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The LASL Lens Design Procedure

Simple, Fast, Precise, Versatile

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THE LASL LENS DESIGN PROCEDURE: SIMPLE, FAST, PRECISE, VERSATILE

by

Berlyn Brixner

ABSTRACT

The Los Alamos Scientific Laboratory general-purpose lens design procedure optimizes specific lens prescriptions to obtain the smallest possible image spots and therefore near-spherical wave fronts of light converging on all images in the field of view. Optical image errors are analyzed in much the same way that they are measured on the optical bench. This lens design method is made possible by using the full capabilities of large electronic computers. First, the performance of the whole lens is sampled with many precisely traced skew rays. Next, lens performance is analyzed with spot diagrams generated by the many rays. Third, lens performance is optimized with a least squares system aimed at reducing all image errors to zero. This statistical approach to lens design uses skew rays and precisely measured ray deviations from ideal image points to achieve greater accuracy than was possible with the classical procedure, which is based on approximate expressions derived from simplified ray traces developed for pencil-and-paper calculation.

1. INTRODUCTION

The Los Alamos Scientific Laboratory (LASL) lens design code has been made much faster since 1962, and present computers are also much faster than 1962 computers. In addition, we have learned that at each stage of lens improvement we are moving in the direction of diffraction-limited performance. This improvement is obtained when the error function includes only those performance errors that improve imaging as they approach zero. Of course, a large computer is needed to process the comprehensive sampling of ray deviations, the flexible weighting of errors, and a least squares minimizing system.

The problem of designing optical lens systems can be divided into four main areas: the specification of

the lens performance requirements, the selection of the starting lens, the procedure for optimizing the particular lens to meet the performance requirements, and the evaluation of the optimized design to find out whether the desired performance was obtained and whether the lens is suitable for manufacture. Lens design computer codes optimize lens performance; people make the other decisions. Most of this report describes the LASL optimization procedure, which is much simpler, more direct, and more precise than the century-old procedure still used by the optical profession.

The LASL general-purpose lens design computer program solves a lens design problem by analyzing optical image errors in much the same way that these errors are measured visually on the optical bench. This analysis is accomplished by providing a

relatively simple and quick method of tracing large numbers of specified skew rays, of determining where the rays are going, of specifying where they should go, and, finally, of getting them to go where they should go. These results are obtained by lens performance optimization directed by the LASL single-number least squares error function. The good results from this procedure are obtained by including in the error function only those performance errors that continually improve the imaging as they approach zero. These performance errors are discovered by, first, multiple skew ray tracing to secure a comprehensive sampling of image spots and, second, a statistical evaluation of the image-spot sizes and positions. Some types of error, such as distortion, may impair imaging if they are included in the error function. Root-mean-square (rms) averaging is used for the statistical evaluation of performance errors obtained from many-ray data, for example, the size of an image spot generated by a bundle of rays. Individual weighting of each type of imaging error enables a designer to emphasize, to balance, or to neglect selected lens performance characteristics to suit the needs of each imaging problem. This image analysis by statistical methods is the foundation of the LASL program.

The lens designing discussed here refers mainly to compound optical systems consisting of combinations of several simple lenses and/or mirrors. The function of a lens, of course, is to produce an image, the optical counterpart of an object. However, the images produced by both simple and compound lenses generally lack a fine focus at *all* parts of the image. To obtain a fine focus, the light wave fronts converging on all image points should be spherical. This means that the optical paths from each object point to its image point should all be equal within a small fraction of a wavelength of light.¹ Expressed another way, every ray of light entering the lens from an object point should go to its image point. The LASL procedure works directly toward the achievement of this result, as is clearly demonstrated by a recent study.² Achieving a fine focus also achieves high-contrast images because less light spreads to adjacent image points. Numerous papers and reports have been written about the LASL program²⁻¹⁷ and about the lens design problems that were solved.¹⁸⁻⁵¹ A brief history of the program development is given in Appendix A.

II. WAVE FRONT OPTIMIZATION

The LASL lens design procedure works to obtain the smallest possible image spots and therefore near-spherical wave fronts of light converging on all the image points in the field of view, which is what lens design is all about. At first, it is not obvious that the LASL procedure does in fact move steadily in the direction of generating spherical wave fronts. The proof of this capability was only recently presented² and the evidence will be repeated here. In a spherical wave front all optical paths from object point to image point will be equal, which means that the optical path differences (OPDs) will be zero. The LASL lens design procedure³ minimizes the lateral deviations of the rays from their ideal image points. This procedure also minimizes the OPDs. The results given here show that there is a linear relationship between the rms image spot size and the rms OPD and that the direction is toward zero.

Figure 1 shows a plot of the rms image spot sizes and the corresponding rms OPDs of images calculated for prescriptions selected from a LASL lens optimization run. The graph indicates an rms-OPD/rms-spot ratio of ~ 0.02 during the run that optimized the lens at $f/4.3$ effective relative aperture. Note that the direction of the linear plot is steadily toward zero as the iterations progress. Progress is fast during the early stages of the 40-iteration run

Fig. 1.
Plot of rms spot sizes and rms OPDs of images calculated for prescriptions selected from a LASL lens optimization run.

and slows markedly toward the end, but the direction is always toward zero. The small deviations from the linear decrease may be caused by variations in the competitive relation of the plotted image point to the other seven image points as the least squares error reduction progresses.

Figure 2 shows the diffraction modulation transfer functions (DMTFs) calculated for prescriptions selected from the same 40-iteration run used for Figure 1. Steady progress of the curves toward the DMTF is clearly indicated. At iteration 40, the curve is slightly below the theoretical limit, possibly because the prescription that was generated by 6-ray bundles is evaluated by a many-ray analysis on the focal plane selected by the least squares system for all eight image spots. It is remarkable that the LASL optimization procedure can discover a near-perfect complex lens design when using only 6-ray image sampling. Examination of a wave aberration map shows the wave front deviations to be well below $1/4 \lambda$ except for a narrow marginal zone occupying about 1% of the pupil area. The rms OPD is 0.034λ , much smaller than the 0.28λ extreme range.

The data for the OPD and DMTF plots were generated by the ACCOS V code, using prescriptions from the best copy-lens search run described in Sec. VII. Those prescriptions were selected from the 40-iteration design run that used eight sets of 6-ray bundles, two colors from each of four object points. A representative set of those images was used to obtain the data presented here, the e -light images from the 2° off-axis object point. Similar results are obtained from each of the other seven sets of images.

