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Conf-830420--1

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LA-UR--83-512

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TITLE: MULTIPLE-SCATTERING CORRECTIONS TO THE BEER-LAMBERT LAW

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SUBMITTED TO: Society of Photo-Optical Instrumentation Engineers, Arlington, VA, April 4-8, 1983.

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FORM NO 836 R4
ST NO 2629 5/81

Multiple-scattering corrections to the Beer-Lambert law

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Abstract

The effect of multiple scattering on the validity of the Beer-Lambert law is discussed for a wide range of particle-size parameters and optical depths. To predict the amount of received radiant power, appropriate correction terms are introduced. For particles larger than or comparable to the wavelength of radiation, the small-angle approximation is adequate; whereas for small densely packed particles, the diffusion theory is advantageously employed. These two approaches are used in the context of the problem of laser-beam propagation in a dense aerosol medium. In addition, preliminary results obtained by using a two-dimensional finite-element discrete-ordinates transport code are described. Multiple-scattering effects for laser propagation in fog, cloud, rain, and aerosol cloud are modeled.

Introduction

Electro-optical and millimeter wave devices incorporated into modern weapon systems are strongly influenced by the battlefield environmental factors which can seriously degrade the system performance. When the amount of an obscurant is relatively low (the optical depth of the medium $\tau<<1$), the dominant sources of degradation include scattering, absorption, turbulence and thermal gradients. Scattering in the atmosphere, however, is actually a multiple scattering process in which the radiation scattered by one element can be scattered again by another element. As shown by Bugnolo[1], even for unit optical thickness there is a significant probability of triple scattering in the forward direction.

To predict the amount of radiant energy available from a beam, the simple Beer-Lambert (B/L) law of exponential attenuation is often used. It is not unusual, as stressed by Tam[2], to see that the B/L law is used without proper justification both for the interpretation of experimental results and the application of transmission data to system evaluation studies. Experimental[3] and theoretical[4,5] investigations were devoted to study the limits of applicability of the B/L law for describing the attenuation of radiation in scattering media.

The purpose of this paper is to describe the multiple scattering of a laser beam within the framework of the radiative transfer theory. Exact solutions to the equation of transfer have not been obtained to date. There are some special cases, however, where simple and useful approximate solutions are available. When the typical size of scattering particles is greater than or comparable to the wavelength, the wave scattered by a particle is largely confined within a small angle in the forward direction and, therefore, by employing the small-angle approximation it is possible to simplify the equation of transfer. This was applied successfully to scattering of a laser beam by hazes, fogs, and clouds[6-8]. For tenuous distribution of particles, the first-order multiple scattering theory can be used, and for dense distribution, when the volume fraction is much greater than 1%, the diffusion approximation is appropriate. When the particles are small compared with the wavelength of radiation the diffusion process is dominant for optical distances greater than unity[9]. The most critical (and difficult) case emerges when the first-order multiple scattering approximation breaks down ($\tau\gtrsim1$) and the particle size falls into an intermediate range neither covered by the diffusion nor the small-angle approximations. It is in this intermediate case that the complexity of the equation of transfer forces one to implement numerical methods of solution.

Small-angle approximation

Under the small-angle approximation and with the z axis chosen along the direction of the incident beam, the equation of transfer satisfied by the radiance (specific intensity) distribution function $I(\underline{\phi},\underline{r},z)$ reads:

$$(\underline{\phi}\cdot\frac{\partial}{\partial\underline{r}} + \frac{\partial}{\partial z} + \sigma)\, I(\underline{\phi},\underline{r},z) = \int p(\underline{\phi}-\underline{\phi}')I(\underline{\phi}',\underline{r},z)d\underline{\phi}, \tag{1}$$

where $I(\underline{\phi},\underline{r},z)$ is the spectral radiance function of a monochromatic beam wave at a point with coordinates $(\underline{r},z)$ along a direction whose unit directional vector has a projection $\underline{\phi}$ transverse to the beam axis taken as the z axis. The volume extinction coefficient is σ, and the normalization condition chosen for the scattering phase function $p(\underline{\phi})$ is such that the integration over the angle variables yields the volume scattering coefficient σ_s, i.e.,

$$\int p(\underline{\phi})d\underline{\phi} = \sigma_s \tag{2}$$

We assume that the incident laser beam can adequately be represented by a Gaussian functional form, both for the spatial distribution and the angular divergence; i.e.

