A Study of Flow and Loss Processes
at the Ends of a Linear Theta Pinch

Progress Report
for the Period

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ABSTRACT

Experimental and analytical studies of end loss from a linear theta pinch have been carried out. Analysis of reduced data on loss from a 25 cm long theta pinch has indicated: rotation at the end of pinch collapse, which appears to persist; ejection of the plasma in two modes, the first of which includes reversed, trapped fields; unique patterns of radial and axial variation of electron density in the end loss flow; substantial inaccuracies in plasma properties indicated by spectroscopy as compared to Thomson scattering. Studies of loss in a 50 cm long pinch with 50 eV, $2 \times 10^{16} \text{ cm}^{-3}$ plasma are underway. Predictive analysis indicates a unique identification of electron thermal conductivity effects with Thomson scattering data. Local magnetic probe, diamagnetic loop, continuum spectroscopy, and axial photography all indicate a highly reproducible loss event. Thomson scattering and impact pressure measurements are underway. Magnetic mirrors have been developed for addition at the ends of the theta pinch coil. A numerical study of magnetic field distributions has been completed.

Analytical studies continue to emphasize computer modeling of experiments. One dimensional codes have successfully described the behavior of Scylla I-C and of Scylla IV-P. A study of cusp confinement has been completed indicating that a potentially interesting experiment could be performed on the Penn State 50 cm pinch experiment. An investigation of thermal conduction loss and viscous heating under nearly collisionless conditions has been started.

I. INTRODUCTION

Studies of the loss of particles and energy from the ends of linear theta pinches have continued during this reporting period. Experimental studies of the transient loss from a 25 cm long theta pinch with reversed, trapped magnetic fields have been completed, and a first, major study of particle and conduction losses from a 50 cm long theta pinch experiment has moved substantially to completion. Both these experiments were conducted without mirrors or crowbar, and progress toward subsequent inclusion of these additions to the experiment has been made. Theoretical studies have continued to be conducted with a substantial base in experimental phenomenology--for purposes of testing theory as well as guiding improvements and alterations in models. Modeling of experiments at Los Alamos Scientific Laboratory has continued, as well as interpretive and predictive modeling of experiments being conducted on this project. The 0-D time dependent code has shown a remarkable ability to model a broad range of experiments. A 1-D time dependent code has been constructed, which already has proven valuable in identifying mechanisms which are active in the loss events. Relating to each of the above topics some brief interpretive comments will now be presented; the details of findings during the reporting period will comprise the body of the progress report.

The laboratory experimental work on the 25 cm long theta pinch study was completed one year ago, since that time efforts have been directed toward reducing and integrating data from the various diagnostics. It will be reiterated that all diagnostics in this experiment had 0.1 μ sec resolution. This accuracy was specified from the conception of the effort in order to study transients with high resolution. Indeed, this experiment has proven to be dominated by transients. Regarding conditions within the coil, there has been identified the simultaneous occurrence of rotation, fluting, trapped field diffusion and annihilation, adiabatic compression and expansion, and end loss. The phenomenology of the particle loss event is identifiable and unique; there appears to be the ejection of a first blast

or front which is then followed by a quasi-steady loss with no trapped fields. A unique contribution in this work is the detailed comparison of spectroscopic indications of plasma properties in the loss plasma as compared to Thomson scattering indications of properties. While this has proven to be a very complex experiment, the diagnostic studies have proven to be capable of identifying the important phenomenology of the loss events.

The studies of loss in the 50 cm long theta pinch experiment have progressed rapidly. An analysis of the experiment and thermal conduction loss effects has identified a clearly identifiable technique for evaluating electron conduction loss using the Thomson scattering diagnosis and a O-D computer code. A new Z-preamplification system has eliminated trapped field effects and has helped to produce a highly reproducible plasma. Without mirrors, and with a primary emphasis on studying conditions in the end region with Thomson scattering, loss of a cool, collisional (50 eV , $2 \times 10^{16} \text{ cm}^{-3}$) plasma is being studied. The short loss times ($6-7 \text{ } \mu\text{sec}$) are compatible with the conduct of the experiment without crowbars on the main bank. Diagnostic studies of the plasma with local magnetic field probe, with diamagnetic loops, with axial photography, with continuum spectroscopy have been completed; Thomson scattering studies are underway and these will be supplemented by local pressure measurements in the end loss region.

Studies of the effects of magnetic mirror additions at the end of the pinch coil have advanced analytically and experimentally. A computer program for the prediction of magnetic field distribution has been used to identify anticipated field strengths in all cases of interest. A separate discharge system to drive additional single-turn coils at the end of the theta pinch has been designed, fabricated and tested. A crowbar design for the mirror coil has been designed and tested; this will be adapted to the main bank.

The experimental effort has continued to result in the development of novel electronic developments; a new SCR pulse generator is reported here.

The theoretical effort during this reporting has been primarily focused on one-dimensional (axial) computer modeling of linear theta pinch experiments. The primary application has been the modeling of hot pinches (Scylla IV-P). A code which includes electron thermal conduction but which does not include ion thermal conduction has shown very good agreement with Scylla IV-P data out to 18 μ sec, after which computed ion temperatures drop faster than measured values. Two different temperature boundary conditions were studied for the exhaust, one with free flow and the other with a cold gas and cold end plate. Although distinctly different temperature behaviors were found in the exhaust region, there was a surprisingly small effect on mid-plane behavior. At 10 μ sec, there was only a 4 percent difference in T_e on-axis.

In a recent finding not detailed in this report, it has been found that for $t > 18 \mu$ sec, non-classical ion viscosity provides the necessary heating to keep the ion temperature at values which agree well with experiment.

The most uncertain part of the modeling of hot pinches ($\lambda_{ii} \geq L$) is the treatment of ion thermal conduction and ion viscosity. An alternative treatment to neglecting them is to use classical collisional expressions combined with limiters to limit the magnitude of the heat flow and the momentum flow. Limiter values were calculated using a flowing Maxwellian. This method has been incorporated into the simulation model, but the results have not been good. The ion temperature drops much too rapidly after reaching peak B field.

An alternative method of determining a limiter is to use a collisionless theory to determine the distribution function. A theory of Dreicer, who obtained a solution of the Vlasov equation for free flow out the end of a pinch, has been adopted. We have not yet used these values in the simulation code.

The 0-D code has been used to simulate the 25 cm pinch and agreement appears to be reasonably good after the trapped field diffuses away. In order to extend the 0-D modeling, post-implosion trapped fields and column rotation are being added.

A study was conducted of cusp confinement of linear theta pinches. The predictions of various theories were examined and applied to experimental parameters of the Penn State pinch. Depending upon the sheath thickness of the sheath, substantially different results were achieved. A collisionally dominated sheath showed little improvement in confinement while an ion Larmor radius thickness indicated a factor of 10 improved confinement.

Additional planned work on implosion modeling was not completed because of the departure from the University of an M.S. student who decided not to pursue doctoral studies.

As in previous years, there has been fruitful cooperation with Professor James Robinson of the Electrical Engineering Department in the completion of this work. The conduct of the detailed experimental and theoretical work has been the responsibility of individual graduate students engaged on this project; their names are noted following the title of identifiable research efforts.

II. TRANSIENT LOSS FROM 25 CM LONG THETA PINCH WITH REVERSED TRAPPED FIELDS (Heidrich, Jacoby, Robinson)

I. Introduction

Previous discussions of this experiment have appeared in earlier progress reports^{1,2}, and concentrated first upon description and operating characteristics of the apparatus, and second upon phenomenological descriptions of the data. During this reporting period, there has been a strong emphasis on precise data reduction, data interpretation and development of a loss model. There has been an effort to carry out complete analyses of the data available from this experiment because of the renewed interest in reversed trapped field linear theta pinch devices^{3,4}. The work by Linford and his colleagues at Los Alamos has resulted in reported plasma confinement times up to 50 μsec in a 1 m long machine, with a dynamic transient period of about 5 μsec at the beginning of confinement. The work to be described below presents data and analyses that will help to understand the dynamic interactions occurring during the transient period of reversed field trapping, trapped field diffusion and annihilation, and the subsequent quasi-steady end loss with no trapped field.

It should be noted that this work forms the basis of a Ph.D. thesis effort by J. Heidrich, and, as such, will be presented in comprehensive report form in the near future. Accordingly, this reporting will only briefly review some of the more important descriptions and results.

2. Description of Experiment

The parameters of the experiment, a figure indicating variation of discharge current and axial magnetic field, and an outline of the diagnostics used in this experiment are given in Table I, Figure 1 and Table II, sequentially. It should be noted that the diagnosis of electron properties (T_e , n_e) in this experiment was originally intended to be exclusively spectroscopic; for this reason helium was chosen as the test gas. With the completed development of the Thomson scattering diagnostic occurring at the end of planned experimental work on the 25 cm pinch, and since numerous questions regarding the accuracy of the

Table I. 25 cm Theta Pinch Parameters

Geometry

Coil Length: 25 cm Coil I.D.: 10.5 cm Tube I.D.: 9 cm

Preionization System: RF Coils Followed by a Theta Discharge

Theta Discharge:

Max theta electric field: 96 volt/cm
 Half cycle time: 2.25 μ sec
 Ring time before ignition of main bank: 9 μ sec

Main Bank:

Charging voltage: 18.5 kV
 Total system capacitance: 100.1 μ F
 Total system inductance: 90.4 nH
 Coil inductance: 32 nH
 Peak current: 548,000 amp.
 Peak driving magnetic field: 19.7 kG
 Half cycle time: 9.45 μ sec
 Maximum theta electric field: (vacuum) 147 volt/cm
 Average over first quarter cycle: (vacuum) 94 volt/cm

Fill Gas: Helium at 100 mT

Figure 1. Current and Magnetic Field Waveform

Table II. Diagnostics Used for 25 cm Theta Pinch Studies

<u>Quantities Measured</u>	<u>Diagnostic Used</u>
Bank Current	Rogowski Coil
Local Magnetic Fields	Pickup Loop Probes
Local Impact Pressure	Specially Designed Piezoelectric Transducer Pressure Probe
Luminosity	Photography Using Kerr Cell Shutter
Plasma Diamagnetism	Balance Signal From a Rogowski Coil and a Flux Loop
Electron Density	Stark Broadening of He^+ 4686Å Line to Underlying Continuum; Thomson Scattering; Absolute Intensity of He^+ 4686Å Line

spectroscopy had developed, Thomson scattering was used to establish conditions at critical points in the experiment lifetime. These data are simply included in the presentation of data to follow, but the specific question of the accuracy of spectroscopy and comparison of its n_e , T_e predictions with those of Thomson scattering is presented in a separate section.

3. General Description of Plasma Behavior

With the comprehensive survey of data taken with the diagnostics outlined above, and without specific discussion of detailed considerations, the phenomenology of the experiment and the loss events can be identified. These are presented in Table III.

Some specific information derived from diagnostics is helpful in understanding the dynamic nature of the plasma being studied. First, from excluded flux measurements with a compensated loop², the parameter, r_{eff} , effective plasma radius, can be determined. While r_{eff} is not equal to column radius, it is proportional, and Figure 2 demonstrates the simultaneous occurrence of bouncing with rotation, fluting and field diffusion. Second, the changing character of this plasma is further identified by spectroscopic studies of the plasma within the coil. Presuming uniform temperature across the column, the absolute intensity

Figure 2. Effective Plasma Radii vs. Time

Table III. Characteristics of Experiment

I. Phenomenology of Experiment

1. Preionization results in trapped field ($\sim 20\%$ at $z = -2.5$, 300 Gauss)
2. Initial period with reversed trapped field in column (2 μsec , 6 KGauss)
3. Initial period with radial column oscillation (2 μsec)
4. Cool, high density plasma ($T_e \sim 13 \text{ eV}$ $n_e \sim 5 \times 10^{16} \text{ cm}^{-3}$)
5. Diffusive elimination of trapped field and oscillations ($\tau_D \sim 1 \mu\text{sec}$)
6. Column rotation with fluting ($\omega \sim 4 \times 10^6 \text{ rad/sec}$)
7. Following period of quasi-steady loss in parallel fields (2-6 μsec)

II. Phenomenology of End Loss Events

1. No loss evident before pinch collapse
2. Initial convection includes reversed trapped field
3. Following period of quasi-steady loss in parallel fields of increasing and decreasing magnitudes

of the $4686\text{\AA}$ He II line was used⁵ to determine T_e at the mid-coil at various times in the column lifetime (Figure 3). The effects of bounce and adiabatic compression are clearly evident, as well as the effects of end loss of particles on mid-coil temperature. An analytic model of magnetic field diffusion in the implosion generated plasma allows the determination of resistivity from measured B field profiles; these T_e indications are uniformly consistent with those from spectroscopy. The dominant influence of diffusion in this low temperature plasma does lead to the annihilation of the trapped field at about 2 μsec ; this behavior is shown in Figure 4, where radial variation n_e and B_z are plotted for two different times at the $1/4$ coil position.

