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TRANSPORT SCALING STUDIES FOR EBT REACTOR*

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ABSTRACT

Transport simulation and modeling studies for the ELMO Bumpy Torus (EBT) reactor are carried out by using 0-D and 1-1/2-D transport calculations. The time-dependent 0-D model is used for global analysis whereas the 1-1/2-D radial transport code is used for accurate determination of density, temperature, and ambipolar potential profiles and of the role of these profiles in reactor plasma performance. Analysis with the 1-1/2-D transport code shows that profile effects near the outer edge of the hot electron ring lead to enhanced confinement by at least a factor of 2-5 beyond the simple scaling that is obtained from the global analysis. The radial profiles of core plasma density and temperatures (or core pressure) obtained from 1-1/2-D transport calculations are found to be similar to those theoretically required for stability.

I. INTRODUCTION

In present ELMO Bumpy Torus (EBT) devices, the core plasma losses are largely governed by the electron transport, leading to a simple scaling.¹⁻⁴

$$n\tau \sim A^2 T_e^{3/2} [1 + (R_B/R_E)(e\phi/T_e)]^2 f(\text{profiles}) \quad (1)$$

where $A = R_T/R_B$ is the magnetic aspect ratio; R_T , R_B , and R_E are the major radius of the torus, magnetic field scale length, and electric field scale length, respectively; and $e\phi$ is the ambipolar potential. Scaling laws governing ion behavior are important in a reactor. At present, ion transport principally determines the radial electric field needed to maintain ambipolarity.

The operating parameter space of a reactor is investigated by using both time-dependent zero-dimensional (0-D) and one- and one-half-dimensional (1-1/2-D) transport calculations.⁵ The 0-D model is used for parametric burn studies to examine the global plasma performance. The 1-1/2-D radial transport code is used to determine density, temperature, and ambipolar potential profiles and the effect of these profiles in energy confinement scaling.^{6,7}

In the experiments, the radial electric field is in a direction opposite to the radial magnetic field gradient inside the ring (e.g., $E_r < 0$). As a result, radial electron transport is dominated by nonresonant processes, whereas ion transport (at low collisionalities, $\nu/\Omega_i < 1$) is dominated by resonant processes. A positive electric field, $E_r > 0$, is also possible if the roles of ions and electrons are interchanged ($T_i > T_e$), which is favored in a reactor

plasma. In this case ions are nonresonant and electrons are resonant (for small ν/Ω_e). Recently, analytic transport coefficients⁸ have been derived for finite electric fields in EBT for both nonresonant and resonant particles, which are used for reactor calculations.

In a global analysis with $E_r < 0$ [which leads to the simple scaling given by Eq. (1)], it is observed that large devices ($A > 20$) are required for ignition. Analysis with the 1-1/2-D transport simulation code provides an important new insight on confinement scaling of EBT in going from present experiments (EBT-S) to a reactor (EBT-R). It is observed that in the EBT-S to EBT-R sequence profile effects near the outer edge of the hot electron ring lead to enhanced confinement beyond the simple scaling obtained from the global analysis (which omits these profile effects). Profile effects suggest that confinement exceeds the simple scaling by at least a factor of 3-5. A similar enhancement factor is also observed⁹ as possible with $E_r > 0$.

II. TRANSPORT EQUATIONS

The particle and energy balance equations solved are of the form (for the j th species, $j = e, D, T, \alpha, \dots$)

$$\partial n_j / \partial t + \nabla \cdot \Gamma_j = S_{pj} \quad (2)$$

$$\partial (3n_j T_j / 2) / \partial t + \nabla \cdot Q_j - q_j \Gamma_j E_r = P_{Ej} \quad (3)$$

where S_{pj} is the net particle source (sink) and P_{Ej} is the net energy source (sink). Here, Γ and Q are the particle and energy (heat) fluxes, respectively, and in one-dimensional (1-D) cylindrical geometry their radial components can be written (for each species j with charge q) as

$$\Gamma_r = -D_r (\nabla n - qn E_r / T) - D_T \nabla T \quad (4)$$

$$Q_r = -K_T \nabla T - K_r (\nabla n - qn E_r / T) \quad (5)$$

where the D and K are transport coefficients. Charge neutrality for electrons and ions ($i = D, T, \alpha, \dots$) is imposed with the equation

