, is that the numerical difficulties associated with taking $T = T_2$, and with the small sample of energetic particles, caused $D(\text{num})$ in Ref. 2 to be much smaller than

D(real).

On the other hand, the results in Fig. 2 obtained using T_2 show that, at least for a monoenergetic distribution, the strong inverse dependence of D_h on v is manifested down to the point where $T_2 \approx \tau_r$, so the possibility must be considered that the difference between the present results and those of Ref. 2 are due, at least in part, to a real physical effect. If so, the difference between the physical situations modeled by the two codes used to obtain these differing results could clearly have major implications for stellarator design.

One possible candidate for this crucial difference is the particular models assumed for the magnetic field. In both the present work and in the classical theory, $B(r, \phi, \theta)$ was assumed to have the simple form given in Eq. (1). Here, because there is only a single helical contribution $e_h \cos(2\theta - m\phi)$, a helically trapped particle will remain helically trapped, for $v = 0$, for radial excursions comparable to those of the idealized superbanana orbits of Ref. 1. For this reason, for such simple fields, the particle detrapping time τ_h is always given by the collisional time $\tau_{hv} = r_h/v$, and the distinction between the classical superbananas, and actual collisionless orbits (for which collisionless detrapping and retrapping can occur), is unimportant.

If instead multiple helicities were present in B , however, this classical picture could change. By the two helicities adding to each other with different phases as a particle drifts in θ , the collisionless detrapping time τ_{ho} could compete with τ_{hv} as the controlling factor on the radial excursion Δr , for the particle energies considered here and in Refs. 1 and 2. Since the fields in Ref. 2 were not stipulated, but were instead computed from a realistic external coil configuration (including vertical field coils), one

may expect that field's decomposition to contain strong contributions from more than one helicity.

Detailed numerical and analytic investigation of this possibility is deferred to later work. Here, a simple estimate is made to see the potential strength of the effect. We assume two helicities of equal strength ϵ_h , and with helical phases $\eta_1 = l\theta - m_1\phi$, $\eta_2 = l\theta - m_2\phi$. Thus, the helical contribution to B is $-\epsilon_h(\cos\eta_1 + \cos\eta_2) = -2\epsilon_h \cos(l\theta - \bar{m}\phi) \cos m\phi$, where $\bar{m} \equiv (1/2)(m_1 + m_2)$, and $m \equiv (1/2)(m_1 - m_2)$. In contrast to the ridges and troughs of the single-helicity form, this perturbation has localized (in ϕ and θ) peaks and wells. Thus, a particle collisionlessly detraps after drifting a distance $\Delta\theta \lesssim \pi/4$ in θ . The collisionless detrapping time is therefore τ_{ho} :

$$\tau_{ho} = \Delta\theta/\dot{\theta} \approx (r/az)^2 (\pi/\epsilon_h E) \quad , \quad (8)$$

so that

$$\tau_{ho}/\tau_{hv} \approx [v(E)/E] (r/az\epsilon_h)^2 \quad . \quad (9)$$

For $\tau_{hv} < \tau_{ho}$, one expects $\Delta r \approx \tau_{hv} \dot{r}$, and D_h should have the $1/v$ -dependence of classical theory. For $\tau_{hv} > \tau_{ho}$, $\Delta r \approx \tau_{ho} \dot{r}$, and D_h should be independent of v (since τ_r should also be collisionlessly determined). For parameters like those in Ref. 2 ($l = 3$, $\epsilon_h = 0.015$, $E = 4T$, $\omega = 1.57 \times 10^{-6}$, $v_R = 0.6 \times 10^{-6}$), one finds $\tau_{hv} = \tau_{ho}$ for $n_R/n \approx 4.6$.

One notes that the model field assumed in Ref. 1 has, in addition to the terms in Eq. (1), a small additional term, proportional to $\cos^2\theta \cos\phi$, so that one might expect multiple-helicity effects to manifest themselves there as well. This may provide an explanation for the fact that the results there do

indeed show a small downturn of $D(\text{num})$, for the single point at lowest collisionality ($v/v_R = 0.2$) computed.

VII. CONCLUSIONS

A principal result of this work is that the classical theory of helical transport has been substantiated numerically, for magnetic fields which are well modeled by the simple form Eq. (1). Nevertheless, stellarators may still be viable confinement devices, for a number of reasons.

First, the present work has ignored the effect of a radial electric field due to the ambipolar potential ϕ , which will arise in a real device. When ϕ is large enough that a particle can make a poloidal transit due to the $E \times B$ drift in a time τ_E less than the detrapping time τ_h , that particle's radial excursion becomes limited by τ_E rather than τ_h . Assuming $e\phi = T$, one has $\tau_h/\tau_E = (a/r)T\epsilon_h/v(E)$ which, for parameters like those in Ref. 1 ($T = 10^{-6}$, $\epsilon_h = 0.93$) equals one for $v(E)/v_R \approx 1/10$. The beneficial effect of ϕ on confinement was numerically observed in Ref. 1, becoming apparent at $v/v_R \sim 1$, appreciably larger than the rough estimate just given.