One specific aspect of this study of the plasma within the pinch coil that is significant, relates to the indications of column rotation. Some fragmentary evidence of rotation was apparent through doppler shift in line radiation studies. Also, the growth rate of the flute

Figure 3. Electron Temperature vs. Time at $z = +12.5$ cm (midcoil)

Figure 4. Radial Distribution of B_z and n_e at $z = +6.25$ cm (1/4 coil)

instabilities has been used to predict a column rotation of $\omega = 3.8 \times 10^6 \text{ sec}^{-1}$. However, by comparison of energy line density derived from excluded flux data and from integrated spectroscopic measurements, the history of rotation can be reconstructed. Specifically, energy line density is defined as

$$E_L = \int_0^R p \frac{3}{2} nkT2\pi r dr$$

which can be evaluated from spectroscopic work. Then, for a rigidly rotating plasma column with a Gaussian density distribution the pressure balance is

$$p + \frac{B^2}{8\pi} = \int_0^r \omega^2 \rho(r) r dr - \frac{a^2 \omega^2 \rho(0)}{2} + \frac{B_\infty^2}{8\pi}$$

which when integrated over the cross-sectional area, multiplied by 3/2 and reduced for low β one finds

$$\omega = \left(\frac{4(E_{LS} - E_L)}{3\pi \rho(r=0) a^4} \right)^{1/2}$$

where E_{LS} is the line density assuming static balance, and a is the column radius. E_{LS} can be derived from diamagnetic loop data, and the difference ($E_{LS} - E_L$) is indicative of ω . Reduced data is presented in Figure 5, where several facts should be noted. First, there is an indicated rotation immediately on collapse; there has been controversy⁶ over the mechanism for initiation of rotation and this is unique evidence of rotation induced during implosion for a collisional, cool theta pinch plasma. Second, the indicated magnitude of the rotation is consistent with that indicated from flute growth rates. Third, in the figure, the drop of loop line density below spectroscopy is an artifice of the experimental technique.

4. Description of Dynamics of the Loss Event

The plasma from within the coil region moves out the ends with a distinct phenomenology. There is the definable progression of a front; there appears to be a first (burst) region which is then followed by quasi-steady loss of decreasing magnitude with time. As

Figure 5. LN (Energy Line Density) vs. Time at $z = +12.5$ cm (midcoil)

the plasma progresses into the end loss region, there is first a steepening and then a diminishing of plasma flow. These events are shown graphically on impact pressure histories at $r = 0$ and at various axial locations in Figure 6. The progression of plasma n_e , T_e from within to outside the coil was derived from spectroscopic and Thomson scattering measurements; the axial variations with time are presented in Figure 7 (a) and (b). One of the more interesting aspects of these figures is that there is an apparent build-up and heating of plasma in the end-loss region as time progresses.

One of the more important intentions of this experiment was to attempt to utilize measurements of n_e , T_e , p_i (impact pressure) to indirectly indicate flow velocities in the end loss event. For the 2.5 cm location outside the coil, a compatible interpretation of velocity is presented at three different times in Figure 8. Detailed comparisons of these indicated velocity magnitudes with sonic speed, cusp speed, and Alfvén speed will be presented as part of a forthcoming report.

One general indication of the loss event in this theta pinch is the time history of particles within the pinch coil. With the local diagnostics that were used (spectroscopy, impact pressure) two independent histories of the mass within the plasma column from mid-coil to 2.5 cm outside the coil can be constructed. Integration of spectroscopic density profiles provides total mass which is presented in Figure 9, along with total mass derived from initial mass minus the pressure-spectroscopy indicated flux at $Z = -2.5$ cm. Considering the error bars on this data, these two schemes are in general agreement. Considering the linear scales used in the plot, however, there does not appear to be loss in the standard $e^{-t/\tau}$ form.

5. Density Profiles in the End-Loss Region; Comparison of Spectroscopic and Thomson Scattering Indications of Plasma Properties

The characteristics of plasma flow into the end loss region include flow with rapid expansion and cooling; these occurrences can lead to nonequilibrium in the population levels of the helium and so

$z = -2.5 \text{ cm}$

P
(Rel. Units)

$z = -5.0 \text{ cm}$

P
(Rel. Units)

$z = -7.5 \text{ cm}$

P
(Rel. Units)

$z = -10.0 \text{ cm}$

P
(Rel. Units)

Figure 6. Impact Pressure History at Various Axial Positions in the End Loss Region

a. Axial Variation of Electron Line Density

b. Axial Variation of Electron Temperature

Figure 7. Axial Variation of Local Line Density and Temperature

Figure 8. Endless Region Radial Profiles of n_e , B_z , Pressure, and Indicated Velocity at $Z = -2.5$ cm (outside coil)

Figure 9. Total Electron Inventory Within Coil to $z = -2.5$
From Integrated Density and From Integrated Flux

perturb the accuracy of the determination of plasma properties. This question was addressed in detail. Also, with a limited amount of experimental effort with a new Thomson scattering diagnostic system, the question of the accuracy of the spectroscopy, in comparison to the Thomson scattering, indicated plasma properties was addressed. With the determination of radial density profiles at two different axial locations at several different times, the spreading of the plasma as it flows into the diverging field lines can be identified.

Considering the expanding plasma, in order to get a measure of electron temperature from the absolute intensity of the He II (4-3) 4686 Å line, both levels of the line must be in local thermodynamic equilibrium. Choosing the $p = 3$ level to be of primary interest, the time scale to maintain equilibrium with the free electrons was determined from rate equations presented in the literature⁷. Specifically, for plasma expanding from $4 \times 10^{16} \text{ cm}^{-3}$ and $T_e = 10 \text{ eV}$ to 2×10^6 and $T_e = 6 \text{ eV}$, the equilibration time for the $p=3$ level is about 10^{-9} sec, so it is in equilibrium. Similarly, significant recombination could perturb the plasma and invalidate analysis of data. Again, following BKM⁷, a recombination time of 25 μsec is indicated. It can be concluded that line intensity measurements are valid in this expanding plasma. Radially uniform temperature was assumed in the plasma column.

The density was first determined spectroscopically from line broadening⁵. Following Abel inversion of the line profiles taken at chords, subtraction of Doppler and instrument broadening, electron densities were determined at 2.5 cm outside the coil at different times. It was not possible to complete these measurements further into the loss region ($Z = -7.5 \text{ cm}$).

Thomson scattering measurements of density and temperature were carried out with a system that has been fully described² at locations of $Z = -2.5 \text{ cm}$, $r = 6.5 \text{ mm}$; $Z = -7.5 \text{ mm}$ and $r = 8 \text{ mm}$. Continuum radiation profiles at 4730 Å were taken to provide relative density variations. A comparison of density profiles from line broadening compared to those from Thomson scattering plus continuum is presented in Figure 10. Clearly, the spectroscopic technique is poor by comparison. This was influenced by scatter in the data as: (1) a single shot

Figure 10. Comparison of Electron Density Profiles from Thomson Scattering and Continuum with Stark Broadening of 4686 Å Line Radiation at $z = -2.5 \text{ cm}$ (outside Coil)

gave intensity at one wavelength on one chord; and (2) resolution of 0.5 cm was used and gradients were evident on that scale.

An improved spectroscopic technique has also been developed to avoid the situation indicated above. Line data on center axis are taken along with continuum radial profiles which are Abel inverted. An iteration scheme is then used where a guess at n_e ($r = 0$) allows one to reconstruct the 4686 Å line profile from the contributions at all radii. Scatter was not severe in this technique as an integration procedure is used. A comparison of Thomson scattering with this reconstruction technique is shown as Table IV; while the agreement is not good, it is better than with simple line-broadening presented above.

Table IV. Comparison of Accuracy of Techniques for Normalizing Profiles

4686 Å Radiation + Continuum Versus Thomson Scatter + Continuum

A. Laser Results

<u>Time</u>	<u>Density at $r = 0$</u>	<u>Percent Error From Mean</u>	<u>Temperature</u>
1.7 μsec	$(0.97 \pm .18) \times 10^{16} \text{ cm}^{-3}$	11.2%	$5.2 \pm .53 \text{ eV}$ (5.6%)
3.0 μsec	$(2.28 \pm .41) \times 10^{16} \text{ cm}^{-3}$	11.9%	$5.2 \pm .39 \text{ eV}$ (4.9%)
4.0 μsec	$(2.17 \pm .66) \times 10^{16} \text{ cm}^{-3}$	19.0%	$5.2 \pm .84 \text{ eV}$ (9.4%)

B. Spectroscopic Results

<u>Time</u>	<u>Density at $r = 0$</u>	<u>Percent Error From Mean</u>	<u>Temperature</u>
1.7 μsec	$(2.2 \pm .67) \times 10^{16} \text{ cm}^{-3}$	30.5%	10.5 eV
3.0 μsec	$(3.6 \pm 1.04) \times 10^{16} \text{ cm}^{-3}$	29.0%	15.0 eV
4.0 μsec	$(2.0 \pm 1.26) \times 10^{16} \text{ cm}^{-3}$	63.0%	9.0 eV

From the Thomson scatter plus continuum technique, the profiles of n_e in the end loss region are plotted in Figure 11 at $t = 2.0 \mu\text{sec}$ and $t = 5.0 \mu\text{sec}$ along with vacuum magnetic field lines. Several observations can be made. At $t = 2 \mu\text{sec}$ the plasma is highly collimated on-axis; at $t = 6 \mu\text{sec}$ the plasma is peaked on-axis at $z = -2.5 \text{ cm}$ but

Figure 11. Electron Density Radial Profiles Compared to Magnetic Field Expansion in End Loss Regi
(Vacuum Field Lines)

is peaked off-axis at $z = -7.5$ cm. There is indicated radial spreading of the loss plasma, but it is not severe or sudden in its axial progression. Indeed, there is an indication of a depressed density on axis in the quasi-steady flow at 5.0 μ sec.

6. Summary of Experiment

General characteristics of the phenomenology of the loss events in this experiment are presented in Table V.

Table V. Summary of Source and End Loss Plasma Behavior

I. Phenomenology of Source Plasma

1. Transient period after collapse $1.15 \leq t \leq 2.0 \mu\text{sec}$
 - a. $T_e \sim 12 \text{ eV}$ $n_e \sim 5 \times 10^{16} \text{ cm}^{-3}$
 - b. Reverse trapped field and column oscillations

$Z = +12.5$ (midcoil)	Reverse Trapped Field = 6 kG
$t = 1.15 \mu\text{sec}$ (pinch)	Driving Field = 7 kG

Field diffusion causes disappearance of reverse trapped field and damping of column oscillations.
 - c. Column rotation with fluting

Fluting starts at $t = 1.5 \mu\text{sec}$, upon second implosion
2. Quasi-steady loss period in parallel fields: $2 \leq t \leq 6 \mu\text{sec}$
 - a. Midcoil's particle inventory decays while the inventory from quarter coil to the end coil is constant: $2 \leq t \leq 4 \mu\text{sec}$
 - b. Particle inventory throughout the coil drops: $t > 4 \mu\text{sec}$
 - c. Flutes disappear: $t \approx 2.8 \mu\text{sec}$
 - d. $m = 2$ profile seen on many shots: $t \geq 2.8 \mu\text{sec}$
 - e. Plasma rotation persists

II. Phenomenology of Plasma in End Region

1. No flow evident before collapse; $t = 1.15 \mu\text{sec}$
2. $t \approx 1.7 \mu\text{sec}$ loss starts and convects some reverse trapped field out of the coil. R_M (Magnetic Reynolds No.) ~ 10
3. Quasi-steady loss in parallel fields: $2 \mu\text{sec} \leq t \leq 6 \mu\text{sec}$
 - a. Typical flow velocities in endloss region $\sim 6 \text{ cm}/\mu\text{sec}$ implying a Mach ≈ 2 flow.
 - b. Local line density tends to increase as the flow expands into the endloss region.
 - c. No strong axial temperature gradients exist.
 - d. The density profiles are more collimated than would be the case if they followed the diverging field line in the endloss region.

III. EXPERIMENTAL STUDIES OF END LOSS WITH THE 50 CM LONG THETA PINCH SYSTEM (Jacoby, Heidrich, Robinson)

1. Introduction

The alteration and enlargement of the energy storage, switching, and discharge apparatus to drive the 50 cm long theta pinch experiment was accomplished during the first part of the contract year. Physical changes in the system included: (1) addition of four new rows of capacitors; (2) design and testing of new switches for each row of capacitors; (3) design and development of a new 2-preionization system; (4) design and fabrication of a new header assembly to handle the expanded bank and possibly a crowbar system; (5) design and testing of a bank crowbar system. All these changes have been satisfactorily accomplished with the exception of the bank crowbar. As will be discussed in detail below, the vacuum arc crowbar that was adapted from published designs did work, but could not operate without continual maintenance for gap cleaning to eliminate breakdown. On that basis it was unsatisfactory for standard operation. The alternative design using a series of high pressure spark gaps has been tested for the magnetic mirror coil crowbars, and it is anticipated that this type of switch will eventually be adapted to the main bank.