$$I_o(\underline{\phi},\underline{r},z = 0) = \frac{\beta^2\gamma^2}{\pi^2} \exp(-\beta^2\underline{\phi}^2-\gamma^2\underline{r}^2) \tag{3}$$

and that the phase function is

$$p(\underline{\phi}) = \frac{\sigma_s^2\alpha^2}{\pi} \exp(-\alpha^2\underline{\phi}^2) \tag{4}$$

With the help of the Fourier transform technique the solution to Eqs. (1-4) can be written in the form of a series expansion in powers of σ_s, cf. Ref. 4. This method is most appropriate to treat the case of an open detector. On the other hand, the finite field of view effects are conveniently quantified when the approach due to Dolin[10] and Fante[11] is employed.

Open detector

In the small-angle approximation, the spectral irradiance $N(\underline{r},z)$ in forward direction, defined by the relation

$$N(\underline{r},z) = \int I(\underline{\phi},\underline{r},z)d\underline{\phi} \quad , \tag{5}$$

leads to the expression for received power

$$P(z,\sigma) = 2\pi \int_0^R N(\underline{r},z)rdr \tag{6}$$

This is the power collected by an open detector of radius R located on the beam axis at a distance z from the source. If we choose the power measured in a vacuum, $P(z,\sigma = 0)$, as a reference power level, $P(z,\sigma)$ can be written as

$$P(z,\sigma) = e^{-\sigma z} [1 + C(z,\sigma)] P(z,\sigma = 0) \tag{7}$$

which defines the correction factor $C(z,\sigma)$. Explicit formulas allowing one to compute $C(z,\sigma)$ are contained in Ref. 4. As an illustration, we show in Fig. 1 the correction factor as a function of optical depth for the advective fog model described by Shettle and Fenn[12]. The different curves in Fig. 1 correspond to the number of terms retained in the truncated power series describing the correction factor. It is evident that the number of scattering events increases with increasing optical depth.

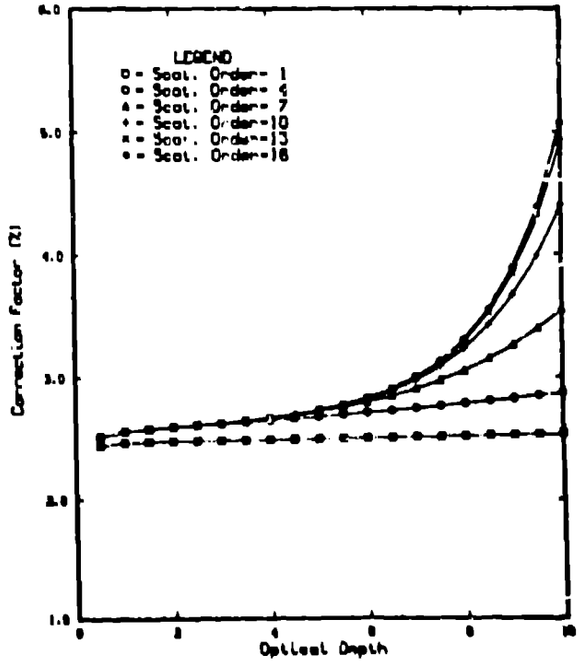

Figure 1. Correction factor for advective fog; $\sigma = 2.5\ km^{-1}$, $\gamma = 0.35\ cm^{-1}$, R = 2 cm. Wavelength $\lambda = 0.55\ \mu m$. Beam divergence $\beta = 2\pi/(\gamma\cdot\lambda)$, $\alpha = 143.9\ rad^{-1}$. Single scattering albedo $\omega = 0.9999$.

When the detector is moved to the source plane what is registered is $P(z = 0,\sigma)$. With this reference power level we define the amplification factor, $A(z,\sigma)$, through the equation

$$P(z,\sigma) = e^{-\sigma z} A(z,\sigma) P(z = 0,\sigma) \tag{8}$$

For a collimated beam, when $\beta \to \infty$, the following simple relation holds

$$A(z,\sigma) = 1 + C(z,\sigma) \tag{9}$$

which shows that the presence of forward scattering has increased the detected power by the factor $A = 1 + C$. In a general case of the beam characterized by a finite value of the parameter β, the quantities $A(z,\sigma)$ and $C(z,\sigma)$ include both the beam divergence and multiple scattering corrections to the B/L law. Figures 2 and 3 show the amplification factor for the advective fog and Deirmendjian's cloud model C1[13].