$$n_e = \sum_i n_i Z_i \quad i = D, T, \alpha, \dots \quad (6)$$

and the ambipolar potential ϕ ($E = -\nabla\phi$) is determined from

$$\Gamma_e = \sum_i Z_i \Gamma_i \quad (7)$$

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A. 1-1/2-D Model

The effect of realistic magnetic geometry is included in 1-1/2-D transport calculations⁹ in that the transport equations are averaged along magnetic field lines computed from a two-dimensional (2-D) solution to the magnetic equilibrium equation in mirror geometry. Defining the geometric form factors,⁹

$$U_1 = [(B_m/L) \oint dl/B]^{-1}, \quad U_2 = \oint dl r^2 B/\rho^2 B_m L, \quad (8)$$

where L is the length of a mirror sector, $B_m = B_{mid}$ is the mid-plane field, l is length measured along the field line, and ρ is the spatial variable that measures the radial position of a constant flux (ψ) surface as it intersects the mirror midplane, we have the following expressions for the transport equations:

$$\begin{aligned} \partial n_j / \partial t + (U_1/\rho) \partial / \partial \rho (\rho U_2 \Gamma_j) &= \langle S_{pj} \rangle_V, \\ \partial / \partial t (3n_j T_j / n_j) + (U_1/\rho) \partial / \partial \rho (\rho U_2 Q_{ej}) & \\ + q_j \Gamma_{ej} U_1 U_2 \partial \phi / \partial \rho &= \langle P_{Ej} \rangle_V. \end{aligned} \quad (9)$$

Here, Γ_j and Q_e are given by Eqs. (4) and (5), respectively, with ρ replacing r , $\langle \rangle_V$ defines the flux surface average over a volume contained within the surface, and

$$V = 2\pi \oint \rho B_m d\rho \oint dl/B \quad (10)$$

is the volume enclosed by a constant ψ surface.

B. 0-D Model

In the 0-D, time-dependent model, equations are averaged over a volume, and the particle and energy confinement times (τ_p and τ_E , respectively) are calculated from

$$\begin{aligned} \tau_{pj} &= \langle n_j \rangle / \langle \nabla \cdot \Gamma_j \rangle = (a/2) \langle n_j \rangle / \Gamma_j(a), \\ \tau_{Ej} &= \left\langle \frac{3}{2} n_j T_j \right\rangle / \langle \nabla \cdot Q_j \rangle = \frac{a}{2} \left\langle \frac{3}{2} n_j T_j \right\rangle / Q_j(a), \end{aligned} \quad (11)$$

where a is the average plasma radius and $\langle \rangle$ represents a volume average.

Note that the charge neutrality [Eq. (6)] and ambipolarity [Eq. (7)] conditions impose

$$\tau_{pe} = \tau_{pi}, \quad (12a)$$

where

$$\tau_{pj}^{-1} = \sum_{j \neq e} (Z_j \langle n_j \rangle / \langle n_e \rangle) \tau_{pj}^{-1}, \quad j = D, T, \dots \quad (12b)$$

Replacing gradients by scale lengths, Eqs. (4) and (5) can be written (for each species) as

$$\begin{aligned} \Gamma_j &= n D_n [l_n^{-1} + (T/n) (D_T/D_n) l_T^{-1} + q E_n/T] \\ &= n D_n L_p, \end{aligned} \quad (13)$$

$$\begin{aligned} Q_j &= n K_n [l_n^{-1} + (T/n) (K_T/K_n) l_T^{-1} + q E_n/T] \\ &= n K_n L_E. \end{aligned} \quad (14)$$

where $l_n = -(\partial \ln n / \partial r)^{-1}$, $l_T = -(\partial \ln T / \partial r)^{-1}$, and L_p and L_E represent the terms (which are functions of scale lengths, electric field, and ratios of transport coefficients) in square brackets. Using Eqs. (13) and (14) in Eq. (11) we have (with the subscript j omitted for brevity)

$$\begin{aligned} \tau_p &= \frac{a}{2} \frac{\langle n \rangle}{n(a)} \frac{1}{D_n(a) L_p(a)} = \frac{a}{2} \frac{f_n(a)}{D_n(a) L_p(a)}, \\ \tau_E &= \frac{a}{2} \frac{\langle 3nT/2 \rangle}{n(a) T(a)} \frac{T(a)}{K_n(a) L_E(a)} = \frac{3a}{4} \frac{f_{nT}(a)}{K_n(a) L_E(a)}, \end{aligned} \quad (15)$$

where $f_n(a) = \langle n \rangle / n(a)$ and $f_{nT}(a) = \langle nT \rangle / [n(a) T(a)]$ are the profile form factors, which can be determined either from assumed density and temperature profiles or from the requirements of stable pressure profiles.^{5,7}