The initial study of loss from the 50 cm long theta pinch was intended to concentrate on detailed mapping of T_e and n_e in the end loss region using the Thomson scattering diagnostic, to consider loss transients in the new configuration without trapped reversed field, and to address the identification of thermal loss effects from the end of the theta pinch. The anticipated loss time from the 50 cm coil for the plasma that is appropriate for complete mapping of the end loss region by Thomson scattering is on the order of 7 μ sec, while half-cycle time is 10.0 μ sec. Accordingly, with the difficulty that was encountered with the bank crowbar, and as this first study can be adequately completed without the crowbar, work has moved forward without a bank crowbar. The addition of mirrors is expected to increase the loss time, and the crowbar system for the main bank, as well as the mirror coils themselves will be developed and installed before those subsequent loss studies are carried out.

Considerable progress has been made in mapping the 50 cm theta pinch plasma in the desired test configuration. Initial studies of the plasma required some changes in experimental conditions, but these have been successfully completed and are detailed below. Thomson scatter diagnosis of plasma within the theta pinch coil and in the end loss region is currently underway.

2. Description of the Theta Pinch Electrical Discharge System

The energy storage capability that is presently operational includes three rows of capacitors from the original system (each row $4 \times 7.7 \mu\text{F}$) and five rows with two Maxwell Lab. capacitors ($14.0 \mu\text{F}$) rated at 20 kV. The charging system and controls remain the same as in the earlier machine. However, all switches for rows of capacitors have been changed from direct overvolted gaps to swinging cascade gaps⁸. This type of triggering requires much less sensitive gap settings, and all eight rows can be reliably discharged within 0.1 μsec of each other. The switch for each row is monitored with a digital clock so that problems and correct settings are easily identifiable.

However, one problem did develop with this system of triggering which resulted in overloading and failure of several capacitors. Less sensitive gap settings would allow a row to remain charged when other rows began their discharge; with the induced voltage excursions, the overvolted gap would then break down and overload capacitors. The installation of safety gaps at the end of each row have eliminated this problem; they are set to discharge when the voltage across them is in the order of 20 kV, thus negating any capacitor overvoltage. The triggering of each switch has also been redesigned to include a resistor capacitor array at the gap itself, which provides a fast rising current pulse on signal from a master gap pulse generator. The previous system had used a single capacitor driving all switches through about 6 ft. of cable. The master gap signal generator has progressed from an induction coil design, to a thyration trigger circuit, to a new design SCR pulse generator. This latest design pulse generator is considered an advance

in state-of-the-art capability and is being prepared for submission as a journal publication; it uses either series or parallel coupled SCR's and is powered by low voltage (DC) batteries. A general discussion of this device will be presented below.

The parameters of the electrical discharge of the theta pinch-capacitor bank are shown in Table V.

Table V. 50 cm Long System Characteristics

Theta Pinch Coil Length	50 cm
Theta Pinch Coil i.d.	10.5 cm
Total Capacitance	232.4 μ F
Charging Voltage	18.0 kV
Bank Inductance	24.5 nH
Coil Inductance	18.6 nH
Peak Field	23 kG
Peak Current	1.05 MA
Quarter Cycle Time	4.7 μ sec
Half Cycle Time	10.0 μ sec

It can be noted from the table that the bank waveform is not precisely sinusoidal. This results from paralleling the four new rows with the old four rows. The new rows use much shorter cables and have less capacitance per row. When these two banks were combined the desirable result for enhanced dynamic pinch effects proved to be a bank that has a slightly faster rise time than fall time during its first half cycle.

3. Z-Preionization System

a. Design criteria

The use of a z-pinch preionization system (ZPI) eliminates such problems as unwanted reversed trapped fields, over- and under-stressing the main bank spark gaps during the P.I. phase, and stringent timing constraints on the main bank firing as can occur when a theta pinch preionization system is used. The ZPI system also holds the

advantage of providing better coupling between available circuit energy and the plasma by means of ohmic heating. A convenient ZPI would provide a discharge of a specific duration which would thereafter be immediately damped with no ringing or residual current. In practice it has been found possible to closely approximate this type of behavior.

The concepts utilized here were developed at Los Alamos Scientific Laboratory and recently documented in an article by DeSilva and Pechacek⁹. The design of such a system can be accomplished through the use of variable resistors or varistors which possess a V-I characteristic of

$$V = kI^n \quad (1)$$

In this case, (these varistors were provided by K. F. McKenna, Los Alamos Scientific Laboratory)

$$V = 500 I^{.26} \quad (2)$$

From the circuit, which is a series RLC configuration, the equation for the current can be written as

$$L \frac{dI}{dt} + kI^n + \frac{1}{C} \int_0^t I dt + \frac{Q_0}{C} = 0 \quad (3)$$

where Q_0 is the initial charge placed on the capacitor, C. Equation 3 can be normalized to

$$\frac{d^2q}{dT^2} + \left(\frac{dq}{dT}\right)^n + q = 0 \quad (4)$$

using

$$\begin{aligned} T &= \omega t \\ \omega &= 1/\sqrt{LC} \\ q &= Z\omega Q \\ I &= dQ/dt \\ Z &= [k\omega C]^{1/n-1} \end{aligned}$$

This equation can be numerically solved for a given n and initial q , q_0 . By iterating a number of different q_0 's through a routine, it can be quickly ascertained when rapid damping or clamping of the current tail occurs. In this system, clamping occurs after the first half cycle. Once the desired q_0 has been selected for a given capacitance and charge

voltage, the quantity Z can be calculated, and from Z the necessary k can be derived. A specific configuration of parallel and series varistors is chosen from

$$k_{\text{eff}} = \left(\frac{s}{pn}\right)k \quad (5)$$

where p is the number of paralleled groups of s varistors in series. A last physical constraint on the varistors dictates the minimum number of paralleled groups, the current handling capability of the varistor. For units used here, it has been found that they can handle around a 100 μsec pulse of 10 kA⁹. This provides sufficient information to adequately design the varistor network.

The choice of inductance, capacitance, and charging voltage were determined by considering plasma and geometry characteristics. Geometry limitations dictate how low an inductance can be achieved since most of the ZPI inductance is contained in the discharge and current return loop. The plasma constraints arise from considerations of best achieving preionization. It is desirable to have the plasma undergo dynamic pinch in order to ensure complete sweeping of the fill gas and the chance for the entire volume to be affected by the discharge. However, to avoid interaction with the discharge chamber walls because of instabilities, the plasma should be allowed to expand after pinch. A pinch at current maximum would be the goal. Maximum current is also a useful feature, since a stronger (hotter) pinch with more ohmic heating is then achieved, which ensures better preionization of the fill gas. Finally, an attempt to minimize inductance should be made; this produces larger peak current from a given circuit and also a faster rise time of the current pulse.

b. Implementation of design criteria

The PI system on the 50 cm theta pinch consists of a Marx¹⁰ circuit containing a 14 μF capacitor rated at 20 kV and a 2.8 μF capacitor rated at 50 kV. This circuit is shown in Figure 12. This Marx circuit is used to create a Z discharge in the fill gas with a half cycle of 5 μsec and a peak current of 24.2 kA. After the first

Figure 12. ZPI Marx Circuit

half cycle the current is immediately damped out by varistors placed in series with the 2 discharge current (Figure 13).

One other aspect of the ZPI system as it presently exists should be noted. Four return cables are mounted in a symmetric manner around the discharge tube. This attempts a cancellation of gross $J \times B$ forces on the plasma in order to keep the plasma centered in the discharge tube. With a single, minimum inductance return path used originally, reproducibility of continuum radiation (at $4726 \text{ \AA}$) from the PI plasma was quite poor. With the symmetrical current return arrangement, however, the ZPI plasma radiation is highly reproducible.

The ZPI performance has shown itself to be quite adequate in generation of sufficient ionization. A magnetic field probe placed at radii of 0 cm, .5 cm, 1 cm, 2 cm, and 3 cm indicates zero field when the main bank is fired, and it remains zero until the plasma sheath passes by it. With the emplacement of the Thomson scattering diagnostics, the precise percentage PI will be identified, but the results of present work have proven satisfactory.

4. Results of Experimental Studies of 50 cm Long Theta Pinch Plasma

a. Introduction

With the discharge circuit and preionization behavior noted above, detailed studies of the plasma generated by the 50 cm long coil were begun. A discharge tube with 7.62 cm i.d. was used, consistent with planned mirror work. The results of an initial study of the theta pinch plasma with a single return strap for preionization with hydrogen gas fill resulted in generally unreproducible data. Further, preliminary evaluation of plasma behavior with diamagnetic loops and continuum spectroscopy indicated that plasma was lost in about 4 μsec . Reconsideration of the plasma properties under test conditions of interest indicated that the temperature-density-gas combination produced a plasma that could be strongly diffusion influenced; as this behavior could be alleviated by use of deuterium, the test gas was changed and deuterium will be used in all subsequent tests. With azimuthally symmetric return PI electrodes and deuterium gas, the theta pinch plasma to be discussed

Figure 13. Discharge Current from ZPI Circuit

below has proven to be well behaved, with a lifetime of about 6.7 μsec based on 10 percent of original energy line density.

One primary constraint that has fixed the fill pressure, through the plasma temperatures and densities required after pinch, is the resolution of the Thomson scattering diagnostic. In order to map densities and temperatures in the end region as completely as possible, and so provide definition data on particle and thermal loss in the end loss region, a relatively high fill pressure of 40 mT deuterium has been chosen. The pinched column plasma is collisional in character, but the loss processes will be available to be fully explored.

The 50 cm theta pinch has been operating from a base pressure of 0.1 mT and a 40 mT fill of D_2 . Three diagnostics have been used on the 50 cm theta pinch: magnetic field probe, diamagnetic loop, and continuum radiation spectroscopy. From these diagnostics applied at $Z = -25$ cm (midcoil) and $Z = +15$ cm (outside coil) the plasma radius, beta on axis, and energy line density have been determined.

Predictions of these diagnostics at $Z = -25$ cm (mid coil) have also been compared with the predictions of the zero dimensional code developed on this project. This code normally is started at experimentally determined conditions. However, with lack of temperature and density information at the present time, this could not be done. Rather, a simple snowplow implosion² driver program was used to provide initial information on plasma radius, density, and temperature. The implosion driver and zero-dimensional code have no provision for trapped fields. However, diagnostics, including axial photography, indicated that a small amount of forward trapped field exists upon implosion, but that about 0.7 μsec later the density profile has smoothed sufficiently to permit a reasonable fitting to a Gaussian profile. Thus, this limitation on the program is not felt to be too significant; in any event, detailed studies to include trapped field are proceeding and do not indicate important changes.

b. Magnetic field probe studies

The axial magnetic field distribution was investigated with a local, multi-turn coil enclosed in a 0.5 cm diameter pyrex tube. This was done for axial positions of $Z = -18.4, -12.1, -5.9, 0$ (coil end), 3, 6, 9, 12, 15 cm and for radii of 0 cm to 3 cm in 0.5 cm increments. It was found that when the probe was inside the pinch coil and inside the plasma column, its presence significantly perturbed the signal of a diamagnetic loop (around the outside of the discharge tube) at the Z position of the probe. At other Z positions the diamagnetic signals were unperturbed. A check was made on the local probe results by the following procedure. The plasma beta on axis at $Z = -25$ cm (mid coil) was determined using the diamagnetic loop and continuum data using the presumption of a Gaussian profile. These results were then compared to the plasma beta determinations derived from the field probe data taken at $r = 0, Z = -18.4$ cm. These two results were in turn also compared to the beta values on axis taken from the zero-dimensional code predictions. All three items are displayed on Figure 14. As can be seen, the probe and loop results are quite close, and both compare reasonably well with the code predictions. Thus, one can reasonably accept the values indicated by the probe even though local diamagnetic loop measurements indicate a perturbation on the plasma from the probe.

Another question regarding local probe measurements related to ablation effects. Using the results given by Lovberg¹², the time without perturbation can be calculated to be about 1 μ sec for pyrex; however, this appears to be a conservative value as indicated by the above results.