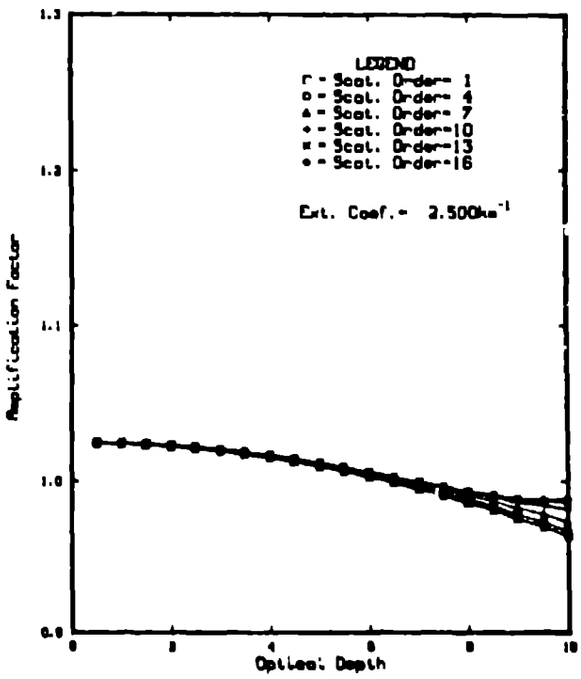

Figure 2. Amplification factor for advective fog; $\gamma = 0.35\ cm^{-1}$, $R = 2$ cm, $\lambda = 0.55\ \mu m$, $\sigma = 143.9\ rad^{-1}$, $\omega = 0.9999$.

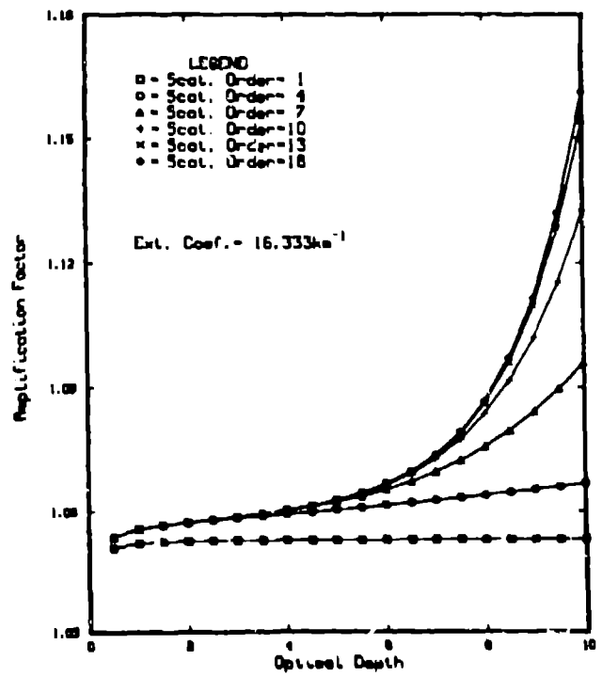

Figure 3. Amplification factor for water cloud C1; $\gamma = 0.35\ cm^{-1}$, $R = 2$ cm, $\lambda = 0.45\ \mu m$, $\sigma = 47.08\ rad^{-1}$, $\omega = 1$.

Detector with a variable field of view

In the work of Fante[11] which applies to sharply peaked phase functions, $I(\underline{\phi}',\underline{r},z)$ in the integrand on the right-hand side of Eq.(1) is expanded in a Taylor series about $\underline{\phi}'=\underline{\phi}$. After truncating the series at the second term, a system of two equations for the unscattered (reduced) radiance $I^{(0)}$ and the scattered (diffuse) portion $I^{(s)}$ is obtained. This theory leads to the expression for $I^{(s)}$ in a finite form of a one-dimensional integral. As the inclusion of the field of view effects requires integrations over both the detector area and the receiving angle, Fante's formulation seems to be most practical. To describe the multiple scattering corrections to the B/L law, one can introduce various transmission functions as was done in Ref. 4. Here, however, we display the multiple scattering effects by evaluating the received power as a function of the detector's field of view. For the advective fog model discussed in Ref. 7, we show, in Fig. 4, the received power in dimensionless units by dividing, for a fixed optical depth, the reduced, diffuse or total power by the largest value of the total power. This choice of normalization enables one to read off directly the percentage deviation from the B/L law as expressed by the ratio of the scattered and total received power.