III. TRANSPORT COEFFICIENTS

A. Nonresonant Particles ($q_j E_r > 0$)

The analytic expressions^{3,10} for nonresonant particles are summarized below:

$$D_n = a_1 \bar{D} [1 + a_2/W_0 - a_3 W_0/(\bar{v}^2 + W_0^2)], \quad (16)$$

$$D_T = b_1 (n/T) \bar{D} [1 + b_2/W_0 - b_3 W_0/(\bar{v}^2 + W_0^2)], \quad (17)$$

$$K_n = c_1 T \bar{D} [1 + c_2/W_0 - c_3 W_0/(\bar{v}^2 + W_0^2)], \quad (18)$$

$$\begin{aligned} K_T &= d_1 n \bar{D} [1 + d_2/W_0 - d_3 W_0/(\bar{v}^2 + W_0^2)] \\ &\approx (7/2) n \bar{D}, \end{aligned} \quad (19)$$

where

$$\bar{D} = 2(r R_B / R_T)^2 v / (\bar{v}^2 + W_0^2), \quad (20)$$

$\bar{v} = v/\Omega_{CB}$, and $W_0 = \Omega_E \times_B / \Omega_{CB} = |q R_B E_r / T|$. The numerical coefficients in Eqs. (16)-(19) are

$$\begin{aligned} a_1 &= 0.068 & b_1 &= 0.137 & c_1 &= 0.239 & d_1 &= 3.517 \\ a_2 &= 8.479 & b_2 &= 12.212 & c_2 &= 12.183 & d_2 &= -0.909 \\ a_3 &= 1.813 & b_3 &= 5.595 & c_3 &= 6.156 & d_3 &= 2.631 \end{aligned}$$

For $E_r < 0$, electron transport is governed by nonresonant processes. For $E_r > 0$, ions would be nonresonant.

B. Resonant Particles ($q_j E_r < 0$)

Neoclassical transport coefficients for the entire range of resonant particle collisionality (ν/Ω) from banana to plateau are given in Refs. 8 and 11. For the plateau regime ($\epsilon^{3/2} < \nu/\Omega_{CB} < 1$, where $\epsilon = a/R_T$ is the inverse plasma aspect ratio), the density diffusion coefficient^{11,12} is (in mks units with T in keV)

$$D_n = 1.25 \times 10^{11} (T/B) (r R_C / R_T^2) e^{-W}. \quad (21)$$

The remaining coefficients are

$$D_T = (n/T) D_n (W_0 + 1), \quad (22)$$

$$K_n = T D_n (W_0 + 5/2), \quad (23)$$

$$K_T = n D_n (W_0^2 + 7/2 W_0 + 5). \quad (24)$$

The more collisionless expressions (banana limit, $\nu/\Omega < e^{1/2}$) depend explicitly on the banana width, and the ambipolar potential scales with the magnetic field as

$$\phi''/\phi' = B''/B' \quad (25)$$

where differentiation is with respect to radius.

In present experiments, $E_r < 0$ and ions are the resonant particles. (Note that for resonant particles, cancellation occurs between the $E \times B$ and ∇B precessional drifts.)

IV. PROFILE EFFECTS IN ENERGY CONFINEMENT SCALING — THE 1-1/2-D MODEL

As indicated before, profile effects near the outer edge of the hot electron ring (see Fig. 1) lead to confinement estimates for reactor-size devices that are significantly better than the estimates obtained from the simple scaling given by Eq. (1). To see this effect and to explain the results of 1-1/2-D calculations, we give the following simple arguments.

Fig. 1. Typical profiles of $U = fdl/B$, the magnetic field B , and pressures (core and ring) as a function of radius at the midplane. Rings modify fdl/B and B to support pressure gradients in regions II and III.