Other parameters that can be derived from the magnetic field probe data include the plasma column radius, plasma diamagnetism, and indications of rotation. The radius and diamagnetism are also indicated by the continuum radiation and diamagnetic loop, respectively. Some preliminary evaluations of data have been made and comparison between these diagnostics have disclosed good agreement. Plasma column radius values from the probe and from the continuum radiation are comparable, as are diamagnetism values determined from the probe and the diamagnetic loop. With reliable magnetic field data, the magnitude of column rotation can be derived from magnetic pressure indicated by the probe in combination with the kinetic pressure of the plasma derived from density

Figure 14. The Beta on Axis as a Function of Time as Derived From the Magnetic Field Probe, From the Diamagnetic Loop, and From the Zero-Dimensional Code Results

and temperature measurements. These latter measurements will be made by a Thomson scattering diagnostic which is presently being installed on the machine.

c. Continuum radiation studies

Measurements of continuum radiation were performed using a Jarell-Ash 0.5 m spectrometer with a 1P21 RCA photomultiplier as a detector. It was determined that the 4726 Å region was relatively clear of line radiation in this system. This was verified by moving off 4726 Å by about 5 Å and failing to make any significant change in signal. Slots cut in the theta pinch coil for Thomson scattering were used for the continuum diagnosis also. Data were taken at $Z = -25$ (mid coil), -18.4, -12.4, -5.8, 3, 6, 9, 12 and 15 cm.

The data at $Z = -25$ cm and $Z = +15$ cm have been Abel inverted¹³ and least squares fitted to a Gaussian distribution. During times shortly after pinch (0.8 μ sec) there was a small amount of forward field imbedded in the center of the plasma column. However, by 1.5 μ sec (or 0.7 μ sec after pinch) this field was distributed sufficiently such that a Gaussian fit could be made. These results for mid coil are shown on Figure 15, where the Gaussian 1/e radius is compared to the parabolic radius results developed by the zero-dimensional code. Once again these two results compare favorably.

By comparison, results from the Abel inversion of continuum radiation at $Z = +15$ cm are not at all steady. The 1/e radius oscillates around a value of 2.2 cm; this is in a tube with internal radius of 3.375 cm. Magnetic field probe results at this position show a flat profile up to a radius of 3 cm, which was the maximum radius probe position. Also, starting near $Z = +12$ cm, a noticeable discoloration of the discharge tube was seen after about 100 shots; this lends credence to the notion that plasma contacted the wall by 15 cm from the end of the pinch coil.

Figure 15. Plasma Radius as a Function of Time as Derived From an Abel Inverted Continuum Profile at the Center of the Machine and From the Zero Dimensional Code Results

d. Diamagnetic loop studies

The diamagnetic loop used is of the type described by Green¹⁴; it utilized one small coil inside the theta pinch coil and a large loop outside the machine. This method of compensated loops was chosen due to its ease of construction and balance. The balancing is accomplished by moving the large outer loop relative to the main pinch coil.

Two variables can be measured with this diagnostic: local energy line density¹⁵ and effective plasma radius. Energy per unit volume is given by

$$E = \frac{3}{2} n(T_i + T_e) = \frac{3}{2} P \quad (6)$$

but from pressure balance and a low beta approximation

$$E_L = \frac{3}{8\pi} B_{ex} S \quad (7)$$

where S is the diamagnetic signal. The semilog variation of E_L provides a measure of energy decay time.

The effective flux exclusion radius is determined from

$$r_{eff}^2 = \frac{S}{\pi B_{ex}} \quad (8)$$

Knowing the plasma pressure profile and using pressure balance as

$$P = \beta \frac{B_{ex}^2}{8\pi} \quad (9)$$

an expression for β_A can be calculated where β_A is the beta value on axis. This is given by

$$S = 2\pi B_z(\infty) \int_0^\infty [1 - \sqrt{1 - \beta(r)}] r dr \quad (10)$$

where

$$\beta(r) = \beta_A f(r) \quad (11)$$

Some common radial distributions for $f(r)$ are sharp boundary, Gaussian, or rigid rotor, and from these, β_A can be determined.

The energy line density was calculated using a Gaussian distribution from the diamagnetic loop signals at Z positions of -25 (mid coil), -18.4, and -12.4 cm (quarter coil); the results are presented in Figure 16. A curious point is that the energy line density at Z = -18.4 is slightly larger than that at Z = -25. A least squares fit to an exponential between the times of 4 μ sec to 7 μ sec gave an average energy decay time of 2.06 μ sec. With this value, calculations can be carried out to establish the order of magnitude of plasma temperatures. With the presumption that the major loss mechanism is convection (neglect conduction), and that plasma heating processes are inactive (which is reasonable at 4 μ sec) then particle loss times can be related to energy loss times. From the fact that $5/2 kT$ is lost per particle from the plasma and

$$\frac{1}{E_L} \frac{\Delta E_L}{\Delta t} = \frac{1}{\frac{3}{2} nkT} \frac{\frac{5}{2} kT \Delta n}{\Delta t} \quad (12)$$

or

$$\tau_p = \frac{5}{3} \tau_E \quad (13)$$

Then, using

$$\tau_p = \frac{L}{2} \sqrt{\frac{m_i}{2 kT_T}} \eta \quad (14)$$

where $T_T = T_i + T_e$, an estimate of the plasma temperature can be arrived at. Using an endloss model developed by Taylor and Wesson¹⁶ for $\beta = 0.25$, $\eta = 1.2$ it is found that $T_e = T_i = 28$ eV. This estimate agrees well with the zero dimensional code result of 28 eV for an average temperature during the time of 4 μ sec to 7 μ sec. As another point of comparison between the data and the zero dimensional code predictions, the energy line density at Z = -12.4 cm and the energy line density predicted by the code are presented in Figure 17. The agreement is again reasonable. The beta on axis results were presented earlier, and they are also in agreement with both diagnostics and code predictions.

Figure 16. The Natural Log of the Energy Line Density as a Function of Time for the Positions of $Z = -25$, $Z = -18.4$, and $Z = -12.4$ cm.

Figure 17. The Natural Log of the Energy Line Density as a Function of Time for the Position of $Z = 12.6$ cm and from the Results of the Zero Dimensional Code

5. Analysis of Thermal Loss Effects for the 50 cm Long Theta Pinch Experiment

The mapping of n_e and T_e in the pinch coil and in the end loss region should allow for a unique specification of conditions related to thermal conduction losses. The general orders of magnitude of loss parameters in this experiment have been established, and the details of a procedure for determining thermal loss effects have been laid out. These will now be presented.

Calculations of conditions after implosion and equilibration indicate densities of $2 \times 10^{16} \text{ cm}^{-3}$ and electron temperatures of 25 eV in a plasma column of radius 1.4 cm at 0.8 μsec after main bank ignition. The peak magnetic field is 23 kG and $w = 3.14 \times 10^5 \text{ sec}^{-1}$. The thermal loss is primarily due to electron conduction, and based on the condition $\lambda_{ei}/L \ll 1$, it is collisional. The electron-ion equilibration time is 0.42 μsec , and so single fluid MHD equations are reasonable.

Some basic concepts of thermal conduction and its measurement were outlined by Green et al.¹⁷. The competing effects of temperature increase due to adiabatic compression and temperature drop due to thermal loss were considered. In that work, it was concluded that conduction cooling at the center should be identifiable when thermal conduction time is less than the time for a rarefaction wave to reach the midcoil. For the plasma in the present experiment, taking $\gamma = \frac{5}{3}$ and $\beta = 0.7$, that relationship is

$$2.5.2 \times 10^{-15} L \ln \Lambda < T_e^2 / n_e \quad (15)$$

where L is the half-length and Λ is the Coulomb logarithm; an electron temperature in excess of 100 eV is necessary to identify this effect. It should be emphasized that these equations presume a quasi-steady process.

The distinction between quasi-steady loss processes, which presume that spatial inhomogeneities have been smoothed out, and the loss during the earlier period when transients can propagate, is made in a recent work by Chu and Johnson¹⁸. There is no heating mechanism or particle loss in this model. The transient solution determines the penetration

of a cooling layer which originates from the end of a plasma column. The time at which the transient similarity solution merges with the quasi-steady solution is given by

$$\tau_c = \frac{3 n L^2}{k_{110} n_c} \quad \text{where } n_c = 2.35$$

where L is the half coil length. In the case of an open-end pinch, temperature is not zero at coil end but, adopting the scaling proposed by Morse¹⁹, temperature is taken as zero at $z = L + 0.4 L = 35$ cm. For the present case then, $\tau_c = 20.8$ μ sec, or the total conduction loss event will occur in the transient loss period. This recognition should provide a satisfactory method of identifying thermal conduction effects, as will be outlined below.

First, consideration will be given to the quasi-steady loss relationships to place them in perspective. The rate of energy loss due to convection from one end is written and evaluated at pinch, as:

$$\begin{aligned} \left. \frac{dE}{dt} \right|_p &= \frac{3}{2} (2 N k T) (\pi R^2) \sqrt{\frac{\gamma P}{\rho}} (1 - \beta) \\ &= 5.67 \times 10^{25} \text{ eV/sec} \end{aligned}$$

The rate of energy loss by electron conduction can be written

$$\left. \frac{dE}{dt} \right|_{zh} = \frac{3}{2} (2 N k) (\pi R^2 L) \frac{dT}{dz}$$

and using the quasi-steady temperature solution developed by Chu and Johnson, at mid coil, with τ_0 defined as an initial cooling time,

$$T(z) = \frac{T_0}{\left(1 + \frac{5}{2} \frac{z^2}{\tau_0}\right)^{2/5}}$$

where T_0 is the temperature at time zero. With the half length replaced by the thermal scaling length, Z , above, the energy loss rate due to conduction can be evaluated at $t = 0$, as:

$$\left. \frac{dE}{dt} \right|_{th} = 2.48 \times 10^{24} \text{ eV/sec}$$

As conduction is an order of magnitude lower, convective losses will dominate the energy loss event. This is consistent with the results of a 2-D code developed by Schneider²⁰, who found that thermal conduction had little effect on energy loss in open ended systems. It should be emphasized that energy loss rates above are proportional to particle and temperature rates of change, but the identification of thermal effects are more easily recognizable when evaluating temperatures rather than temperature rates.

The transient solution developed by Chu and Johnson⁶ is carried out in dimensionless variables, with an equivalent heat flux expressed as

$$S = .725 e^{-n/1.4}$$

which can be used to establish

$$K_{11e} \frac{\partial T}{\partial Z} = \frac{2}{7} S T_o \left(\frac{3n}{\tau} \right)^{1/2}$$

where K_o is the classical thermal conductivity at $T = T_o$. The energy lost per unit volume is then

$$\frac{\partial}{\partial Z} (K_{11} \frac{\partial T}{\partial Z}) = \frac{2}{7} T_o \frac{3n}{\tau} \frac{0.725}{1.4} \exp\left[-\frac{Z}{1.4} \left(\frac{3n}{K_o \tau}\right)^{1/2}\right]$$

The thermal loss time $\left(\frac{1}{\tau_{th}} = \frac{1}{T} \frac{dT}{dt}\right)$ can be evaluated during the transient period at a time and location when it would be maximum. With a loop at the end of the coil, the transient period is viewed for the largest time; the transient period ends when the heat flux has penetrated to the center coil. Accordingly, the thermal loss time at coil end is written

$$\frac{1}{T} \frac{dT}{dt} = \frac{2}{7} \frac{T_o}{T} \frac{0.725}{1.4} \exp\left[-\frac{10}{1.4} \left(\frac{3n}{K_o \tau}\right)^{1/2}\right]$$

with a maximum value of $0.172 \times 10^5 \text{ sec}^{-1}$ at $1.17 \mu\text{sec}$. This is a value of $\frac{1}{T} \frac{dT}{dt}$ resulting without convection. To estimate the net energy loss time, it should be added to the energy loss time due to particles only, without conduction

$$\frac{5}{3} \frac{1}{N} \frac{dN}{dt} = \frac{5}{3} \frac{1}{L} \left[\frac{Yp}{\rho} (1 - \rho) \right]^{1/2} = 3.33 \times 10^5 \text{ sec}^{-1}$$

Accordingly, the energy loss time is

$$\frac{1}{E_t} \frac{dE_t}{dt} = 3.50 \times 10^5 \text{ sec}^{-1}$$

As the energy loss time can be determined from diamagnetic loop data¹⁵ with a coil at the end of the theta pinch and the particle loss time determined from Thomson scattering n_e data, the thermal conduction loss event could be generally evaluated in this experiment.

A more specific method of determining conduction effects and the magnitude of electron thermal conduction losses is related to Thomson scattering measurements of T_e , coupled with the use of the O-D computer code. As demonstrated above, and in a published work¹¹, this model of theta pinch behavior agrees quite well with experiment. In order to gain a feel for the effect of thermal conduction in temperature when adiabatic compression, end loss, and resistive diffusion are included, the O-D code developed was run for a number of values of

$$K_{11e} = C \times K_{11e \text{ classical}}$$

where C is a constant that was varied between 0 and 10. The results are shown on Figure 18 for discharge tube radius of 3.810 cm. Note that three items in particular stand out as being affected by this variation: peak temperature, time to peak temperature, and the temperature at 6.0 microsec. Experience with the 25 cm long theta pinch has shown that a variation of ± 5 eV around a particular temperature should be measureable.