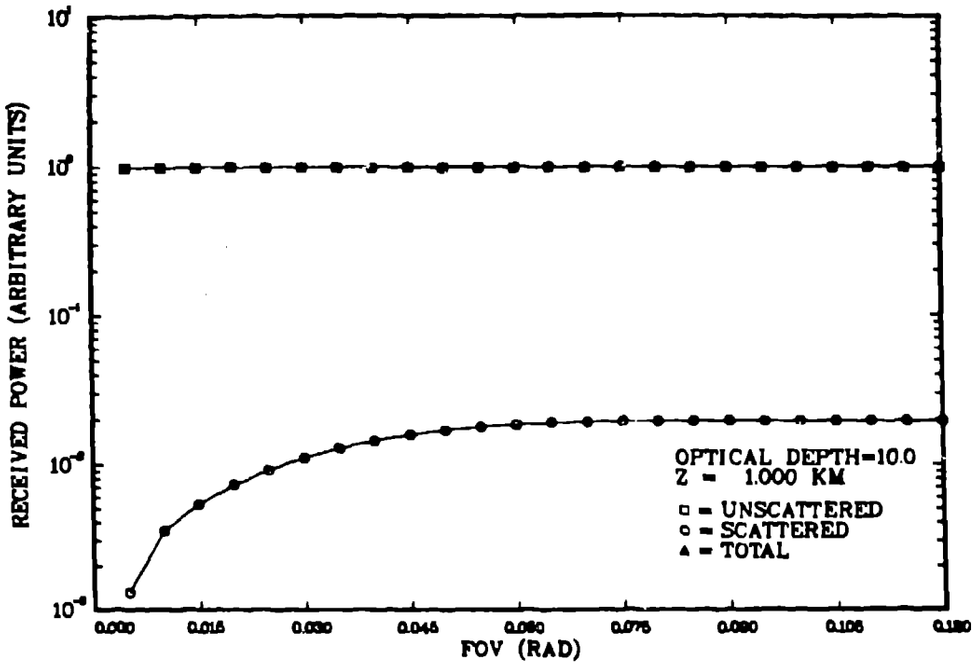

Figure 4. Received power for advective fog; $\lambda = 1.06$ μm, $\gamma = 0.28$ cm^{-1}, R = 1 cm, $\alpha = 40.41$ rad^{-1}. $\omega = 1$.

A striking feature of Fig. 4 is a rapid saturation of the scattered power as the detector's field of view exceeds 0.045 rad (2.5 deg). This result implies that except when the receiver's field of view is very small the calculations referring to the open-detector case can safely be employed. In Figs. 5 and 6, the received power is shown for rain particles at wavelength $\lambda = 1.06$ μm. In this extreme case of large (millimeter) particles, characterized by the value of $\alpha = 2648.9$ rad^{-1}, the diffuse power dominates over the reduced power for optical depth $\tau = 4$.

Diffusion approximation

The diffusion approximation, which corresponds to the lowest-order truncation in the spherical harmonics expansion of the radiance distribution function, was employed by Tam and Zardecki[14] to examine the role of non-small-angle scattering for off-axis beam propagation. More specifically, if the diffuse part of the radiance is written in the form

$$I_d(\underline{s},\underline{r},z) = \frac{1}{4\pi}\left[(\rho(\underline{r},z) + 3\underline{s}\cdot\underline{J}(\underline{r},z)\right] \quad , \tag{10}$$

where $\underline{s}$ is a unit direction vector, the function $\rho(\underline{r},z)$, proportional to the energy density, satisfies a diffusion equation:

$$\nabla^2\rho - \kappa^2\rho = -\frac{1}{D}\, q_0 + 3\underline{\nabla}\cdot\underline{q}_1 \tag{11}$$