Fig. 2.

Diffraction MTFs calculated for prescriptions selected from a LASL lens optimization run.

III. LASL LENS DESIGN PROCEDURE

The LASL lens design procedure is made possible by the availability of large electronic computers, which enable us to analyze, statistically and simultaneously, many bundles of skew rays traced through the lens to form geometric images. Rays are geometric lines that simulate the propagation of light through the lens. Each ray samples the focusing ability of the part of the lens through which it passes. Bundles of rays are used so that the focusing ability of all parts of the lens will be sampled. The bundles of rays from each object point intersect the focal surface to form a spot that represents the optical image. The smaller the spot the better the image. The ultimate image quality, called diffraction-limited resolution, is obtained when the spots are so small that the optical paths of the rays from object to image are all equal within a small fraction of a wavelength of light.¹ When the optical paths of the rays in the image-forming bundle are all nearly equal, a smooth, near-spherical, concave wave front of light will be converging on the image point. Although most lens designs cannot be improved enough to give diffraction-limited performance, the LASL procedure achieves the nearest approach to that condition and therefore yields the best lens for most purposes.

Our computer procedure works quickly and precisely to obtain the smallest image spots from the particular lens type that is being studied. Image-spot size is directly proportional to the optical path differences in the ray bundle² and, therefore, our procedure leads to the most nearly spherical wave fronts at all points in the field of view of the lens. This image-quality improvement, called optimization, is obtained by evaluating the individual ray deviations (image errors) from their ideal image points on the focal surface and minimizing those deviations by varying the lens specifications, called parameters, in a least squares system of mathematical analysis. The parameters to be varied are the curvatures of the surfaces, the thicknesses of the simple lenses, the distances between the simple lenses or mirrors, the distance to the focal surface, the distance to the entrance pupil, and the size of the entrance pupil. The entrance pupil is the aperture through which the light appears to enter the lens. Infrequently used variable parameters are the

conic eccentricity that specifies any conic surface, the polynomial coefficients that specify an aspheric surface, the translation and tilt numbers that specify an off-axis optical surface, and the refractive indexes of the glass. At each iteration of the least squares system, the optical path differences of the rays in each image spot become proportionately smaller as the lateral deviations become smaller. Because the least squares analysis is nonlinear, the optimization cycle is repeated until the lens performance gains are insignificant. These are the main features of the LASL lens design program, which is simple, fast, and precise in its calculation capabilities.

While improving the image quality, the LASL procedure automatically minimizes and balances the seven primary optical aberrations at the same time that the lateral deviations in the image spots are minimized. All seven aberrations (spherical, coma, astigmatism, field curvature, longitudinal chromatic, distortion, and lateral chromatic) are minimized simultaneously because all cause lateral deviations of the rays from their ideal image points on the desired focal surface. The aberrations are balanced as far as possible because large aberrations cause large lateral deviations and therefore become prime targets for the least squares minimizing system. See Appendix B for additional information about these relationships.

Both old and new computation procedures for lens designing can work only with a specific lens prescription. They can calculate the optical performance indicators for the specified lens, and can improve performance by suggesting changes of the prescription parameters, or can even change the parameters automatically. But the selection of a suitable starting lens prescription depends on the knowledge, experience, and skill of the person directing the search for the best lens. And the designing procedure can change the starting lens parameters only to obtain the best performance of that particular lens for a specific use. However, a designing procedure that is simple, fast, and precise makes it possible to explore new regions of lens construction that have not yet been investigated in detail. Exploration of these new regions has proved fruitful at LASL.¹⁹⁻⁵¹

IV. HOW THE PROGRAM WORKS

The *size* and *position* of the *spot diagram* are the main sources of information for lens performance analysis and optimization by the LASL lens design program. The *size* and *direction* of the *cone of rays* that generate the spot diagram are additional sources of information. Rays are traced accurately through the lens because there is a close correlation between the behavior of geometric rays and the focusing of light by the lens. The spot diagram is the geometric image or pattern of points formed by the intersection of a bundle of rays traced from the object point, through the lens, and to the focal surface of interest, usually a plane perpendicular to the optical axis. Spot-diagram images are used because they closely resemble actual images of point light sources. These spot diagrams are used to make the error function, which has performance targets that coincide with the preferences of the designer and the user. The aim of our system is to make all rays coincide with their specified target positions. In this way we try to reduce the image errors to zero. The *error* is simply the distance between the ray and its target point. Because many rays are traced from each object point, an rms average is calculated to find the average ray error. Each error in the position of the ray is multiplied by an assigned weight, which determines the relative importance of that error in the performance of the lens as a whole. The sum of all squared weighted ray errors is the *error function*, which measures the failure of the lens to make perfect images—after the defects have been balanced by weighting to best fit the user's needs. To improve lens performance, the error function is driven towards zero by a least squares system that is activated by making lens parameter changes. The lens *parameters* are the numbers that specify the lens prescription. The performance of the improved lens is measured by the remaining ray errors, all of which will usually be greater than zero because lens systems are generally imperfect imaging devices. Because of the inherent imaging difficulty, some of the remaining ray errors are usually large enough to cause detectable image deterioration, even for the most important performance characteristics.

Because spot-diagram images closely resemble the actual images of point light sources, the spot-diagram errors are analyzed by the LASL program in much the same way that they are measured visually on the optical bench. To explain our method of measuring the ray errors, we first define the unusual meanings of technical terms used in this report.

Figure 3 shows the *focus*, which is the image spot diagram on the focal plane of interest, at lower left. The *best focus* is found by moving the focal plane of interest to the beam waist, the region of smallest size along the ray bundle. That spot diagram is illustrated at the right.

Figure 4 shows how the *image size* is found by measuring the rms radius of the rays in the spot from its *centroid*. The centroid is the mean position of all the rays in the spot-diagram image. Note that 70% of the rays are inside the rms radius circle.

Figure 5 shows how the *image position* is found by measuring from the optical axis to the position of the spot's centroid on the focal plane of interest.

Figure 6 shows how the *distortion* is found by measuring the variation of the centroid position from the position required by rectilinear geometry or by other considerations.

Figure 7 shows how the *lateral chromatism* is found by measuring the lateral variation of the centroid positions over the selected spectral range.

Figure 8 shows how the *longitudinal chromatism* is found by measuring the longitudinal variation of the smallest focal-spot positions over the selected spectral range.