It is easy to see that the temperature behavior at late times appears to be the most sensitive to variations in thermal conductivity as one might expect. Accordingly, a computer simulation of events appears to be a valid means of determining the thermal conductivity effect, and in the final analysis may perhaps be the best method of determining electron thermal conductivity.

Figure 18. Plasma Temperature Versus Time for Variable Electron Thermal Conductivity

IV. MAGNETIC MIRROR FIELD STUDIES -- (Veglia, Robinson)

1. Introduction

Work that had been described in an earlier progress report² had concentrated on order of magnitude estimates of mirror performance and testing of mirror coil circuits. One result of these efforts was the decision to utilize a discharge tube of smaller bore (7.62 cm i.d., 8.56 cm i.d.) than the theta pinch coil (10.5 cm i.d.) so that the mirror fields at the ends could be built-up by reducing the area of the 50 cm long coil at its ends, as well as adding separate mirror coils at the ends with their own electrical discharge circuitry. Just as with the main capacitor bank, there is a need for a mirror crowbar system in the add-on coils to retain high magnetic field strengths over the duration of the loss events. Accordingly, the efforts during the present reporting period have concentrated on: development of a computer program to predict magnetic field distributions with and without mirrors; design, fabrication and testing of mirror coil circuits; and development of an effective, reproducible crowbar system.

2. Magnetic Mirror System for the 50 cm Long Theta Pinch

The physics of magnetic mirror interaction has been reviewed in an earlier report², and this subject will not be reconsidered here. Two points will be noted, however; (1) the effectiveness of the mirror and the stability are related to the criterion that $\lambda_{ii}/L_m < 0.1$ where L_m is the length of the coil; and (2) the time of initiation and rise time of the mirror fields as compared to those of the main theta pinch fields is an important experimental variable for mirror effects and stability. Regarding the first point, the mirror length was chosen to be 2-5 cm long; inserts for the reduction of the diameter to 8-9 cm at the end of the coil have been evaluated for 2.54 and 5.08 cm lengths and add-on coils have been also evaluated for diameter of 8.9 cm with 2.54 and 5.08 cm lengths. Primarily because of rise-time considerations, single turn add on coils have been chosen; multi-turn coils are capable of generating higher magnetic fields but with long cycle time, so the mirror fields would interfere with main coil pinch dynamics. With single turn coils, there is

a problem with the distribution of circuit inductance. Due to the relatively low inductance (~ 75 nH) of the single turn coil, significant voltage division occurs between the coil and the remainder of the discharge circuit. To have efficient energy transfer to the mirrors, the coil inductance must be dominant in the circuit. To accomplish this, the inductance of the current distribution system must be low.

In a lumped LRC discharge circuit, inductance is attributed to four sources: (1) capacitors, (2) switch, (3) mirror coil, and (4) transmission line. A swinging cascade gap⁸ is used to discharge the mirror capacitors. Eight gaps of this type are presently in use on the main capacitor banks, and are known to function reliably. The inductance of each of these gaps was found experimentally to be 40 nH. The inductance of the mirror coil is determined from standard analyses²¹. The only variable inductance in the system is that of the transmission line and connections. Two types of lines were considered: coaxial cable and parallel plate. Component inductance values are presented in Table VII.

Table VII. Inductance Values for Mirror System

	2.54 cm Cable	5.02 cm Cable	2.54 cm Plate	5.08 cm Plate
Capacitors @ 40 nH	20	20	20	20
Swinging Cascade Gap	40	40	40	40
RG8/U Cable (6ft, 10 parallel)	12.9	12.9		
Parallel Plate (6 ft, 6 in. wide)			15.4	15.4
Connectors	54	54	10	10
Mirror Coil (8.9 cm i.d.)	<u>123</u>	<u>77</u>	<u>123</u>	<u>77</u>
System Inductance (nH)	249.9	203.9	208.4	162.4

These figures show that there is an advantage to using parallel plate since it gives both a lower total system inductance and has a higher proportion of the inductance in the coil.

The electrical discharge characteristics of the mirror coil circuitry are summarized in Table VIII.

Table VIII. Mirror System Parameters

	2.54 cm Coil Length	5.08 cm Coil Length
Capacitance (μF)	28	28
System Inductance (nH)	208	153
Charging Voltage (kV)	18	18
Coil Inductance (nH)	123	77
Quarter Cycle Time (μsec)	3.72	3.3
Maximum Current (A)	2.02×10^5	2.28×10^5
Maximum B Field (kG)	27.5	28

3. Crowbar Development for Mirror Circuitry

As the loss times for the 50 cm theta pinch with mirrors is anticipated to be longer than without mirrors, it will be necessary to maintain main bank fields and mirror fields by a crowbar circuit. The requirements for an effective crowbar short include: (1) closing at voltage zero; and (2) inductance lower than 40 nH capacitor inductance. Several different types of crowbar designs (vacuum arc, rail gap, pressure gap) have been fabricated and tested at this point, none has shown the desirable operational characteristics of precision timing and absolute repeatability.

Present development has centered on a design with pairs of hemispherical electrodes for crowbar gaps in atmospheric air being triggered by a high voltage pulsed center pin. This pin is driven by a pair of 40 kV, 2 μF ceramic capacitors (Maxwell Labs.) in a self-triggering Marx generator configuration which can be set to automatically fire at voltage zero on the main bank. This design has operated reasonably successfully, but is somewhat unreproducible, with delays typically of 1-2 μsec after current peak. A typical record of the mirror field circuit operating with crowbar, with a single pair of electrodes for the crowbar switch is shown in Figure 19. The intended

Main Bank Current →
(Rel. Units)

Crowbar Current →
(Rel. Units)

← 2V/div

← 1V/div

2 μsec

Figure 19. Mirror Circuit Current and Crowbar Current With Single Pair of Electrodes (2, 14 μ F @ 18 kV)

design will operate with two pairs of electrodes per switch, thus reducing the switch inductance.

Present efforts are being directed toward providing the trigger pin with a high voltage (70 kV) pulse with higher current to induce multiple breakdown to the electrodes.

4. Numerical Modeling of 50 cm Coil Magnetic Fields With and Without Magnetic Mirrors

To aid in the design of the mirror coils and evaluation of their effects, a computer program has been written to predict the magnetic field map for coil configurations of interest. The program has been specialized for five geometries:

- (1) straight theta pinch coil (with axially non-uniform current distribution)
- (2) theta pinch coil with reduced area (end inserts)
- (3) theta pinch coil with add-on mirror coils
- (4) theta pinch coil with reduced area (end inserts) and add-on mirror coils
- (5) up to 20 mirror pairs.

The method of solution is basically a numerical integration of the Biot-Savart integral for each of the different cases. The coil is broken up into a number of current loops at various Z positions, and each loop is divided into equal azimuthal segments. Using the differential form of the Biot-Savart Law, the contribution of each segment to the vector B field at the field point is determined. The basic form of the differential B-field vector is

$$\vec{dB}(\vec{r}_p) = \frac{\mu_0}{4\pi} \frac{I_c \vec{dL} \times (\vec{r}_p - \vec{r}_c)}{|\vec{r}_p - \vec{r}_c|^3} \quad (1)$$

where $\vec{r}_p$ is the vector to the field point, $\vec{r}_c$ is the vector to the differential current element, $\vec{dL}$ is the differential vector length of the element, and I_c is the current flowing in the element. The above expression is transformed into cylindrical coordinates by

$$\vec{r}_p = r \cos \theta_p \hat{i} + r \sin \theta_p \hat{j} + z_p \hat{k} \quad (2)$$

$$\vec{r}_c = a \cos \theta_c \hat{i} + a \sin \theta_c \hat{j} + z_c \hat{k} \quad (3)$$

and

$$d\vec{L} = a d\theta_c (-\sin \theta_c \hat{i} + \cos \theta_c \hat{j}) \quad (4)$$

where r , θ_p and z_p are the radial, azimuthal, and axial positions of the field point; a , θ_c , and z_c are the radial, azimuthal and axial positions of the current element; and $d\theta_c$ is the differential angle subtended by the element. Using the above three equations in equation (1) gives the following expression for $d\vec{B}(\vec{r}_p)$

$$d\vec{B}(\vec{r}_p) = \frac{\mu_0 I_c}{4\pi} a d\theta_c \frac{[\cos \theta_c (z_p - z_c) \hat{i} + \sin \theta_c (z_p - z_c) \hat{j} + (a - r \cos \theta_c) \hat{k}]}{[(r - a \cos \theta_c)^2 + (a \sin \theta_c)^2 + (z_p - z_c)^2]^{3/2}} \quad (5)$$

This equation presumes azimuthal symmetry, so that θ_p was set equal to zero. This then gives the vector B-field at any field point with coordinates r and z_p .

The total vector field is found by summing $d\vec{B}$'s for all current segments and all loops. Hence

$$\vec{B}(\vec{r}_p) = \frac{\mu_0 a}{4\pi} \sum_{\text{Loops}} I_c \sum_{\text{Segments}} d\vec{B}(\vec{r}_p) \quad (6)$$

It should be noted that the loop current I_c is a function of axial position (z_c), and as such, must be included in the summation over the loops. An expression for the axial current distribution in the coil is given by Bardotti²² as

$$I_c(z_c) = \frac{C}{(h^2 - z_c^2)^{1/2}} + j^*(z_c) \quad (7)$$

where $j^*(z_c)$ was found numerically to be approximated by

$$j^*(z_c) = \left[\frac{1}{\pi} - C \right] \frac{2}{h^2} (h^2 - z_c^2)^{1/2} \quad (8)$$

Here h is the coil length and C is a function of h which was numerically evaluated. This current distribution was incorporated into the code in tabular form for a number of coil aspect ratios (length/diameter) ranging from 0.5 to 5. Since the shape of the distributions are similar for all aspect ratios, a linear interpolation routine is used to find intermediate distributions.

The code was run for a number of coil configurations which are to be used in the mirror theta pinch effort. The results are shown in Figures 20 and 21. Figure 20 is a comparison of the B_z profiles on-axis for a number of mirror configurations along with that for the straight 50 cm pinch coil. The straight coil profile agrees well with experimental results. Figure 21(a) is a comparison of B_z profiles at various radii for a 50 cm pinch with 2.54 cm long end inserts and 2.54 cm long add-on mirror coils. Both inserts and add-ons have an inside radius of 4.4 cm while the pinch coil radius is 5.2 cm. Figure 21(b) is a comparison of B_z profiles at different radii considering only 5.08 cm add-on coils. It should be noted that the addition of add-ons tends to smooth out the transition from center coil field to peak mirror field. This should increase plasma stability in the machine.

Figure 20. Variation of B_z for Different Mirror Configurations with 50 cm Long Theta Pinch Coil

Figure 21. Variation of B_z with Axial Position for Fixed Radius

V. BANK ENGINEERING -- (Jacoby)

1. SCR Trigger Circuit

In order to provide initial triggering for spark gap switches, an SCR Marx circuit¹⁰ was constructed that provides an output pulse of 13 kV with a full width at half maximum (FWHM) of some 0.16 μ sec that has a risetime of about 0.12 μ sec. Thyratrons and krytrons are traditionally used for spark gap triggering²³. Both tubes have fast risetimes, on the order of some tens of nanoseconds; however, there are some disadvantages. Both tubes require high voltage power transformers, are thus tied to power outlets, and both are expensive. The thyatron also requires a large amount of heater power, and accordingly is subject to a certain warm-up time. A krytron requires no heater power, but has a limited life. SCR's, on the other hand, are inexpensive, reliable, and require no warm-up time or heater power. However, they are slower in risetime than the tubes and possess a much lower value of hold-off voltage. As will be shown, however, this low hold-off voltage value can be turned to an advantage and the risetime, while slower, is quite adequate.

The circuit used for the SCR's is a Marx arrangement (Figure 22). Other means of avoiding the subkilovolt hold-off voltage of SCR's such as stacking one SCR on top of another²⁴ have been used, but the Marx permits the generation of high voltages from a low voltage source. Essentially, the voltages on all the capacitors, in this case of value 0.05 μ F each, add together to produce a high voltage pulse. This arrangement of parallel charging and series discharging permits the use of common ceramic disc capacitors rather than capacitors with a high voltage rating, or cable, as the energy storage medium. It also obviates the need for a high voltage transformer to charge the capacitor or cable, and in its place permits the use of batteries or transformers that are much cheaper, much more common, and easily available. The choice of resistor values in the Marx stems from the RC discharge time of the ladder network. Once the SCR's are triggered, the tops of the charged capacitors are connected to the bottoms of the capacitors in front of them and also with the resistors on the bottom of the circuit leading

Figure 22. Marx SCR Circuit Diagram

back to ground. For the period of time of the high voltage pulse, the resistors should be chosen so as to provide isolation from the ground connection through them. In this case the main pulse lasts some 0.5 μsec . An RC discharge time of 50 μsec should be more than sufficient to provide the desired ground isolation and accordingly a value of $1\text{K}\Omega$ was used. The capacitor value was simply chosen from appropriate stock. Sufficient energy must be provided on the high voltage side of the output step-up transformer to ensure arcing of the trigger pulse. An effective capacitance when viewed on that side of the output transformer of around 0.1 nF has in the past proven quite adequate. Another consideration that could affect the RC value is repetitive pulsing of the Marx unit. For that application, the resistive and capacitive values would have to be properly chosen to permit full charging between pulses and still provide adequate ground isolation upon discharge.