In Eq. (11), the source terms q_0 and $\underline{q}_1$ are determined by the known distribution of the reduced radiance; D is the diffusion coefficient given in terms of the absorption coefficient σ_a, the mean value of the scattering angle $\langle\mu\rangle$ and the filling factor[9] H as

$$D = \{3[\sigma_s(1 - H)(1 - \langle\mu\rangle) + \sigma_a]\}^{-1} \tag{12}$$

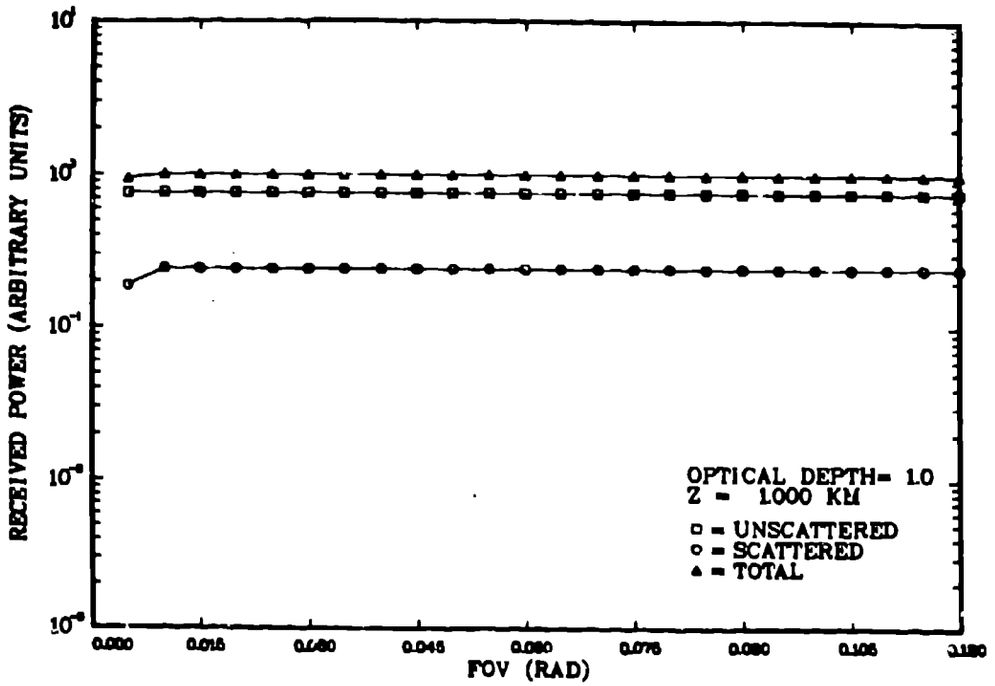

Figure 5. Received power for rain; $\lambda = 1.06$ μm, $\gamma = 2.8$ cm^{-1}, R = 0.25 cm. $\omega = 0.9848$.

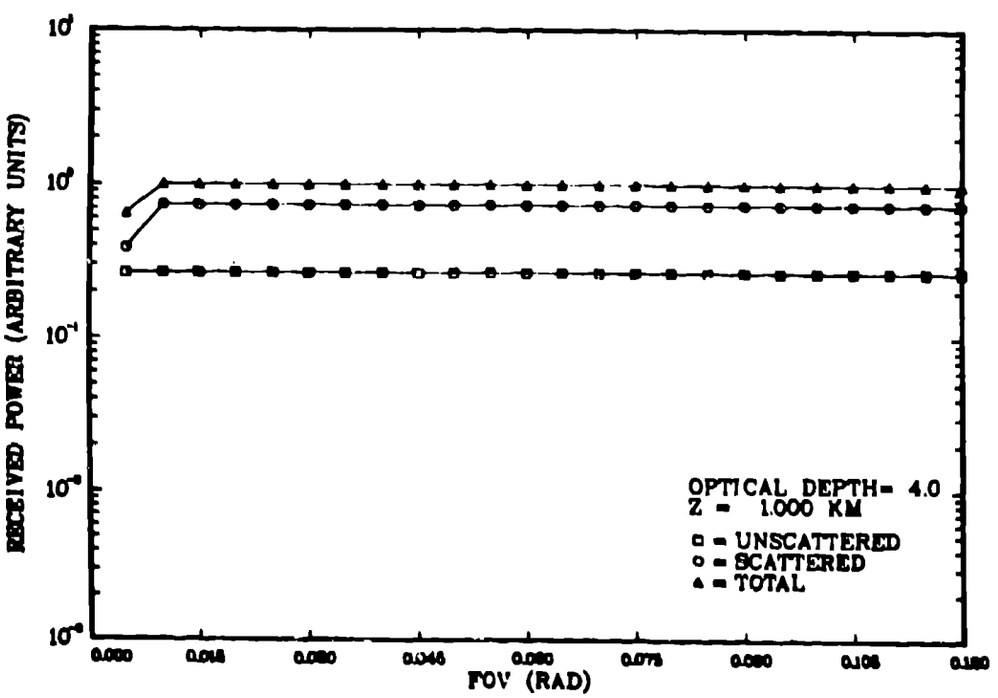

Figure 6. Received power for rain; $\lambda = 1.06$ μm, $\gamma = 2.8$ cm^{-1}, R = 0.25 cm. For optical depth of 4, the scattered power dominates over the unscattered power. $\omega = 0.9848$.