Figure 9 shows how the *f/number* is found by measuring the ray-to-axis angle of each ray in the axial image-forming cone and then scaling it to the entrance pupil diameter by a Gaussian approximation.

Figure 10 shows how the *entrance* and *exit pupils* are found by measuring with edge rays the size and location of the apertures through which the ray bundles appear to enter and leave the lens.

Figure 11 shows how *ray bundles* from the object points are generated by specifying each ray position in a unit entrance pupil, which the code then scales to the actual entrance pupil size. Two examples are shown, the zone and the grid ray distributions. The rays can be positioned to fit any desired shape.

Fig. 3.
Spot diagrams at two focal positions along ray bundle.

Fig. 4.
Spot diagram measured by rms radius of rays from centroid.

Fig. 5.
Image position measured by centroid distance
from optical axis.

Figure 12 shows the *violations*, which are rays lost within the lens because of any of the following conditions: total reflection, failure of the ray to intersect the surface, an aperture obstruction (vignetting), and another type of obstruction called lens-element featheredge, which occurs between adjacent surfaces.

To summarize, the designing process is concerned mainly with optimizing the image spot diagrams; first, by obtaining the smallest image spots on the focal surface of interest; second, by obtaining rectilinear images, or images at the specified positions; third, by obtaining the closest clustering of the colored images at each lateral image position so that image sharpness is maintained for an extended spectral range; fourth, by maintaining the specified *f*/number in the bundles that generate the spots; and

Fig. 6.
Distortion measured from ideal image position.

fifth, by finding the entrance and exit pupil positions that enhance the overall performance of the lens system.

After the lens has been optimized for optical performance, the lens prescription is searched for mechanical defects that stand between the calculated design and the constructed lens. Usually, some lens parameters are beyond practical engineering boundaries, a condition that can often be corrected by reoptimization with restraints on the troublesome parameters. Finally, after the mechanical features and the optical performance have been optimized to our satisfaction, we find that our least squares procedure has simultaneously and automatically minimized the sensitivity to optical performance errors caused by manufacturing errors

Fig. 7.
Lateral chromatism measured by range of centroids.

Fig. 8.
Longitudinal chromatism measured by range of smallest focal-spot positions.

Fig. 9.
The $f/\#$ measured by scaling equation for all axial rays.

Fig. 10.
Entrance and exit pupils measured by position and size.

Fig. 11.
Two types of ray patterns used for image analysis.

Fig. 12.
Four types of violations evaluated by program.

in the prescription parameters. This extra benefit is obtained because we are now near the minimum of each parameter's gradient, which means that manufacturing errors will cause the least amount of image degradation.

If errors are made in manufacturing the lens elements, performance of the as-made lens prescription is reoptimized by varying the air-space parameters. In this way we usually obtain the full optical performance predicted for the error-free design when we test the lens on the optical bench.

V. HOW LENSES ARE DESIGNED

The procedure for designing lenses with the LASL program consists of four relatively simple steps.

- (1) The designer selects the optical performance specifications to be achieved: optical resolution, f /number, image magnification, angle of view, allowable distortion, spectral range, allowable lateral chromatism, and entrance and exit pupil positions.
- (2) A starting lens prescription is selected with the expectation that, after redesigning, it will give the desired optical performance.
- (3) The starting lens design is optimized for the desired application by selecting the parameters that are to be allowed to vary in the iterative least squares program.
- (4) The resulting optimized design is studied to find out whether the desired optical perfor-

mance was obtained and whether the lens is suitable for manufacturing.

If the improved lens is not acceptable, steps (2), (3), and (4) are repeated with variations until the unacceptable defects are corrected.

There are four versions of the LASL lens design program in active use, each with a few different capabilities. All four achieve the same results, excellent lens designs. The four versions are described briefly in Appendix A.

A. Performance Specifications

The lens designer imposes lens performance specifications so that the optimization program will have performance targets to achieve. The LASL program makes available simultaneous optimization for several conditions of lens use, to meet the needs of modern optical engineering. However, most lenses are designed for one condition of use because that is the most frequent imaging requirement and also because it was the only procedure available before invention of the LASL code.

1. One Condition of Use. The optical performance of a lens is measured at target points on the focal surface. The image spots should coincide in size and position with these points. To discover how well the lens design meets the performance specifications, we evaluate the errors in the size of the image spots and in the distance of the spots from the target points. The performance approaches perfection when the image spots are no larger than the Airy-disk diffraction limit and when the spot centroids deviate from the target points by less than the Airy-disk radius. For images larger than the diffraction limit, the monochromatic spatial resolution of the lens approximates the rms radius of the rays from the spot's centroid. The f /number (relative-aperture) specification for the image is usually determined by considering the resolution and the image brightness needed by the lens user. The image-magnification specification is the ratio of the required image size to the object size. The image-target positions on the focal surface are specified by the magnification. The angle of view is then specified by the object distance. The distortion

specification determines the allowable lateral deviation of the image-spot centroid from its target point. The specified spectral range of imaging is determined by the sets of monochromatic refractive indexes selected for ray tracing. The lateral-chromatism specification is the maximum allowable lateral deviation of the colored spot centroids at each image position. Both the entrance and exit pupil positions may be specified. During optimization, the pupils will move to positions that give the best overall optical performance if no particular positions are required.

2. Several Conditions of Use. In the preceding section, the optical performance targets were considered for a lens with a single condition of use, the most frequent imaging requirement. However, lenses frequently have several conditions of use, and it is often desirable to optimize performance for all these conditions simultaneously. A common example of different conditions of use is an ordinary camera lens used to make images of objects at long, at medium, and at short distances. A more complex example is a zoom lens with moving lens elements and a range of focal lengths in addition to a range of focal distances.^{20-22,28} Two versions of the LASL program^{6,16} have a substitution option that can simultaneously optimize the lens for as many as five conditions of use. This simultaneous optimization for several conditions of use greatly improves the overall lens performance with little loss of performance at the position used initially for one-condition-of-use optimization.³² The substitution option enables parameters, imaging targets, and error weights to be changed for each condition of use.