Second, by building stellarators with large aspect ratio, and by operating in a regime of higher-collisionality (density), the higher- v results of Ref. 2, as well as theory, indicate that better confinement can be obtained than for a tokamak of like parameters.

Finally, by appropriate tailoring of the magnetic field, either by the introduction of multiple helicity of comparable magnitude, or perhaps by the attainment of fields which are nearly omnigenous locally,⁶ one may be able to achieve favorable transport in the lower-collisionality regime as well. This would be particularly crucial for the confinement of α and beam-injected particles.

ACKNOWLEDGMENTS

I am grateful to Rob Goldston, Goietta Kuo-Petravic, Russell Kulsrud, Carl Oberman, and especially to Allen Boozer for numerous enlightening discussions.

This work was supported by the United States Department of Energy Contract No. DE-AC02-76-CH03073.

REFERENCES

1. A. H. Boozer, G. Kuo-Petravic, PPPL Report-1703 (1980), (submitted to Phys. Fluids).
2. R. C. Potok, P. A. Politzer, L. M. Lidsky, Phys. Rev. Lett. 45, 1328 (1980).
3. A. A. Galeev, R. Z. Sagdeev, H. P. Furth, M. N. Rosenbluth, Phys. Rev. Lett. 22, 511 (1969).
4. J. W. Connor, R. J. Hastie, Phys. Fluids 17, 114 (1974).
5. A. Pytte, A. H. Boozer, Phys. Fluids 24, 88 (1981).
6. P. J. Catto, R. D. Hazeltine, SAI Report UC-20g (1981).

APPENDIX A. On the Scaling of $D(\text{num})$ With T

Here we consider how the ratio $D(\text{num})/D(\text{real})$ of the numerically obtained to the true diffusion coefficient may be expected to vary, as the run time T used for each particle in the ensemble is varied, for orbits of the sort considered here.

One such orbit, computed for $v/v_R = 1/3$, $E = 10^{-6}$, is shown in Figs. 4(a) and 4(b). In Fig. 4(a), θ is plotted versus t . Times when the particle is helically trapped are characterized by small, rapidly oscillatory excursions in θ . During these times, one sees in Fig. 4(b) that large steps are taken in the radial variable $\psi(t)$ or $r(t)$.

For simplicity, we idealize this process, by dividing time into intervals of length τ_h , during which a particle's radial velocity $\dot{r}$ is assumed constant. Thus, in a fraction $f_h/2$ of the intervals, a particle has $\dot{r} = \Delta_h/\tau_h$, in a fraction $f_h/2$, it has $\dot{r} = -\Delta_h/\tau_h$ and in a fraction $1 - f_h$, it has $\dot{r} = 0$ (untrapped). For $T \gg \tau_h$, there are more than one interval, and so one finds

$$D(\text{num}) = \langle [\Delta r(T)]^2 \rangle / 2T = (1/2) f_h \Delta_h^2 / \tau_h = D_0. \quad (\text{A1})$$

The fact that D_0 is independent of T indicates that T is large enough that a truly diffusive process is being considered, so we identify D_0 with $D(\text{real})$.

For $T < \tau_h$, only a fraction T/τ_h of a single interval occurs for each particle in the ensemble, so the radial motion for each particle is of a convective, nondiffusive nature, with step size $\Delta r(T) = \dot{r}T$. Therefore,

$$D(\text{num}) = \langle (\dot{r}T)^2 \rangle / 2T = \langle \dot{r}^2 \rangle T / 2 = D(\text{real})(T/\tau_h). \quad (\text{A2})$$

Here, $D(\text{num}) \propto T$, decreasing linearly with T from $D(\text{real})$ at $T = \tau_h$. Thus, transport coefficients computed for too short a run time will indicate more favorable confinement than is physically valid, as is argued in Section IV.

FIGURE CAPTIONS

Fig. 1. Diffusion coefficient D versus collisionality for a device with $l = 2$, $m = 6$, $\epsilon_a^{-1} = 7$, $\delta_a^{-1} = 9.43$, $q^{-1} = 0.5$, for particles with energy $E = 10^{-6}$, and using run time $T_1 \equiv 2/\nu$. The approximate agreement between the theoretical and numerical values for the helical contribution D_h to D is evident.

Fig. 2. Same as Fig. 1, but using $E = 1.57 \times 10^{-6}$, and run time $T_2 = 3.75/\nu_R$, independent of collisionality.

Fig. 3. Same as Figs. 1 and 2, but using $l = 2$, $m = 40 = \epsilon_a^{-1}$, $\delta_a^{-1} = 4$, $q^{-1} = 0.25$, $E = 1.57 \times 10^{-6}$, and run time T_1 .

Fig. 4. Plot of the orbit of a typical particle versus time, with the same parameters as in Fig. 1, and at collisionality $\nu = \nu_R/3$. Portions when the particle is helically trapped are characterized by small, rapid oscillations in poloidal angle $\theta(t)$ [Fig. 4(a)], and large steps in radial variable $\psi(t)$ [Fig. 4(b)]. These large, randomly directed radial steps are the origin of the helical contribution D_h to the transport.

8IT0090

Figure 1

81T0091

Figure 2

81T0092

Figure 3

B10089

Figure 4