Finally,

$$\kappa = (\sigma_a/D)^{1/2} \tag{13}$$

Equation (11) should be solved with the boundary condition demanding that the total diffuse flux directed inward the scattering boundary be zero. The radiant flux $\underline{J}$ in Eq. (10) is then determined through the equation

$$\underline{J}(\underline{r},z) = D[3\underline{q}_1(\underline{r},z) - \underline{\nabla}\rho(\underline{r},z)] \tag{14}$$

We have applied the diffusion approximation to analyze the propagation of a collimated laser beam incident normally upon a slab containing smoke particles. The White Phosphorous (WP) smoke characteristics were generated with the aid of the AGAUSX[15] computer code, as developed by the Atmospheric Sciences Laboratory. Figure 7 shows the amplification factor as a function of optical depth for two smoke concentrations.

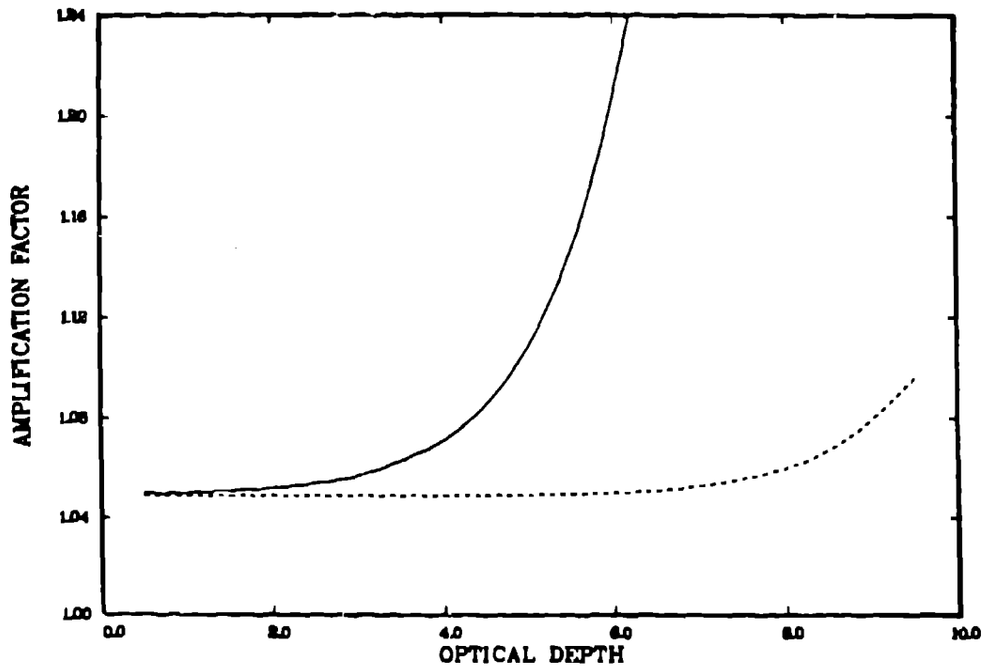

Figure 7. Amplification factor for WP smoke; $\lambda = 1.06$ μm, $\gamma = 1.4$ cm^{-1}, $R = 1$ cm, $\langle\mu\rangle = 0.6620$, $H = 0$. The continuous line refers to extinction coefficient $\sigma = 1.76 \times 10^3$ km^{-1} and the dashed line refers to $\sigma = 1.76 \times 10^2$ km^{-1}. Single scattering albedo: 0.9993. Open detector.

Since the scattered radiation is spread over wide angles when the diffusion approximation applies, the significance of multiple scattering will tend to become smaller and smaller as the detector is moved away from the boundary of the medium. This should be contrasted with the small-angle scattering case where the radiation is confined to narrow angles and the multiple scattering corrections are essentially insensitive to the detector's location with respect to the medium boundary.