B. Starting Lens Prescription

The starting lens prescription can range from an existing complex design to no more than a series of glass sheets with plane surfaces. Because the LASL code takes so little computer time, a designer can afford to explore new regions of lens construction. However, if an existing lens design is thought to be adequate for the desired imaging problem, it can be scaled to the required size and then entered in the designing code for performance evaluation in the

proposed condition of use. If the performance is not entirely adequate, the optimizing program will quickly find small changes in the surface curvatures and the spacings of the elements to give the best performance in the specified condition of use. If still better performance is required, the designer can let the optimizing program find the lens design most suitable for the immediate problem. A design is started from scratch by making monochromatic runs on the smallest number of air-spaced glass sheets needed to make the simplest lens that could meet the imaging requirements.^{7,28,29,35-38} After making runs on a series of increasingly complex lenses, the designer chooses the lens arrangement that best suits the need. The monochromatic lens design is achromatized by substituting crown and flint glasses in the prescription and readjusting by additional runs. Sometimes we start with bichromatic runs when a quick evaluation of competing lens types is desired for a particular problem.⁴⁰ (See Sec. VII.) The designer's ingenuity will devise the procedure to solve a particular lens design problem.

The starting lens prescription is specified by a set of parameters, all of which must be given before calculations can be started. The entrance pupil is specified by its radius and its distance along the optical axis relative to the first optical surface. Each optical surface is specified by six kinds of parameters. The first surface specification is its type: rotationally symmetrical or cylindrical. The second surface specification is the circular radius of curvature at the vertex of the surface. The third surface specification is the eccentricity, which modifies the circular section to generate a conic section. A further modification of the conic section is the aspheric section, which is specified by four polynomial coefficients. The fourth surface specification gives the position of the surface vertex relative to the optical axis. This specification requires the translations and tilts of the surface in the x- and y-directions, measured from the vertex and from a normal to the tangent plane at the vertex. The fifth surface specification is the position of the surface along the optical axis, which is the distance to the next optical surface. The sixth surface specification is the set of refractive indexes for the optical material between the specified surface and the next surface.

C. Optimizing Prescription Performance

The optical performance of the starting prescription is improved by reducing the sizes of the image errors by the least squares optimization procedure that changes the prescription parameters at the direction of the error function. The error function contains all image errors as a sum of the squared weighted errors. The larger the image error, the harder the program will work to reduce its size. The apparent size of the error is controlled by a weight, a numerical coefficient assigned to that error to express its relative importance in the problem. Important image errors are given large weights if the errors are difficult to reduce in size. Although an error given a weight of zero will not be minimized by the program, the error usually changes in size as a result of lens changes that reduce the sizes of the weighted image errors. Until the designer discovers which errors are difficult to correct, a starting weight of 1 is usually assigned to all the important errors. There are seven classes of weights available for each image point and each color. These weights are on the image spots with the x- and y-components weighted separately, on the image heights, on the f /number, on the exit pupil distance, on the exit pupil radius, and on the violations.

The designer starts optimization by indicating which of the following lens parameters are to be changed: the spherical surface curvatures, the distances between surfaces, the conic eccentricities, the polynomial coefficients that generate aspheric surfaces, the surface translations and tilts, or the refractive indexes. The spherical surface curvature parameters are usually the most effective variables for reducing the sizes of the image errors. However, some variable distance parameters are usually essential for obtaining an excellent complex design. The size and the position of the entrance pupil may be especially important. The translation and tilt coordinates that determine the lateral position of each surface are important in some problems. Conic or aspheric surfaces are not usually very helpful, and their fabrication cost is high. Varying the refractive index usually leads to imaginary high-index glasses.

A very important problem during optimization is minimizing violations, the loss of rays within the lens. When a ray is lost, a new error is generated, weighted, and then used in place of the usual ray-deviation error in the error function. A heavy weight

on violations helps in the generation of useful lens designs by reducing the number of lost rays. Lost rays indicate a vignetted design, which distorts the overall lens performance evaluation.

Another helpful control during optimization is the ability to assign upper and lower boundaries to all variable parameters. However, this capability slows the rate of convergence when the parameters are trying to exceed their boundaries. To speed convergence, these parameters should be fixed at their boundaries before optimization is continued. These boundary controls quickly indicate the troublesome parameters that try to exceed their boundaries and thereby slow the designing progress.

By using a scale factor to change the prescription size, the designer can control the mobility of the axial-distance parameters in relation to the surface-curvature parameters. Changing the scale factor changes parameter mobility because the derivatives of the error function relative to the axial distances vary as the first power, whereas these derivatives relative to the surface curvatures vary as the third power. The mobility of each type of parameter depends on its derivative. In addition to the variation of parameter gradient with lens size and parameter type, there is also a small gradient variation depending on the parameter's position in the lens system. The relative size of the parameter gradient indicates the parameter's relative ability to move the rays to the desired positions on the focal plane of interest. In a group of parameters that are varied simultaneously, parameters with the largest gradients show the largest change. In that group, parameters that have relatively very small gradients will not change significantly. The gradients of the variable parameters are printed at each iteration because each gradient indicates that parameter's capability for continued reduction of the error function. The error function varies as the scale factor squared.

D. Evaluating the Optimized Design

First, the code's analysis of the image errors must be examined to find out how well the desired performance specifications are met. The design is acceptable if no error degrades lens performance below requirements. Usually some errors will be too large. Large errors are reduced by additional runs with

larger weights on the unacceptable errors. If changing the weights does not yield an acceptable design, another arrangement of the glasses may help or a more complex design may be required. After the performance specifications have been met, the next step is to see if the lens prescription is suitable for construction. Some elements may be too thick or too thin, too large or too small, or the whole lens assembly may be too long or too close to the focus. These defects can often be corrected by reruns that hold the troublesome parameters within tolerable limits. Again, a different arrangement of glasses or a more complex design may be helpful.

VI. SAMPLE PROBLEM 1

A. Optimizing Lens Performance

For the first sample problem, I chose the Symposium Lens run on our fastest program because that lens design is well documented.⁴⁷ This new design, LASL-3, gives performance practically indistinguishable from that of the 1963 LASL-2 design.²³⁻²⁴ After a series of preliminary runs to find the proper scale factor, the LASL-3 design was discovered in two runs totaling 22 s of computation.

During all iterations, 16 variables were used simultaneously—8 surface curvatures and 8 axial distances. To make the axial-distance parameters change a modest amount during designing, the lens size was scaled up by a factor of 2.5. During all iterations, the entrance-pupil size and the first air space were held constant. The first run used 6-ray patterns for each object point and each color; the second run used 26-ray patterns. Figure 13 shows the starting lens and the new design, which has only a modest thickening of the lens elements and no unusual shape features.