The use of SRC's as the switch elements requires little by way of explanation with the exception of triggering. The SRC's in the Marx are triggered at the same time through separate windings on a common ferrite core. This is done to ensure simultaneous turn-on of all SCR's. This is important since if an SCR delays, it will have to hold off the combined voltage of all those that have turned on behind it, and it will probably fail. The resistor value in the trigger circuit of each SCR is chosen to provide adequate turn-on current to the gate. The devices used are a Motorola HEP R1223 SCR with a 600 V hold-off rating. They require 30 mA minimum to trigger and consequently the value of $6\text{K}\Omega$ was chosen for the resistor in order to provide three times the minimum trigger current. This ensures a quick and definite turn on of the device. The trigger transformer is a 1:1 arrangement that provides about 500 V across each secondary. The resistor is necessary in the gate circuit of each SCR in order to ensure uniformity of triggering of each device, since each SCR cannot be expected to have the same gate to cathode impedance. Consequently, without that resistance the SCR's with the lower gate impedance could drain off all the trigger energy and so prevent the others from firing at all.

The master trigger SCR is used to provide a rapidly rising high energy pulse that is split up among the five Marx SCR's to provide triggering of these units. This permits a low energy pulse to trip the entire unit. The trigger transformer consists of a toroidal ferrite core using about seven turns of wire for each winding. A seven volt zener diode is placed in series with the gate of the master trigger SCR to provide a measure of noise immunity for the circuit since it is intended to sit out in the open near the spark gap that it is supposed to trigger. This appears to work quite well since no pretriggering of the unit has been observed in its operational time of three months.

The circuit provides a 2.5 kV pulse into a high impedance which rises in 0.11 μ sec. The load used in this case is a high voltage Tektronix probe P6013; response is shown in Figure 23(a). The value of 2.5 kV is expected since there are five capacitors each charged to 500 V. When a 1:7 turns ratio step-up output transformer is used as the load, the peak voltage drops to some 2.0 kV due to the voltage division between the impedance of the transformer and the combined forward resistances of the SCR's; this is displayed in Figure 23(b). The output pulse of the transformer reaches 13.2 kV with a risetime of 0.12 μ sec and has a FWHM of 0.16 μ sec; this can be seen in Figure 23(c). This 13.2 kV output pulse is then used to trip a spark gap of the direct overvolted variety⁸. This spark gap holds off 18 kV in the present application. The jitter in its triggering by the Marx circuit has been observed to be less than ± 50 nsec; this is determined through an examination of records showing the timing of the pre-ionization system which the triggered spark gap sets off with respect to the initial command pulse to the Marx circuit.

Previously a thyatron unit was used to fire the spark gap and the jitter in its performance was comparable to the Marx SCR circuit now in use. The thyatron unit, though, required a high voltage transformer, high voltage components, a large amount of heater power, and was rather bulky. The SCR unit by contrast uses a 500 V battery, low voltage components, and is quite compact.

Figure 23. Pulse Generation Capability of SCR Trigger Circuit
 (a) Into Tektronix P6013 Load; (b) Into Output Transformer with 1:7 Turns Ratio as Load; (c) Output Pulse from Transformer with 1:7 Turns Ratio

VI. COMPUTER MODELING OF PLASMA COLUMN BEHAVIOR -- (Stover)

1. Introduction

Development of the 1-D fluid plasma end loss code² has continued during this reporting period. In an effort to incorporate more relevant physics into the code for simulating both hot and cold θ pinch experiments, the following effects have been added: (1) thermal interaction with the background plasma, (2) finite plasma electrical conductivity, (3) magnetic field curvature, (4) viscous pressure and (5) ion flux limiting of ion viscosity and ion heat conduction for machines with $\lambda_i/L \geq 1$. This code, with use of the appropriate modifications, was used to conduct detailed computer simulations of the Scylla I-C experiment²⁵ and the Scylla IV-P²⁶ experiment.

The equations comprising the 1-D end loss model will be reviewed in section 2 along with a discussion of the above added effects. Detailed computer simulation results will be discussed in section 3.

2. 1-D Plasma Model

A sketch of the open ended linear theta pinch and the coordinate system adopted here is presented in Figure 24, along with useful notation. It should be remembered that in developing the 1-D time dependent code that the z coordinate was considered the most relevant in describing plasma column loss processes, thus the code is both z and t dependent.

The governing two-fluid (electron and ion) equations include an axial momentum equation which is used to obtain the mass averaged plasma velocity $U(z,t)$

$$m_i n \frac{dU}{dt} = - \frac{\partial}{\partial z} (nkT_e + nkT_i + P_v) + \frac{1}{A_p} \int^{A_p} (\vec{J} \times \vec{B})_z dA \quad (1)$$

We have added a more generalized description for viscous pressure P_v which includes both classical ion viscosity and a flux limiter expression for ion viscosity, as well as pseudoviscosity used for dealing with hydrodynamic shocks². For collisional ions, $\lambda_i/L \ll 1$ we use the Spitzer²⁷ expression for classical viscous pressure

Figure 24. Linear Theta Pinch Geometry

$$P_v = -\frac{4}{3} n_{11} \frac{\partial U}{\partial z} \quad \text{ergs/cm}^3 \quad (2)$$

where

$$n_{11} = 1.013 \times 10^3 \frac{A_i^{1/2} T_i^{5/2}}{\ln A}$$

with T_i the ion temperature in keV, A_i the atomic weight and $\ln A$ the coulombic logarithm. When the ions are collisionless, $\lambda_i/L \geq 1$ an ion viscous flux limiter was used to determine a limiting functional form for the viscous pressure obtained from

$$P_v = nm \left\langle v_z' v_z' - \frac{1}{3} (v')^2 \right\rangle$$

where v' represents a rest frame velocity defined by $v' = v - u$ with v the velocity in the lab frame. A displaced Maxwellian distribution function is used to evaluate P_v with the result

$$P_v = -6.024 \times 10^{-10} n T_i S^2 \quad \text{ergs/cm}^3 \quad (3)$$

where

$$S = 2.285 \times 10^{-8} A_i^{1/2} U/T_i^{1/2}$$

and U has units cm/sec. The limiter was implemented by choosing the functional dependence for P_v , either Equation (2) or (3), which yields the smallest value for P_v . For the collisionless ion experiment discussed in the next section, P_v determined from Equation (3) always yielded the smallest P_v thus it was used for all mesh points. Implementation of pseudoviscous P_v was discussed in reference 2.

The $\vec{B}$ curvature $(\vec{J} \times \vec{B})_z$ term in Equation (1) was evaluated assuming weak $\vec{B}$ curvature. Using the slender plasma column scaling of Weitzner²⁸, which is reasonable for the θ pinch experiments modeled here, we can write

$$\frac{\partial B_z}{\partial r} \gg \frac{\partial B_r}{\partial z}$$

Furthermore, assuming B_z inside the plasma has a parabolic radial distribution¹¹ and also assuming B_r inside the plasma has a linear radial distribution with $B_r = 0$ at $r = 0$, then

$$\frac{1}{A_p} \int^{A_p} (\vec{J} \times \vec{B})_z dA = \frac{B_{re} B_{ze}}{\mu_0 r_p} [1 - (1 - \beta_a)^{1/2}]$$

where B_{re} and B_{ze} are found from the Biot Savart Law and the shape of the plasma boundary ($\partial r_p / \partial z$). Here r_p is the plasma radius and β_a is the on-axis ratio of plasma pressure to vacuum field magnetic pressure.

A Joule heating term $(\gamma - 1)P_j/n$ has been added to the electron energy equation, for the plasma can now be considered to have finite electrical conductivity. Including this term, the electron thermal energy equation becomes

$$\frac{dkT_e}{dt} = -\frac{(\gamma - 1)}{n} kT_e \frac{dn}{dt} + \frac{(\gamma - 1)}{nA_p} \frac{\partial}{\partial z} (K_e A_p \frac{\partial T_e}{\partial z}) + \frac{k(T_i - T_e)}{\tau_{eq}} + \frac{(\gamma - 1)}{n} P_j$$

Using Amperes Law, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ and again assuming: (1) small $\vec{B}$ curvature; and (2) B_z parabolic in r , the Joule heating of the electrons

$$P_j = \frac{n}{A_p} \int^{A_p} J^2 dA$$

becomes

$$P_j = 2.483 B_e^2 n_A [1 - (1 - \beta_a)^{1/2}]^2 \text{ keV}/(\text{cm}^3\text{-sec})$$

where n_A is the strong magnetic field limit for plasma electrical resistivity²⁷ and B_e is the driving vacuum magnetic field.

The ion thermal energy equation is written as

$$\frac{d}{dt} kT_i = -\frac{(\gamma - 1)}{n} kT_i \frac{dn}{dt} + \frac{(\gamma - 1)}{nA_p} P_v \frac{\partial}{\partial z} (UA_p) + \frac{(\gamma - 1)}{nA} \frac{\partial}{\partial z} (A_p q_i) + \frac{k(T_e - T_i)}{\tau_{eq}}$$

In the case where the ions are collisional, $\lambda_i/L \ll 1$ the ion heat flux by conduction is modeled by

$$q_i = K_{11}^i \frac{\partial T_i}{\partial z}$$

where

$$K_{11}^i = 2.49 \times 10^{27} \frac{T_i^{5/2}}{A_i^{1/2} \ln \Lambda}$$

An ion energy flux limiter due to conduction was also used to determine a limiting form for q_i when $\lambda_i/L \geq 1$. For this case the limiter q_i was found from the velocity space integral definition

$$q_i = \frac{nm_i}{2} \langle v_z' (v')^2 \rangle$$

with the distribution function defined as above for the P_v limiter. The result is

$$q_L = 6.45 \times 10^{-9} (A_i T_i)^{1/2} (U^2 + 1.92 \times 10^{15} \frac{T_i}{A_i}) \exp(-5.22 \times 10^{-16} \frac{A_i U^2}{T_i}) \quad (6)$$

Implementation of the limiter ($\lambda_i/L \geq 1$) is the same as outlined above for the viscous flux limiter. For the experiment discussed here with $\lambda_i/L \geq 1$, a critical location, z_{crit} , was identified where q_i from Equation (5) and q_L from Equation (6) intersect. For $z > z_{crit}$, Equation (6) was used to determine q_i and for $z < z_{crit}$, Equation (5) was used.

The radial momentum equation remains unchanged from the simple radial pressure balance expression reported the previous reporting period,

$$nkT_e + nkT_i + \frac{1}{2\nu_0} \left(\frac{\phi}{A_p} \right) = \frac{B_e^2}{2\nu_0} \quad (7)$$

If the plasma resistivity is chosen to be non-zero, the magnetic flux, ϕ , for a given fluid element

$$\phi(z,t) = \int^{A_p} B_z dA$$

will change with z and t due to radial diffusion of driving magnetic field into the plasma. The rate of change of magnetic flux for a given

Lagrangian element of plasma can be found by recalling the magnetic field diffusion equation

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\frac{n_A}{\mu_0} \nabla \times \vec{B} - \vec{u} \times \vec{B} \right)$$

Again invoking the small B curvature approximation, taking $B_z = B_z(r)$ to be parabolic inside the plasma, and integrating over the plasma cross section you get

$$\frac{d\phi}{dt} = \frac{4\pi n_A}{\mu_0} B_e [1 - (1 - \beta_s)^{1/2}] \quad (8)$$

Note that if $n_A \rightarrow 0$ then $\phi \rightarrow \text{constant}$. The integrated flux from Equation (8) is substituted into Equation (7), and the set of energy equations, Equation (7) and a mass conservation² expression are solved iteratively as outlined in reference 1.

Treatment of the plasma boundary condition where the plasma is assumed not to thermally interact with the background plasma was outlined in reference 2. In this case, we had set $\frac{\partial T_e}{\partial z} = \frac{\partial T_i}{\partial z} = 0$ at the exhaust front or at a prescribed end wall location where Lagrangian elements are truncated. The capability to model heat conduction to a cool thermal plasma reservoir of infinite extent was added during this reporting period. This enables us to evaluate the impact of significantly varying exhaust boundary conditions on midplane plasma temperature. This will be discussed in more detail in section 3.