From conservation of energy, the total power absorbed by the toroidal core plasma P_{total} must equal the integral of the heat flux $\bar{Q}$ over the plasma surface S ,

$$P_{\text{total}} = \int \bar{Q}_{\text{se}} \cdot d\vec{S} = 4\pi^2 R r_a U_2 Q_{\text{se}}(a) \quad (26)$$

where all the quantities (a , R , U_2 , ρ) have been defined earlier. Defining the energy containment time as energy density times volume (V) divided by total power absorbed [see Eq. (11)],

$$\tau_E = \langle (3/2)nT_e \rangle V / P_{\text{total}} \quad (27)$$

we have

$$n\tau_E = (3a/4U_1 U_2) [n^2 T_e / Q_{\text{se}}(a)] \quad (28)$$

Here, Eqs. (10) and (26) are used and the $\langle \rangle$ notation (representing averages) has been dropped for brevity. The electron heat flux [see Eq. (5)] is on the order of (ignoring density gradients for simplicity)

$$Q_{\text{se}} \sim K_{Te} (\partial T_e / \partial \rho) (1 + e\phi/T_e) \quad (29)$$

where $E_r < 0$ is assumed (i.e., electrons are nonresonant) and the thermal conductivity K_{Te} is given by Eq. (19). Substituting Eqs. (19) and (29) into Eq. (28) in the collisionless limit ($\bar{\nu}_e \ll W_{\text{se}}$) and using $\nu_e = \nu_{\text{se}} + \nu_{\text{ei}} = 3.79 \times 10^{-15} n T_e^{-3/2}$ (in mks units with T in keV) and $\partial T_e / \partial \rho \sim T_e / \Delta \rho \sim T_e / l_T$, we have (in mks units)

Fig. 2. Confinement parameter $n\tau_E$ vs collisionality $\bar{\nu}_e = \nu_e / \Omega_{ce}$ for sequence of EBT devices. Dashed curves represent values of $n\tau$ calculated from Eq. (1). Solid curves (solid circles for a reactor) are results from 1-1/2-D transport calculations.

$$\pi r_E \approx 1.3 \times 10^{14} (\Delta \rho / a) (R_T / a)^4 (a^2 W_{\infty} / 2 R_{\theta} \rho)^2 T_e^{3/2} . \quad (30)$$

Conventional scaling assumes parabolic profiles valid near $\rho \ll a$. In this case $\Delta \rho / a \sim 1/2$ and $(a^2 W_{\infty} / 2 R_{\theta} \rho) \sim e \phi / T_e \sim 0(1)$ so that Eq. (30) reduces to Eq. (1). [Note that the functional form of Eq. (30) is the same as if density gradients were not ignored.]

In Fig. 2, the dashed curves represent the values of πr calculated from Eq. (1) for the sequence of EBT devices: EBT-S, -P, -Q, and -R. The solid curves show the results of 1-1/2-D transport simulations of these increasingly larger devices. Note that the confinement is better than predicted by the simple $A^{1/2} T_e^{3/2}$ scaling of Eq. (1). To understand this, we must first consider the detailed geometry for these cases, which is shown in Fig. 3.

Figure 3 shows the 2-D mod- B contours along with normalized magnetic field profiles in the midplane for the devices in the EBT-S to EBT-R sequence. Note that the depression in B due to the hot electron ring occurs progressively closer to the magnetic axis for the larger machines. This results in relatively larger gradients in B on the outer edge of the magnetic well for these cases. The ambipolar electric field is calculated from the ordinary differential equation [see Eq. (25)]

$$E'(\rho)/E(\rho) = B''(\rho)/B'(\rho) . \quad (31)$$

Note that relatively larger gradients in B on the outer edge of the magnetic well lead to larger electric fields in this same region. This in turn improves confinement for the larger machines and

allows steeper pressure gradients near the outer edge. A typical pressure profile is shown in Fig. 4. Note from Figs. 4 and 1 that the radial profile of core pressure obtained from 1-1/2-D transport simulations is similar to those theoretically required for stability.^{5,7}

These profile effects enter the simple point model of Eq. (30) through the electric field term, $W_{\infty} a^2 / 2 R_{\theta} \rho$. Global confinement time is determined not by the magnitude of this term near the magnetic axis but by its magnitude near the outer edge of the magnetic well, where diffusion is minimum. For example, near the magnetic axis, $W_{\infty} a^2 / 2 R_{\theta} \rho$ is on the order of unity for all cases in Fig. 2. However, near the outer edge it reaches progressively larger peak values of $\sim 2.2 \rightarrow 4.5$ for the EBT-S to EBT-R sequence. Using these values in Eq. (30) with $\Delta \rho / a \sim 0.5-1$ gives a reasonably close approximation to the 1-1/2-D results of Fig. 2.