The changes of the lens parameters during designing are shown in the next two illustrations, with each design in the series giving better performance than the preceding design. Figure 14 shows the radius of curvature of each lens surface at each iteration for both runs. As is expected, the greatest changes occur during the first few iterations. The small amount of change during the last two iterations indicates that a stable design has been achieved. Figure 15 shows the axial distances between adjacent lens surfaces at each iteration for both runs. Again, the greatest

Fig. 13.
Starting and improved Symposium Lens designs.

changes occur during the first few iterations, and there is little change during the last two iterations. Also shown is the movement of the entrance pupil during designing. The graph shows all the surface positions measured from the focal plane. Table I gives the prescriptions for four lens designs.

Figure 16 shows design progress in the first run, during which the error function drops 3 orders of magnitude in 12 iterations. In each of the above least squares iterations the automatic search procedure tries a series of damping numbers that produce an improved lens prescription, which in turn is used as the starting prescription for the next least squares iteration. Although the first 11-s run gave a very good lens design, the second run gave slightly better overall image quality without making much change in the lens shape.

Figure 17 analyzes in detail one damping-number search curve that was selected from another series because it shows the random occurrence of local minimums when small damping numbers are used on a lens near convergence. The numbered data points are the ones found by the program during one

Fig. 14.

Surface curvature changes during Symposium Lens design.

Fig. 15.

Axial distance changes during Symposium Lens design.

iteration. To get the data for this illustration, the code was later directed to evaluate the error function for a large number of additional damping numbers. At least four local minimums exist in the region explored. We usually find that if the interval is explored in still finer detail, additional minimums are discovered.

Table II gives the performance characteristics of the initial design (SYM-0) and of the 6-ray and 26-ray (LASL-3) designs described here, as well as those of LASL-2, all as evaluated by the Doyle program.¹⁸ Although the LASL-2 and LASL-3 designs are quite different in appearance (Fig. 18), they have similar performance characteristics. Closer performance agreement could probably be obtained by additional weight adjustment.

B. Discussion

The general appearance of an optimized lens design varies considerably with the freedom of movement allowed for the axial-distance parameters. This variation is well illustrated by the LASL-2 and LASL-3 lenses, which have very similar performance characteristics. Also, when the different parameters of a lens system are varied simultaneously, their effectiveness in reducing the size of the error function differs greatly. Generally, the surface-curvature parameters are most effective, and the axial distances are least effective. However, designs with long axial distances generally show the best performance characteristics if there is no need to consider such other factors as convenience, practicability, or difficulty of construction.

TABLE I
PRESCRIPTIONS FOR SYMPOSIUM LENS DESIGNS*

<u>Glass</u>	<u>Param.</u>	<u>SYM-0</u>	<u>6-ray</u>	<u>LASL-3</u>	<u>LASL-2</u>
620603	EP	-1.250	-1.286	-1.2868	-2.860
	R1	+2.174	+2.670	2.7278	+28.100
	D1	0.300	0.3887	0.3892	0.625
	R2	∞	-6.9055	-6.4116	-3.7923
(air)	D2	0.0025	0.0025	0.0025	0.036
620603	R3	+1.390	+1.7183	+1.7167	+1.6422
	D3	0.300	0.2937	0.2930	0.475
	R4	+5.000	+8.1637	+8.1349	+21.033
(air)	D4	0.525	0.1415	0.1349	0.142
649338	R5	-1.667	-2.6231	-2.6914	-2.8286
	D5	0.125	0.2191	0.2374	0.625
	R6	+1.667	+1.3425	+1.3176	+1.0763
(air)	D6	0.625	0.4933	0.4909	0.174
620603	R7	-5.000	+7.2588	+7.0110	+2.4209
	D7	0.300	0.2358	0.2352	0.7353
	R8	-1.390	-1.4655	-1.4357	-1.3463
(air)	BF	1.188	1.5534	1.5589	1.3513

*All prescriptions are scaled up by a factor of 2.5.

By enlarging the lens being designed from a 1.0-unit to a 2.5-unit size, the changes in the axial-distance parameters were increased from insignificant to significant amounts. These changes led to the LASL-3 design. This size control was not used for LASL-2, and it was not possible to test all parameters operating simultaneously. The size control apparently was not used for the other Symposium Lenses, designed by other programs.

In an attempt to find out if there is something unique about the LASL-2 lens design, I restarted designing with eight plane surfaces, using the LASL-2 axial distances, and then designed on the surface curvatures only. In 28 iterations, the program found curvatures approximating the LASL-2 curvatures, with an average difference of about 3%. The error function of this design is slightly smaller than the LASL-2 error function as evaluated by the Doyle code.

Because no rays were lost within the lens and no boundaries were passed, these program controls were not tested during the design of LASL-3.

VII. SAMPLE PROBLEM 2

For the second sample problem, I chose the design of a copying lens for microfilm photography because that problem is well documented,⁴⁹⁻⁵¹ illustrates important features of the LASL program, and is typical of the searching necessary when performance specifications go beyond existing technology. A new design was needed because no commercial lens could be found that met all the precise imaging requirements. The excellent performance of the new lens illustrates the success of the rms error function as a suitable criterion for the design of high-resolution and high-contrast lenses.

Fig. 16.
Design progress during first Symposium Lens run.

Fig. 17.
Local minimums on damping-number search curve.

Fig. 18.
Shapes of LASL-2 and LASL-3 Symposium Lens designs.

TABLE II
CHARACTERISTICS OF SYMPOSIUM LENS DESIGNS*

<u>Performance Feature</u>	<u>SYM-0</u>	<u>6-ray</u>	<u>LASL-3</u>	<u>LASL-2</u>
Ray pattern	6	6	26	26
f/number	1.96	1.41	1.41	1.42
Error function $\times 10^{-9}$	8140.	7.9	6.4	4.7
Spot radius, = 0.0625	0.0234	0.00076	0.00076	0.00071
avg rms for = 0.1625	0.0259	0.00080	0.00063	0.00064
image hgt = 0.2625	0.0314	0.00135	0.00114	0.00086
Lateral = 0.0625	0.0010	0.00002	0.00003	0.00009
chromatism = 0.1625	0.0028	0.00005	0.00008	0.00022
for hgt = 0.2625	0.0048	0.00007	0.00011	0.00032
Distortion = 0.0625	+0.0200	-0.00005	-0.00012	-0.00018
error for = 0.1625	+0.0536	-0.00006	-0.00025	-0.00030
image hgt = 0.2625	+0.0919	+0.00012	+0.00016	+0.00008

*All characteristics are scaled up by a factor of 2.5.