Discrete-ordinates procedure

The most direct method to solve numerically the equation of transfer is the discrete-ordinates approach in which the dependent variable $I(\underline{s},\underline{r},z)$ is replaced by a discrete set of values at a discrete set of points $I(\underline{s}_i,\underline{r}_j,z_k)$. The derivatives and integrals in the transfer equation must also be replaced by a corresponding discrete representation. In this way one arrives at a set of algebraic equations for the discrete representation of the dependent variable.

In this section, we describe the results pertaining to a simple scenario in which the scattering medium is formed by a homogeneous distribution of 99% relative humidity rural aerosol[12]. The computations have been performed by adapting the two-dimensional general purpose transport code TWOTRAN[16] (originally designed to solve neutron and gamma-ray transport problems) to analyze the propagation of a laser beam for a more complex geometric configuration[17]. In cylindrical r-z coordinates, in the plane $z_0 = 3$ km, a collimated laser beam has the following radiance distribution function:

$$I(\underline{s},\underline{r},z) = \frac{F}{\pi w^2} \exp(-\underline{r}^2/w^2)\, \delta(\underline{s} - \underline{z}) \tag{15}$$

where F = 1 MW is the total power emitted, w = 0.5 m is the half-width of the spatial beam profile, and δ denotes the Dirac delta function. An isotropically reflecting target with the reflectivity coefficient of 0.5 is described by the inequalities $0 \leq r \leq 0.002$ km and $4.0 \leq z \leq 4.006$ km. The target thus generates a diffuse radiation field in the r-z plane. We choose the concentration of the aerosol particles to be given by the number density of 50,000 particles per cm^3. This corresponds to the extinction coefficient $\sigma = 1.32 km^{-1}$ at the wavelength $\lambda = 1.06$ μm. The visual range, inversely proportional to the extinction coefficient at $\lambda = 0.55$ μm, is V = 1.52 km.

Slightly modifying our previous discussion, we introduce a correction factor C' for the isotropically emitted radiation. C' is entirely due to multiple scattering. As the target reflects isotropically, the reduced radiance at a distance ℓ from the target is

$$I_r = \frac{P_o \exp(-\sigma\ell)}{4\pi\ell^2} \tag{16}$$

where P_o is the power emitted by the target. The total radiance, a sum of the reduced and diffuse parts, is written in terms of C' as

$$I = \frac{P_o \exp(-\sigma\ell)}{4\pi\ell^2} (1 + C') \tag{17}$$

Table 1 lists both the received radiance and the correction factors when a detector is moving along a line parallel to the z-axis at r = 500 m pointing toward the target.

Table 1. TWOTRAN code results in r-z geometry for radiance and correction factor. Target coordinates r = 0.0 km, z = 4.000 km. Detector height r = 0.500 km. Visual range V = 1.52 km.

Detector's z Coordinate (km)	Optical Depth	Radiance ($W.m^{-2}.sr^{-1}$)	Beer/Lambert Correction Factor P5	Delta-4
1.398	3.496	2.6520×10^{-3}	1.66	2.78
2.898	1.596	3.8658×10^{-2}	0.85	1.92
3.247	1.193	2.5389×10^{-1}	0.69	0.82
3.437	0.993	2.2313×10^{-1}	0.55	0.65
3.571	0.869	1.6951×10^{-1}	0.50	0.61
3.682	0.782	2.2435×10^{-1}	0.36	0.44
3.787	0.717	2.2986×10^{-1}	0.35	0.41

As the TWOTRAN code requires the coefficients of the spherical harmonics expansion of the phase function, we have used the P5 expansion (five terms) and then checked the consistency of this truncation with the aid of the delta-4 approximation[16]. A more detailed account of this procedure will be published elsewhere[17].

Conclusions

In this paper, we have discussed the limits of applicability of Beer-Lambert's law and the extensions of its range of usefulness in discrete random media, such as fog, rain, and smoke. The essential parameters, which determine a method of solution of the multiple scattering problem, are the optical depth and the particle size parameter. We have shown that in several limiting cases the radiative transfer equation yields simple approximate solutions. Modern neutron transport techniques can define the limits of applicability of the approximate solutions and can be of practical value when the various approximations break down. As the discrete-ordinates transport codes can solve problems with more involved geometries and nonhomogeneities, their advantages should be further explored.

Acknowledgement

The author is grateful to Dr. S. A. W. Gerstl for helpful discussions and comments.

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