V. PROFILE FORM FACTORS USED IN THE 0-D MODEL

The radial profile of stable core pressure p_c can be derived from^{5,7}

$$p_c'(r) + \alpha(r) p_c(r) = 0 \quad (32)$$

to give

$$p_c(r) = C \exp[-\int \alpha(r) dr] , \quad (33)$$

where C is constant and expressions for $\alpha(r)$ are given by^{5,7}

Fig. 3. Two-dimensional mod- B contours (a) and normalized magnetic field profiles (b) for various EBT devices. Note that depression in B due to hot electron ring occurs progressively closer to the magnetic axis for larger machines.

Fig. 4. Typical pressure profile obtained from 1-1/2-D transport simulations.

$$\alpha(r) = \begin{cases} \frac{2\gamma r}{\langle r_0 R_c(r_0) \rangle}, & 0 < r < r_0 - \Delta \\ \frac{2\gamma r}{\langle r_0 R_c(r_0) \rangle} \left(1 + \frac{\beta_h \langle R_c \rangle}{4\Delta} \right), & r_0 - \Delta < r < r_0 \\ \frac{1}{\Delta}, & r_0 < r < r_0 + \Delta \end{cases}$$

Here, Δ is the ring half-width, R_c is the radius of curvature, r_0 is the ring radius, β_h is the ring beta, and $\gamma = 5/3$ is the ratio of specific heats (see Fig. 1 for geometry).

Considering, for example, the central core plasma region in Fig. 1, $0 < r < r_0 - \Delta$,

$$p_c(r) = p_c(0) \exp[-\gamma r^2 / r_0 R_c(r_0)], \quad (34)$$

where to a first-order approximation $r_0 R_c(r_0) \approx \text{constant}$. Because $p'/p = (n'/n) + (T'/T)$, several combinations of density and temperature profiles can satisfy Eq. (34). For example, if $T'/T \ll n'/n$,

$$n(r) = n(0) \exp\left[-\frac{\gamma r^2}{r_0 R_c(r_0)}\right] = n(0) \exp(-\epsilon r^2), \quad (35)$$

where $\epsilon = \gamma / r_0 R_c(r_0)$. The density scale length is

$$l_n = -[(1/n)(dn/dr)]^{-1} = R_c(r_0)/2\gamma, \quad (36)$$

and the profile form factor is

$$\begin{aligned} f_n(r_0) &= \langle n \rangle / n(r_0) = (1/\epsilon r_0^2) [\exp(\epsilon r_0^2) - 1] \\ &= 2(l_n/r_0) [\exp(\epsilon r_0^2) - 1]. \end{aligned} \quad (37)$$

Confinement times are determined from Eq. (15) by using the scale length and profile form factor given by Eqs. (36) and (37), respectively. Here, the ambipolar electric field is calculated from Eq. (7). The resulting ambipolarity condition (ignoring the alpha particle contribution and taking $\partial T/\partial r \approx 0$ for simplicity) is given by

$$e^{W_0} = \frac{15}{\bar{\nu}} (\bar{\nu}^2 + W_0^2) \frac{(R_B/l_n) - W_0}{(R_B/l_n) + W_0}. \quad (38)$$

Here, $\bar{\nu}$ is the collisionality parameter of the nonresonant species and $W_0 = R_B |eE_e|/T$. In obtaining Eq. (38) it is assumed that the diffusion coefficient for the nonresonant particle (e or i) is given by Eq. (16) and that the resonant particle is in the plateau regime and the associated diffusion coefficient is in the form of Eq. (21). The solution for Eq. (38) is displayed in Fig. 5.

Fig. 5. Ambipolarity condition: W_0 vs collisionality parameter $\bar{\nu} = \nu/\Omega_e B$ for nonresonant particle (solution to Eq. (38)).

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