The LASL 1972 program (LENSED¹⁶) was used to find the lens.

A. Performance Specifications

The first specification, and the most difficult to satisfy, was high resolution, preferably diffraction limited (400 line pairs/mm), throughout the image field at $f/4$ relative aperture. The second specification was a 30-mm-diam flat image field, needed for photographing flat art work on plates and film. The third specification was excellent rectilinearity, 0.01-mm maximum error at any point in the field. This error limitation also includes the lateral chromatism. Fourth, we needed high-contrast images to minimize image spread with increasing film exposure. This last specification meant that image flare had to be small. The fifth requirement was the ability to copy at a 20-to-1 reduction, a size convenient for our application. The sixth specification was red-green achromatization to permit the lens to be used with monochromatic or incandescent light illumination. Red-blue achromatization, un-

necessary in this problem, would lower resolution, because the red-blue chromatic errors are much larger than the red-green.

B. The Lens That Solved the Problem

Figure 19 shows the new copy lens design that satisfies all the above specifications. Note that both doublets have assumed a meniscus shape with the rear doublet near the image, and note that in the rear doublet the conventional construction for the powers of the glasses has been reversed. The design presents no manufacturing problems, as indicated by the performance of the three lenses we made. On the optical bench, when the as-manufactured lens elements were assembled without adjustment into the cells we had built, they all gave diffraction-limited resolution. The incandescent-light images appeared to be nearly as good as the monochromatic images; no color was visible in the 400-line/mm images. The excellent incandescent-light visual resolution was confirmed by the 320-line/mm resolution

C. Search for the New Design

The first step in the search for a new design is to identify the key features of the existing design. In this case, the key features are the horizontal stripes and the circular shape.

Fig. 2
A circular object with horizontal stripes, showing a pattern of alternating light and dark bands.

Figure 23 shows these two characteristics for each of the several lens arrangements that were tried. In most cases, the maximum lateral error is larger than the maximum spot size, an indication that distortion and lateral chromatism are relatively difficult to correct with the weights that were used. The starting triplet (C-F-C), which has the largest spot sizes, is at the right. A flint glass was added to the triplet to make the starting quadruplet (C-F-F-C), which gave only slightly better performance than the triplet. Next, the starting pair of cemented doublets (CF-FC), seen at the top, was made by removing the first and last air spaces of the quadruplet. This design has the largest lateral chromatism. The starting compound triplet (C-CF-FC), seen at the upper left, was made by adding a crown singlet to the two doublets. Below these four starting designs, the effect of changing the sequence of glasses is shown. When the last two glasses were reversed (CF-CF), the two-doublet design was greatly improved. The compound triplet design was also improved by reversing the glasses in the central doublet (C-FC-FC). The quadruplet's performance began to look useful after the first flint was replaced by a crown (C-C-F-C). The best performance was obtained with the glass sequence shown at the bottom (C-CF-CF), where a crown singlet was added to the best two-doublet design. I selected this best two-color design

Fig. 23.

Selected performance characteristics of lens types evaluated during search for LASL copy lens.

as the one most likely to yield a satisfactory final design.

When this choice had been made, the design was refined by continued designing after changing from bundles of 6 rays in d and e to bundles of 26 rays in C , d , and e light. Performance of the resulting three-color design met all six specifications. (The most important lens performance characteristics of this design are given in Table III under Catalog Glass.) Next, I replaced the catalog refractive indexes with the melt indexes of the glasses to be used for lens construction and made a short run to adjust the prescription for optimum performance. The adjusted design gave approximately the same performance as the catalog-glass design did, except for rather large lateral chromatism, which could have been reduced by additional iterations had I planned to make this melt design. Finally, by a series of five short design runs to make small changes in three air spaces and one glass thickness, I adjusted the melt design to fit a set of test glasses selected from our stock. The performance of the final design is indistinguishable from that of the catalog-glass design, as shown in Table III.

The prescriptions for the final lens designs are shown in Table IV. Shown under Catalog Glass is the prescription that ended my search for a good design. Under Melt Glass, the prescription was optimized for the measured refractive indexes of the glass melts used to make the lens. Note the small parameter changes that were needed to regain almost completely the performance that was obtained with catalog indexes. The last column shows the prescription readjusted to fit the available test glasses. Again note that the good performance of the catalog-glass design was recovered by making small changes in three air spaces and one glass thickness. I was able to use one test glass for surfaces 1, 4, and 8.

D. Accessory Performance Characteristics

Figure 24 shows the rms radius of the 52-ray image spots across the focal plane for C , d , and e light. The average spot size is about $2 \mu\text{m}$, even at the field edge, 15 mm off axis. About 70% of the rays are contained in the rms radius circle. Note that although the greatest variation in spot size is in d light, even in the region of the largest spots there is no noticeable loss of resolution when aerial or

TABLE III
CHARACTERISTICS OF MICROFILM LENS DESIGNS*

Performance Features for C-d-e Light		Catalog Glass	Melt Glass	+Test Glasses
Error function $\times 10^{-11}$		5.2	4.3	4.8
Spot radius,	= 15:2	0.0019	0.0017	0.0017
average rms	= 10:2	0.0020	0.0019	0.0021
radius for	= 5:1	0.0019	0.0016	0.0018
image hgt	= 0:0	0.0020	0.0018	0.0019
Lat. chrom.	= 15:2	0.0005	0.0055	0.0005
range for	= 10:2	0.0002	0.0035	0.0002
image hgt	= 5:1	0.0001	0.0017	0.0001
Distortion	= 15:2	+0.0005	+0.0041	-0.0028
error for	= 10:2	-0.0004	-0.0018	+0.0009
image hgt	= 5:1	-0.0001	-0.0009	+0.0007

*Dimensions in mm.

photographic images are being viewed. If it had been important, the large size of the *d* light spots, at around 11 mm, could have been reduced by using a larger weight on that color and image position. However, some other spots would then probably have increased in size.

Figure 25 shows the spot sizes at the optimum focal positions throughout the image field. Note how similar the curves are for *C*, *d*, *e*, and *F*. The position and the shape of the *F* curve indicate that at the blue-light focus the lens should still make diffraction-limited images, probably with some loss of contrast. Visual tests qualitatively confirm that conclusion.