Computer simulations to be discussed here were based on an assumed spatially uniform driving magnetic field inside the compression coil. Initial conditions for A_p , T_e , T_i , n and ϕ immediately following implosion were likewise taken to be independent at z .

3. Simulation Results

Simulation results for two recent θ pinch experiments are presented here and compared with experimental results. The two experiments discussed are: (1) the Scylla 1-C experiment²⁵, a "cold" experiment where $\lambda_i/L \ll 1$ and $\lambda_i/r_p \ll 1$ and (2) the Scylla IV-P

experiment²⁶, a "hot" experiment, where T_i is sufficiently high so that $\lambda_i/L \geq 1$. In Scylla I-C, where λ_i is much smaller than the smallest characteristic length of the experiment, a fluid plasma description is appropriate and serves as a good check for the code. Computer simulation results for this case agreed rather well with experimental results. In contrast to Scylla I-C, Scylla IV-P ions are collisionless and $T_e/(\partial T_e/\partial z) \sim \lambda_e$ in the exhaust thus the applicability of a fluid plasma description is not clear. Despite those apparent difficulties, the fluid code adequately modeled Scylla IV-P plasma behavior over a significant fraction of the experiment. A possible explanation for this agreement is discussed.

Scylla I-C results are discussed first. In this experiment the particle confinement time, τ_p , is approximately 15 μsec whereas the characteristic thermal conduction times for the electrons and ions, τ_{th}^e and τ_{th}^i , are on the order of 150 μsec and 5 millisecc, respectively. Consequently, thermal conduction effects are not important for this experiment. The energy equilibration time between Maxwellian ions and electrons²⁷ is 0.3 μsec thus electron and ion temperatures are nearly equal throughout the experiment. As a result $T_e = T_i$ was assumed²⁵ in reducing the inferred midplane plasma temperature.

Both experimental results and computer simulation results are presented in Figures 25 and 26. Results presented in Figure 25 include integrated on-axis number density, n_a , obtained from end-on laser interferometry; single species plasma temperature, obtained from n_a , midplane diamagnetic loop measurements and radial pressure balance; and the applied magnetic field waveform. Computationally predicted n_a agrees well with experimental results for $t < 4 \mu\text{sec}$ with $n_{i0} = 0$. Assuming magnetic field diffusion into the Scylla I-C plasma column is governed by classical resistivity, $\eta = \eta_{\perp}$, excellent agreement in n_a is obtained during the entire experiment. Sheath broadening resulting from excitation of the lower hybrid drift instability²⁷ in post-implosion theta pinches was found earlier² to be negligible in this experiment. Midplane temperatures agree overall quite well for times less than 7 μsec . The rapid dropoff in midplane T is coincident with the arrival of the leading end loss rarefaction wave at the midplane. The simulation predicted a drop in T at midplane at 7 μsec but not the sharp drop

Figure 25. Time History of Scylla I-C On-Axis Electron Density, Single Component Temperature and Applied Magnetic Field

Figure 26. Time History of Scylla I-C Total Plasma Column Electron Inventory

observed. To date a satisfactory explanation for this discrepancy has not been advanced but it has been suggested that this may be a two dimensional effect. It should be noted that the difference between T for $\eta = 0$ and $\eta = \eta_1$ simulation results was negligible thus only the $\eta = 0$ results are presented. This small difference, despite the presence of Joule heating of the electrons when $\eta = \eta_1$ is attributed to a balance between Joule heating of the electrons and a relative cooling of the total plasma due to smaller compression of the plasma arising from an increase of magnetic flux in the plasma.

Total plasma column electron inventory results are presented in Figure 26. Computer simulated particle inventory agrees well with measured values falling within shot-to-shot data scatter. The axial $(\vec{J} \times \vec{B})_z$ body force was found to have a relatively small effect on total particle inventory. At $t = 9 \mu\text{sec}$ total particle inventory was found to be approximately 5 percent greater than when $(\vec{J} \times \vec{B})_z$ is set equal to zero. This results from a $(\vec{J} \times \vec{B})_z$ deceleration of plasma inside the compression coil which is accelerated by passage of the inward traveling rarefaction wave.

As a check on the 1-D code, rarefaction wave propagation as predicted by the code was compared with the leading characteristic wave speed, V_c , obtained from the isentropic plasma equations³⁰.

$$V_c = V_a V_s / (V_a^2 + V_s^2)^{1/2}$$

where V_a is the Alfven speed defined by

$$V_a^2 = B^2 / (\rho \mu)$$

and V_s is the plasma acoustic speed defined by

$$V_s^2 = \gamma p / \rho$$

The speed V_c is commonly referred to in the literature as the cusp speed. Presented in Figure 27 are z dependent on-axis number density profiles normalized to their midplane values for $t = 0.28 \mu\text{sec}$ (implosion),

Figure 27. Rarefaction Wave Propagation in Scylla I-C

2 μsec , 4 μsec , and 6 μsec . Also presented are leading wave surface locations for V_a , V_c , and V_s at those times. Both V_a and V_s generated wave fronts significantly precede the arrival of the rarefaction wave predicted by the 1-D code. The V_c wavefront locations, however, do agree well with the computational rarefaction wave locations thus providing a level of confidence for accurate modeling of plasma end loss.

Considering the Scylla IV-P experiment next, electron thermal conduction and particle confinement scale as $\tau_{th}^e < \tau_p$. Thus, in contrast to the Scylla I-C experiment, electron thermal conduction effects are important in this experiment. Peak ion temperatures exceed 3 keV and peak electron temperature is about 500 eV. The ions appear to be collisionless over the length of the machine for $\lambda_i/L \geq 1$ whereas the electrons appear to be collisional because $\lambda_e/L \ll 1$. The electrons are not collisional, however, for all locations in the exhaust. This point will be discussed later. The electron-ion energy equilibration time is on the order of 40 μsec thus large differences between electron and ion temperature can be supported in the plasma.

Computer simulation and experimental time histories of both electron and total plasma temperature at the coil midplane are presented in Figure 28. First consider the two simulations conducted where the boundary conditions on the exhaust front and/or end wall were varied. In one simulation $\frac{\partial T_e}{\partial z} = 0$ at the exhaust boundary whereas for the other simulation, electron heat was allowed to be conducted to a 5 eV background plasma at the exhaust boundary. In both simulations, the ions were assumed to have zero viscosity and zero thermal conductivity. Note that regardless of the boundary condition selected, good agreement is obtained between simulation and experiment for $t \leq 20 \mu\text{sec}$, a significant fraction of the experiment. For $t > 20 \mu\text{sec}$, the simulation predicts plasma temperatures which are too low. Simulation electron temperatures agree quite well with Thomson scattering inferred results for $t > 5 \mu\text{sec}$ where it should also be noted that the exhaust boundary condition has a relatively small influence on midplane T_e . For $t < 5 \mu\text{sec}$, simulated T_e were too low assuming classically collisional electron thermal conduction.

Figure 28. Time History of the Electron and Total Plasma Temperature at the Coil Midplane, and Results from MHD Code. Data Points from: (●) Thomson Scattering, (○) Pressure Balance, (Δ) Neutron Emission Rate Plus Thomson Scattering.

By allowing $K_e \rightarrow 0$ it was found that T_e would lie inside the error bars but for $t > 5$ μsec , the energy gained via transfer from the hot ions resulted in unreasonably high electron temperatures. This indicates that the electrons conduct heat into the exhaust by classical heat conduction for $t > 5$ μsec but for $t < 5$ μsec electron heat conduction is limited, perhaps by a non fluid kinetic effect. Identification of electron energy flow limiting processes which may explain this behavior are currently under investigation by Gary³¹. The relatively small influence of the exhaust boundary condition on midplane T_e was initially somewhat surprising. The reason for this behavior can be seen by examining Figure 29 where electron temperatures are presented at $t = 10$ μsec for plasma in the vicinity of the end of the compression coil. It is interesting to note that electron temperatures at the end wall of the exhaust bubble differ by approximately 150 eV whereas the midplane temperature differs by less than 15 eV. Electron thermal heat fluxes were calculated at the end of the compression coil for both boundary conditions and found to differ very little.

The explanation of this result is found by noting that $q \sim K_e(\partial T_e / \partial z)$ and $K_e \sim T_e^{5/2}$. If the condition $(\partial T_e / \partial z) = 0$ at the end wall was chosen, the T_e at the end coil was large while $\partial T_e / \partial z$ was small. Conversely, for the case $T_e = 5$ eV at the end wall, T_e at the coil was smaller than the previous case, while $\partial T_e / \partial z$ was larger. The changes balanced each other, resulting in similar heat fluxes. These results indicate that although the plasma electron temperature inside the compression coil is strongly dependent on the plasma being able to conduct heat to the exhaust, it is only weakly dependent on the exhaust boundary condition.

It was noted above that although $\lambda_e / L \ll 1$, the electrons may not necessarily be locally collisional in regions of the exhaust. This can be seen by examining Figure 30 where the electron gradient scale length, $T_e / (\partial T_e / \partial z)$, is compared with the electron mean free path, λ_e . For the description for electron thermal conduction adopted here to be accurate, we must require that

Figure 29. Scylla IV-P Exhaust Electron Temperature at $t = 10 \text{ usec}$

Figure 30. Scylla IV-P Electron Thermal Conduction Transport Check for $t = 10 \mu\text{sec}$

$$\frac{\lambda_e}{(T_e / \frac{\partial T_e}{\partial z})} \ll 1 \quad (9)$$

From Figure 30 it can be seen that for points inside the compression coil, relation (9) is satisfied but as you proceed into the exhaust region this relation begins to break down thus some uncertainty is involved in using the classical small-mean-free-path expression for K_e at these points.

Presented in Figure 31 are on-axis number density, n_a , and total electron inventory as measured by an end-on laser interferometer. Because of the negligible effect of electron boundary condition on these parameters, only the $T_e = 5$ eV boundary condition results are presented. Simulated and experimental n_a agree in general quite well over the duration of the experiment. A slight discrepancy does exist, however, for $t \sim 8$ μ sec where the code predicts that n_a should follow the post crowbar driving magnetic field. Total electron inventory, N , as predicted by the simulation agrees reasonably well with the measured inventory. Simulated N falls within the shot to shot scatter of data but lies on the low side of the data. It is felt that this can be attributed to the presence of exhaust plasma which is included in the electron inventory as seen by the interferometer. The code calculates N based on the electron inventory only inside the compression coil. Most importantly, however, the predicted rate of loss of plasma, dN/dt , agrees well with that measured, indicating that although the measured inventory includes plasma in the exhaust region, the rate of loss of plasma from the compression coil, considered to be more important, is being properly simulated.

Returning to Figure 28, it was noted above that simulated total plasma temperature was too low for $t \geq 20$ μ sec. Examination of the T_e results indicates that this discrepancy results largely from simulated T_i being too low. It was felt that additional ion heating at these later times resulting from viscous heating of the ions could supply the additional heating required for good T_i agreement. Due to the fact that Scylla IV-P ions have $\lambda_i/L \geq 1$ it was felt that the method

Figure 31. Time History of Scylla IV-P On-Axis Electron Density and Total Electron Inventory

described in section 2 for handling viscous pressure with a Maxwellian limiter would be reasonable. Furthermore, if viscous ion pressure is considered, it was also felt that for consistency, ion thermal conduction should also be implemented into the simulation using a flux limiter as was done with ion viscosity. This method was also described in section 2. Conducting a simulation with non zero P_v and q_i limited by the appropriate limiter functions described in section 2, it was observed that total plasma temperature, or ion temperature, dropped significantly following peak magnetic field, see Figure 28, giving poorer agreement than the simulations where $P_v = q_i = 0$. This result suggests that any one of three statements can be made: (1) $P_v = q_i = 0$ is appropriate; (2) the limiter description is reasonable and either (a) a poor choice for the ion distribution function has been made or (b) the limiter coefficients need to be adjusted; (3) an alternative formulation for P_v and q_i should be developed which should always be used when $\lambda_i/L \geq 1$. Simulations attempted for classical P_v and q_i indicated more than sufficient viscous heating of the ions was available due to the strong $T_i^{5/2}$ dependence in the ion viscosity coefficient. This indicates that by properly adjusting the limiter coefficients, higher simulation T_i at $t > 20$ μsec should in principle be realizable. Flux limiter simulation results noted above indicate that the limiter for ion viscosity is too low and the limiter for ion thermal conduction is probably too high. As a result, the sensitivity of midplane T_i to limiter cutoff coefficient is currently under investigation. Such a prescription, however, may not necessarily be applicable to similar machines thus alternate descriptions for P_v and q_i as outlined above in statement (3) are also under consideration.