Figure 26 shows the 52-ray sagittal and tangential focal surfaces throughout the field for *C*, *d*, and *e* light. On the optical bench, this amount of astigmatism is not noticeable. The *F* light curves are of similar shape, but at 6.4-mm back focus. Note that these curves were obtained by statistical analysis of the spot diagrams, which gives a precise evaluation of the astigmatism as it is measured visually on the optical bench.

VIII. DISCUSSION

There have been many questions about the existence of local minima in the error function that could cause the LASL program to fail to find the optimum prescription. Figure 27 shows a possible variation of the error function along the correction path chosen by the program. The concern is that a minimum like that shown at A might exist; if it did and if there were no bypass around the hump, the program would stop there and fail to achieve a satisfactory prescription. No such local minimum has been observed during the long history of optimizing lenses with the LASL program, except under two extraordinary circumstances. Apparently the correction space is so complicated that there is always a bypass around a hump. By testing the effect of varying all the parameters the program finds a bypass route. The two extraordinary conditions, readily detected because they cause abrupt, large increases in the error function, are encountering violations or encountering the boundaries of the

TABLE IV
PRESCRIPTIONS FOR MICROFILM LENS DESIGNS*

<u>Glass Type</u>	<u>Parameter</u>	<u>Catalog Glass^b</u>	<u>Melt Glass^c</u>	<u>Melt + Test-Glass Radius</u>
(air)	D0	3048.0	3048.0	3048.0
	EP	-30.480	-30.480	-20.320
	R1	+94.549	+96.261	+96.495
517642	D1	25.400	25.400	25.400
	R2	-396.21	-385.275	-370.84
(air)	D2	0.254	0.254	0.254
517642	R3	+56.137	+55.430	+54.229
	D3	20.284	19.822	18.771
	R4	-91.186	-94.219	-96.495
617366	D4	22.860	22.860	22.860
	R5	+35.657	+35.583	+35.174
(air)	D5	54.092	54.806	55.321
517642	R6	+80.625	+80.594	+83.591
	D6	24.130	24.130	24.130
	R7	+56.220	+58.130	+57.937
617366	D7	24.130	24.130	24.130
	R8	+93.896	+94.087	+96.495
(air)	BF	6.350	6.350	6.198

*Dimensions in mm.

^bCatalog indexes are 1.51431, 1.51680, 1.51871, 1.61164, 1.61659, 1.62058.

^cMelt indexes are 1.51480, 1.51728, 1.51921, 1.61199, 1.61694, 1.62094.

parameters. These two conditions distort the performance evaluation in a discontinuous manner and probably should not be considered local minima in the present context.

Another common question is whether the lens prescription at the end of an optimization is unique. For the usual many-parameter lens design problem, there does not seem to be a unique prescription. There are many similar, but slightly different, prescriptions that all give about the same optical performance. Apparently, a slight change in one parameter can be compensated by other slight changes in the remaining ones. In Fig. 27, the error function is drawn to have constant value at late stages of the correction process, indicating this region of compensating errors. In addition, progress from a slight local minimum like B in the diagram to the real minimum C can be very slow. The operator may well choose to stop at B if the error function is

already within system requirements, shown by the dashed line in the diagram.

Some special problems do have unique solutions where the error really goes to zero (within the computer resolution). The unique solution is reached whatever the starting prescription. Examples of problems with unique prescriptions are the aplanatic lens, the conic-section mirrors, and the Cassegrainian telescope.

ACKNOWLEDGMENTS

The conversion of these ideas into the very successful Brixner-LASL lens design code has been made possible by the cooperation of many colleagues during the program's 28-year development. Special thanks go to John C. Holladay, Charles A. Lehman (d. 1966), and Thomas C. Doyle, whose enthusiastic

Fig. 24.

The 52-ray image spot sizes at focal plane of LASL copy lens.

Fig. 26.

The 52-ray astigmatic surfaces of LASL copy lens.

Fig. 25.

The 52-ray image spot sizes at optimum focal points of LASL copy lens.

Fig. 27.

Schematic of error function vs correction path, showing the existence of local minima. The dashed line indicates the maximum value of the error function permitted by the system requirements.

application of mathematical know-how to my proposals has made codes out of ideas. More than a little help along the way was given me by A. J. Lipinski (d. 1953), R. H. Stark, R. E. Von Holdt, W.

A. Allen, J. S. Courtney-Pratt, W. C. Davis, I. J. Cherry, K. T. Imamura, William Anderson, G. W. Wilkerson, M. A. Winkler, M. M. Klein, and J. V. Kline. In another important area, making and testing lenses designed by the code, thanks go to R. E. Lapp, W. J. Wynne, C. G. Phelps (d. 1978), and the commercial manufacturers who made so many lens elements to our tight engineering specifications.

APPENDIX A

HISTORY OF LENS DESIGN AT LASL

The Optics Group at LASL has maintained an active program of lens design during the past 30 years. Because an analytic solution for the constructional data of a lens in terms of the desired performance has not been found with today's mathematics, iterative approximations must still be used. From 1948 to 1958, the main tool used by LASL lens designers F. A. Lucy (d. 1978), A. J. Lipinski, and W. A. Allen was an electric desk calculator that aided pencil-and-paper calculation of aberration coefficients. Those calculations followed the century-old procedure that used paraxial ray tracing because this method was well established. Although accurate trigonometric ray tracing was discovered in the seventeenth century, it was rarely used in lens design because so much time was needed to make the calculations. Several simplifications have made ray tracing practical as a lens design tool. First, ray tracing was limited to meridional rays rather than to skew rays, which were excessively time consuming. Second, about a century ago, these meridional rays were limited to those *very close to the optical axis* (paraxial). This limitation makes it possible to substitute the ray angle in radians for the sine of the angle and to substitute unity for the cosine of the ray angle. In 1953, F. A. Lucy began calculating the aberration coefficients by electronic computer;⁵³ in 1954, he used an analog computer to minimize and balance the third-order aberration coefficients;^{54,55} in 1955, he used skew rays for spot-diagram analysis;⁵⁶ and later, he wrote a complete lens design handbook.⁵⁷

Starting in 1951, A. J. Lipinski and I used trigonometric skew ray tracing⁵⁸ to make lens performance evaluations and began analyzing lens designs by multiple ray tracing. I began designing complex lenses in 1959 when large electronic computers made possible the invention of an entirely new procedure, which *simultaneously minimizes large numbers of ray deviations from their ideal positions*. I have continued to use this procedure, which optimizes the sizes and the positions of spot-diagram images in a least squares system of analysis.⁹

Briefly noted here are the steps that have brought the LASL lens design program to its present state of

development. During the course of its 20-year history, the program has been designated as the Holladay code,^{3,12} the Lehman code,^{4,9,11,13} the JPL code,¹⁴ the Brixner code,^{2,5-8,10,15,17,20,27} the Doyle code,¹⁶ the Kline code,¹⁶ and the Klein code.¹⁷ Morris Klein's version is the latest in a series that goes back to 1951 when W. A. Allen made me an easy-to-use skew-ray-tracing code⁵⁶ that employed for the first time the now essential floating decimal calculation, which had just been invented at LASL by D. B. MacMillan and R. H. Stark.^{59,60}

While we were analyzing and designing lenses with this program on the IBM Card Programmed Electronic Computer, making manual changes of the lens parameters and at the same time getting data for drawing the rays intersecting the focal plane, we discovered that a simple graphic analysis helped reduce the spot sizes. This discovery, in turn, made us envision the possibility of automating the process through a least squares minimizing technique. Even after computers with large fast memories became available, however, there was the usual conservative resistance to change.⁶³⁻⁶⁷ It was some time before two mathematicians interested in innovative schemes—J. C. Holladay and C. A. Lehman—developed my proposals into an effective code.^{3,4}

That code, devised by Holladay in 1958-1959 and further developed by Lehman until his death in 1966, is the only basic program that designs many-ray image-spot size and position data, making no use of aberration theory. Since 1960, that code has been widely distributed and sometimes modified by other people to meet their special requirements.^{10,34,37,39,41,43-46,48} An unreported 1967 modification at LASL is a grating subroutine that was used to design a small auxiliary spectrograph and to optimize a large Czerney-Turner spectrograph.⁴⁰ Until the Holladay-Lehman FAP-language code was rewritten in FORTRAN by P. J. Firnett and L. A. Wilson in 1967 (FOLDP), under JPL-NASA contract,¹⁴ the code could be run only on the IBM 704-7090 machines.

The Doyle code (LENSEDES)¹⁶ incorporates improvements that have speeded up the design

program as a result of T. C. Doyle's 1964-1973 investigation of nonlinear least squares optimizing procedures for continuous many-parameter systems.^{41,42} The most important improvement is the use of Doyle's refined increment-vector damping technique to speed optimization. Other program improvements include the use of (1) analytic differentiation to calculate the rate of change of the image errors with respect to the variable parameters; (2) a refined calculation of the parameter increment vector so that all variables can be calculated simultaneously; (3) a procedure that minimizes the loss of rays within the lens, called violations; (4) a lens prescription scale factor to control the relative mobility of the various parameters; and (5) enlarged convergence criteria and weighting procedures as well as another type of increment-vector search technique. Doyle's code now solves

more than 80% of our lens design problems, although it runs only centered optical systems for one condition of use.

The John Kline code (GTRACE)¹⁶ is a very specialized program that traces bundles of skew rays through a two-mirror plane-grating spectograph. The program was written by J. V. Kline in 1975 to replace the capabilities of the grating subroutine written by I. J. Cherry in 1967 as a subroutine for the now inactive Holladay-Lehman FAP-language code.⁴⁰

The Morris Klein code (CLIMB)¹⁷ is a versatile program that employs a Monte Carlo random selection procedure to sample the object intensity, the object space, the entrance pupil, and the color response. The 1978 CLIMB code has the greatest versatility for evaluation and design of the most complex optical systems.

APPENDIX B

SEIDEL ABERRATIONS AND THE LASL ANALYSIS

Our statistical analysis of spot diagrams takes into consideration all seven image errors analyzed over a century ago by Petzval, Abbe, and Seidel with their paraxial aberration coefficients. This procedure was a short cut suitable for repetitive evaluation by pencil-and-paper calculation, the only kind available at that time. Because the LASL procedure uses precisely traced skew rays to make spot diagrams, there is no need to consider the first few in the infinite series of polynomial coefficients that are derived from simplified paraxial ray traces. Those coefficients, called aberrations, are used widely for designing lenses even though, by computer standards, they are only approximate indications of lens performance defects. In addition, reducing the (third-order) aberration coefficients to near-zero values does not produce the best lens. Also, acceptable values for the seven individual aberration coefficients are not known in absolute terms because their interactions vary greatly with the complexity of the lens type being studied and with the specific imaging requirement as well. We describe below the

way that the image errors represented by the Seidel aberrations may be observed in our spot-diagram analysis. The behavior of a paraxial ray is statistically of little significance in the spot-diagram image because it has very little effect on the focal position. Analysis of imaging by a concave mirror and an achromatic lens provided the data for the following examples.

Figure B-1 shows how the spherical error is observed in the axial spot diagram of a centered system when the monochromatic spot diagram at the best focus is larger than the spot obtained from a perfect lens. The character of the defect is shown by the way that the rays are distributed in the spot diagram. Note that the edge rays move toward or past the center of the large image spot.

Figure B-2 shows how coma is observed as a comet-shaped image formed by rays in a series of radial zones in off-axis spot diagrams.

Figure B-3 shows how astigmatism is observed by noting the longitudinal distance between the radial

Fig. B-1.
Spherical error displayed in spot diagram.

Fig. B-2.
Coma blur displayed in spot diagram.

Fig. B-3.
Astigmatic images along ray bundle.

and tangential focal positions along the off-axis ray bundle.

Figure B-4 shows how curvature of field is observed by noting the longitudinal distance between the best focal positions of off-axis and on-axis ray bundles.

Figure B-5 shows how distortion is observed by noting the lateral variation of the image spot from the rectilinear image height, using the centroid instead of the principal ray.

Figure B-6 shows how the lateral chromatic error is observed by noting the maximum lateral distance between the monochromatic off-axis images over the spectral range, again using the centroid instead of the principal-ray intersections.

Figure B-7 shows how the longitudinal chromatic error is observed by noting the maximum distance between the smallest monochromatic axial images over the spectral range, instead of using the paraxial-ray intersection distances.

Curvature of Field

Fig. B-4.
Curvature of field measured by positions of smallest focal spots.

Focal Plane of Interest

Fig. B-5.

Distortion measured by error in centroid position.

Focal Plane of Interest

Fig. B-6.

Lateral chromatism measured by range of monochromatic centroid positions.

Fig. B-7.

Longitudinal chromatism measured by range of smallest monochromatic spots.

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