In summary, good agreement between simulation and experimental results was obtained for the Scylla I-C experiment assuming classical η . Overall good agreement between simulation and experimental results was also obtained for the Scylla IV-P experiment. Electron thermal conduction was found to be adequately described assuming classical small mean free path transport for $t \geq 5$ μsec . Midplane electron temperature

was also found to be weakly dependent on the exhaust thermal boundary conduction. The fluid ion description appears to be reasonable in simulating the Scylla IV-P ions for $t < 20 \mu\text{sec}$ despite $\lambda_i/L \geq 1$. A possible explanation for this fluid like behavior may result from excitation of an ion cyclotron electromagnetic instability³² driven by ion energy anisotropy, $T_\perp > T_{\parallel}$, which tends to isotropize the plasma. The growth rate for this instability is roughly equal to ω_{ci}^{-1} where ω_{ci} is the ion cyclotron frequency. For Scylla IV-P, ω_{ci}^{-1} is approximately 3×10^{-9} seconds, thus assuming excitation of this instability, the plasma may be considered to be nearly isotropic over the duration of the experiment. Finally in reference to the ion limiter results discussed above, more work is needed in modeling ion transport.

VII. CUSP CONFINEMENT STUDIES -- (Parvazian)

The geometry for cusp-contained linear theta pinches is shown in Figure 32.

Figure 32. Cusp-Ended Theta Pinch Geometry

The sheath thickness is δ while the ring cusp has a thickness of 2δ . The coil radius is r_c , the plasma radius is r_p and the cusp radius is R .

The particle confinement time τ is defined by $\tau = N/F$ where N is the total number of ions in a volume V and F is the particle loss rate. The volume is taken to be $V = \pi r_p^2 L$ since the cusp volume is small compared to the pinch. Assuming a high β collisional plasma, and that the loss through the point and ring cusps is identical, Phillips³³ obtained the result

$$F = 8\pi R n_0 v_{Thi} \delta g$$

where g is a numerical factor approximately equal to unity, n_0 is plasma density in the pinch, and $v_{Thi} = (2kT_i/m_i)^{1/2}$. The maximum loss rate is assumed to be the free flow loss $F_{max} = \pi r_p^2 L / v_{Thi}$. The end loss reduction factor R_f is defined by $R_f = F_{max}/F$ so that $R_f = \pi r_p^2 / 8\pi R \delta$. To obtain R we use the B field for a cusp $B^2 = a^2(r^2 + 4z^2)$ where $z = 0$ is the center of the cusp. At $z = 0$ and $r = r_c$,

$B^2 = a^2 r_c^2 = M^2 B_o^2$ where B_o is the B field confining the pinch column and M is the mirror ratio. Thus $a = MB_o/r_c$. For $r \approx R$ the value $B = B_o$ occurs at $z = \delta$ so

$$B_o^2 = a^2 (R^2 + 4\delta^2) = \frac{M^2 B_o^2}{r_c^2} (R^2 + 4\delta^2) \approx \frac{M^2 B_o^2 R^2}{r_c^2}$$

Thus $R \approx r_c/M$ and

$$R_f = \frac{r_p^2 M}{8 r_c \delta}$$

The crucial unknown is the sheath thickness δ .

Various theories for δ have been reviewed by Haines³⁴. A range of δ values are possible, depending upon whether δ is determined from charge separation, collisions or some type of instability. Typical values range from $\delta = (a_i a_e)^{1/2}$, where a_i and a_e are the ion and electron Larmor radii in the cusp region, to twice the ion Larmor radius. For a collisional sheath

$$\delta = (a_e a_i)^{1/2} \left(\frac{L_{vpi}}{v_{Thi}} \right)^{1/2}$$

Noting that a_i and a_e are each a factor M^{-1} smaller than in the pinch, i.e.

$$a_i = \frac{a_{oi}}{M} = \frac{1}{M \Omega_{oi}} \left(\frac{kT}{m_i} \right)^{1/2}$$

$$a_e = \frac{1}{M \Omega_{oe}} \left(\frac{kT}{m_e} \right)^{1/2}$$

where $\Omega_{oi} = eB_o/m_i c$, we can write $\delta = \frac{\delta_o}{M}$

and

$$R_f = \frac{r_p^2 M^2}{r_c \delta_o}$$

where δ_o is the sheath thickness for the plasma column.

To identify the implications of this analysis for the experiments being done in this laboratory, the following set of parameters were chosen using hydrogen as a test gas:

$$r_p = 1.5 \text{ cm}, \quad r_c = 7.6 \text{ cm}, \quad B_0 = 25 \text{ kG and } 30 \text{ kG}$$

$$M = 2.5 \text{ and } 3, \quad T = T_e = T_i = 20 \text{ eV, and } L = 25 \text{ cm.}$$

In Table IX, the results of calculations of R_f are shown for the several different choices for the functional form of δ , the sheath thickness. The first form, a_{io} , is associated with a turbulent sheath; the values appear to agree with some of the early cusp experiments, although it is unclear if one should have $\delta = a_{io}$ or $2a_{io}$. The second possible δ expression results from electrostatic separation or an ion-acoustic wave instability, and a sheath thickness of this value has been observed in some recent low density cusp confinement experiments. The last values are associated with the theory of a high beta collisional sheath.

Values for R_f vary from unity to over 100, depending upon which theory, if any, is applicable. None of the theories do a very good job of including finite β , but an examination of the effect of finite β suggests that R_f would be reduced. A very interesting experiment appears to be possible for somewhat higher temperatures (50 eV) and densities of the order of $10^{15}/\text{cm}^3$.

Table IX. Reduction Factor of a Cusp-Ended Theta Pinch

Sheath Thickness (δ_0 cm)	B_0 (kG)	M (Mirror Ratio)	R_f
a_{io}	25	2.5	13
	30	3	18
$(a_{io} a_{eo})^{1/2}$	25	2.5	83
	30	3	119
$(a_{io} a_{eo})^{1/2} \left(\frac{Lv_{ei}}{v_{Thi}}\right)^{1/2}$ $n = 10^{15}/\text{cm}^3$	25	2.5	2.4
	30	3	4.5
$(a_{io} a_{eo})^{1/2} \left(\frac{Lv_{ei}}{v_{Thi}}\right)^{1/2}$ $n = 10^{16}/\text{cm}^3$	25	2.5	1.0
	30	3	1.2

VIII. COLLISIONLESS PLASMA END LOSS -- (Klevans)

If the ion-ion mean free path is longer than the length of a linear theta pinch device, the ions are generally considered to be collisionless, and classical fluid concepts such as thermal conductivity and viscosity become unclear in both concept and evaluation. This has led to ambiguities in the modeling of the Scylla IV-P experiment when an attempt was made to include both ion thermal conduction and ion viscosity. In order to place an upper limit on these quantities, a limiter concept was employed where, assuming a flowing half Maxwellian, an upper limit on heat flow and viscous energy flow was determined.

An alternative way to get insight into the collisionless case is to use the solution to the Vlasov equation developed by Dreicer³⁵ for free flow from a collisionless plasma column. Defining $s = z/L$, where z is the axial distance from the center of a column of length L , and $\tau = (\frac{2kT}{M})^{1/2} t/L$, where T is the initial temperature and M is the ion mass, Dreicer obtained

$$f(s, u, t) = h(s - u\tau) \frac{e^{-u^2}}{\sqrt{\pi}} \quad (1)$$

where

$$f = \frac{1}{n_0} \left(\frac{2kT}{M} \right)^{1/2} \int_{-\infty}^{\infty} \int F(v_x, v_y, v_z) dv_y dv_x$$

$$u = \left(\frac{M}{2kT} \right)^{1/2} v_z$$

$$h(s - u\tau) = \begin{cases} 1 & u_{\min} < u < u_{\max} \\ 0 & u_{\max} < u \text{ or } u < u_{\min} \end{cases}$$

$$u_{\max} = \frac{s}{\tau} + \frac{1}{2\tau} ; \quad u_{\min} = \frac{s}{\tau} - \frac{1}{2\tau} \quad (2)$$

and n_0 is the density at $t = 0$.

The above distribution function can be used to obtain an upper limit on the heat flow q_z . We obtained the result

$$q_z = n_0 kT \left(\frac{2kT}{M} \right)^{1/2} \left\{ \frac{1}{2\pi^{3/2}} \frac{\alpha^3}{\tilde{n}} + \frac{\gamma}{\pi^{1/2}} - \frac{\alpha}{2\pi^{1/2}} - \frac{3}{2\pi} \frac{\alpha\beta}{\tilde{n}} \right\} \quad (3)$$

where

$$\alpha = \exp(-u_{\min}^2) - \exp(-u_{\max}^2) \quad (4)$$

$$\beta = u_{\min} \exp(-u_{\min}^2) - u_{\max} \exp(-u_{\max}^2) \quad (5)$$

$$\gamma = u_{\min}^2 \exp(-u_{\min}^2) - u_{\max}^2 \exp(-u_{\max}^2) \quad (6)$$

$$\tilde{n} = 0.5 [\phi(u_{\max}) - \phi(u_{\min})] = \frac{n(z)}{n_0} \quad (7)$$

$$\phi(u) = \frac{2}{\pi^{1/2}} \int_0^u e^{-t^2} dt \quad (8)$$

Plots of $\tilde{q}(s,t) = q_z / [n_0 kT (2kT/M)^{1/2}]$ for several locations in the system are shown in Figure 33. The quantity $\tilde{q}$ is essentially the numerical coefficient for the limiter solution. Not only does it not exceed 0.09, but it drops off very rapidly after a half-transit time.

The evaluation of viscosity was achieved by calculating the pressure tensor, and by developing an energy equation from the Vlasov equation. The result is an energy equation of the form

$$\frac{dE}{dt} = -E \frac{\partial \bar{v}_z}{\partial z} - P_z \frac{\partial \bar{v}_z}{\partial z} - \frac{\partial q_z}{\partial z} \quad (9)$$

with

$$\begin{aligned} E &= \frac{M}{2} \int (v_z - \bar{v}_z)^2 F d^3v + nkT \\ &= \frac{nkT_z}{2} + nkT \end{aligned} \quad (10)$$

$$P_z = M \int (v_z - \bar{v}_z)^2 F d^3v = nkT_z \quad (11)$$

$$\bar{v}_z = \frac{1}{n} \int v_z F d^3v \quad (12)$$

Figure 33. Normalized Axial Heat Flow as a Function of Time at Various Axial Locations

The energy and momentum equations both indicate that no viscosity is present in the description. From this viewpoint, there would be no viscosity.

It is not clear that this description is appropriate for computing a limiter to be used in a conventional fluid theory. In the collisionless theory there is no mixing of parallel and perpendicular energy, so that the initial temperature T describes the perpendicular temperature throughout the loss event, while T_z drops significantly. In the conventional fluid theory, the assumption of isotropy in temperature is employed in lowest order. In the fluid analysis, one defines the stress tensor as

$$\pi_{\alpha\beta} = nm \langle v'_\alpha v'_\beta - (v'^2/3) \delta_{\alpha\beta} \rangle \quad (13)$$

so that

$$\begin{aligned} \pi_{zz} &= (2/3) nm \langle v'_z v'_z \rangle - (1/3) nm \langle v'^2_x + v'^2_y \rangle \\ &= - (2/3) nk (T - T_z) \end{aligned} \quad (14)$$

Using Dreicer's distribution function, we obtain

$$T_z = T \left(1 + \frac{\beta}{\tilde{n}\pi^{1/2}} - \frac{\alpha^2}{2\pi\tilde{n}} \right) \quad (15)$$

The behavior of T_z is shown in Figure 34. After a short transient period it drops to small values so that

$$\pi_{zz} \approx - 2/3 nkT$$

The use of equations (14) and (15) with (7) are being evaluated for use as a limiter.

Figure 34. Normalized Axial Temperature as a Function of Time at Various Axial Locations

IX. REVIEW OF FACULTY AND STUDENT PARTICIPATION

T. M. York Associate Professor of Aerospace Engineering	July 1978 FT 1 week Sept. 1978-May 1979 25%
E. H. Klevans Professor of Nuclear Engineering	June 1978 FT 1 week July 1978 FT 4 weeks Sept. 1978-May 1979 25%
John Heidrich Graduate Assistant, Physics	June 1978-May 1979 50%
Elmer Stover Graduate Assistant, Nuclear Engineering	June 1978-May 1979 50%
Barry Jacoby Graduate Assistant, Nuclear Engineering	June 1978-May 1979 50%
Vincenzo Veglia Graduate Assistant, Nuclear Engineering	June 1978-May 1979 0% ERDA Traineeship
Akbar Parvazian Graduate Assistant, Nuclear Engineering	June 1978-Aug. 1978 50% Dec. 1978-May 1979 50%
Frank Mead Graduate Assistant, Aerospace Engineering	Sept. 1978-May 1979 50%

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